Bonds, Currencies and Expectational Errors

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Abstract

We propose a model in which sticky expectations concerning short-term interest rates generate *joint* predictability patterns in bond and currency markets. Using our calibrated model, we quantify the effect of this channel and find that it largely explains why short rates and yield spreads predict bond and currency returns. The model also creates a downward sloping term structure of carry trade returns, difficult to replicate in a rational expectations framework. Including a sticky short rate expectations channel into a standard affine term structure model improves its fit and allows the model to better capture the drift patterns in the data.

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1 Introduction

This paper presents the first unified theory of bond and currency markets based on expectational errors. According to this theory forecast errors concerning short-term interest rates give rise to joint predictability patterns in bond and currency markets. These predictability patterns nest, and can explain, many of the predictability puzzles documented in the previous literature.

Lustig et al. (2019) argue that the literature's key findings concerning currency and bond return predictability are related: while a high short-term interest rate predicts high returns for a currency, it predicts low returns for long-term bonds denominated in this currency. Similarly, a steep slope of the yield curve predicts low returns for a currency but high returns for corresponding long-term bonds. Such negative correlation between the currency and bond premia represents a puzzle for rational expectations macrofinance models. The model presented in this paper explains this correlation.

Our model is based on the well-documented finding that forecasters update their short rate predictions sluggishly (Coibion and Gorodnichenko, 2015). We do not offer an explanation for this pattern, though we note that it can be caused indirectly due to slow updating concerning factors driving interest rates.[1] However, the key assumption of our approach is that currencies and bonds are priced consistently with such biased

[1] There are various possible explanations, for example D'Acunto et al. (2019) argue that household forecast errors are related to *cognitive frictions*. Sticky expectations are also consistent with *inattention* (see e.g., Gabaix, 2019). Moreover, Ilut (2012) notes that similar effects follow from models with ambiguity averse preferences.

expectations concerning short rates.

Then, the return on a bond or currency can be decomposed into a rational risk premium, a short rate misperception effect and a risk premium misperception effect. This decomposition is an identity, it holds in all models in which subjective expectations are given by a probability measure. We argue that under relatively weak and realistic conditions the contribution of short rate forecast errors to return variation can be identified econometrically.

We use our calibrated model to quantify the effect of the interest rate misperception channel. We find that it can account for most of the variation in bond and currency premia driven by changes in short rates and yield spreads. The channel generates coefficients in predictability regressions similar to those found in the data.

Various authors, including Gourinchas and Tornell (2004), Cieslak (2017) and Piazzesi et al. (2015), have explored the effects of expectational errors on bond and currency returns separately. However, what has heretofore been unnoticed is that expectational errors concerning short rates provide a natural candidate for a *joint* theory of bond and currency markets.

The economic intuition behind our key results is simple. The current home and foreign short-term interest rates are known but agents must forecast their future values. The value of a foreign currency is increasing in expected foreign short-term interest rates and the value of foreign long-term bond decreasing in expected (foreign) short-term interest rates. When agents underpredict the path of future foreign interest rates, the value of the foreign currency is lower than under rational expectations but the value of the foreign bond higher than under rational expectations. This implies high actual returns for the currency but low returns for the corresponding bond.

In the data this underprediction is associated with *sticky expectations*. When short-term interest rates increase, for example due to a contractionary monetary policy shock, it takes time for forecasters to revise their future short rate expectations up. This leads forecasters to underpredict the future path of short rates. As the forecasters slowly increase their expectations over future foreign short-term interest rates, the foreign currency appreciates but the value of the foreign bond falls. Before the forecasters have updated their expectations closer to rational values, the returns for a currency will be high but the returns for the bond low.

Note that sticky expectations give rise to a relation between the *level* of short-term interest rates and the degree of underprediction concerning future interest rates. When short-term interest rates are high, they have on average increased recently. Therefore high short-term interest rates are associated with larger underprediction concerning future interest rates. This implies that a high short-term interest rate predicts high returns for a currency but low returns for the corresponding long-term bond.

We now demonstrate this intuition further with a simplified version of the model. Assume that the currencies are subject to similar perceived risk premia. Denote the short-term interest rate differential between the foreign and home country by $x_t \equiv i_t^* - i_t$ and the log FX rate by s_t, where an increase in s_t implies an appreciation of the foreign currency. The logarithmic perceived uncovered interest rate parity condition is:

$$\mathbb{E}_t^S[s_{t+1}] - s_t + x_t = 0, \tag{1}$$

where S denotes the subjective probability measure of the agents. Roughly, this states that the perceived expected return from borrowing in the home currency and investing in the foreign currency is zero. For simplicity assume a stationary nominal exchange rate and a long-run expected log exchange rate of 0 (e.g. due to symmetric countries).[2] From this one can solve:

$$s_t = \sum_{i=0}^{\infty} \mathbb{E}_t^S[x_{t+i}]. \tag{2}$$

[2]We discuss the role of the permanent component of the FX rate later.

Given persistent interest rates, the foreign currency is strong after shocks that raise foreign interest rates above home interest rates: $x_t > 0$. The violations of uncovered interest parity are due to the fact that now under subjective expectations the interest rate differential tends to remain lower than under rational expectations $\mathbb{E}_t[x_{t+1}] - \mathbb{E}_t^S[x_{t+1}] > 0$. This is because the forecasters are slow at increasing their interest rate forecasts after the positive interest rate shocks. On the other hand, this implies that $\mathbb{E}_t[s_{t+1}] - \mathbb{E}_t^S[s_{t+1}] > 0$. That is, the foreign currency will be stronger on average the next period than predicted by forecasters.

The relative log price of a zero coupon bond of maturity n is:

$$q_t^*(n) - q_t(n) = -\sum_{i=0}^{n-1} \mathbb{E}_t^S\left[x_{t+i}\right]. \tag{3}$$

When $x_t > 0$ the price of the foreign bond, $q_t^*(n)$, that is known by all agents, is relatively low and the yield high. However, because this is due to a recent interest rate shock the forecasters believe $\mathbb{E}_t[x_{t+1}] - \mathbb{E}_t^S[x_{t+1}] > 0$ and therefore $\mathbb{E}_t[q_{t+1}^*(n-1) - q_{t+1}(n-1)] - \mathbb{E}_t^S[q_{t+1}^*(n-1) - q_{t+1}(n-1)] < 0$. The misestimation of the interest rate process therefore creates variation in bond risk premia, measured under rational expectations, as high interest rate currencies have long-term bonds that are overpriced compared to prices under rational expectations.

Why does this type of model explain the joint behaviour of bonds and currencies? When $x_t > 0$, foreign currency short-term securities have high returns. At the same time the long-term bond of the same currency is relatively overpriced and yields low actual returns. Higher maturity increases the sensitivity of a bond to predictions about future interest rates, so this effect is stronger the longer the maturity of the bond. One can see that these effects partly offset each other so that a strategy that buys a long-term bond of the foreign currency and sells a similar bond of the home currency yields small domestic currency returns. This explains why the term structure of carry trade premia is downward sloping.

We provide strong empirical evidence that supports the importance of short rate forecast errors for bond and currency returns. Survey data is inconsistent with a time-varying subjective risk premium as implied by standard risk-based models. Because variables that predict bond and currency returns appear unrelated to their subjective expectations, the predictability can only be due to expectational errors. Currency returns also tend to be particularly high and bond returns low following recent short rate hikes. This is exactly as predicted by a model with sticky short rate expectations.

Finally, we discuss the results in the context of affine term structure models. We show that such models can be amended to incorporate sticky short rate expectations. The sticky expectations version of a standard affine term structure model is more consistent with survey data and gives a more accurate match to the predictability patterns observed in the data.

Related Literature This paper contributes to the vast literature on markets for currencies and government bonds. Special attention is given to explaining predictability patterns in bond and currency returns. The seminal paper for currencies is Fama (1984) which finds that currencies with high short-term interest rates appreciate rather than depreciate as predicted by uncovered interest rate parity. On the other hand, Fama and Bliss (1987) and Cochrane and Piazzesi (2005) find that high bond yields are associated with high bond returns, a violation of the expectations hypothesis. Lustig et al. (2019) argue that these two findings are related as high relative bond yields predict low returns for the corresponding currency.

A large literature in the tradition of rational expectations consumption based asset pricing has attempted to explain the predictability patterns in bond and currency markets. Examples include applications of the habit model for the bond market (Wachter, 2006) and those for the currency market (Verdelhan, 2010). Moreover, e.g. Bansal and Shaliastovich

(2012) apply the long-run risk model to both bonds and currencies.

A second literature in the tradition of no-arbitrage term structure models (see e.g., Duffie and Kan, 1996) has taken a more reduced form approach to modeling bonds. Similar models have been applied to currencies (Backus et al., 2001; Lustig et al., 2011). Moreover, e.g. Sarno et al. (2012) study the joint performance of a four factor affine model in pricing bonds and currencies. Note that Lustig et al. (2019) argue that neither the standard structural models nor these no-arbitrage models are able to replicate the term structure of carry trade returns.

A key alternative to the risk-based approach is to relax the assumption of rational expectations. This choice is motivated by the systematic expectational errors documented in surveys (Bacchetta et al., 2009; Coibion and Gorodnichenko, 2012; Greenwood and Shleifer, 2014; Nagel and Xu, 2021). The idea that currency returns are driven by mispricings has been explored by Froot and Frankel (1989), McCallum (1994), Gourinchas and Tornell (2004) and Burnside et al. (2011). Similarly, the effects of belief distortions on interest rates have been studied by, for example, Froot (1989), Xiong and Yan (2010), Hong and Sraer (2013), Piazzesi et al. (2015) and Cieslak (2017). However, to our best knowledge this is the first paper that offers a *joint* explanation for bond and currency markets based on expectational errors.

Perhaps the closest papers to our are Gourinchas and Tornell (2004) and Ilut (2012), who starting from different assumptions[3] have arrived at similar FX dynamics as those in this paper, but do not apply their model to the bond market. Furthermore, we tie short

[3] Gourinchas and Tornell (2004) assume that agents' overweight the importance of transitory short rate shocks, which gives rise to similar FX mechanisms as in this paper. Relative to this paper their approach, however, gives rise to some additional learning effects but imposes further restrictions on conditional variances. Ilut (2012) assumes agents possess ambiguity averse preferences. Here underreaction is an optimal response against misspecification of the short rate process. As mentioned before, we do not take a stance on the source of sticky expectations.

rate forecast errors to a more general framework, and test additional implications for FX, bond and survey data.

The risk-based models discussed above are based on the assumption of frictionless markets. Jylhä and Suominen (2010) and Gabaix and Maggiori (2015) argue that financial frictions can explain currency carry trade returns. In contemporaneous work Greenwood et al. (2019) posit that asset market frictions can explain both the properties of bonds and currencies, including the downward sloping term structure of carry trade returns. While these effects might complement the ones presented in this paper, they are inconsistent with our survey evidence.

In concurrent work Stavrakeva and Tang (2020) decompose the variance of exchange rate changes into variance of changes in short rate expectations, risk premia and the permanent component of the exchange rate as well as respective covariance terms. Contrary to the view that exchange rates are best explained by financial variables they argue that macroeconomic news explain most of exchange rate volatility. Their methodology differs from ours in several respects. First, they focus on decomposing FX rate variances rather than predictability coefficients. Second, in contrast to our simple but parsimonious sticky expectations specification, they estimate a VAR(2)-model using restrictions based on survey data. This specification, however, does not nest a sticky expectations model, that is the short rate process we apply is excluded a priori. Third, they do not study the implications of their model to bond market puzzles.

2 Determinants of Bond and Currency Premia

2.1 A General Framework

We first introduce the general model structure. Let there be two probability measures P and S. Here P corresponds to objective probabilities as viewed by a rational econometrician. On the other hand, S represents the subjective beliefs of an investor. The standard assumption in the literature is that $P = S$, but this paper argues that it is better to view these two as separate. If different investors have heterogeneous beliefs, we can also define S as a weighted average of the individual probabilities of the agents.[4] For simplicity we omit the P-symbol from expectations taken under rational beliefs.

Without loss of generality, consider the case of two countries, home and foreign, where the latter variables are denoted by stars. As in the introduction, the home and foreign log prices of nominal zero coupon bonds of maturities n are $q_t(n)$ and $q_t^*(n)$. Moreover, the short rate difference between the two countries is $x_t \equiv i_t^* - i_t$ and the log nominal exchange rate is s_t.

