THE REGULATORY IMPACT ON EFFICIENCY OF A FIRM

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1. Introduction

- Different forms of regulation in order to prevent the monopoly power over the customers and to guarantee the reliability and quality of supply at economically or politically desired prices: return of return regulation, price-cap, revenue – cap,…

- Environmental regulation (Helfand (1991), Luptacik (2010)

- How to find the “right” price level in the case of price – cap regulation? “too high a price ceiling makes the firm an unregulated monopolist, too low a cap conflicts with viability and in between the “right” price level is difficult to compute” (Laffont – Tirole (1994), p.17)

2. The rate of return regulation and its welfare implications

- 2.1. Averch –Johnson effect

Averch –Johnson (1962): the effect of overcapitalization; the firm " has incentive to substitute between the factors in an uneconomic fashion" (p.1068).

Proposition 1:

The firm does not equate the marginal rate of factor substitution to the ratio of the input prices. The firm has an incentive to increase its investment: the amount of capital used with the regulatory constraint is not less than the amount used without a constraint.

Proof: We define the firm's production function as:

$$q = f(x_1, x_2) \quad \text{where } f_1 = \frac{\partial f}{\partial x_1} > 0,\ f_2 = \frac{\partial f}{\partial x_2} > 0$$
$$f(0, x_2) = f(x_1, 0) = 0.$$

That is, marginal products are positive, and production requires both inputs.

The inverse demand function can be written:

$$p = p(q) \quad \text{where } p'(q) = \frac{dp}{dq} < 0.$$

Profit Π is defined by (II.55).

Let x_1 denote the physical quantity of plant and equipment in the rate base, b_1 the acquisition cost per unit of plant and equipment in the rate base, β_1 the value of depreciation of plant and equipment during a time period in question, and B_1 the cumulative value of depreciation.

The regulatory constraint of AVERCH - JOHNSON (1962) is:

$$\frac{pq - c_2 x_2 - \beta_1}{b_1 x_1 - B_1} \leqq s \tag{II.59}$$

where the profit net of labor cost and capital depreciation constitutes a percentage of the rate base (net depreciation) no greater than a specified maximum s.

For simplicity, AVERCH - JOHNSON (1962) assumed that depreciation (β_1 and B_1) is zero and the acquisition cost b_1 is equal 1, (i.e. the value of the rate base is equal to the physical quantity of capital). The price or the "cost of capital" c_1 is the interest cost involved in holding plant and equipment (to be distinguished from the acquisition cost b_1). The regulatory constraint (II.59) can be then rewritten as,

$$\frac{pq - c_2x_2}{x_1} \leqq s$$

or

$$pq - sx_1 - c_2x_2 \leqq 0. \tag{II.60}$$

The "fair rate of return" s is the rate of return allowed by the regulatory agency on plant and equipment in order to compensate the firm for the cost of capital.

If $s < c_1$, the allowable rate of return is less than the actual cost of capital and the firm would withdraw from the market. Therefore, we shall assume $s \geqq c_1$; the allowable rate of return must at least cover the actual cost of capital.

The problem of the firm is to maximize the profit described by the function (II.55) subject to (II.60) and $x_1 \geqq 0$, and $x_2 \geqq 0$. The Lagrange function is defined as:

$$\Phi(x_1, x_2, u) = p(q)q - c_1x_1 - c_2x_2 - u(p(q)q - sx_1 - c_2x_2)$$

$$MR_1^0 = c_1 - \frac{(s - c_1)}{1 - u^0} u^0. \tag{II.64}$$

Under the assumption that $s > c_1$ and $u^0 < 1$ (as claimed by AVERCH-JOHNSON (1962)), it follows from (II.64) that $MR^0 < c_1$.

If the revenue function $G \equiv pf(x_1, x_2)$ is concave (this assumption is not mentioned in AVERCH - JOHNSON (1962); it was introduced by TAKAYAMA (1969)) then the marginal revenue product of capital MR_1 is a non-increasing function of capital used and consequently the amount of capital used under the regulatory constraint (x_1^0) is not less than the amount used without a constraint (x_1^*). If G is assumed to be strictly concave, then $\partial MR_1 / \partial x_1 < 0$; hence $x_1^0 > x_1^*$. Furthermore, it follows from (II.61) and (II.62)

$$\frac{MR_1}{MR_2} = \frac{c_1}{c_2} - \frac{(s - c_1)}{c_2} \frac{u^0}{(1 - u^0)} < \frac{c_1}{c_2}.$$

2.2 The rate of return regulation under profit and revenue maximization

Essential assumption in the Averch-Johnson model is that the firm maximizes profit. BAILEY – MALONE (1970) argue that if the firm maximizes either revenue or output, then it will tend to undercapitalize. The following question arises: what ist the impact of the rate of return regulation for the firm maximizing revenue as well as profit?

The multiobjective optimization problem yields:

$$\frac{MR_1}{MR_2} = \frac{f_1}{f_2} = \frac{\alpha_1 c_1 - us}{(\alpha_1 - u)c_2}$$

For the unregulated monopoly maximizing profit
($u = 0$ and $\alpha_1 = 1$, $\alpha_2 = 0$), the marginal rate of substitution of capital for labor is equal to the ratio of their prices.

For the revenue maximizing firm under regulatory constraint ($\alpha_1 = 0$; $u > 0$) the form yields:

$$\frac{f_1}{f_2} = \frac{s}{c_2} > \frac{c_1}{c_2}$$

i.e. undercapitalization effect shown by BAILEY – MALONE (1970). What kind of result will be obtained if the firm maximizes profit as well as revenue?

The answer depends on the relation between α_1 and u. If $\alpha_1 > u$ (the preference for the profit maximization is relativ high or the regulatory constraint is not very tight, then under the basic assumption $s > c_1$ it can be shown that Averch-Johnson effect or overcapitalization occurs. In the opposite case, $\alpha_1 < u$, the result is undercapitalization, the firm has an incentive to substitute labor for capital.

