# A Generalized Nash Equilibrium Model for Post-Disaster Humanitarian Relief

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EURO HOpe mini-conference
Vienna University of Economics and Business, Austria
June 29-30, 2017

# Acknowledgments

Thanks are provided to All Souls College at Oxford University in England for support through my Visiting Fellowship during the 2015-2016 trinity term and also to the Radcliffe Institute for Advanced Study at Harvard University for the Summer Fellowship in 2017.

Support from the John F. Smith Memorial Fund at the University of Massachusetts Amherst is also acknowledged.

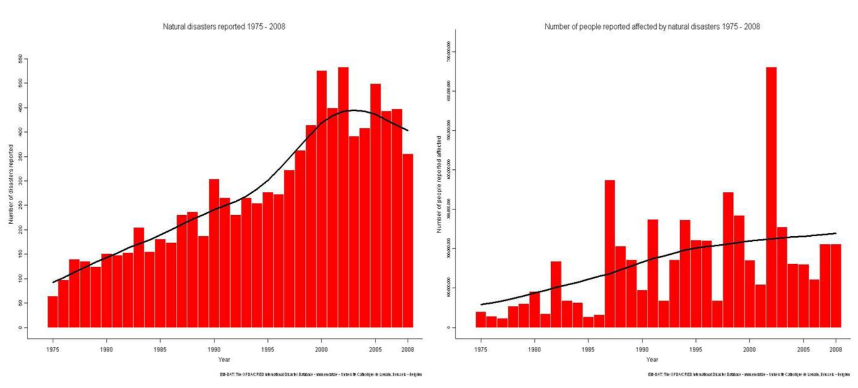
Special appreciation is extended to the organizers of this conference for the invitation to present.

# Outline

- Background and Motivation
- The Game Theory Model for Post-Disaster Humanitarian Relief
- The Algorithm
- A Case Study on Hurricane Katrina
- Summary and Conclusions

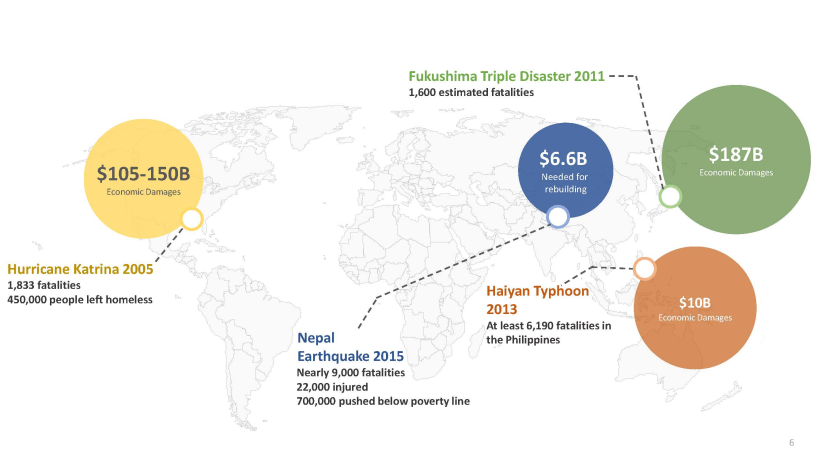
# Background and Motivation

# Natural Disasters (1975–2008)




Disasters have a catastrophic effect on human lives and a region's or even a nation's resources. A total of 2.3 billion people were affected by natural disasters from 1995-2015 (UN Office of Disaster Risk (2015)).

# Some Recent Disasters



# Hurricane Katrina in 2005

Hurricane Katrina has been called an *"American tragedy,"* in which essential services failed completely.

# The Triple Disaster in Japan on March 11, 2011

# Superstorm Sandy and Power Outages

Manhattan without power October 30, 2012 as a result of the devastation wrought by Superstorm Sandy.

# Challenges Associated with Disaster Relief

• Timely delivery of relief items is challenged by damaged and destroyed infrastructure (transportation, telecommunications, hospitals, etc.).

• Shipments of the wrong supplies create congestion and materiel convergence (sometimes referred to as the second disaster).