Similarly to Lustig et al. (2019) we define the excess foreign currency return as

$$r_{t+1}^{FX} \equiv x_t + s_{t+1} - s_t. \tag{4}$$

This also corresponds to the return from a carry trade strategy of borrowing short term in the home currency, investing the proceeds to foreign short term bills and finally converting the proceeds back to home currency. We define the relative excess return from n maturity zero-coupon bonds as

$$r_{t+1}^{B}(n) \equiv q_{t+1}^*(n-1) - q_{t+1}(n-1) - (q_t^*(n) - q_t(n)) - x_t. \tag{5}$$

This corresponds to the excess local currency return difference between foreign and home bonds. Finally, we define the dollar return difference between foreign and home bonds as

$$r_{t+1}^{FX}(n) \equiv q_{t+1}^*(n-1) - q_{t+1}(n-1) - (q_t^*(n) - q_t(n)) + s_{t+1} - s_t = r_{t+1}^{B}(n) + r_{t+1}^{FX}. \tag{6}$$

[4]We discuss such aggregation more formally in the appendix. To rule out ill-defined cases we assume the probability measures are equivalent, that is they agree on zero probability events.

This is mechanically the sum of the currency excess return and local currency bond return difference. We can view $r_{t+1}^{FX}(n)$ as excess returns from a modified version of the standard carry trade: here an investor sells short home n-maturity bonds and buys foreign bonds of the same maturity. The standard carry trade is then a special case of such a strategy $r_{t+1}^{FX} = r_{t+1}^{FX}(1)$.

The conditional rational expectations for the above returns, or relative currency and bond premia are:

$$\Theta_t^{FX} \equiv \mathbb{E}_t[r_{t+1}^{FX}] = x_t + \mathbb{E}_t[s_{t+1}] - s_t \tag{7}$$

$$\Theta_t^{B}(n) \equiv \mathbb{E}_t[r_{t+1}^{B}(n)] = \mathbb{E}_t[q_{t+1}^{*}(n-1) - q_{t+1}(n-1)] - (q_t^{*}(n) - q_t(n)) - x_t \tag{8}$$

$$\Theta_t^{FX}(n) \equiv \mathbb{E}_t[r_{t+1}^{FX}(n)] = \mathbb{E}_t[q_{t+1}^{*}(n-1) - q_{t+1}(n-1)] - (q_t^{*}(n) - q_t(n)) + \mathbb{E}_t[s_{t+1}] - s_t \tag{9}$$

Note that Lustig et al. (2019) call $\Theta_t^{FX}(n)$ *the term structure of carry trade premia*. Similarly we define the subjective relative currency and bond premia as

$$\zeta_t^{FX} \equiv \mathbb{E}_t^S[r_{t+1}^{FX}] = x_t + \mathbb{E}_t^S[s_{t+1}] - s_t \tag{10}$$

$$\zeta_t^{B}(n) \equiv \mathbb{E}_t^S[r_{t+1}^{B}(n)] = \mathbb{E}_t^S[q_{t+1}^{*}(n-1) - q_{t+1}(n-1)] - (q_t^{*}(n) - q_t(n)) - x_t \tag{11}$$

$$\zeta_t^{FX}(n) \equiv \mathbb{E}_t^S[r_{t+1}^{FX}(n)] = \mathbb{E}_t^S[q_{t+1}^{*}(n-1) - q_{t+1}(n-1)] - (q_t^{*}(n) - q_t(n)) + \mathbb{E}_t^S[s_{t+1}] - s_t \tag{12}$$

These represent the subjective conditional expectations of the above return returns. In a standard model, these would represent compensation for risk. However, more broadly they can also include ”convenience yields” (see e.g., Jiang et al., 2018) necessary to explain violations from covered interest parity type no-arbitrage conditions (see e.g., Du et al., 2018).

For simplicity we assume that all of the components of the above three equations, relative premia, short rate differentials, relative bond price changes[5] and exchange rate changes (but not the level of the FX rate) are stationary under the subjective measure. For notational convenience we assume these have been demeaned with their unconditional mean. We can iterate the first equation to provide an expression for the level of the FX rate:

$$s_t = \mathbb{E}_t^S \sum_{j=0}^{\infty} x_{t+j} - \mathbb{E}_t^S \sum_{j=0}^{\infty} \zeta_{t+j}^{FX} + \lim_{j\to\infty} \mathbb{E}_t^S [s_{t+j}].$$

This states that the level of FX rate reflects the subjectively expected path of short rate differentials and risk premia as well as a permanent component of the FX rate. A similar decomposition under the objective measure has been considered e.g. by Engel (2014) and Jiang et al. (2018); for an early application for the real exchange rate see Clarida and Gali (1994). Note that $\lim_{j\to\infty} \mathbb{E}_t^S [s_{t+j}]$ is generally time-varying and hence s_t is non-stationary. In particular it may feature a unit root.

We can solve an analoguous expression for the relative bond price:

$$q_t^*(n) - q_t(n) = -\mathbb{E}_t^S \sum_{j=0}^{n-1} x_{t+j} - \mathbb{E}_t^S \sum_{j=0}^{n-1} \zeta_{t+j}^{B}.$$

The permanent component does not appear in this expression due to finite maturity. Note that, holding other terms constant, the expected path of short rate differentials affects both the level of FX rate and the relative value of bonds. However, the value of the foreign currency is increasing in expected (foreign - home) short rate differentials but the relative value of the foreign bond is decreasing in expected short rate differentials. This

[5]Note that bond prices, not only bond price changes, might be stationary though this is not required.

fundamental property will be important for the later results. Plugging these expressions in to the formulas for the rational expectations of currency returns we obtain

$$\underbrace{\Theta_t^{FX}}_{\text{Currency premium}} = \underbrace{\zeta_t^{FX}}_{\text{Risk premium differential}} \underbrace{+\mathbb{E}_t\left[\mathbb{E}_{t+1}^S\sum_{j=0}^{\infty}x_{t+1+j}-\mathbb{E}_t^S\sum_{j=0}^{\infty}x_{t+1+j}\right]}_{\text{Interest rate misperception effect}}$$
$$\underbrace{-\mathbb{E}_t\left[\mathbb{E}_{t+1}^S\sum_{j=0}^{\infty}\zeta_{t+1+j}^{FX}-\mathbb{E}_t^S\sum_{j=0}^{\infty}\zeta_{t+1+j}^{FX}\right]}_{\text{Risk premium misperception effect}} + \underbrace{\mathbb{E}_t[\lim_{j\to\infty}\mathbb{E}_{t+1}^S[s_{t+j}]-\lim_{j\to\infty}\mathbb{E}_t^S[s_{t+j}]]}_{\text{Permanent component misperception effect}}$$

$$\equiv \zeta_t^{FX}+\Theta_t^{IRM}+\Theta_t^{RPM}+\Theta_t^{PCM}, \tag{13}$$

where the second equality is simply naming a letter for each component. Similarly for bonds we have

$$\underbrace{\Theta_t^B(n)}_{\text{Bond premium differential}} = \underbrace{\zeta_t^B(n)}_{\text{Risk premium differential}} \underbrace{+\mathbb{E}_t\left[\mathbb{E}_{t+1}^S\sum_{j=0}^{n-2}x_{t+1+j}-\mathbb{E}_t^S\sum_{j=0}^{n-2}x_{t+1+j}\right]}_{\text{Interest rate misperception effect}}$$
$$\underbrace{-\mathbb{E}_t\left[\mathbb{E}_{t+1}^S\sum_{j=0}^{n-2}\zeta_{t+1+j}^B(n-j-1)-\mathbb{E}_t^S\sum_{j=0}^{n-2}\zeta_{t+1+j}^B(n-j-1)\right]}_{\text{Risk premium misperception effect}}$$

$$\equiv \zeta_t^B(n)+\Theta_t^{B,IRM}(n)+\Theta_t^{B,RPM}(n).$$

A similar decomposition is obtained for $\Theta_t^{FX}(n)$. Now consider, the standard simple linear return forecasting model

$$r_{t+1} = \alpha + \beta f_t + \epsilon_{t+1},$$

where f_t is a forecasting factor, ϵ_{t+1} is a zero mean error term and in place of r_{t+1} we can have either r^{FX}_{t+1}, $r^{FX}_{t+1}(n)$ or $r^{B}_{t+1}(n)$. In any sample T the OLS estimate of β is given by

$$\beta = \frac{Cov(r_{t+1}, f_t)}{Var(f_t)} = \frac{Cov(\Theta_t, f_t)}{Var(f_t)},$$

where $\Theta_t = \mathbb{E}_t[r_{t+1}]$ and the second equality follows from the fact that $r_{t+1} = \Theta_t + r_{t+1} - \Theta_t$, where $r_{t+1} - \Theta_t$ is independent from time t information. The above formula for the OLS estimate of β holds also when the true relationship between r_{t+1} and f_t is not linear. The linearity of covariance and the above decompositions for the bond and currency premia then imply the following decomposition for β

$$\begin{aligned}\underbrace{\beta}_{\text{Predictability coefficient}} = \underbrace{\beta^{RP}}_{\text{Risk premium differential effect}} + \underbrace{\beta^{IRM}}_{\text{Interest rate misperception effect}} + \\ \underbrace{\beta^{RPM}}_{\text{Risk premium misperception effect}} + \underbrace{\beta^{PCM}}_{\text{Permanent component misperception effect}} .\end{aligned} \quad (14)$$

For example $\beta^{FX,IRM}$ is given by

$$\beta^{FX,IRM} = \frac{\mathbb{C}ov(\Theta^{FX,IRM}_t, f_t)}{\mathbb{V}ar(f_t)} = \frac{\mathbb{C}ov(\mathbb{E}_t\left[\mathbb{E}^S_{t+1}\sum_{j=0}^{\infty} x_{t+1+j} - \mathbb{E}^S_t \sum_{j=0}^{\infty} x_{t+1+j}\right], f_t)}{\mathbb{V}ar(f_t)}.$$

Moreover, for the bond premium we naturally have $\beta^{B,PCM}(n) = 0$. This implies that the forecasting power of a factor f_t comes from correlation with the rational risk premium, from correlation with the different misperception parts of the bond or currency premium or both. A similar decomposition can be obtained for a linear model with multiple predicting factors. In this paper we are particularly interested in β^{IRM}, the effect of interest rate misperceptions on bond and currency returns. However, identifying β^{IRM} requires further assumptions about the short rate process.

2.2 Identifying Assumption

We now describe conditions under which the coefficients described in the previous section can be identified using data on short rates, bond and currency returns and survey expectations. We take the short rate process under the subjective and objective measure as exogeneous and later estimate these processes from the data.

The following condition describes the key assumption of the paper:

Condition SE

Under the objective measure the short rate differential x_t follows an AR(p) - process. However, under the subjective measure S, the conditional expectation is given by a sticky expectations process $\mathbb{E}_t^S[x_{t+h}] = k\sum_{j=0}^{\infty}(1-k)^j \mathbb{E}_{t-j}[x_{t+h}]$.

As in Coibion and Gorodnichenko (2015), we further focus on the case $p = 1$, that is assume that under the objective measure the short rate difference x_t follows an AR(1) process. This gives a good fit to observed data on short rates but we study the robustness of the results to alternative specifications in the appendix. We can rewrite the sticky expectations process as follows:

$$\mathbb{E}_t^S[x_{t+h}] = k\mathbb{E}_t[x_{t+h}] + (1-k)\mathbb{E}_{t-1}^S[x_{t+h}].$$

As argued by Coibion and Gorodnichenko (2015) this process gives a good fit to survey data on short rates. If beliefs are rational $k = 1$ and the subjective and objective expectations coincide. However, typically $0 < k < 1$ so that the subjective expectation is a weighted average of the last period expectation and the current value for the state. Effectively, the biased measure underreacts to interest rate shocks.

Note that under the assumption that the objective data is given by an AR(1)-process we have

$$\mathbb{E}_t[x_{t+h}] = \lambda^h x_t.$$

Here $-1 < \lambda < 1$; there is no constant because the variables are demeaned. From the initial definition of a sticky expectations process it then follows:

$$\mathbb{E}_t^S[x_{t+h+j}] = \lambda^j \mathbb{E}_t^S[x_{t+h}]$$

and hence also

$$\mathbb{E}_t^S[x_{t+h}] = k\mathbb{E}_t[x_{t+h}] + (1-k)\lambda\mathbb{E}_{t-1}^S[x_{t+h-1}],$$

this expression is used for deriving some of the following results. How could k be estimated? As in Coibion and Gorodnichenko (2015), it is useful to consider the following regression:

$$x_{t+h} - \mathbb{E}_t^S[x_{t+h}] = \alpha^{FR} + \beta^{FR}[\mathbb{E}_t^S[x_{t+h}] - \mathbb{E}_{t-1}^S[x_{t+h}]] + e_{t+h}. \tag{15}$$

Here we regress the forecast error for the short rate differential on the corresponding forecast revision. As explained in the appendix, the model implies that $\alpha^{FR} = 0$ and $\beta^{FR} = \frac{1-k}{k}$. In a rational model $k = 1$, $\beta^{FR} = 0$ and forecast errors are not predictable. More generally, a positive (negative) coefficient for the regression indicates underreaction (overreaction).