Proposition 2

In the firm maximizing revenue as well as profit and underlying regulatory constraint ($u > 0$) the cost minimizing allocation of production factors in the sense

$$\frac{f_1}{f_2} = \frac{c_1}{c_2}$$

cannot be achieved independently of the firm's objective preferences. The overcapitalization effect of the profit maximization cannot be compensated by the undercapitalization effect of the revenue maximization.

$$\frac{d\left(\frac{f_1}{f_2}\right)}{d\alpha_1} > 0 \qquad \frac{d\left(\frac{f_1}{f_2}\right)}{d\alpha_2} < 0$$

2.3. Welfare aspects of a regulation

- Shesinski (1971):

  The fair rate of return leads to a non-optimal state in the sense of Pareto

  The basic question is whether it improves the performance of the economy, from a welfare point of view, as compared with the unregulated monopoly situation (where output is too small).

  “ second best problem” or multi-objective optimization problem, where we have to choose between two situations, each deviating in one way or another from optimality.

The utility or social welfare function is

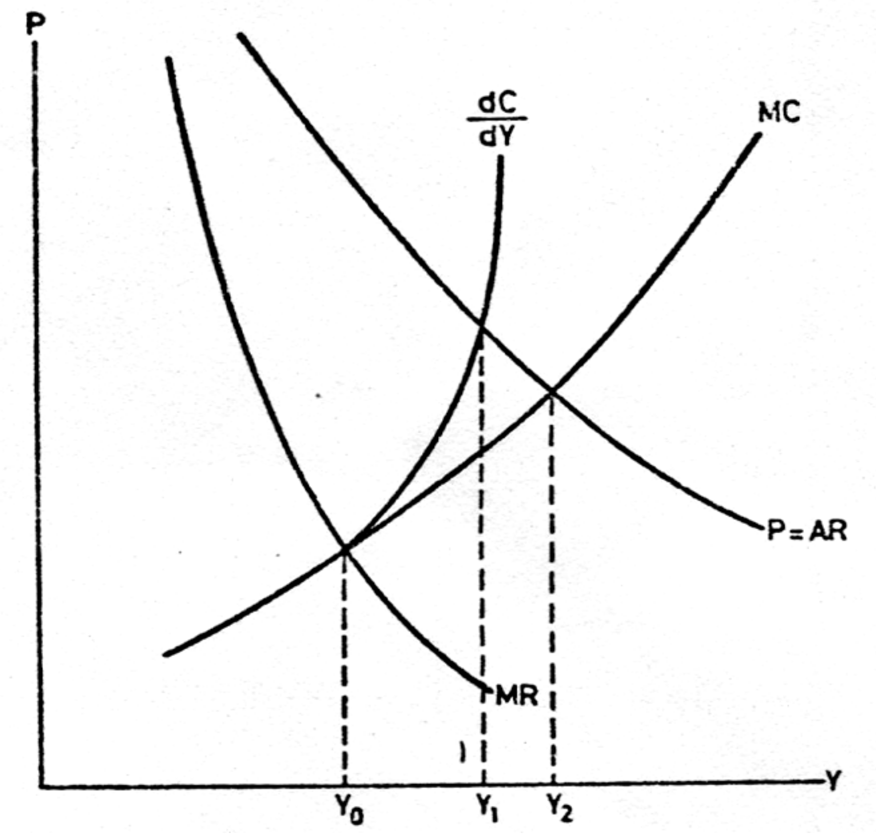
$U = U(Y,K,L)$

where $U_1 > 0$, $U_2 < 0$ and $U_3 < 0$

Decreasing **s** from the ineffective level **s'** (unconstrained point), always raises utility. Therefore, "*some regulation via the fair rate of return criterion is always worthwhile*". (p.177)

Optimal degree of regulation:

Since regulation can always improve welfare it is interesting to find the level of s that maximizes utility. In the constraint region (r < s < s‘), the necessary condition for maximum of U $\left(\frac{dU}{ds}=0\right)$ implies: $P\frac{dY}{ds}=\frac{dC}{ds}$