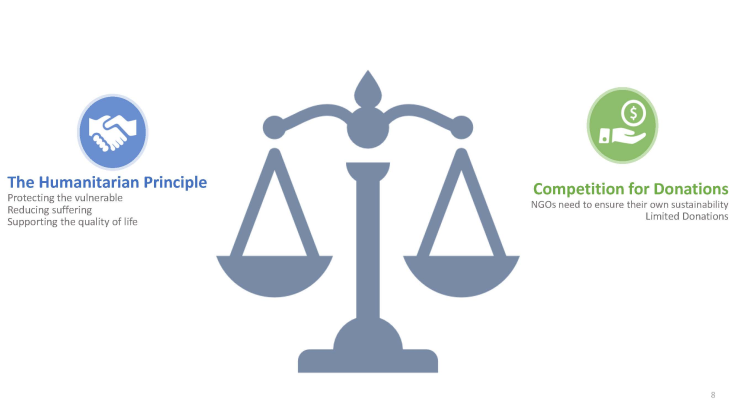
• • Within three weeks following the 2010 earthquake in Haiti, 1,000 NGOs were operating in Haiti. News media attention of insufficient water supplies resulted in immense donations to the Dominican Red Cross to assist its island neighbor. Port-au-Price was saturated with both cargo and gifts-in-kind.

• • After the Fukushima disaster, there were too many blankets and items of clothing shipped and even broken bicycles.

• • After Katrina, even tuxedos were delivered to victims.

Better coordination among NGOs is needed.

# Challenges Associated with Disaster Relief - The NGO Balancing Act




There were 1.5 million registered NGOs in the US in 2012. $300 billion in donations given yearly to US charities.

# Challenges Associated with Disaster Relief - Driving Forces

# Need for Game Theory Network Models for Disaster Relief

## Therefore

there is a need to *develop appropriate analytical tools* that can assist NGOs, as well as governments in modeling the complex interactions in disaster relief to improve outcomes.

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# Game Theory and Disaster Relief

We developed the first Generalized Nash Equilibrium (GNE) model for post-disaster humanitarian relief, which contains both a financial component and a supply chain component. The Generalized Nash Equilibrium problem is a generalization of the Nash Equilibrium problem (cf. Nash (1950, 1951)).

"A Generalized Nash Equilibrium Network Model for Post-Disaster Humanitarian Relief," Anna Nagurney, Emilio Alvarez Flores, and Ceren Soylu, *Transportation Research E* **95** (2016), pp 1-18.

# Some Literature

Our disaster relief game theory framework entails competition for donors as well as media exposure plus supply chain aspects. We now highlight some of the related literature on these topics.

• Natsios (1995) contends that the cheapest way for relief organizations to fundraise is to provide early relief in highly visible areas.

• Balcik et al. (2010) note that the media is a critical factor affecting relief operations with NGOs seeking visibility to attract more resources from donors. They also review the challenges in coordinating humanitarian relief chains and describe the current and emerging coordination practices in disaster relief.

# Some Literature

- Olsen and Carstensen (2003) confirmed the frequently repeated argument that media coverage is critical in relation to emergency relief allocation in a number of cases that they analyzed.

- Van Wassenhove (2006) also emphasizes the role of the media in humanitarian logistics and states that following appeals in the media, humanitarian organizations are often flooded with unsolicited donations that can create bottlenecks in the supply chain.

- Zhuang, Saxton, and Wu (2014) develop a model that reveals the amount of charitable contributions made by donors is positively dependent on the amount of disclosure by the NGOs. They also emphasize that there is a dearth of existing game-theoretic research on nonprofit organizations. Our model attempts to help to fill this void.

# Game Theory and Disaster Relief

Although there have been quite a few optimization models developed for disaster relief there are very few game theory models.

Toyasaki and Wakolbinger (2014) constructed the first models of financial flows that captured the strategic interaction between donors and humanitarian organizations using game theory and also included earmarked donations.

Muggy and Stamm (2014), in turn, provide an excellent review of game theory in humanitarian operations and emphasize that there are many untapped research opportunities for modeling in this area.

Additional references to disaster relief and humanitarian logistics can be found in our paper.