This condition fully pins down the effect of short rate forecast errors on bond and currency returns. In particular we have the following proposition:

Proposition 1 (Condition SE and the Term Structure of Carry Trade Returns)**.** *Assume condition SE holds (under $p = 1$). Now the interest rate misperception parts of the FX premia under objective beliefs are given by ($n \geq 2$)*

$$\Theta_t^{FX,IRM} = \left[1 + \frac{\lambda k}{1-\lambda}\right]\left[\mathbb{E}_t[x_{t+1}] - \mathbb{E}_t^S[x_{t+1}]\right] \tag{16}$$

$$\Theta_t^{FX,IRM}(n) = \frac{k\lambda^n}{1-\lambda}\left[\mathbb{E}_t[x_{t+1}] - \mathbb{E}_t^S[x_{t+1}]\right]. \tag{17}$$

Similarly the interest rate misperception part of the (population OLS estimate of) slope coefficient in a predictability regression with the short rate differential, $f_t = x_t$ is

$$\beta^{FX,IRM} = \left[1 + \frac{\lambda k}{1-\lambda}\right]\frac{(1-k)(\lambda-\lambda^3)}{1-(1-k)\lambda^2} > 0 \tag{18}$$

$$\beta^{FX,IRM}(n) = \frac{k\lambda^{n-1}}{1-\lambda}\frac{(1-k)(\lambda-\lambda^3)}{1-(1-k)\lambda^2} > 0. \tag{19}$$

$\beta^{FX}(n)$ decays at rate λ^n and approaches zero as $n \to \infty$. Similarly $\Theta_t^{FX,IRM}(n) \to 0$ as $n \to \infty$. Moreover $\beta^{B,IRM}(n) = \beta^{FX,IRM}(n) - \beta^{FX,IRM} < 0$.

Note that assuming condition SE, $\Theta_t^{FX,IRM}(n)$ and $\beta^{FX,IRM}(n)$ tend to zero as $n \to \infty$. That is, the term structure of the interest rate component of FX premia is downward sloping. On the other hand, as explained by Lustig et al. (2019), standard models typically do not imply that the rational risk premium component $\zeta_t^{FX}(n)$ is downward sloping. This suggests that allowing for sticky short rate expectations might be key to understanding why the actual term structure of FX premia is downward sloping. In particular if the other components are numerically small this property will hold for the actual FX premia and carry trade returns.

2.3 Possible Additional Assumptions

Proposition 1 implies that condition SE alone is sufficient to determine the contribution of short rate forecast errors to bond and currency return predictability. Later in the empirical section we find that this forecast error component can explain most of bond and currency return predictability.

In this section we describe further assumptions that can be used to derive stronger results by effectively shutting down other channels. These exercises are particularly useful for illustrating the mechanisms created by sticky short rate expectations. However, because in the empirical part we apply survey data to measure subjective risk premia, these assumptions are not required.

We first consider the following condition:

Condition CRP

The relative risk premia are constant in time under the subjective measure: ζ_t^{FX}, $\zeta_t^{FX}(n)$ *and* $\zeta_t^{B}(n)$ *are constant.*[6]

Note that in our general framework this also implies that the risk premium misperception components are zero. This means that all time-variation in returns under objective beliefs is due to misperceptions about future short rates and the permanent component of the FX rate. This latter effect can be muted using the following assumption:

Condition NLRM

The permanent component misperception effect is zero $\mathbb{E}_t[\lim_{j\to\infty}\mathbb{E}_{t+1}^{S}[s_{t+j}] - \lim_{j\to\infty}\mathbb{E}_t^{S}[s_{t+j}]] = 0.$

This condition is naturally satisfied for example when the investors have correct long-

[6] Assuming any two conditions implies the third, as $\zeta_t^{B}(n) = \zeta_t^{FX}(n) - \zeta_t^{FX}$.

run beliefs. Assuming both condition CRP and NLRM now implies that all time variation in objectively expected returns is due to misperceptions about relative short rates. This implies the following proposition:

Proposition 2 (Condition CRP and the Term Structure of Carry Trade Returns)**.** *Assume conditions SE holds (under $p = 1$). Furthermore assume conditions CRP and NLRM hold. Now the FX premia are given by ($n \geq 2$)*

$$\Theta_t^{FX} = \left[1 + \frac{\lambda k}{1-\lambda}\right]\left[\mathbb{E}_t[x_{t+1}] - \mathbb{E}_t^S[x_{t+1}]\right] \tag{20}$$

$$\Theta_t^{FX}(n) = \frac{k\lambda^n}{1-\lambda}\left[\mathbb{E}_t[x_{t+1}] - \mathbb{E}_t^S[x_{t+1}]\right]. \tag{21}$$

Similarly the (population OLS) estimate of the slope coefficient in a predictability regression with the short rate differential, $f_t = x_t$ is

$$\beta^{FX} = \left[1 + \frac{\lambda k}{1-\lambda}\right]\frac{(1-k)(\lambda - \lambda^3)}{1-(1-k)\lambda^2} > 0. \tag{22}$$

$$\beta^{FX}(n) = \frac{k\lambda^{n-1}}{1-\lambda}\frac{(1-k)(\lambda - \lambda^3)}{1-(1-k)\lambda^2} > 0. \tag{23}$$

$\beta_1^{FX}(n)$ decays at rate λ^n and approaches zero as $n \to \infty$. Similarly $\Theta_t^{FX}(n) \to 0$ as $n \to \infty$. Moreover $\beta^B(n) = \beta^{FX}(n) - \beta^{FX} < 0$.

The results of this proposition look similar to those of the previous one. However, there is a crucial difference. The results of the previous proposition concern the interest rate misperception component of the currency premia and predictability coefficient. Proposition 2 instead shows that by further imposing conditions CRP and NLRM, the bond and currency risk premia and the related predictability coefficients correspond to these same interest rate misperception components. This is because these conditions imply that the other components are zero. This also implies that the actual term structure of FX premia is downward sloping.

The above conditions are less restrictive than they might initially sound. Note that they do not imply that risk premia are constant under the objective measure, but rather that all time variation in objectively measure risk premia are caused by misperceptions concerning short term interest rates. Hassan and Mano (2017) argue that a full model of the FX premium needs to have both a persistent cross-currency component as well as time-varying part that explains why increases in relative foreign interest rates lead to higher foreign currency returns. The above framework satisfies these requirements. However, note that we only provide a theory about time-variation in bond and currency premia but not about persistent cross-currency differences in these premia.

Note that many papers using models with belief distortions to explain return predictability such as Gourinchas and Tornell (2004), Bouchaud et al. (2018) and Brooks et al. (2019) and Cieslak (2017) make the stronger assumption that agents are risk neutral. This implies zero and hence constant risk premia. Constant subjective risk premia also follow naturally from widely used models where arbitraugers possess mean variance or constant absolute risk aversion preferences, which eliminate wealth effects, and asset supplies are constant.[7]

Condition CRP is consistent with our later empirical findings that subjective risk premia do not vary with the same variables that predict returns. For example in our notation, variables affecting the objective FX risk premium Θ_t^{FX} seem largely unrelated

[7] In such models an arbitrauger's carry trade position is $\frac{\mathbb{E}_t(R^c)}{\gamma \mathbb{V}ar_t(R_t^c)}$, where R_t^c is the return from carry trade and γ is risk aversion. Assuming a constant foreign bond excess supply $\bar{B}$, the equilibrium is determined by $\frac{\mathbb{E}_t(R^c)}{\gamma \mathbb{V}ar_t(R_t^c)} = \bar{B}$. Approximating $R^c \approx s_{t+1} - s_t + x_t$ as in Ilut (2012) or Greenwood et al. (2019) gives a solution $s_{t+1} - s_t = \zeta_{FX} - x_t$, where the FX premium is $\zeta^{FX} = \bar{B}\gamma \mathbb{V}ar_t^S(\sum_{i=1}^{\infty} \mathbb{E}_{t+1}^S[x_{t+i}])$. As in Ilut (2012) and Greenwood et al. (2019) assume the short rate process is exogeneous. Our specification for the subjective short rate process then implies that this premium is constant (see the derivations for the affine term structure model).

to its subjective counterpart ζ_t^{FX}. That is we largely cannot reject the null that the risk premium component β^{RP} in decomposition 14 is zero. In the appendix we also explain that the assumption of constant risk premia could be replaced with the assumption that the subjective risk premium is uncorrelated with short rates.

Conditions SE, CRP and NLRM imposed in Proposition 2 can also be used to identify predictability coefficients related to alternative predictors than just the relative level of short rates. In particular they imply that the slope coefficient in a regression with a relative yield spread predictor has the opposite sign than the slope coefficient in a regression with relative short rate. This is because, assuming constant risk premia, the yield spread tends to be low when short rates are high.

To illustrate the logic behind the results, we first show the evolution of the yield curve and exchange rate after a shock that increases the foreign interest rate. Figure I plots the impulse responses to an interest rate shock assuming conditions SE, CRP and NLRM[8]. When foreign interest rates increase above home interest rates, forecasters update their relative short rate forecasts upward but not as much as a rational forecaster would do. Because long term interest rates are averages over expected short rates, they increase but less than short rates, so the relative yield curve becomes downward sloping. The price of a long-term bond falls but by less than according to rational expectations. The foreign currency appreciates but by less than predicted by rational expectations. However, in the long-run expectations converge to rational values. During the interim period, a high interest rate predicts positive returns for the foreign currency but low relative returns for long-term foreign bonds.

Given conditions SE, CRP and NLRM, a positive interest rate differential predicts positive carry trade returns for any maturity bonds. However, the effect is declining in the bond maturity n and there is no predictability in the limit $n \to \infty$. Figure II shows

[8]The figure assumes the long-run log-exchange rate is 0 so here $s_t = \sum_{i=0}^{\infty} \mathbb{E}_t^S [x_{t+i}]$ and $q_t^*(n) - q_t(n) = -\sum_{i=0}^{n-1} \mathbb{E}_t^S [x_{t+i}]$. The impulse responses are computed using the benchmark calibration derived later.

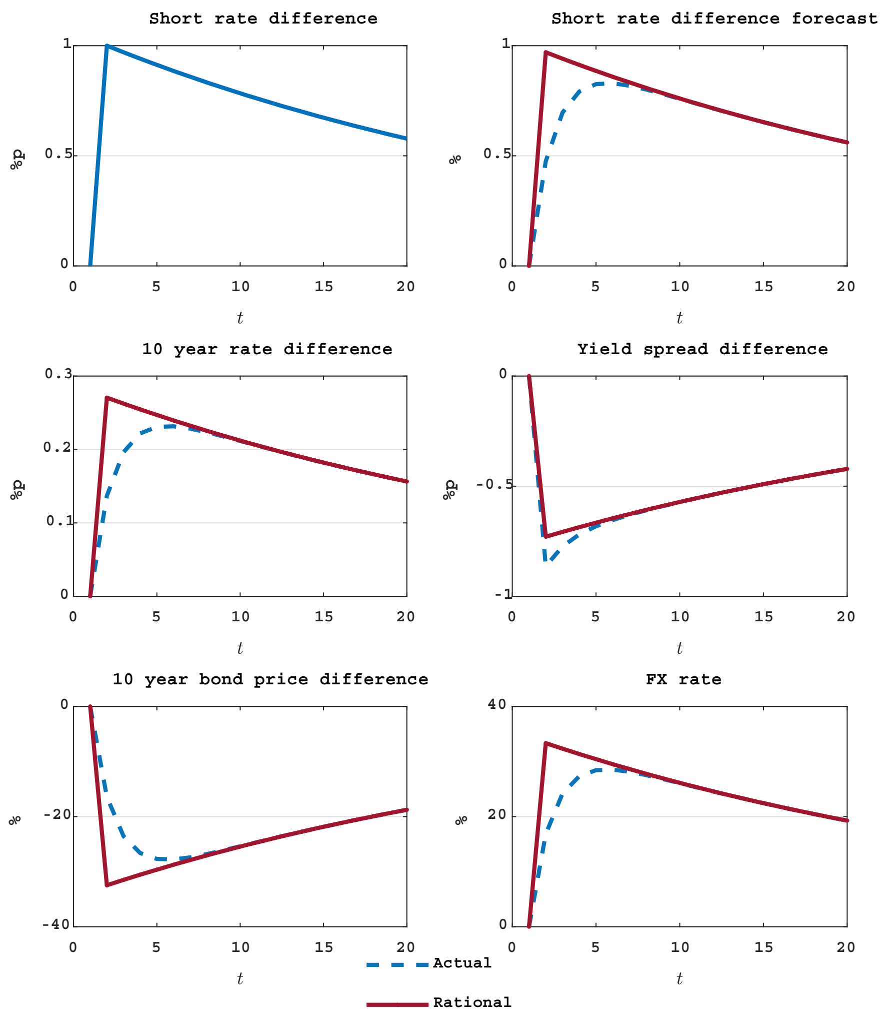

Figure I Impulse responses to a shock to the foreign interest rate when short rate forecast errors drive all variation in objective premia. Time is measured in months.

the decay pattern for relative carry trade returns for different values of the persistence parameter λ[9]. As explained before, the downward sloping term structure emerges because variation in expected bond returns offsets variation in expected currency returns.