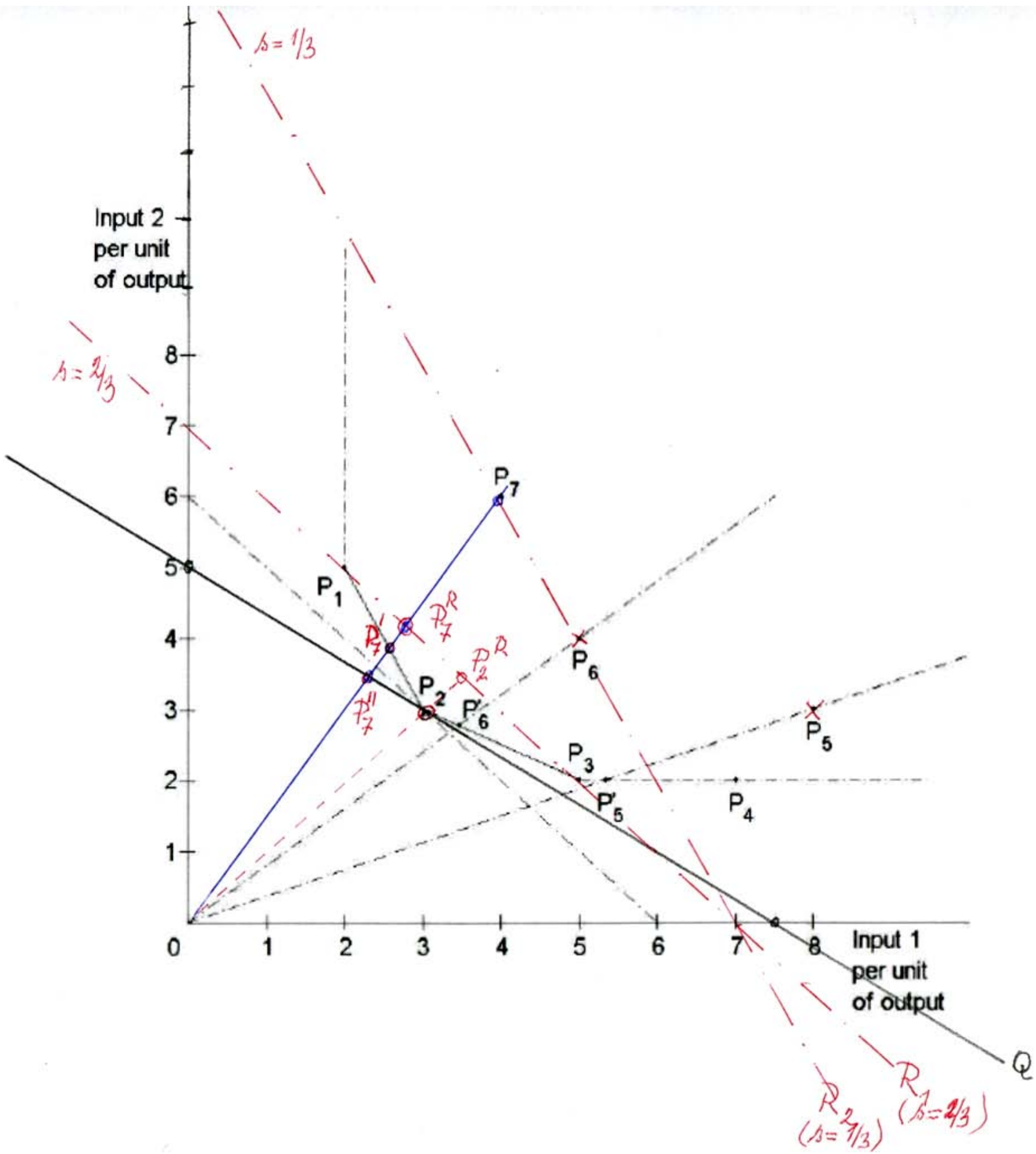
3. The rate of return regulated version of Farrell efficiency

- Färe – Logan (1992)

How to measure the efficiency under rate of return regulation using data envelopment analysis and what is the regulatory impact on efficiency.

DMU	1	2	3	4	5	6	7
Input 1	2	3	5	7	8	5	4
Input 2	5	3	2	2	3	4	6
Output	1	1	1	1	1	1	1

- The regulatory constraint is $s \geq \frac{pq - c_1 x_1}{c_2 x_2}$ or $\frac{c_1}{s} x_1 + c_2 x_2 \geq \frac{pq}{s}$
- In the regulated case, technical efficiency is price – dependent which is not the case for unregulated case.
- For $c_1 = 2$, $c_2 = 3$, $p = 1$, $q = 14$ and
- $s = 2/3$ we have: $3x_1 + 3x_2 \geq 21$
- $s = 1/3$ we have: $6x_1 + 3x_2 \geq 42$



Input 2
per unit
of output
Input 1
per unit
of output
0
1
2
3
4
5
6
7
8
P1
P2
P3
P4
P5
P6
P7
P'5
P'6
Q

$$TE = \frac{OP_7^{'}}{OP_7} \qquad RTE = \frac{OP_7^{R}}{OP_7}$$

$$AE = \frac{OP_7^{''}}{OP_7^{'}} \qquad RAE = \frac{OP_7^{''}}{OP_7^{R}}$$

$$OE = \frac{OP_7^{''}}{OP_7} \qquad ROE = \frac{OP_7^{''}}{OP_7}$$

The addition of a rate of return constraint to the usual technology constraints is required.

Regulatory impact:

$$\frac{AE}{RAE}, \frac{TE}{RTE}, \frac{OE}{ROE}$$

$$\frac{TE}{RTE} \leq 1, \frac{OE}{ROE} \leq 1$$

4. The impact of environmental regulation on the eco-efficiency of firms

- Extension of DEA models for measuring of the eco-efficiency
- Färe et al. (1989), Färe et al. (1995), Tyteca (1997) and others.
- Korhonen – Luptácik (2004)

Model A:

$$max\ \ h_A = \frac{\sum_{r=1}^{k} \mu_r y_{r0} - \sum_{s=k+1}^{p} \mu_s y_{s0}}{\sum_{i=1}^{m} \nu_i x_{i0}}$$

subject to:

$$\frac{\sum_{r=1}^{k} \mu_r y_{rj} - \sum_{s=k+1}^{p} \mu_s y_{sj}}{\sum_{i=1}^{m} \nu_i x_{ij}} \leq 1,\ \ j = 1,2,\ ...,\ n$$

$\mu_r\ ,\ \nu_i \geq \varepsilon,\ \ r = 1,\ 2,\ ...,\ p;\ \ i = 1,\ 2,\ ...,\ m$

$\varepsilon > 0$ ("Non-Archimedean").