# Game Theory Model for Post-Disaster Humanitarian Relief

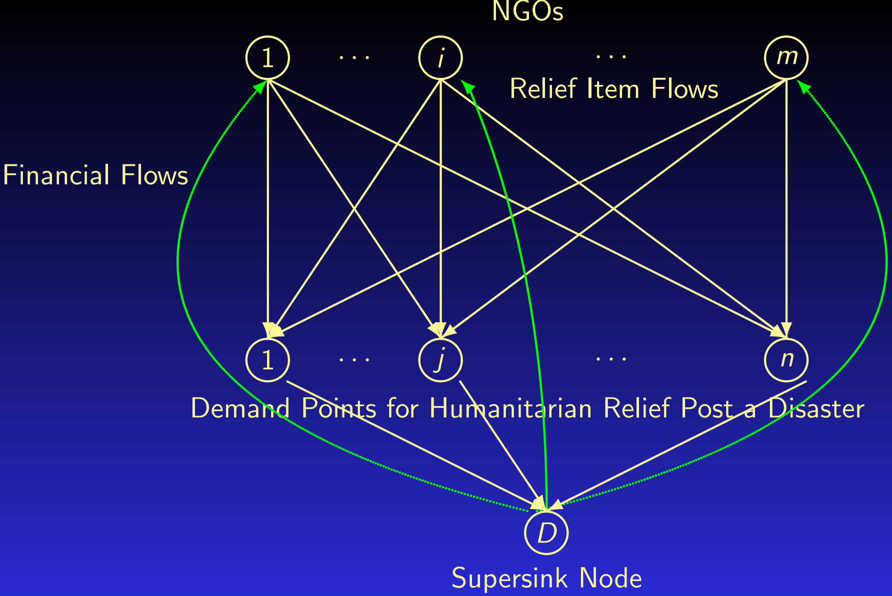


Figure 1: The Network Structure of the Game Theory Model

We assume that each NGO $i$ has, at its disposal, an amount $s_i$ of the relief item that it can allocate post-disaster. Hence, we have the following conservation of flow equation, which must hold for each $i$; $i = 1, \ldots, m$:

$$\sum_{j=1}^{n} q_{ij} \leq s_i. \tag{1}$$

In addition, we know that the product flows for each $i$; $i = 1, \ldots, m$, must be nonnegative, that is:

$$q_{ij} \geq 0, \quad j = 1, \ldots, n. \tag{2}$$

Each NGO $i$ encumbers a cost, $c_{ij}$, associated with shipping the relief items to location $j$, denoted by $c_{ij}$, where we assume that

$$c_{ij} = c_{ij}(q_{ij}), \quad j = 1, \ldots n, \tag{3}$$

with these cost functions being strictly convex and continuously differentiable.

In addition, each NGO $i$; $i = 1, \ldots, m$, derives satisfaction or utility associated with providing the relief items to $j$; $j = 1, \ldots, n$, with its utility over all demand points given by $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$. Here $\gamma_{ij}$ is a positive factor representing a measure of satisfaction/utility that NGO $i$ acquires through its supply chain activities to demand point $j$.

Each NGO $i$; $i = 1, \ldots, m$, associates a positive weight $\omega_i$ with $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$, which provides a monetization of, in effect, this component of the objective function.

# The Game Theory Model

Finally, each NGO $i$; $i = 1, \ldots, m$, based on the media attention and the visibility of NGOs at location $j$; $j = 1, \ldots, n$, acquires funds from donors given by the expression

$$\beta_i \sum_{j=1}^{n} P_j(q), \tag{4}$$

where $P_j(q)$ represents the financial funds in donation dollars due to visibility of all NGOs at location $j$. Hence, $\beta_i$ is a parameter that reflects the proportion of total donations collected for the disaster at demand point $j$ that is received by NGO $i$.

Expression (4), therefore, represents the financial flow on the link joining node $D$ with node NGO $i$.

# The Game Theory Model

Each NGO $i$ seeks to maximize its utility with the utility corresponding to the financial gains associated with the visibility through media of the relief item flow allocations, $\beta_i \sum_{j=1}^n P_j(q)$, plus the utility associated with the supply chain aspect of delivery of the relief items, $\omega_i \sum_{j=1}^n \gamma_{ij} q_{ij} - \sum_{j=1}^n c_{ij}(q_{ij})$.

The optimization problem faced by NGO $i$; $i = 1, \ldots, m$, is, hence,

$$\text{Maximize} \quad \beta_i \sum_{j=1}^n P_j(q) + \omega_i \sum_{j=1}^n \gamma_{ij} q_{ij} - \sum_{j=1}^n c_{ij}(q_{ij}) \tag{5}$$

subject to constraints (1) and (2).