Figure III shows the slope coefficient β^{FX} as a function of both k and λ given conditions SE, CRP and NLRM. The coefficient is positive. For typical parameter values β^{FX} is decreasing in k and increasing in λ. The benchmark calibration used later predicts $\beta_1^{FX} \approx 0.99$

Figure IV shows the slope coefficient of a regression of relative returns of 10 year bonds on short rate differential x_t assuming conditions SE, CRP and NLRM. This is also given by $\beta^{FX}(n) - \beta^{FX}$. The coefficient is negative. For typical parameter values the slope coefficient is increasing in k and λ. The benchmark calibration discussed later predicts $\beta_1^{FX}(n) - \beta_1^{FX} \approx -0.99$. This opposite predictability in bond returns largely offsets the predictability in currency returns so that there is little predictability in the returns of carry trades implemented with 10 year bonds.

It can be shown that the model predicts the opposite patterns when relative yield spreads are used as predictors. A high slope of the yield curve predicts low currency returns but high bond returns. This occurs because the slope of the yield curve tends to be high when interest rates are low.

Finally, under conditions SE, CRP and NLRM, the model implies that foreign currency returns tend to be particularly high and bond returns low when foreign short rates have recently increased relative to past values. This is formalized in the following proposition:

Proposition 3. *Assume conditions SE, CRP and NLRM hold. Define the average past short rate difference as:* $\bar{x}_t \equiv x_t + (1-k)\lambda x_{t-1} + (1-k)^2\lambda^2 x_{t-2} + \ldots$. *Consider the regressions*

[9] This shows the relative profitability / predictability coefficient. That is the coefficient for the short maturity carry trade is normalized to 1.

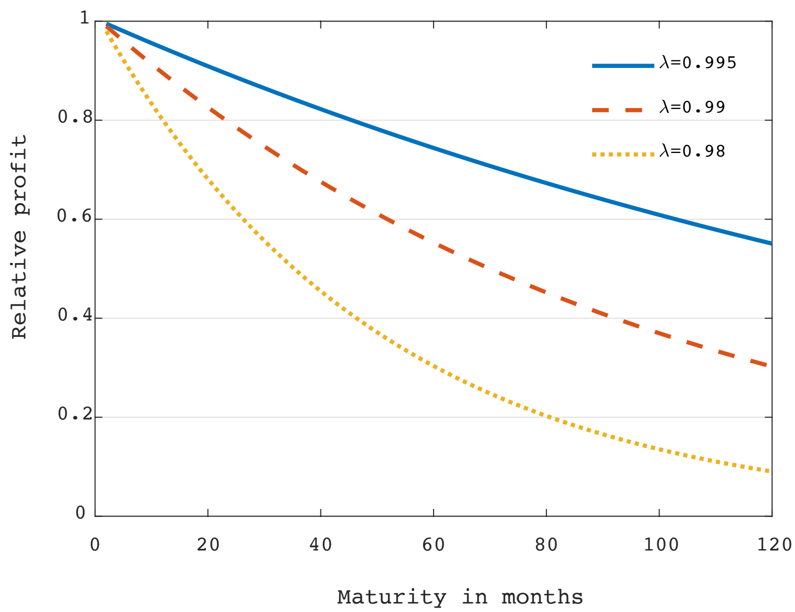

Figure II The term-structure of carry trade when short rate forecast errors drive all variation in objective premia.

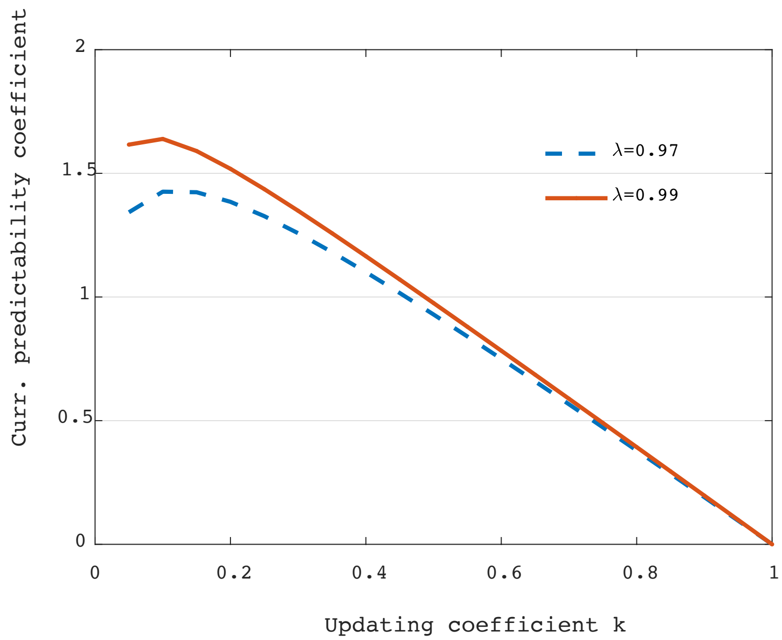

Figure III The currency return predictability coefficient as a function of k and λ when short rate forecast errors drive all variation in objective premia.

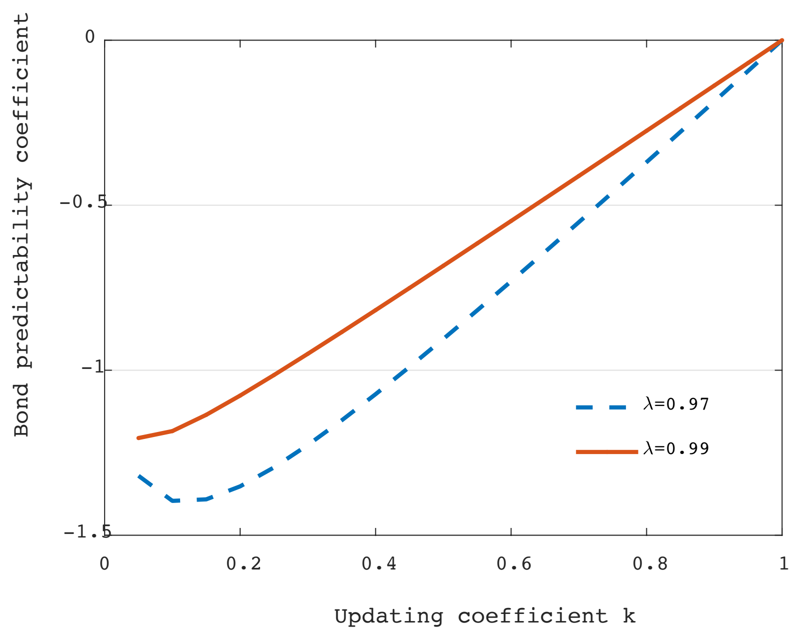

Figure IV The relative bond return predictability coefficient as a function of k and λ when short rate forecast errors drive all variation in objective premia.

$$r_{t+1}^{FX} = \alpha^{FX} + \beta_1^{FX} x_t + \beta_2^{FX} \bar{x}_{t-1} + \varepsilon_{t+1} \tag{24}$$

and

$$r_{t+1}^{B}(n) \equiv r_{t+1}^{FX}(n) - r_{t+1}^{FX} = \alpha^{B}(n) + \beta_1^{B}(n) x_t + \beta_2^{B}(n) \bar{x}_{t-1} + \varepsilon_{t+1}^{n} \tag{25}$$

The (population OLS) estimate of β_1^{FX} is positive, of β_2^{FX} is negative, of $\beta_1^{B}(n)$ is negative and of $\beta_2^{B}(n)$ is positive.

Proof: see appendix.

Expectational errors concerning short rates are particularly large after recent short rate shocks. On the other hand, given our conditions, the rationally expected currency return is strictly increasing in these errors and the expected bond return is decreasing. This implies that high short-term interest rates relative to past short rates should predict high returns for a currency but low returns for the corresponding long-term bond. This explains why the slope coefficient on the past average short rate difference $\bar{x}_{t-1}$ has the opposite sign than the slope coefficient on the short rate difference x_t. Imposing only condition SE, the above result holds for the interest rate misperception component of bond and currency.

3 Empirical Evidence

We now turn to empirically test the model predictions as well as quantifying the effect of interest rate misperceptions on bond and currency returns.

3.1 Data

We first briefly describe the data used. We focus on the G10 currencies of Australia, Canada, Germany, Japan, Norway, New Zealand, Sweden, Switzerland, U.K. and U.S. We utilize FRED to obtain data on end of month FX rates and interest rates on 3 month and 10 year government securities.

We calibrate the agents' expectations using survey data. Consensus economics provides a monthly report of forecasts for 3 month and 10 year interest rates as well as FX rates obtained by surveying professional forecasters. Following Coibion and Gorodnichenko (2012), we average over the forecasts provided by different financial institutions.[10] Forecasts are available for all countries except Australia and New Zealand.

We calculate bond returns using Citigroup government bond local currency 10 year indices available for all countries except Norway.[11]

The start and end dates for the bond indices and survey data are given in table I, where we also report the number of observations. We choose US as the home country and express all returns and rates as differences to US.

3.2 Time Variation in Bond and Currency Premia

We start by replicating the four key predictability regressions in Lustig et al. (2019). The main difference is that we focus on a quarterly horizon instead of monthly. This is since our survey data for long term rates and exchange rates is also quarterly. However, the results are fairly similar for the monthly horizon.

Table II, panel A, gives the results for currency excess returns and panel B for relative bond returns. A high short rate difference is associated with high currency returns, a result

[10] See the appendix for a formal discussion about aggregation.

[11] The downside of bond indices is that they are based on coupon bonds, while the theoretical results are for zero-coupon bonds. However, the theoretical predictions hold qualitatively for coupon bonds. Moreover, in unreported robustness checks we obtain similar results using the zero-coupon yield curve data set of Wright (2011), for market data (see also the results in Lustig et al. (2019)). Moreover, the difference between the predictions for coupon and zero-coupon bonds is small according to simulations. The benefit of using bond indices is that they are free from approximation error in common interpolation procedures and corresponding returns are tradable.

	AUS	CAN	GER	JAP	NOR	NZ	SWE	CH	UK	US
					Bonds Index					
Start	85M1	85M1	85M1	85M1	NA	85M1	85M1	85M1	85M1	85M1
End	19M2	19M2	19M2	19M2	NA	19M2	19M2	19M2	19M2	19M2
Obs	410	410	410	410	NA	410	410	410	410	410
					Consensus					
Start	NA	91M5	91M5	91M5	98M6	NA	94M12	98M6	91M5	89M10
End	NA	19M4	19M4	19M4	19M4	NA	19M4	19M4	19M4	19M4
Obs	NA	336	336	336	251	NA	293	252	336	355

Table I Start and end dates for the bond index and survey data.

also known as the forward premium puzzle (Fama, 1984). However, it implies low returns for the country's long term bonds relative to those in the US. The signs of these coefficients are therefore as predicted by Proposition 2.

Similarly, a steep yield spread difference predicts low currency excess returns but high relative bond returns. This is since the yield spread tends to be high during times of high interest rates. These results are also in line with our sticky expectations model.