Input-Oriented CCR Primal $(CCR_P - I)$	Input-Oriented CCR Dual $(CCR_D - I)$
$min\ g_A = \theta - \varepsilon \mathbf{1}^T(\mathbf{s}^b + \mathbf{s}^g + \mathbf{s}^-)$ s.t. $\mathbf{Y}^g\boldsymbol{\lambda} - \mathbf{s}^g = \mathbf{y}_0^g$ $\mathbf{Y}^b\boldsymbol{\lambda} + \mathbf{s}^b = \mathbf{y}_0^b$ $\mathbf{X}\boldsymbol{\lambda} - \theta \mathbf{x}_0 + \mathbf{s}^- = \mathbf{0}$ $\boldsymbol{\lambda},\ \mathbf{s}^-,\ \mathbf{s}^g,\ \mathbf{s}^b \geq \mathbf{0}$ $\varepsilon > 0$ ("Non-Archimedean")	$max\ h_A = \boldsymbol{\mu}_g^T \mathbf{y}_0^g - \boldsymbol{\mu}_b^T \mathbf{y}_0^b$ s.t. $\boldsymbol{\nu}^T \mathbf{x}_0 = 1$ $\boldsymbol{\mu}_g^T \mathbf{Y}^g - \boldsymbol{\mu}_b^T \mathbf{Y}^b - \boldsymbol{\nu}^T \mathbf{X} \leq \mathbf{0}$ $\boldsymbol{\mu}_g,\ \boldsymbol{\mu}_b, \boldsymbol{\nu} \geq \varepsilon \mathbf{1}$ $\varepsilon > 0$ ("Non-Archimedean")

Model B:

$$\max\ h_B = \frac{\sum_{r=1}^{k} \mu_r y_{r0}}{\sum_{i=1}^{m} \nu_i x_{i0} + \sum_{s=k+1}^{p} \mu_s y_{s0}}$$

subject to:

$$\frac{\sum_{r=1}^{k} \mu_r y_{rj}}{\sum_{i=1}^{m} \nu_i x_{ij} + \sum_{s=k+1}^{p} \mu_s y_{sj}} \leq 1,\ \ j = 1,2,\ \ldots,\ n$$

$\mu_r\ ,\ \nu_i \geq \varepsilon,\ \ r = 1,\ 2,\ \ldots,\ p;\ \ i = 1,\ 2,\ \ldots,\ m$

$\varepsilon > 0$ ("Non-Archimedean").

Input/Undesirable Output-Oriented CCR Primal (CCR_P - I/UO)	Input/Undesirable Output-Oriented CCR Dual (CCR_D - I/UO)
$min\ g_B = \theta - \varepsilon \mathbf{1}^T(\mathbf{s}^b + \mathbf{s}^g + \mathbf{s}^-)$ s.t. $\mathbf{Y}^g\boldsymbol{\lambda} - \mathbf{s}^g = \mathbf{y}_0^g$ $\mathbf{Y}^b\boldsymbol{\lambda} - \theta\mathbf{y}_0^b + \mathbf{s}^b = \mathbf{0}$ $\mathbf{X}\boldsymbol{\lambda} - \theta\mathbf{x}_0 + \mathbf{s}^- = \mathbf{0}$ $\boldsymbol{\lambda},\ \mathbf{s}^-,\ \mathbf{s}^g,\ \mathbf{s}^b \geq \mathbf{0}$ $\varepsilon > 0$ ("Non-Archimedean")	$max\ h_B = \boldsymbol{\mu}_g^T\mathbf{y}_0^g$ s.t. $\boldsymbol{\nu}^T\mathbf{x}_0 + \boldsymbol{\mu}_b^T\mathbf{y}_0^b = 1$ $\boldsymbol{\mu}_g^T\mathbf{Y}^g - \boldsymbol{\mu}_b^T\mathbf{Y}^b - \boldsymbol{\nu}^T\mathbf{X} \leq \mathbf{0}$ $\boldsymbol{\mu}_g, \boldsymbol{\mu}_b, \boldsymbol{\nu} \geq \varepsilon\mathbf{1}$ $\varepsilon > 0$ ("Non-Archimedean")

Model C:

$$max\ \ h_C = \frac{\sum_{r=1}^{k} \mu_r y_{r0} - \sum_{i=1}^{m} \nu_i x_{i0}}{\sum_{s=k+1}^{p} \mu_s y_{s0}}$$

subject to:

$$\frac{\sum_{r=1}^{k} \mu_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij}}{\sum_{s=k+1}^{p} \mu_s y_{sj}} \leq 1,\ \ j = 1,2,\ \ldots,\ n$$

$\mu_r\ ,\ \nu_i \geq \varepsilon,\ \ r = 1,\ 2,\ \ldots,\ p;\ \ i = 1,\ 2,\ \ldots,\ m$

$\varepsilon > 0$ ("Non-Archimedean").

Undesirable Output-Oriented CCR Primal (CCR_P - UO)	Undesirable Output-Oriented CCR Dual (CCR_D - UO)
$min\ g_C = \theta - \varepsilon \mathbf{1}^T(\mathbf{s}^b + \mathbf{s}^g + \mathbf{s}^-)$ s.t. $\mathbf{Y}^g\boldsymbol{\lambda} - \mathbf{s}^g = \mathbf{y}_0^g$ $\mathbf{Y}^b\boldsymbol{\lambda} - \theta\mathbf{y}_0^b + \mathbf{s}^b = \mathbf{0}$ $\mathbf{X}\boldsymbol{\lambda} + \mathbf{s}^- = \mathbf{x}_0$ $\boldsymbol{\lambda},\ \mathbf{s}^-,\ \mathbf{s}^g,\ \mathbf{s}^b \geq \mathbf{0}$ $\varepsilon > 0$ ("Non-Archimedean")	$max\ h_C = \boldsymbol{\mu}_g^T\mathbf{y}_0^g - \boldsymbol{\nu}^T\mathbf{x}_0$ s.t. $\boldsymbol{\mu}_b^T\mathbf{y}_0^b = 1$ $\boldsymbol{\mu}_g^T\mathbf{Y}^g - \boldsymbol{\mu}_b^T\mathbf{Y}^b - \boldsymbol{\nu}^T\mathbf{X} \leq \mathbf{0}$ $\boldsymbol{\mu}_g,\ \boldsymbol{\mu}_b,\ \boldsymbol{\nu} \geq \varepsilon\mathbf{1}$ $\varepsilon > 0$ (“Non-Archimedean”)

It can be shown (Theorem 1 in the paper by Korhonen – Luptácik (2004) p. 442) that to analyze the eco-efficiency any of the models A – C can be used.