# The Game Theory Model

We also have that, at each demand point $j$; $j = 1, \ldots, n$:

$$\sum_{i=1}^{m} q_{ij} \geq \underline{d}_j, \tag{6}$$

and

$$\sum_{i=1}^{m} q_{ij} \leq \bar{d}_j, \tag{7}$$

where $\underline{d}_j$ denotes a lower bound for the amount of the relief items needed at demand point $j$ and $\bar{d}_j$ denotes an upper bound on the amount of the relief items needed post the disaster at demand point $j$.

We assume that

$$\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} \underline{d}_j, \tag{8}$$

so that the supply resources of the NGOs are sufficient to meet the minimum financial resource needs.

# The Game Theory Model

Each NGO $i$; $i = 1, \ldots, m$, seeks to determine its optimal vector of relief items or strategies, $q_i^*$, that maximizes objective function (5), subject to constraints (1), (2), and (6), (7).

**Theorem: Optimization Formulation of the Generalized Nash Equilibrium Model of Financial Flow of Funds**

*The above Generalized Nash Equilibrium problem, with each NGO's objective function (5) rewritten as:*

$$Minimize \quad -\beta_i \sum_{j=1}^{n} P_j(q) - \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} + \sum_{j=1}^{n} c_{ij}(q_{ij}) \qquad (9)$$

*and subject to constraints (1) and (2), with common constraints (6) and (7), is equivalent to the solution of the following optimization problem:*

$$Minimize \quad -\sum_{j=1}^{n} P_j(q) - \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\omega_i \gamma_{ij}}{\beta_i} q_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{1}{\beta_i} c_{ij}(q_{ij}) \quad (10)$$

*subject to constraints: (1), (2), (6), and (7).*

**Variational Inequality (VI) Formulation**

The solution $q^*$ with associated Lagrange multipliers $\lambda_k^*$, $\forall k$, for the supply constraints; Lagrange multipliers: $\lambda_l^{1^*}$, $\forall l$, for the lower bound demand constraints, and Lagrange multipliers: $\lambda_l^{2^*}$, $\forall k$, for the upper bound demand constraints, can be obtained by solving the VI problem: determine $(q^*, \lambda^*, \lambda^{1^*}, \lambda^{2^*}) \in R_+^{mn+m+2n}$:

$$\sum_{k=1}^{m}\sum_{l=1}^{n}\left[-\sum_{j=1}^{n}(\frac{\partial P_j(q^*)}{\partial q_{kl}}) - \frac{\omega_k \gamma_{kl}}{\beta_k} + \frac{1}{\beta_k}\frac{\partial c_{kl}(q_{kl}^*)}{\partial q_{kl}} + \lambda_k^* - \lambda_l^{1^*} + \lambda_l^{2^*}\right] \times [q_{kl} - q_{kl}^*]$$

$$+\sum_{k=1}^{m}(s_k - \sum_{l=1}^{n} q_{kl}^*) \times (\lambda_k - \lambda_k^*) + \sum_{l=1}^{n}(\sum_{k=1}^{n} q_{kl}^* - \underline{d}_l) \times (\lambda_l - \lambda_l^{1^*})$$

$$+\sum_{l=1}^{n}(\bar{d}_l - \sum_{k=1}^{m} q_{kl}^*) \times (\lambda_l^2 - \lambda_l^{2^*}) \geq 0, \quad \forall (q, \lambda, \lambda^1, \lambda^2) \in R_+^{mn+m+2n}, \tag{11}$$

# The Algorithm

# The Algorithm

We utilize the Euler Method,which is one of the algorithms induced by the general iterative scheme of Dupuis and Nagurney (1993).

**Explicit Formulae for the Euler Method Applied to the Game Theory Model**

We have the following closed form expression for the product flows $k = 1, \ldots, m; l = 1, \ldots, n$, at each iteration:

$$q_{kl}^{\tau+1}$$

$$= \max\{0, \{q_{kl}^{\tau} + a_{\tau}(\sum_{j=1}^{n}(\frac{\partial P_j(q^{\tau})}{\partial q_{kl}}) + \frac{\omega_k \gamma_{kl}}{\beta_{kl}} - \frac{1}{\beta_k}\frac{\partial c_{kl}(q_{kl}^{\tau})}{\partial q_{kl}} - \lambda_k^{\tau} + \lambda_l^{1^{\tau}} - \lambda_l^{2^{\tau}})\}\}$$

the following closed form expressions for the Lagrange multipliers associated with the supply constraints, respectively, for $k = 1, \ldots, m$:

$$\lambda_k^{\tau+1} = \max\{0, \lambda_k^{\tau} + a_{\tau}(-s_k + \sum_{l=1}^{n} q_{kl}^{\tau})\}.$$