3.3 Time Variation in Subjective Bond and Currency Risk Premia

We can measure the subjective risk premium component β^{RP} in decomposition 14 directly using survey data. This is because, unlike the other components, β^{RP} depends only on subjective expectations over the next period.[12] In a rational expectations model, this component would account for all of return predictability: $\beta^{RP} = \beta$. However, this does not appear to be the case empirically.

Using a quartely horizon, we construct the subjective currency risk premia using survey expectations concerning exchange rate changes. We form the subjective bond risk premia

[12]Note that the other components depend on the whole term structure of subjective expectations, which is not directly observable.

PANEL A: Currency Excess Returns										
	3 month rate					yield spread				
	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1(3)$	$s.e$	R^2	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1$	$s.e$	R^2
Panel			1.36***	0.49	0.0256			-1.33**	0.63	0.0124
AUS	-0.004	0.006	1.52***	0.57	0.0368	0.004	0.008	-0.74	1.08	0.0004
CAN	0.005	0.004	1.32**	0.57	0.0024	0.007	0.005	-0.45	0.79	-0.0006
GER	0.016***	0.005	-0.53	1.03	0.0001	0.023**	0.011	1.70	1.90	0.0046
JAP	0.014**	0.006	2.83**	0.91	0.056	-0.018	0.012	-4.24*	2.40	0.0220
NZL	-0.006	0.006	1.87	0.43	0.0929	0.009*	0.005	-2.61***	0.47	0.0958
SWE	-0.005	0.005	0.42	0.99	0.0006	-0.003	0.005	-0.64	0.65	0.0032
CH	0.009*	0.005	1.80*	1.03	0.0232	-0.006	0.010	-2.11	1.73	0.0123
UK	-0.001	0.005	1.37	1.03	0.0199	0.003	0.006	-0.72	1.12	0.0010

PANEL B: Bond Local Currency Return Differences										
	3 month rate					yield spread				
	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1(3)$	$s.e$	R^2	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1$	$s.e$	R^2
Panel			-0.86***	0.22	0.028			1.56***	0.49	0.0342
AUS	0.004	0.003	-0.93*	0.50	0.0437	0.004	0.005	1.34*	0.80	0.0297
CAN	0.002**	0.002	-1.36***	0.42	0.0672	0.005**	0.003	1.67***	0.46	0.0744
GER	-0.003	0.002	-0.58*	0.35	0.0135	0.006*	0.003	1.92***	0.64	0.048
JAP	-0.007***	0.003	-0.85**	0.38	0.0152	0.012**	0.005	3.01***	1.01	0.0389
NZL	0.005	0.004	-1.15**	0.57	0.0317	-0.005**	0.002	-3.09***	1.13	0.0297
SWE	0.002	0.002	-0.68	0.50	0.0212	0.000	0.003	1.02	0.84	0.016
CH	-0.007***	0.003	-0.80	0.52	0.0151	0.011***	0.004	3.05***	0.68	0.1032
UK	0.001	0.002	-0.95**	0.44	0.0269	0.003	0.003	1.17**	0.53	0.023

Table II shows the results from regressing the currency and relative bond (foreign minus US) returns on short rate and yield spread differences. The standard errors of the panel regression (country fixed effects) are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

similarly using survey expectations of quarterly 10 year interest rates changes and by applying a duration approximation as in Bacchetta et al. (2009). We then explain these subjective premia using short rate and yield spread differentials. The slope coefficients in these regressions correspond to the risk premium components β^{RP} in decomposition 14.

The results are given in table III. There is no significant relation between short rates and subjective premia neither at the country nor panel level. Hence we find no support for the hypothesis that time-varying subjective premia would explain our key results concerning predicting bond and currency returns with short rates. At the panel level there is also no significant relation between expected currency premia and yield spread differences. The only case where we find some evidence for a risk premium channel is when predicting subjective bond premia with yield spread differences. However, even here the panel slope coefficient (0.71) is smaller than in the data (1.56) so that a subjective risk premium could explain less than half of the predictability observed in the data.

This evidence is broadly in line with Nagel and Xu (2021) who argue that such subjective premia do not explain bond and currency predictability. Like our paper they argue that survey expectations are consistent with a model with time-varying beliefs about interest rates, or more generally cash flows, rather than time-varying risk aversion or time-varying perceptions of risk.

3.4 Calibrating the Short Rate Process

To quantify the importance of interest rate misperceptions, we need to calibrate the process for short rate differentials x_t. Given condition SE, we only need to find the persistence parameter λ and the underreaction coefficient k.

We estimate the persistence parameter for interest rate differentials using OLS. We consider the process separately for each country as well as for a panel with all the countries. Note that taking differences removes the common secular downward trend in interest

PANEL A: Expected Currency Excess Returns										
	3 month rate					yield spread				
	$\hat{\beta}_0$	*s.e*	$\hat{\beta}_1(3)$	*s.e*	R^2	$\hat{\beta}_0$	*s.e*	$\hat{\beta}_1$	*s.e*	R^2
panel			-0.16	0.68	0.0001			0.00	0.91	0
CAN	0.004	0.003	0.47	-1.04	0.001	0.007***	0.002	2.12*	1.02	0.001
EUR	0.000	0.005	-0.33	1.61	-0.0035	0.000	0.005	0.14	2.76	-0.0035
JPY	-0.001	0.006	2.82	1.80	0.0088	-0.026**	0.013	-5.68*	3.01	0.0088
NOK	0.014**	0.007	-1.50	1.03	0.0156	0.013*	0.007	1.66	1.30	0.0156
SEK	0.009	0.006	-1.52	1.16	0.0225	0.009	0.006	2.16	1.70	0.0225
CHF	0.004	0.004	1.44	1.01	0.0149	-0.003	0.004	-1.83	1.08	0.0149
UK	-0.002	0.005	-1.02	1.42	0.0084	-0.002	0.005	1.33	1.96	0.0084

PANEL B: Expected Bond Local Currency Return Differences										
	3 month rate					yield spread				
	$\hat{\beta}_0$	*s.e*	$\hat{\beta}_1(3)$	*s.e*	R^2	$\hat{\beta}_0$	*s.e*	$\hat{\beta}_1$	*s.e*	R^2
panel			0.10	0.26	0.0005			0.71**	0.32	0.0282
CAD	0.001	0.001	0.25	0.45	-0.0007	0.003*	0.002	1.07**	0.53	0.0225
EUR	0.007***	0.003	0.29	0.34	0.0015	0.006***	0.002	0.29	0.51	-0.0011
JPY	0.005***	0.002	-0.37	0.52	0.0004	0.008***	0.003	1.30	0.86	0.0236
NOK	0.006	0.002	-0.11	0.35	0.0007	0.008***	0.002	0.47	1.61	0.0118
SEK	0.000	0.004	0.56	1.03	0.0201	0.004	0.005	0.59	1.14	0.0006
CHF	0.001	0.001	-0.30	0.34	0.0048	0.003	0.002	0.61	0.42	0.0165
UK	0.004**	0.002	0.70	0.60	0.0154	0.005 **	0.002	1.07*	0.63	0.0155

Table III shows the results from regressing survey expectations of currency and relative bond (foreign minus US) returns on short rate and yield spread differences. The standard errors of the panel regression (country fixed effects) are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

	$\hat{\lambda}_0$	$s.e$	$\hat{\lambda}_1$	$s.e$	R^2
panel			0.987***	0.008	0.979
CAN	0.020	0.023	0.957***	0.016	0.932
GER	-0.016	0.022	0.994***	0.013	0.988
JAP	-0.032	0.017	0.989***	0.008	0.992
NOR	0.014	0.038	0.989***	0.016	0.975
SWE	-0.017	0.027	0.991***	0.011	0.980
CH	-0.014	0.021	0.988***	0.011	0.974
UK	0.005	0.022	0.981***	0.015	0.975

Table IV shows the results from regressing the monthly short rate differential (foreign minus US) on its first lag. The standard errors of the panel regression are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

rates. The resulting persistence parameters are given in table IV. Interest rate differentials are highly persistent with estimates ranging between 0.96 and 0.99. We choose the panel estimate $\lambda \approx 0.99$ as the baseline calibration.

We then need an estimate of the underreaction coefficient k. For this purpose we regress the forecast error for the short rate differential on the corresponding forecast revision using the 12 month forecasts. As explained before and in greater detail in the appendix, the model implies that the slope coefficient in this regression is $\beta^{FR} = \frac{1-k}{k}$.

Table V shows the results from this regression along with the implied values for k. We use the panel estimate $k \approx 0.49$ as the baseline calibration.

With a k above one, indicating overreaction, Canada seems to be an outlier but we still include it in the panel regression. Most of the country specific coefficient values are close to each other. Indeed with the exception of Canada, none of the country-level values are statistically different from the panel estimate.

	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1$	$s.e$	R^2	implied k
panel			1.059**	0.391	0.035	0.486
CAN	0.282	0.164	-0.230	0.217	0.003	1.299
GER	0.304*	0.186	1.628**	0.684	0.074	0.380
JAP	0.331*	0.190	1.771***	0.535	0.083	0.361
NOR	0.351	0.279	1.995***	0.720	0.089	0.334
SWE	-0.319	0.238	1.461***	0.582	0.055	0.406
CH	0.305	0.185	0.972*	0.530	0.028	0.507
UK	0.200	0.171	0.564	0.373	0.008	0.639

Table V shows the results from regressing the difference in forecast error (foreign minus US) when forecasting short rates 12 months ahead on the difference in short rate forecast revisions. The standard errors of the panel regression are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

3.5 Explaining Bond and Currency Returns

Table VI summarizes the results for the panel regressions in Table II as well as the corresponding values for the 1 month horizon. Here we also show the slope coefficients from a regression of relative bond dollar returns on short rate and yield spread differentials. As explained before, these are mechanically the sum of the slope coefficients in the bond and currency regressions and detailed also in the appendix. However, because of opposite predictability patterns for currency and bond returns, these coefficients are fairly small and statistically indifferent from zero.

The second column shows the interest rate misperception components β^{IRM} obtained under the sticky short rate expectations specification. One can see that, except for the 3 month horizon regression for excess currency returns on short rate differences, this component accounts for more than half of the size of the total coefficient. For yield spread

PANEL A: 1 month horizon				
Regression	β (Data)	β^{IRM}	β^{RP}	$\beta^{RPM}+\beta^{PCM}+\text{error}$
LHS: Currency Excess Return , RHS: short rate difference (x_t)	1.489	0.99 (66 %)	NA	NA
LHS: Currency Excess Return , RHS: yield spread difference	-1.943	-2.56 (132 %)	NA	NA
LHS: Local Currency Bond Return Difference, RHS: short rate difference (x_t)	-1.259	-0.99 (79 %)	NA	NA
LHS: Local Currency Bond Return Difference, RHS: yield spread difference	1.250	2.34 (187%)	NA	NA
LHS: Dollar Bond Return Difference, RHS: short rate difference (x_t)	0.23	0.29	NA	NA
LHS: Dollar Bond Return Difference, RHS: yield spread difference	-0.69	-0.22	NA	NA

PANEL B: 3 month horizon				
Regression	β (Data)	β^{IRM}	β^{RP}	$\beta^{RPM}+\beta^{PCM}+\text{error}$
LHS: Currency Excess Return , RHS: short rate difference (x_t)	1.36	0.6 (44 %)	−0.16	0.92
LHS: Currency Excess Return , RHS: yield spread difference	-1.33	-1.53 (115%)	0	0.2
LHS: Local Currency Bond Return Difference, RHS: short rate difference (x_t)	-0.86	-0.68 (79 %)	0.10	-0.28
LHS: Local Currency Bond Return Difference , RHS: yield spread difference	1.56	1.62 (104 %)	0.71	-0.77
LHS: Dollar Bond Return Difference , RHS: short rate difference (x_t)	0.50	-0.08	−0.06	0.64
LHS: Dollar Bond Return Difference, RHS: yield spread difference	0.23	0.08	0.71	-0.56

Table VI shows the key slope coefficients measured from the data (panel regressions) decomposed into interest rate misperception and subjective risk premia parts as well as a residual component.

Coefficient	Data	Simple Sticky Expectations Model
Volatility ratio, 10 year rate, 3 month rate	0.67	0.57
Autocorrelation, 10 year rate	0.97	0.99
Autocorrelation currency returns	0.32	0.51
Volatility ratio, exchange rate changes, short rate changes	93	42

Table VII shows additional statistics measured from the data (panel regressions) as well as those predicted by the model under sticky expectations and constant risk premia and permanent component.

differences this component even accounts for more than 100% of the predictability. We can conclude that sticky short rate expectations go a long way in explaining bond and currency predictability patterns and explain the puzzling opposite signs for bonds and currencies. As shown by Lustig et al. (2019) standard rational expectations macrofinance models have trouble replicating these facts. Moreover, these models lack a channel that seems to quantitatively explain most of the predictability of bond and currency returns.