The efficient units are efficient – no matter which model is used.

Production data of the French cement industry

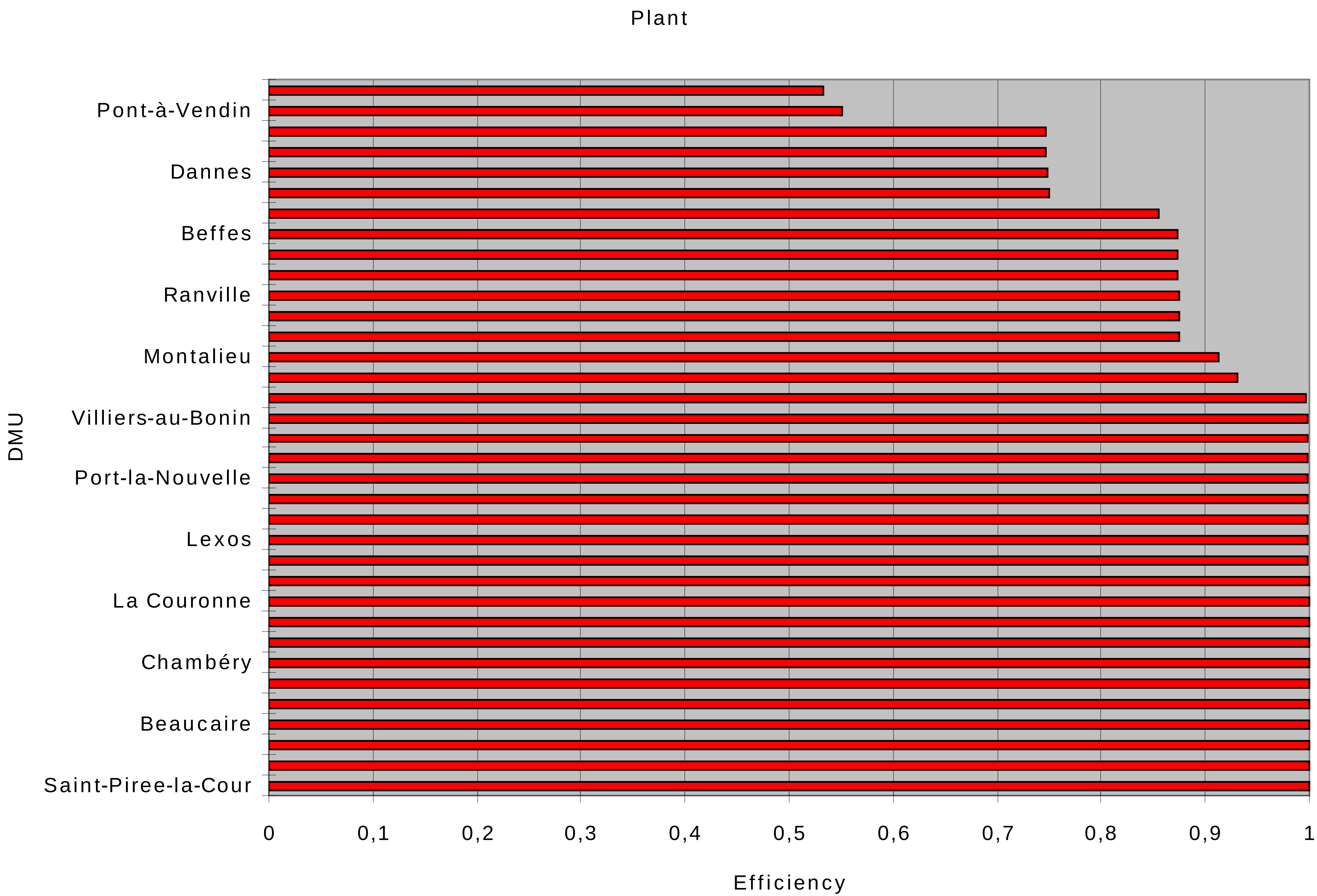
Harrison - Radow (2002), p. 130

Plant	Emissions in 1000t	Heat Input (MMBtu)	Clinker Production in 1000 t	Cement Production in 1000t
Saint-Piree-la-Cour	755	3027	1014	1230
Heming	549	2109	706	844
Couvrot	541	2354	736	1074
Montalieu	518	2239	685	870
Airvault	516	2343	687	1074
Le Teil	462	1844	617	745
La Malle	441	1999	586	662
Martres	407	1965	494	599
Origny	399	1843	528	623
Cormeilles	389	1720	431	541
Lexos	375	1466	491	583
Beaucaire	354	1465	490	614
Lumbres	349	2107	377	468
La Couronne	345	1283	430	523
Grave-de-Peille	344	1460	456	580
Bussac	331	1373	460	307
Havre-Saint-Vigor	290	1086	364	430
Port-la-Nouvelle	256	955	320	388
Rochefort	255	979	328	375
Xeuilley	251	1088	319	405
Beffes	249	1130	331	479
Altkirch	248	954	319	375
Contes	242	1173	294	340
Saint-Egreve	235	940	315	400
Gargenville	221	915	307	614
Dannes	210	953	239	33
Val-d'Azergues	201	901	264	334
Crechy	196	728	244	310
Frangey	190	800	235	272
Villiers-au-Bonin	177	732	245	276
Ranville	170	774	227	338
Pont-a-Vendin	101	594	106	135
Chambery	92	376	126	160
Cruas	80	330	110	135
La Perelle	52	236	59	75