## The Algorithm

The following closed form expressions are for the Lagrange multipliers associated with the lower bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{1^{\tau+1}} = \max\{0, \lambda_l^{1^{\tau}} + a_\tau(-\sum_{k=1}^{n} q_{kl}^{\tau} + \underline{d}_l)\}.$$

The following closed form expressions are for the Lagrange multipliers associated with the upper bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{2^{\tau+1}} = \max\{0, \lambda_l^{2^{\tau}} + a_\tau(-\bar{d}_l + \sum_{k=1}^{m} q_{kl}^{\tau})\}.$$

# Hurricane Katrina Case Study

# Hurricane Katrina Case Study

Making landfall in August of 2005, Katrina caused extensive damages to property and infrastructure, left 450,000 people homeless, and took 1,833 lives in Florida, Texas, Mississippi, Alabama, and Louisiana (Louisiana Geographic Information Center (2005)).

Given the hurricane's trajectory, most of the damage was concentrated in Louisiana and Mississippi. In fact, 63% of all insurance claims were in Louisiana, a trend that is also reflected in FEMA's post-hurricane damage assessment of the region (FEMA (2006)).

# Hurricane Katrina Case Study

The total damage estimates range from $105 billion (Louisiana Geographic Information Center (2005)) to $150 billion (White (2015)), making Hurricane Katrina not only a far-reaching and costly disaster, but also a very challenging environment for providing humanitarian assistance.

We consider 3 NGOs: the Red Cross, the Salvation Army, and Others and 10 Parishes in Louisiana.

# Hurricane Katrina Case Study

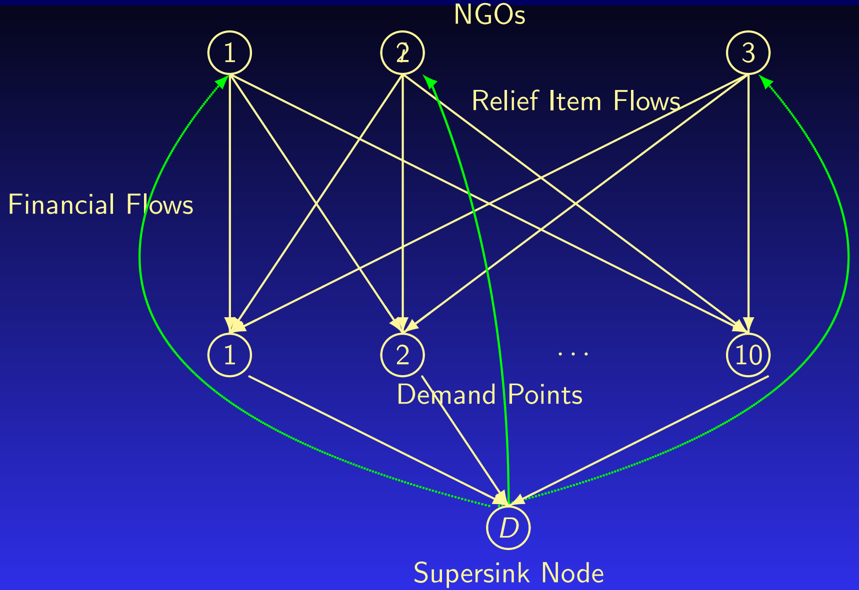


Figure 2: Hurricane Katrina Relief Network Structure

The structure of the $P_j$ functions is as follows:

$$P_j(q) = k_j \sqrt{\sum_{i=1}^{m} q_{ij}}.$$

The weights are:

$$\omega_1 = \omega_2 = \omega_3 = 1,$$

with $\gamma_{ij} = 950$ for $i = 1, 2, 3$ and $j = 1, \ldots, 10$.