For the 3 month horizon we can use survey data to measure the subjective risk premium components β^{RP} as in table III. The sample values based on panel regressions are plotted on the third column. However, as mentioned before these are fairly small and mostly statistically indifferent from zero. Finally using these values we can then use decomposion 14 to solve for the residual component that represents the risk premium and permanent component misperception channels as well as possible measurement error.[13] This residual component is shown on the last column.

Table XIII also shows the ratio between the volatilities of 10 year rate differentials and 3 month rate differentials,[14] as well as the autocorrelation in ten year rates. Assuming sticky short rate expectations alone does not pin down theoretical values for these parameters. However, the table shows the predicted values if we further impose condition CRP, i.e. constant risk premia. Here the theoretical values are fairly close to those in the data.

The table also shows the ratio of volatility of exchange rate changes to the volatility of short rate changes. Further assuming a constant permanent component for the exchange rate, we obtain a value of 42, somewhat smaller than in the data. As explained in the appendix some further shocks are required to increase this volatility and in particular to solve the exchange rate disconnect puzzle. This issue is shared by key competing risk-based theories of FX return predictability. Finally, the table shows the autocorrelation

[13]For bonds the residual component represents only the risk premium misperception channel and measurement error.

[14]We focus on the ratio because short rates are effectively taken as exogeneous.

currency returns (0.32) as well as their implied theoretical value (0.51).

Bond and Currency Returns and Past Short Rates In a sticky expectations model, short rate forecast errors tend to be particularly high after recent short rate changes. This is because it takes time for forecasters to update their predictions. This implies that high short-term interest rates relative to past short rates should predict high returns for a currency but low returns for the corresponding long-term bond, as explained in Proposition 3.

We now test this implication of the sticky expectations model. We construct the past average short rate difference $\bar{x}_t$ using our estimates of k and λ.[15] We then regress relative bond and currency returns on x_t and $\bar{x}_{t-1}$. The results are given in table VIII. The slope coefficients on short rate differences are as before though larger in magnitude. However, as predicted by the model the slope coefficient on the average past short rate is negative in the currency regression but positive in the bond regression. Foreign currency returns tend to be particularly high when the foreign short rate has recently increased. Similarly foreign bond returns tend to be particularly high when the foreign short rate has recently decreased.

Strictly speaking these predictions require conditions CRP and NLRM. However, they hold more generally for the interest rate misperception components of the slope coefficients. Moreover, they hold for the actual slope coefficients if there is no mechanism large enough that would offset these effects.

These results further support the model mechanism depicted in figure I and the idea that bond and currency return predictability patterns are largely *drift* patterns. Here a positive shock to the foreign short rate leads to a slow appreciation of the foreign currency

[15] Because we weight the past rates with our estimates of k and λ, this regression is generally subject to a generated regressor problem. However, alternative weighting schemes that do not depend on these estimates yield similar results.

PANEL A: Currency Excess Returns							
	$\hat{\beta}_0$	*s.e*	$\hat{\beta}_1$ (x_t)	*s.e*	$\hat{\beta}_2$ $(\bar{x}_{t-1})$	*s.e*	R^2
panel			4.961**	1.717	-1.838*	0.827	0.015
AUS	-0.001	0.002	7.265***	2.813	-2.797**	1.322	0.045
CAN	0.001	0.001	0.544	3.675	0.397	1.734	0.052
GER	0.005**	0.002	2.201	5.064	-1.233	2.436	0.038
JAP	0.004*	0.002	3.626	6.349	-0.417	3.159	0.022
NZL	-0.001	0.002	6.894**	2.899	-2.582*	1.481	0.032
SWE	-0.001	0.002	3.810*	2.287	-1.796	1.312	0.064
CH	0.003	0.002	7.355	5.604	-2.645	2.802	0.046
UK	0.000	0.002	2.423	3.574	-0.717	1.676	0.036
PANEL B: Bond Local Currency Return Differences							
	$\hat{\beta}_0$	*s.e*	$\hat{\beta}_1$ (x_t)	*s.e*	$\hat{\beta}_2$ $(\bar{x}_{t-1})$	*s.e*	R^2
panel			-1.322***	1.172	3.917***	0.564	0.044
AUS	0.000	0.001	-8.962***	2.118	4.061***	1.038	0.045
CAN	0.000	0.001	-8.228**	3.678	3.560**	1.773	0.052
GER	-0.001	0.001	-9.132***	2.451	4.232***	1.185	0.038
JAP	-0.002**	0.001	-10.880***	3.171	4.975***	1.562	0.022
NZL	0.000	0.001	-7.513***	2.822	3.263**	1.401	0.032
SWE	0.000	0.001	-7.419***	1.373	3.369***	0.722	0.064
CH	-0.002***	0.001	-10.698***	3.359	4.879***	1.589	0.046
UK	-0.001***	0.001	-8.783***	2.339	4.017***	1.164	0.036

Table VIII shows the results from regressing the currency and relative bond (foreign minus US) returns on short rate differences and an average of past short rate differences. The standard errors of the panel regression are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

and a sluggish fall in the value of foreign bond.

The above findings are related, but not identical,[16] to the delayed overshooting (Eichenbaum and Evans, 1995) and post-FOMC announcement drift (see e.g Brooks et al., 2019) patterns documented in the previous literature. Delayed overshooting refers to the fact that the response of the FX rate to interest rate shocks is hump-shaped. A contractionary shock to US monetary policy induces a gradual appreciation of the US dollar followed by depreciation.[17]

A pattern similar to the delayed overshooting puzzle of currencies is the post-FOMC announcement drift in bond markets. Here the yields of long-maturity treasuries respond sluggishly to changes in the Federal Funds Rate. In concurrent work, Brooks et al. (2019) argue that slow updating concerning short-term interest rates can explain the FOMC post announcement drift. This pattern is also generated by our model as can be seen in the FX impulse response plotted in figure I.

Could the above findings be generated in a fully risk-based model? This would be problematic. First, we have not found evidence that changes in short rates would predict survey-based expectations of bond and currency returns. Second, this would be inconsistent with the mathematical form of leading risk based models.[18]

[16]There is a clear correlation between the short-term bill rates applied in this paper and central bank policy rates. However, this correlation is far from perfect. Also these literatures first attempt to identify an unexpected monetary policy *shock* and then study its effect.

[17]Gourinchas and Tornell (2004) argue that the misspecification of the short rate proccess can explain the delayed overshooting puzzle.There is also some more recent evidence that interest rate changes predict currency returns. Dahlquist and Hasseltoft (2020) find evidence that recent trends in variables such as short rates explains currency returns and subsume the information in short rate levels.

[18]For example the habit model by Verdelhan (2010) implies that past short rates should not predict future bond and currency returns after controlling for current short rates as opposed to the results documented in this section. This model features a single state variable. No other variable should be able to predict returns after controlling for this state variable. However, short rate differentials are a simple function of this state variable (see e.g., Engel, 2016) . Hence controlling for short rate differentials is equivalent to controlling for

4 An Affine Term Structure Model

An alternative to estimating subjective risk premia directly from survey data is to estimate them to match the predictability patterns in the data. We now estimate a sticky expectations version of a standard term structure model and argue that a) incorporating sticky expectations helps in matching the data, b) this implies smaller estimates for time varying subjective risk premia. This exercise is similar in spirit to that in Piazzesi et al. (2015).

We now assume that markets are complete. We also consider the case of symmetric countries. This is useful for illustrative purposes and because we focus on time series predictability, rather than explaining persistent cross-country differences between returns.[19]

Under the subjective measure S, the home and foreign nominal stochastic discount factors (SDFs), $M_{t,t+1}$ and $M^*_{t,t+1}$, follow symmetric (conditionally) log-normal processes

$$log(M_{t,t+1}) \equiv m_{t,t+1} = -logR - \frac{\bar{\sigma}_\epsilon^2 \bar{\varphi}_t^2}{2} - \bar{z}_t - \frac{\sigma_\epsilon^2 \varphi_t^2}{2} - z_t - \bar{\varphi}_t \bar{\epsilon}_{t+1} - \varphi_t \epsilon_{t+1} \tag{26}$$

$$log(M^*_{t,t+1}) \equiv m^*_{t,t+1} = -logR - \frac{\bar{\sigma}_\epsilon^2 \bar{\varphi}_t^{*2}}{2} - \bar{z}_t - \frac{\sigma_\epsilon^2 \varphi_t^{*2}}{2} - z^*_t - \bar{\varphi}^*_t \bar{\epsilon}_{t+1} - \varphi^*_t \epsilon^*_{t+1}. \tag{27}$$

The shocks $\boldsymbol{\epsilon}_t = (\epsilon_t, \epsilon^*_t, \bar{\epsilon}_t)$ are independent and follow a (joint) normal distribution with mean zero and variances[20] σ_ϵ^2, σ_ϵ^2 and $\bar{\sigma}_\epsilon^2$. z_t and z^*_t are country specific states and $\bar{z}_t$ is a state shared by both countries. These states can represent either deep structural state variables or reduced form factors often used in term structure models.

Under the objective measure, the states $\mathbf{z}_t = [z_t, z^*_t, \bar{z}_t]'$ follow the process

$$\mathbf{z}_t = \Lambda \mathbf{z}_{t-1} + \boldsymbol{\epsilon}_t, \tag{28}$$

where

this state variable.

[19] Sticky expectations do not naturally generate persistent cross country differences in returns.

[20] Note that we assume countries are symmetric and the shocks ϵ_t and ϵ^*_t have the same variance σ_ϵ^2.

$$\Lambda = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \bar{\lambda} \end{bmatrix}.$$

Here $0 < \lambda < 1$ and $0 < \bar{\lambda} < 1$. The home short rate is simply $logR + z_t + \bar{z}_t$ and the foreign rate $logR + z_t^* + \bar{z}_t$. This implies that under the objective measure the short rate differential x_t evolves as

$$x_t = \lambda x_{t-1} + \tilde{\epsilon}_t,$$

where $\tilde{\epsilon}_t \equiv \epsilon_t - \epsilon_t^*$. Hence as in condition SE, the actual short rate differential process follows an AR(1) process.

On the other hand, the investors form expectations concerning the country specific shocks using a sticky expectations specification. This implies the key assumption of this paper:

$$\mathbb{E}_t^S[x_{t+1}] = \lambda(1-k)\mathbb{E}_{t-1}^S[x_t] + \lambda k x_t$$

The market prices of risk are given by

$$\varphi_t = \varphi_0 + \varphi_1 z_t + \varphi_2 \bar{z}_t \quad \bar{\varphi}_t = \bar{\varphi}_0 + \bar{\varphi}_1 z_t + \bar{\varphi}_2 \bar{z}_t$$

$$\varphi_t^* = \varphi_0^* + \varphi_1^* z_t + \varphi_2^* \bar{z}_t \quad \bar{\varphi}_t^* = \bar{\varphi}_0^* + \bar{\varphi}_1^* z_t + \bar{\varphi}_2^* \bar{z}_t$$

Bond prices and the FX rate can be solved using the following standard pricing conditions:

$$e^{q_t(n)} = \mathbb{E}_t^S[M_{t+1} e^{q_{t+1}(n-1)}],$$

$$e^{q_t(n)^*} = \mathbb{E}_t^S[M_{t+1}^* e^{q_{t+1}(n-1)^*}],$$

$$m_{t+1} + s_{t+1} - s_t = m_{t+1}^*,$$

where the currency pricing equation follows from complete markets. Because we focus on currency and relative bond returns, beliefs concerning the common shock are less relevant for the results. However, we assume these are also given by a sticky expectations specification.[21]

The specification for the SDFs then implies a closed form expression for the bond prices as argued by the following proposition:

Proposition 4 (The yield curve)**.** *Denote the state variable* $\boldsymbol{Y}_t = [z_t, \bar{z}_t, \mathbb{E}_t^S[z_{t+1}], \mathbb{E}_t^S[\bar{z}_{t+1}]]'$*. The home logarithmic prices of zero coupon bonds are affine functions of* $\boldsymbol{Y}_t$ *and given by*

$$q_t(n) = A(n) + \boldsymbol{B}(n)'\boldsymbol{Y}_t, \tag{29}$$

where $A(n)$ *and* $\boldsymbol{B}(n)$ *are given in the appendix. The foreign prices of zero coupon bonds take an analogous form but the state variables are* $\boldsymbol{Y}_t^* = [z_t^*, \bar{z}_t, \mathbb{E}_t^S[z_{t+1}^*], \mathbb{E}_t^S[\bar{z}_{t+1}]]'$*.*

Proof: see appendix.