No.	DMU	Score	Rank
1	Saint-Piree-la-Cour	1	1
2	Heming	0,999014126	14
3	Couvrot	1	1
4	Montalieu	0,912991567	22
5	Airvault	1	1
6	Le Teil	0,99840225	18
7	La Malle	0,874661517	24
8	Martres	0,749933598	30
9	Origny	0,854686221	29
10	Cormeilles	0,747338081	32
11	Lexos	0,999084385	13
12	Beaucaire	1	1
13	Lumbres	0,533511586	35
14	La Couronne	0,999559025	10
15	Grave-de-Peille	0,931577571	21
16	Bussac	1	1
17	Havre-Saint-Vigor	0,999331859	11
18	Port-la-Nouvelle	0,998779594	16
19	Rochefort	0,998705489	17
20	Xeuilley	0,87394048	26
21	Beffes	0,873180959	28
22	Altkirch	0,996695222	20
23	Contes	0,74704799	33
24	Saint-Egrève	0,998827831	15
25	Gargenville	1	1
26	Dannes	0,747641933	31
27	Val-d'Azergues	0,873418731	27
28	Créchy	0,999165199	12
29	Frangey	0,87573895	23
30	Villiers-au-Bonin	0,99777345	19
31	Ranville	0,874377204	25
32	Pont-à-Vendin	0,550254441	34
33	Chambéry	1	1
34	Cruas	1	1
35	La Pérelle	1	1

Model A

Average score: 0,9144

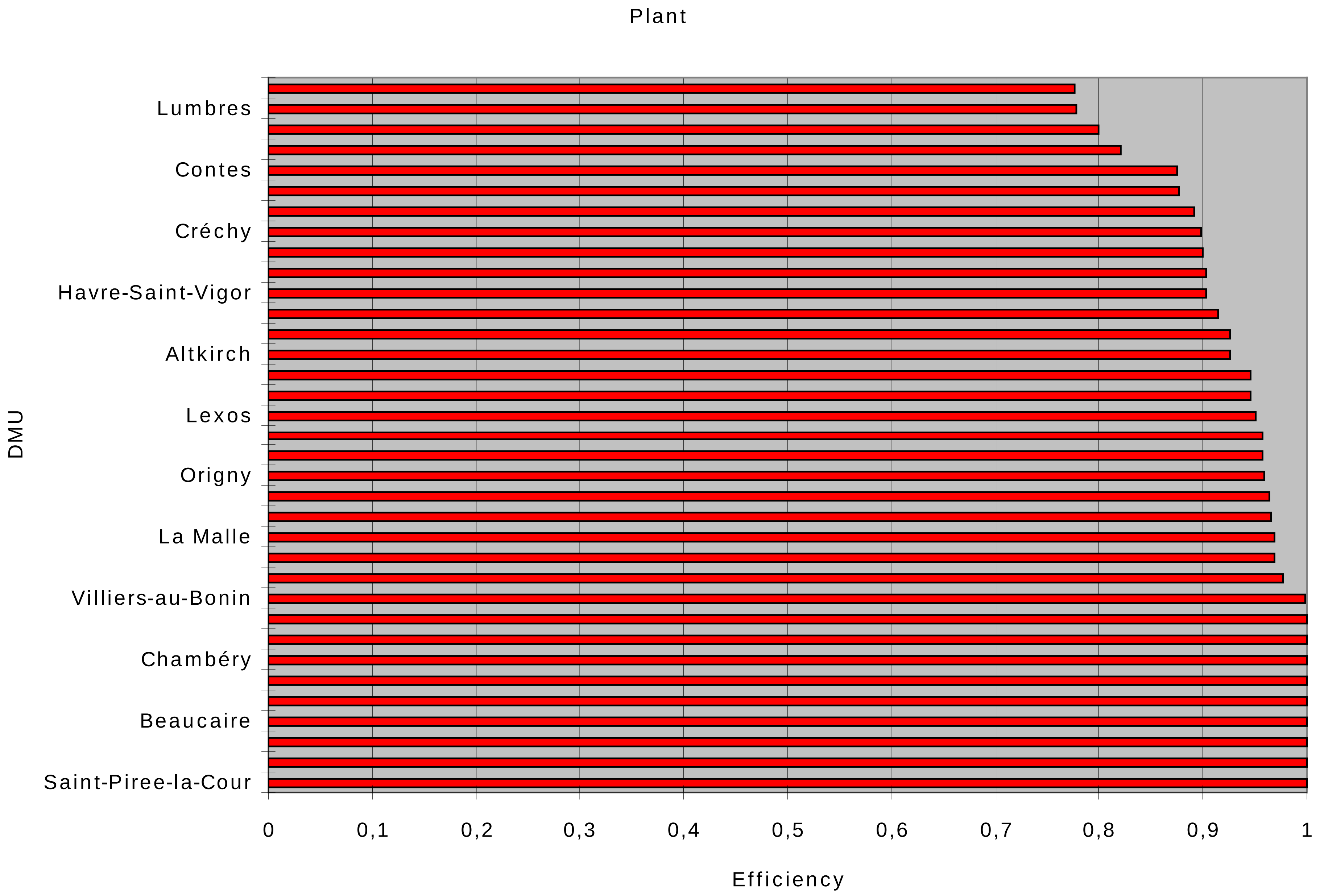


Plant
Pont-à-Vendin
Dannes
Beffes
Ranville
Montalieu
Villiers-au-Bonin
Port-la-Nouvelle
Lexos
La Couronne
Chambéry
Beaucaire
Saint-Piree-la-Cour
DMU
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1
Efficiency