# Hurricane Katrina Case Study

| Hurricane Katrina Demand Point Parameters | | | | | |
|---|---|---|---|---|---|
| Parish | Node $j$ | $k_j$ | $\underline{d}_j$ | $\bar{d}_j$ | $p_j$ (in %) |
| St. Charles | 1 | 8 | 16.45 | 50.57 | 2.4 |
| Terrebonne | 2 | 16 | 752.26 | 883.82 | 6.7 |
| Assumption | 3 | 7 | 106.36 | 139.24 | 1.9 |
| Jefferson | 4 | 29 | 742.86 | 1,254.89 | 19.5 |
| Lafourche | 5 | 6 | 525.53 | 653.82 | 1.7 |
| Orleans | 6 | 42 | 1,303.99 | 1,906.80 | 55.9 |
| Plaquemines | 7 | 30 | 33.28 | 62.57 | 57.5 |
| St. Barnard | 8 | 42 | 133.61 | 212.43 | 78.4 |
| St. James | 9 | 9 | 127.53 | 166.39 | 1.2 |
| St. John the Baptist | 10 | 7 | 19.05 | 52.59 | 6.7 |

Table 1: Demand Point Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

We then estimated the cost of providing aid to the Parishes as a function of the total damage in the area and the supply chain efficiency of each NGO. We assume that these costs follow the structures observed by Van Wassenhove (2006) and randomly generate a number based on his research with a mean of $\hat{p} = .8$ and standard deviation of $s = \sqrt{\frac{.8(.2)}{3}}$.

We denote the corresponding coefficients by $\pi_i$. Thus, each NGO $i$; $i = 1, 2, 3$, incurs costs according the the following functional form:

$$c_{ij}(q_{ij}) = \left(\pi_i q_{ij} + \frac{1}{1 - p_j}\right)^2.$$

# Hurricane Katrina Case Study

| Data Parameters for NGOs Providing Aid | | | | | |
|---|---|---|---|---|---|
| NGO | $i$ | $\pi_i$ | $\gamma_{ij}$ | $\beta_i$ | $s_i$ |
| Others | 1 | .82 | 950 | .355 | 1,418 |
| Red Cross | 2 | .83 | 950 | .55 | 2,200 |
| Salvation Army | 3 | .81 | 950 | .095 | 382 |

Table 2: NGO Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

# Hurricane Katrina Case Study

| Generalized Nash Equilibrium Product Flows | | | |
|---|---|---|---|
| Demand Point | Others | Red Cross | Salvation Army |
| St. Charles | 17.48 | 28.89 | 4.192 |
| Terrebonne | 267.023 | 411.67 | 73.57 |
| Assumption | 49.02 | 77.26 | 12.97 |
| Jefferson | 263.69 | 406.68 | 72.45 |
| Lafourche | 186.39 | 287.96 | 51.18 |
| Orleans | 463.33 | 713.56 | 127.1 |
| Plaquemines | 21.89 | 36.54 | 4.23 |
| St. Barnard | 72.31 | 115.39 | 16.22 |
| St. James | 58.67 | 92.06 | 15.66 |
| St. John the Baptist | 18.2 | 29.99 | 4.40 |

Table 3: Flows to Demand Points under Generalized Nash Equilibrium

# Hurricane Katrina Case Study

The total utility obtained through the above flows for the Generalized Nash Equilibrium for Hurricane Katrina is $9,257,899$, with the Red Cross capturing 3,022,705, the Salvation Army 3,600,442.54, and Others 2,590,973. It is interesting to see that, despite having the lowest available supplies, the Salvation Army is able to capture the largest part of the total utility. This is due to the fact that the costs of providing aid grow at a nonlinear rate, so even if the Salvation Army was less efficient and used all of its available supplies, it will not be capable of providing the most expensive supplies.

In addition, we have that the Red Cross, the Salvation Army, and Others receive 2,200.24, 1418.01, and 382.31 million in donations, respectively. Also, notice how the flows meet at least the lower bound, even if doing so is very expensive due to the damages to the infrastructure in the region.

# Hurricane Katrina Case Study

Furthermore, the above flow pattern behaves in a way that, after the minimum requirements are met, any additional supplies are allocated in the most efficient way. For example, only the minimum requirements are met in New Orleans Parish, while the upper bound is met for St. James Parish.

# The Nash Equilibrium Solution

If we remove the shared constraints, we obtain a Nash Equilibrium solution, and we can compare the outcomes of the humanitarian relief efforts for Hurricane Katrina under the Generalized Nash Equilibrium concept and that under the Nash Equilibrium concept.