As argued in the appendix the currency premium depends on these same state variables. Hence, we can view our specification as a six factor affine model with non-standard

[21] The assumption concerning this shock only affects the risk premium misperception channels and this effect appears fairly small. The sticky expectations specification also does not pin down the subjective conditional variance of the shock. This variance only affects the constant parts of bond prices that are not the focus of this paper. However, when solving for these constant parts we assume beliefs are formed by a noisy information model, which given the noisiness of signals, implies a specification for this variance.

factor dynamics and special restrictions between the three "true" state variables and their subjective expectations.

Rational vs Sticky Expectations Model: An Estimation Exercise To further demonstrate that accounting for sticky short rate expectations helps in matching the data, we now estimate the above models. We estimate the market price of risk parameters φ_0, φ_1, φ_2 and $\bar{\varphi}_0$, $\bar{\varphi}_1$, $\bar{\varphi}_2$ but calibrate all other parameters. Similarly to the previous section, the persistence parameters of country-specific and common shocks λ and $\bar{\lambda}$ can be estimated directly using short rate data and we set $\lambda = \bar{\lambda} = 0.99$. We consider both a rational calibration with $k = \bar{k} = 1$ and sticky expectations calibration with $k = \bar{k} = 0.49$[22].

We estimate the market price of risk parameters as follows. We target 4 of the 6 slope parameters in table VI[23]. In particular, we consider the regressions with relative bond returns and relative FX returns. We also target the volatility ratio between 3 month rates and 10 year rates. The appendix gives closed form expression for the model implied coefficients. We use numerical optimization to find the parameters that minize the equally weighted sum of squared deviations between the model implied coefficients and those in the data.

The rational model yields a sum of squared deviations of 0.97. The sum of the absolute values of the four parameters that determine the time-variation in market prices of risk is 16.1.

We then consider the sticky expectations calibration but estimate the same market prices of risk parameters. The model yields a sum of squared deviations of 0.24. Hence the pricing errors of the model fall by 75%. The sum of the absolute values of the four parameters that determine the time-variation in market prices of risk is smaller at 2.9.

[22] We set these two parameters to be equal because forecast revisions predict short rate revisions and short rate differential revisions in roughly the same way.

[23] Note that these slopes fully determine the remaining two.

This is because the sticky expectations model attributes a smaller part of time variation in objective premia to time-variation in market price of risk. One can also see this by running the baseline regression of explaining currency returns by short rate differentials under the sticky expectations measure. Here the slope coefficient is close to zero and the subjective FX risk premium small. However, the effects of this small risk premium have some compounding effects due to the risk premium misperception channel.

We conclude that accounting for sticky expectations in a standard affine term structure model helps in matching data on bond and currency returns. Note that the sticky expectations version of the model is also broadly more consistent with the data. In particular it explains why forecast errors about interest rates and currencies are predictable and why short rate changes rather than short rates seem to predict bond and currency returns.

On Consumption Based Asset Pricing Models The above affine model does not give a direct economic interpretation about the sources of bond or currency risk. However, it nests several consumption based specifications. To see this, assume each contry is populated by a representative agent with CRRA prefrences $\beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$. The real SDF is given by

$$m_{t+1} = log\beta - \gamma \Delta c_{t+1}$$

Consider an endowment economy where log-consumption follows:

$$\Delta c_{t+1} = -z_t - \bar{z}_t + \epsilon_{t+1}$$

and the factors have the law of motion specified in the beginning of the section. This would be a simple example of a model that is of our affine form and satisfies condition

CRP.[24]

Note that the above example abstracts away from inflation. We could distinguish between real and nominal pricing kernels by making an assumption on the inflation process (see e.g., Lustig et al., 2011, 2014).[25] Empirically shocks to expected inflation contribute much less to the variation in nominal yields than would be predicted by many structural models (Duffee, 2018; Haubrich et al., 2012).[26]

5 Conclusion

We show that well-documented sluggish updating concerning short rates creates joint predictability patterns in bond and currency markets. These predictability patterns explain most of the variation in expected bond and currency returns driven by variation in short rates and yield spreads.

Importantly, the biases work in opposite directions for bonds and currencies. The relative prices of currencies are increasing and the relative prices of long-term bonds decreasing in expected short rates. Therefore, high interest rate currencies tend to be underpriced but the long-term bonds of these same currencies overpriced. This provides a novel explanation for the fact that the term structure of expected carry trade returns is downward sloping.

The analysis bears important policy implications. Monetary policy that affects short rates transmits to bond yields and FX rates at a lag. Including sticky expectations to standard term structure models allows them to better capture the predictability patterns

[24] For the relationship between affine term structure models and consumption based models (see e.g., Creal and Wu, 2020).

[25] Alternatively one could formulate the theoretical predictions for real pricing kernels and use data on real interest rates and exchange rates for the empirical part.

[26] We would therefore expect our state variables to have higher correlations with real variables rather than inflation rates. However, our approach allows for different interpretations concerning these variables.

in the data.

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6 Appendix

6.1 Proof of Proposition 1

We have

$$\Theta_t^{FX,IRM} = \mathbb{E}_t\left[\mathbb{E}_{t+1}^S \sum_{j=0}^{\infty} x_{t+1+j} - \mathbb{E}_t^S \sum_{j=0}^{\infty} x_{t+1+j}\right]$$

Note that

$$\mathbb{E}_{t+1}^S \sum_{j=0}^{\infty} x_{t+j+1} = x_{t+1} + \frac{1}{1-\lambda}\mathbb{E}_{t+1}^S[x_{t+2}]$$

and

$$\mathbb{E}_t^S \sum_{j=0}^{\infty} x_{t+j+1} = \mathbb{E}_t^S[x_{t+1}] + \frac{1}{1-\lambda}\mathbb{E}_t^S[x_{t+2}]$$

Hence

$$\Theta_t^{FX,IRM} = \mathbb{E}_t\left[x_{t+1} + \frac{1}{1-\lambda}\mathbb{E}_{t+1}^S[x_{t+2}] - \mathbb{E}_t^S[x_{t+1}] - \frac{1}{1-\lambda}\mathbb{E}_t^S[x_{t+2}]\right]$$

$$= \mathbb{E}_t\left[x_{t+1} + \frac{1}{1-\lambda}\mathbb{E}_{t+1}^S[x_{t+2}] - \mathbb{E}_t^S[x_{t+1}] - \frac{\lambda}{1-\lambda}\mathbb{E}_t^S[x_{t+1}]\right]$$

$$= \mathbb{E}_t\left[x_{t+1} - \mathbb{E}_t^S[x_{t+1}] + \frac{\lambda k}{1-\lambda}[\lambda x_t - \mathbb{E}_t^S[x_{t+1}]]\right] = \left[1 + \frac{\lambda k}{1-\lambda}\right]\left[\mathbb{E}_t x_{t+1} - \mathbb{E}_t^S x_{t+1}\right]$$

.

Similarly

$$\Theta_t^{B,IRM}(n) = -\mathbb{E}_t\left[\mathbb{E}_{t+1}^S \sum_{j=0}^{n-2} x_{t+1+j} - \mathbb{E}_t^S \sum_{j=0}^{n-2} x_{t+1+j}\right]$$

Note that

$$\mathbb{E}^S_{t+1}\sum_{j=0}^{n-2} x_{t+j+1} = x_{t+1} + \frac{1-\lambda^{n-2}}{1-\lambda}\mathbb{E}^S_{t+1}[x_{t+2}]$$

and

$$\mathbb{E}^S_{t}\sum_{j=0}^{n-2} x_{t+j+1} = \mathbb{E}^S_t[x_{t+1}] + \frac{1-\lambda^{n-2}}{1-\lambda}\mathbb{E}^S_{t}[x_{t+2}]$$

Hence

$$\Theta_t^{B,IRM}(n) = -\mathbb{E}_t\left[x_{t+1} + \frac{1-\lambda^{n-2}}{1-\lambda}\mathbb{E}^S_{t+1}[x_{t+2}] - \mathbb{E}^S_t[x_{t+1}] - \frac{1-\lambda^{n-2}}{1-\lambda}\mathbb{E}^S_t[x_{t+2}]\right]$$

$$= -\left[1 - \frac{\lambda k(1-\lambda^{n-2})}{1-\lambda}\right]\left[\mathbb{E}_t x_{t+1} - \mathbb{E}^S_t x_{t+1}\right]$$

and therefore

$$\Theta_t^{FX,IRM}(n) = \Theta_t^{B,IRM}(n) + \Theta_t^{FX,IRM} = \left[\mathbb{E}_t x_{t+1} - \mathbb{E}^S_t x_{t+1}\right]\frac{k\lambda^{n-1}}{1-\lambda}$$

We then solve for the expressions for the interest rate misperception components of the predictability coefficients. Moreover,

$$\beta^{FX,IRM} = \frac{\mathbb{C}ov(\Theta_t^{FX,IRM}, x_t)}{\mathbb{V}ar(x_t)} \tag{30}$$

On the other hand

$$cov(x_t, \Theta_t^{FX,IRM}) = \left[1 + \frac{\lambda k}{1-\lambda}\right] cov(x_t, \mathbb{E}_t[x_{t+1}] - \mathbb{E}^S_t[x_{t+1}]) =$$
$$Var(x_t)\lambda\left[1 + \frac{\lambda k}{1-\lambda}\right] - \left[1 + \frac{\lambda k}{1-\lambda}\right] cov(x_t, \mathbb{E}^S_t[x_{t+1}]).$$

Also

$$\mathbb{E}_t^S[x_{t+1}] = k\lambda x_t + k(1-k)\lambda^2 x_{t-1} + k(1-k)^2\lambda^3 x_{t-2} + \dots$$

$$\begin{aligned} cov(x_t, \mathbb{E}_t^S[x_{t+1}]) &= Var(x_t)[\lambda k + k(1-k)\lambda^3 + k(1-k)^2\lambda^5 + \dots] \\ &= \frac{\lambda k}{1-(1-k)\lambda^2} Var(x_t). \end{aligned}$$

Hence

$$cov(x_t, \Theta_t^{FX,IRM}) = -Var(x_t)\left[1 + \frac{\lambda k}{1-\lambda}\right]\left[\frac{\lambda k - \lambda + (1-k)\lambda^3}{1-(1-k)\lambda^2}\right]$$

and

$$\beta^{FX,IRM} = \left[1 + \frac{\lambda k}{1-\lambda}\right]\frac{(1-k)(\lambda - \lambda^3)}{1-(1-k)\lambda^2}.$$

The expression for $\beta^{FX,IRM}(n)$ follows similarly.

6.2 Proof of Proposition 2

We have

$$\begin{aligned} \underbrace{\Theta_t^{FX}}_{\text{Currency premium}} = \underbrace{\zeta_t^{FX}}_{\text{Risk premium differential}} &+ \underbrace{\mathbb{E}_t\left[\mathbb{E}_{t+1}^S \sum_{j=0}^{\infty} x_{t+1+j} - \mathbb{E}_t^S \sum_{j=0}^{\infty} x_{t+1+j}\right]}_{\text{Interest rate misperception effect}} \\ &- \underbrace{\mathbb{E}_t\left[\mathbb{E}_{t+1}^S \sum_{j=0}^{\infty} \zeta_{t+1+j}^{FX} - \mathbb{E}_t^S \sum_{j=0}^{\infty} \zeta_{t+1+j}^{FX}\right]}_{\text{Risk premium misperception effect}} + \underbrace{\mathbb{E}_t[\lim_{j\to\infty} \mathbb{E}_{t+1}^S[s_{t+j}] - \lim_{j\to\infty} \mathbb{E}_t^S[s_{t+j}]]}_{\text{Permanent component misperception effect}} \end{aligned}$$

Condition CRP implies that the risk premium misperception effect is zero. Condition NLRM implies that the permanent component misperception effect is zero. Moreover, because each component in the expression is demeaned, condition CRP implies that risk premium differential is zero. Hence we then have

$$\underbrace{\Theta_t^{FX}}_{\text{Currency premium}} = \underbrace{\mathbb{E}_t\left[\mathbb{E}_{t+1}^S \sum_{j=0}^{\infty} x_{t+1+j} - \mathbb{E}_t^S \sum_{j=0}^{\infty} x_{t+1+j}\right]}_{\text{Interest rate misperception effect}}$$

Similarly, condition CRP and NLRM also imply that the interest rate misperception component drives all variation in bond premia. Now assuming condition the expressions for currency and bond premia follow directly from the same manipulations as in the proof of Proposition 2.