No.	DMU	Score	Rank
1	Saint-Piree-la-Cour	1	1
2	Heming	0,945885451	21
3	Couvrot	1	1
4	Montalieu	0,969489676	12
5	Airvault	1	1
6	Le Teil	0,976582548	11
7	La Malle	0,967966747	13
8	Martres	0,877206802	30
9	Origny	0,95949353	16
10	Cormeilles	0,799063384	33
11	Lexos	0,950827363	19
12	Beaucaire	1	1
13	Lumbres	0,77744068	34
14	La Couronne	0,902173353	26
15	Grave-de-Peille	0,956851981	17
16	Bussac	1	1
17	Havre-Saint-Vigor	0,903380663	25
18	Port-la-Nouvelle	0,899790645	27
19	Rochefort	0,925874664	23
20	Xeuilley	0,914850402	24
21	Beffes	0,956846996	18
22	Altkirch	0,925917141	22
23	Contes	0,87477451	31
24	Saint-Egrève	0,96490057	14
25	Gargenville	1	1
26	Dannes	0,820618806	32
27	Val-d'Azergues	0,946384827	20
28	Créchy	0,897493007	28
29	Frangey	0,891931606	29
30	Villiers-au-Bonin	0,997877771	10
31	Ranville	0,96318304	15
32	Pont-à-Vendin	0,776525333	35
33	Chambéry	1	1
34	Cruas	1	1
35	La Pérelle	1	1

Model C

Average score: 0,935


Plant
DMU
Lumbres
Contes
Créchy
Havre-Saint-Vigor
Altkirch
Lexos
Origny
La Malle
Villiers-au-Bonin
Chambéry
Beaucaire
Saint-Piree-la-Cour
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1
Efficiency

No.	DMU	Model A	Model B	Model C
1	Saint-Piree-la-Cour	1	1	1
2	Heming	0,999014126	0,999014126	0,945885451
3	Couvrot	1	1	1
4	Montalieu	0,912991567	0,969519644	0,969489676
5	Airvault	1	1	1
6	Le Teil	0,99840225	0,99840225	0,976582548
7	La Malle	0,874661517	0,967966747	0,967966747
8	Martres	0,749933598	0,877206802	0,877206802
9	Origny	0,854686221	0,95949353	0,95949353
10	Cormeilles	0,747338081	0,799063384	0,799063384
11	Lexos	0,999084385	0,999084385	0,950827363
12	Beaucaire	1	1	1
13	Lumbres	0,533511586	0,77744068	0,77744068
14	La Couronne	0,999559025	0,999559025	0,902173353
15	Grave-de-Peille	0,931577571	0,956851981	0,956851981
16	Bussac	1	1	1
17	Havre-Saint-Vigor	0,999331859	0,999331859	0,903380663
18	Port-la-Nouvelle	0,998779594	0,998779594	0,899790645
19	Rochefort	0,998705489	0,998705489	0,925874664
20	Xeuilley	0,87394048	0,914850402	0,914850402
21	Beffes	0,873180959	0,956846996	0,956846996
22	Altkirch	0,996695222	0,996695222	0,925917141
23	Contes	0,74704799	0,87477451	0,87477451
24	Saint-Egrève	0,998827831	0,998827831	0,96490057
25	Gargenville	1	1	1
26	Dannes	0,747641933	0,820618806	0,820618806
27	Val-d'Azergues	0,873418731	0,946384827	0,946384827
28	Créchy	0,999165199	0,999165199	0,897493007
29	Frangey	0,87573895	0,891931606	0,891931606
30	Villiers-au-Bonin	0,99777345	0,998239784	0,997877771
31	Ranville	0,874377204	0,96318304	0,96318304
32	Pont-à-Vendin	0,550254441	0,776525333	0,776525333
33	Chambéry	1	1	1
34	Cruas	1	1	1
35	La Pérelle	1	1	1

$$\hat{x}_o = x_o - s^-$$

$$\hat{y}_o^g = y_o^g + s^g \quad (s^g = Y^g\lambda - y_o^g)$$

$$\hat{y}_o^b = y_o^b - s^b \quad (s^b = y_o^b - Y^b\lambda)$$

The regulatory constraint defined as emission per unit of input is:

$$\frac{\hat{y}_o^b}{\hat{x}_o} \leq \alpha_1$$

If the regulatory constraint is fullfilled:

$$\rho_o^r = \rho_o$$

The violation of this constraint is described by the slack variable:

$$s_o^r = \hat{y}_o^b - \alpha_1 \hat{x}_o$$

The new efficiency score is:

$$\rho^r = 1 - \frac{1}{m}\left(\sum_{i=1}^{m} \frac{s_i^- + s^r}{x_{io}}\right) \leq \rho_0$$