# The Nash Equilibrium Solution

| Nash Equilibrium Product Flows | | | |
|---|---|---|---|
| Demand Point | Others | Red Cross | Salvation Army |
| St. Charles | 142.51 | 220.66 | 38.97 |
| Terrebonne | 142.50 | 220.68 | 38.93 |
| Assumption | 142.51 | 220.66 | 38.98 |
| Jefferson | 142.38 | 220.61 | 38.74 |
| Lafourche | 142.50 | 220.65 | 38.98 |
| Orleans | 141.21 | 219.59 | 37.498 |
| Plaquemines | 141.032 | 219.28 | 37.37 |
| St. Barnard | 138.34 | 216.66 | 34.59 |
| St. James | 142.51 | 220.65 | 38.58 |
| St. John the Baptist | 145.51 | 220.66 | 38.98 |

Table 4: Flows to Demand Points under Nash Equilibrium

# The Nash Equilibrium Solution

Under the Nash Equilibrium, the NGOs obtain a higher utility than under the Generalized Nash Equilibrium. Specifically, of the total utility $10,346,005.44$, 2,804,650 units are received by the Red Cross, 5,198,685 by the Salvation Army, and 3,218,505 are captured by all other NGOs.

Under this product flow pattern, there are total donations of 3,760.73, of which 2,068.4 are donated to the Red Cross, 357.27 to the Salvation Army, and 1,355 to the other players.

# The Nash Equilibrium Solution

It is clear that there is a large contrast between the flow patterns under the Generalized Nash and Nash Equilibria. For example, the Nash Equilibrium flow pattern results in about $500 million less in donations.

While this has strong implications about how collaboration between NGOs can be beneficial for their fundraising efforts, the differences in the general flow pattern highlights a much stronger point.

# Additional Insights

Under the Nash Equilibrium, NGOs successfully maximize their utility. Overall, the Nash Equilibrium solution leads to an increase of utility of roughly 21% when compared to the flow patterns under the Generalized Nash Equilibrium. But they do so at the expense of those in need. In the Nash Equilibrium, each NGO chooses to supply relief items such that costs can be minimized. On the surface, this might be a good thing, but recall that, given the nature of disasters, it is usually more expensive to provide aid to demand points with the greatest needs.

# Additional Insights

With this in mind, one can expect oversupply to the demand points with lower demand levels, and undersupply to the most affected under a purely competitive scheme. This behavior can be seen explicitly in the results summarized in the Tables.

For example, St. Charles Parish receives roughly 795% of its upper demand, while Orleans Parish only receives about 30.5% of its minimum requirements. That means that much of the 21% in 'increased' utility is in the form of waste.

In contrast, the flows under the Generalized Nash Equilibrium guarantee that minimum requirements will be met and that there will be no waste; that is to say, *as long as there is a coordinating authority that can enforce the upper and lower bound constraints, the humanitarian relief flow patterns under this bounded competition will be significantly better than under untethered competition.*

# Additional Insights

In addition, we found that changes to the values in the functional form result in changes in the product flows, but the general behavioral differences are robust to changes in the coefficients: $\beta_i$, $\gamma_{ij}$, $k_j$, $\forall i, j$, and the bounds on upper and lower demand estimates.

# Additional Research on Game Theory and Disaster Relief

At the Dynamics of Disasters conference in Kalamata, Greece, July 5-9, 2017, we will present the following papers:
"A Variational Equilibrium Network Framework for Humanitarian Organizations in Disaster Relief: Effective Product Delivery Under Competition for Financial Funds," A. Nagurney, P. Daniele, E. Alvarez Flores, and V. Caruso (2017),

A Multitiered Supply Chain Network Equilibrium Model for Disaster Relief with Capacitated Freight Service Provision," A. Nagurney (2017).

# Additional Research on Game Theory and Disaster Relief

The previous paper is an extension of: "Freight Service Provision for Disaster Relief: A Competitive Network Model with Computations," A. Nagurney, in *Dynamics of Disasters: Key Concepts, Models, Algorithms, and Insights*, I.S. Kotsireas, A. Nagurney, and P.M. Pardalos, Eds., Springer International Publishing Switzerland (2016), pp. 207-229.

# Summary and Conclusions

- We presented a Generalized Nash Equilibrium model, with a special case being a Nash Equilibrium model, for disaster relief with supply chain and financial fund aspects for each NGO's objective function.

- Each NGO obtains utility from providing relief to demand points post a disaster and also seeks to minimize costs but can gain in financial donations based on the visibility of the NGOs in terms of product deliveries to the demand points.

- A case study based on Hurricane Katrina was discussed.

- All the models were network-based and provide new insights in terms of disaster relief and management.

# THANK YOU!


For more information, see:

http://supernet.isenberg.umass.edu