Cementáreň	bez regulácie	$\alpha_1 = 0.3$	$\alpha_1 = 0.25$	$\alpha_1 = 0.2$	$\alpha_1 = 0.15$	$\alpha_1 = 0.1$
Saint-Piree-la-Cour	0.9984	0.9984	0.9984	0.9487	0.8987	0.8488
Heming	0.9977	0.9977	0.9868	0.9370	0.8871	0.8372
Couvrot	0.9319	0.9319	0.9319	0.8884	0.8418	0.7952
Montalieu	0.9118	0.9118	0.9084	0.8629	0.8173	0.7717
Airvault	0.8739	0.8739	0.8722	0.8285	0.7848	0.7411
Le Teil	0.9973	0.9973	0.9960	0.9462	0.8963	0.8464
La Malle	0.8737	0.8737	0.8715	0.8278	0.7842	0.7405
Martres	0.7493	0.7493	0.7295	0.6920	0.6546	0.6171
Origny	0.8539	0.8539	0.8508	0.8081	0.7655	0.7228
Cormeilles	0.7468	0.7447	0.7074	0.6701	0.6327	0.5954
Lexos	0.9982	0.9982	0.9920	0.9421	0.8922	0.8423
Beaucaire	0.9969	0.9969	0.9969	0.9546	0.9048	0.8549
Lumbres	0.5333	0.5276	0.5010	0.4743	0.4476	0.4210
La Couronne	0.9989	0.9989	0.9797	0.9298	0.8798	0.8299
Grave-de-Peille	0.9309	0.9309	0.9280	0.8814	0.8349	0.7884
Bussac	1.0000	1.0000	1.0000	0.9589	0.9089	0.8589
Havre-Saint-Vigor	0.9990	0.9990	0.9817	0.9317	0.8818	0.8318
Port-la-Nouvelle	0.9987	0.9987	0.9803	0.9304	0.8804	0.8305
Rochefort	0.9986	0.9986	0.9877	0.9378	0.8879	0.8379
Xeuilley	0.8739	0.8739	0.8616	0.8179	0.7742	0.7306
Beffes	0.8730	0.8730	0.8709	0.8273	0.7836	0.7400
Altkirch	0.9966	0.9966	0.9858	0.9360	0.8861	0.8363
Contes	0.7470	0.7470	0.7275	0.6901	0.6528	0.6154
Saint-Egreve	0.9988	0.9988	0.9985	0.9485	0.8986	0.8486
Gargenville	1.0000	1.0000	1.0000	0.9585	0.9085	0.8585
Dannes	0.7475	0.7475	0.7140	0.6766	0.6392	0.6018
Val-d'Azergues	0.8733	0.8733	0.8685	0.8249	0.7812	0.7375
Crechy	0.9989	0.9989	0.9794	0.9295	0.8796	0.8296
Frangey	0.8755	0.8755	0.8569	0.8131	0.7693	0.7256
Villiers-au-Bonin	0.9976	0.9976	0.9976	0.9553	0.9054	0.8555
Ranville	0.8741	0.8741	0.8730	0.8293	0.7856	0.7419
Pont-a-Vendin	0.5319	0.5214	0.4948	0.4682	0.4416	0.4150
Chambery	0.9988	0.9988	0.9988	0.9538	0.9039	0.8540
Cruas	0.9935	0.9935	0.9935	0.9498	0.9001	0.8504
La Perelle	0.7451	0.7451	0.7111	0.6738	0.6365	0.5993
PRIEMER	0.9004	0.8999	0.8895	0.8458	0.8008	0.7558

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The results of model C with the regulatory constraints:

$$\frac{\hat{y}_0^b}{\hat{x}_0} \leq \alpha_1$$

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Cementáreň	bez regulácie	$\alpha_1 = 0.25$	$\alpha_1 = 0.2$	$\alpha_1 = 0.15$	$\alpha_1 = 0.1$
Saint-Piree-la-Cour	0.9666	0.9666	0.8019	0.6014	0.4009
Heming	0.9255	0.9255	0.7683	0.5762	0.3842
Couvrot	0.9792	0.9792	0.8702	0.6527	0.4351
Montalieu	0.9517	0.9517	0.8645	0.6484	0.4322
Airvault	0.9583	0.9583	0.9081	0.6811	0.4541
Le Teil	0.9611	0.9611	0.7983	0.5987	0.3991
La Malle	0.9563	0.9563	0.9066	0.6799	0.4533
Martres	0.8735	0.8735	0.8735	0.7242	0.4828
Origny	0.9524	0.9524	0.9238	0.6929	0.4619
Cormeilles	0.7974	0.7974	0.7974	0.6632	0.4422
Lexos	0.9423	0.9423	0.7819	0.5864	0.3909
Beaucaire	0.9962	0.9962	0.8277	0.6208	0.4138
Lumbres	0.7774	0.7774	0.7774	0.7774	0.6037
La Couronne	0.8970	0.8970	0.7438	0.5578	0.3719
Grave-de-Peille	0.9540	0.9540	0.8488	0.6366	0.4244
Bussac	1.0000	1.0000	0.8296	0.6222	0.4148
Havre-Saint-Vigor	0.9033	0.9033	0.7490	0.5617	0.3745
Port-la-Nouvelle	0.8996	0.8996	0.7461	0.5596	0.3730
Rochefort	0.9257	0.9257	0.7678	0.5759	0.3839
Xeuilley	0.9147	0.9147	0.8669	0.6502	0.4335
Beffes	0.9568	0.9568	0.9076	0.6807	0.4538
Altkirch	0.9257	0.9257	0.7694	0.5770	0.3847
Contes	0.8743	0.8743	0.8743	0.7271	0.4847
Saint-Egreve	0.9647	0.9647	0.8000	0.6000	0.4000
Gargenville	1.0000	1.0000	0.8281	0.6210	0.4140
Dannes	0.8191	0.8191	0.8191	0.6807	0.4538
Val-d'Azergues	0.9453	0.9453	0.8965	0.6724	0.4483
Crechy	0.8960	0.8960	0.7429	0.5571	0.3714
Frangey	0.8901	0.8901	0.8421	0.6316	0.4211
Villiers-au-Bonin	0.9962	0.9962	0.8271	0.6203	0.4136
Ranville	0.9611	0.9611	0.9106	0.6829	0.4553
Pont-a-Vendin	0.7553	0.7553	0.7553	0.7553	0.5881
Chambery	0.9857	0.9857	0.8174	0.6130	0.4087
Cruas	0.9896	0.9896	0.8250	0.6188	0.4125
La Perelle	0.8166	0.8166	0.8166	0.6808	0.4538
PRIEMER	0.9231	0.9231	0.8252	0.6396	0.4313

Comparison of the results for the models A, B, C with regulatory constraint defined by emission per unit of input:

	bez regulácie	$\alpha_1 = 0.25$	$\alpha_1 = 0.2$	$\alpha_1 = 0.15$	$\alpha_1 = 0.1$
Model A	0.9004	0.8895	0.8458	0.8008	0.7558
Model B	0.9119	0.9119	0.8932	0.8707	0.8482
Model C	0.9231	0.9231	0.8252	0.6396	0.4313

Silanic (2013)

Further research:
undesirable outputs as weak disposable outputs
(C. Bremberger, F. Bremberger, M.Luptacik, S.Schmitt, 2013)

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