6.3 Proof of Proposition 3

Given conditions SE, CRP and NLRM, the currency risk premium is given by

$$\Theta_t = \left[1 + \frac{\lambda k}{1-\lambda}\right]\left[\mathbb{E}_t[x_{t+1}] - \mathbb{E}_t^S[x_{t+1}]\right]$$

Here

$$\mathbb{E}_t[x_{t+1}] = \lambda x_t$$

and

$$\mathbb{E}_t^S[x_{t+1}] = k\lambda x_t + k(1-k)\lambda^2 x_{t-1} + k(1-k)^2\lambda^3 x_{t-2} + \dots$$

Therefore

$$\Theta_t = \left[1+\frac{\lambda k}{1-\lambda}\right]\left[\lambda(1-k)x_t - k(1-k)\lambda^2 x_{t-1} - k(1-k)^2\lambda^3 x_{t-2} - \dots\right] =$$
$$\left[1+\frac{\lambda k}{1-\lambda}\right]\lambda(1-k)\left[x_t - k\lambda\bar{x}_{t-1}\right]$$

Hence

$$r_{t+1}^{FX} = \left[1+\frac{\lambda k}{1-\lambda}\right]\lambda(1-k)\left[x_t - k\lambda\bar{x}_{t-1}\right] + \epsilon_{t+1}$$

This implies

$$\beta_1^{FX} = \left[1+\frac{\lambda k}{1-\lambda}\right]\lambda(1-k).$$

and

$$\beta_2^{FX} = -\left[1+\frac{\lambda k}{1-\lambda}\right]\lambda(1-k)k\lambda.$$

The signs are as predicted by the proposition. The proof for the bond regression is similar.

6.4 Proof of Proposition 4

The standard bond pricing equation is

$$P_t(n) = \mathbb{E}_t^S[M_{t+1}P_{t+1}(n-1)],$$

which can be expressed using our previous log notation as

$$q_t(n) = log\left(\mathbb{E}_t[exp(m_{t,t+1} + q_{t+1}(n-1))]\right),$$

Let us conjecture that this has a solution of the form

$$q_t(n) = A(n) + B_1(n)z_t + B_2(n)\bar{z}_t + B_3(n)E_t^S[z_{t+1}] + B_4(n)E_t^S[\bar{z}_{t+1}] = A(n) + \mathbf{B}'(n)\mathbf{y}_t$$

Given this conjectured form $q_{t+1}(n-1)$ and $m_{t,t+1}$ are conditionally jointly normal. Hence we obtain

$$q_t(n) = \mathbb{E}_t^S[m_{t,t+1} + q_{t+1}(n-1)] + \frac{1}{2}\mathbb{V}ar^S[m_{t,t+1} + q_{t+1}(n-1)]$$

The initial values must be such that $A(0) = B_1(0) = B_2(0) = B_3(0) = B_4(0) = 0$. Note that

$$\mathbb{E}_t^S[m_{t,t+1} + q_t(n-1)] = -logR - z_t - \bar{z}_t - \frac{\sigma_\epsilon^2\varphi_t^2}{2} - \frac{\bar{\sigma}_\epsilon^2\bar{\varphi}_t^2}{2} + A(n-1) + \mathbf{B}'(n-1)\mathbb{E}_t^S[\mathbf{y}_{t+1}]$$

$$\mathbb{V}ar_t^S[m_{t,t+1} + q_{t+1}(n-1)] = \sigma_\epsilon^2\varphi_t^2 + \bar{\sigma}_\epsilon^2\bar{\varphi}_t^2 + B_1(n-1)\mathbb{V}ar_t^S(z_{t+1}) + B_2(n-1)\mathbb{V}ar_t^S(\bar{z}_{t+1}) +$$
$$B_3(n-1)\mathbb{V}ar_t^S(\mathbb{E}_{t+1}^S[z_{t+2}]) + B_4(n-1)\mathbb{V}ar_t^S(\mathbb{E}_{t+1}^S[\bar{z}_{t+2}]) + 2\varphi_t B_3(n-1)Cov_t^S(\epsilon_{t+1}, \mathbb{E}_{t+1}^S[z_{t+2}]) +$$
$$2\bar{\varphi}_t B_4(n-1)Cov_t^S(\bar{\epsilon}_{t+1}, \mathbb{E}_{t+1}^S[\bar{z}_{t+2}])$$

So we obtain the following equation

$$A(n) + \mathbf{B}'(n)\mathbf{y}_t = -logR - z_t - \bar{z}_t + A(n-1) + \mathbf{B}'(n-1)\mathbb{E}_t^S[\mathbf{y}_{t+1}] + \frac{1}{2}B_1(n-1)^2\mathbb{V}ar_t^S(z_{t+1}) +$$
$$\frac{1}{2}B_2(n-1)^2\mathbb{V}ar_t^S(\bar{z}_{t+1}) + \frac{1}{2}B_3(n-1)^2\mathbb{V}ar_t^S(\mathbb{E}_{t+1}^S[z_{t+2}]) + \frac{1}{2}B_4(n-1)^2\mathbb{V}ar_t^S(\mathbb{E}_{t+1}^S[\bar{z}_{t+2}]) +$$
$$\varphi_t B_3(n-1)Cov_t^S(\epsilon_{t+1}, \mathbb{E}_{t+1}^S[z_{t+2}]) + \bar{\varphi}_t B_4(n-1)Cov_t^S(\bar{\epsilon}_{t+1}, \mathbb{E}_{t+1}^S[\bar{z}_{t+2}]) +$$
$$B_1(n-1)B_3(n-1)Cov_t^S(z_{t+1}, \mathbb{E}_{t+1}^S[z_{t+2}]) + B_2(n-1)B_4(n-1)Cov_t^S(\bar{z}_{t+1}, \mathbb{E}_{t+1}^S[\bar{z}_{t+2}])$$

Recall that

$$\varphi_t = \varphi_0 + \varphi_1 z_t + \varphi_2 \bar{z}_t \quad \bar{\varphi}_t = \bar{\varphi}_0 + \bar{\varphi}_1 z_t + \bar{\varphi}_2 \bar{z}_t$$

Hence we have

$$A(n) = A(n-1) - logR + \frac{1}{2}B_1(n-1)^2 \mathbb{V}ar_t^S(z_{t+1}) +$$
$$\frac{1}{2}B_2(n-1)^2 \mathbb{V}ar_t^S(\bar{z}_{t+1}) + \frac{1}{2}B_3(n-1)^2 \mathbb{V}ar_t^S(\mathbb{E}_{t+1}^S[z_{t+2}]) + \frac{1}{2}B_4^2(n-1)\mathbb{V}ar_t^S(\mathbb{E}_{t+1}^S[\bar{z}_{t+2}]) +$$
$$\varphi_0 B_3(n-1)Cov_t^S(\epsilon_{t+1}, \mathbb{E}_{t+1}^S[z_{t+2}]) + \bar{\varphi}_0 B_4(n-1)Cov_t^S(\bar{\epsilon}_{t+1}, \mathbb{E}_{t+1}^S[\bar{z}_{t+2}]) +$$
$$B_1(n-1)B_3(n-1)Cov_t^S(z_{t+1}, \mathbb{E}_{t+1}^S[z_{t+2}]) + B_2(n-1)B_4(n-1)Cov_t^S(\bar{z}_{t+1}, \mathbb{E}_{t+1}^S[\bar{z}_{t+2}])$$

$$B_1(n) = -1 + \varphi_1 B_3(n-1)Cov_t^S(\epsilon_{t+1}, \mathbb{E}_{t+1}^S[z_{t+2}]) + \bar{\varphi}_1 B_4(n-1)Cov_t^S(\bar{\epsilon}_{t+1}, \mathbb{E}_{t+1}^S[\bar{z}_{t+2}])$$

$$B_2(n) = -1 + \varphi_2 B_3(n-1)Cov_t^S(\epsilon_{t+1}, \mathbb{E}_{t+1}^S[z_{t+2}]) + \bar{\varphi}_2 B_4(n-1)Cov_t^S(\bar{\epsilon}_{t+1}, \mathbb{E}_{t+1}^S[\bar{z}_{t+2}])$$

$$B_3(n) = B_1(n-1) + \lambda B_3(n-1) \quad B_4(n) = B_2(n-1) + \bar{\lambda} B_4(n-1).$$

The sticky expectations process does not pin down expressions for the conditional covariance and variance terms. Given the noisy information model discussed in the appendix they are constant and given by:

$$Cov_t^S(\epsilon_{t+1}, \mathbb{E}_{t+1}^S[z_{t+2}]) = Cov_t^S(\epsilon_{t+1}, \lambda k z_{t+1} + (1-k)\lambda \mathbb{E}_t^S[z_{t+1}]) = \lambda k \sigma_\epsilon^2$$

and similarly

$$Cov_t^S(\bar{\epsilon}_{t+1}, \mathbb{E}_{t+1}^S[\bar{z}_{t+2}]) = \bar{\lambda}\bar{k}\bar{\sigma}_\epsilon^2$$

.

$$\mathbb{V}ar_t^S(z_{t+1}) = \sigma^2 + \sigma_v^2 \quad \mathbb{V}ar_t^S(\bar{z}_{t+1}) = \bar{\sigma}^2 + \bar{\sigma}_v^2$$

$$\mathbb{V}ar_t^S(\mathbb{E}_{t+1}^S[z_{t+2}]) = \lambda k(\sigma^2 + \sigma_v^2) \quad \mathbb{V}ar_t^S(\mathbb{E}_{t+1}^S[\bar{z}_{t+2}]) = \bar{\lambda}\bar{k}(\bar{\sigma}^2 + \bar{\sigma}_v^2)$$

However, these terms are not used for the main results of this paper, because we focus on relative bond prices. Note that symmetry implies

$$q_t^*(n) - q_t(n) = B_1(n)x_t + B_3(n)\mathbb{E}_t^S[x_{t+1}]$$

The Rational Case The rational model is a special case of the above model. Here the solution is

$$q_t(n) = A_r(n) + B_{1,r}(n)z_t + B_{2,r}(n)\bar{z}_t$$

The coefficients can also be solved from

$$A_r(n) + B_{1,r}(n)z_t + B_{2,r}\bar{z}_t = -logR - z_t - \bar{z}_t + A_r(n-1) + B_{1,r}(n-1)\lambda z_t + B_{2,r}(n-1)\bar{\lambda}\bar{z}_t + \\ + \frac{1}{2}B_{1,r}(n-1)^2\sigma_\epsilon^2 + \frac{1}{2}B_{2,r}(n-1)^2\bar{\sigma}_\epsilon^2 + \varphi_t B_{1,r}(n-1)\sigma_\epsilon^2 + \bar{\varphi}_t B_{2,r}(n-1)\bar{\sigma}_\epsilon^2$$

So that we have the solution

$$A_r(n) = \\ A_r(n-1) - logR + \frac{1}{2}B_{1,r}(n-1)^2\sigma_\epsilon^2 + \frac{1}{2}B_{2,r}(n-1)^2\bar{\sigma}_\epsilon^2 + \varphi_0 B_{1,r}(n-1)\sigma_\epsilon^2 + \bar{\varphi}_0 B_{2,r}(n-1)\bar{\sigma}_\epsilon^2$$

$$B_{1,r}(n) = -1 + B_{1,r}(n-1)\lambda + \varphi_1 B_{1,r}(n-1)\sigma_\epsilon^2 + \bar{\varphi}_1 B_{2,r}(n-1)\bar{\sigma}_\epsilon^2$$

$$B_{2,r}(n) = -1 + B_{2,r}(n-1)\bar{\lambda} + \varphi_2 B_{1,r}(n-1)\sigma_\epsilon^2 + \bar{\varphi}_2 B_{2,r}(n-1)\bar{\sigma}_\epsilon^2$$

Note that here

$$q_t(n)^* - q_t(n) = B_{1,r}(n)x_t$$

6.5 Closed Form Solutions for the Predictability Coefficients in the Affine Model

We now derive analytical expressions for all the predictability coefficients in the context of our affine model. These closed form expressions greatly simplify and speed up model estimation. The relative spread is

$$-\frac{B_1(n)}{n}x_t - \frac{B_3(n)}{n}\mathbb{E}_t^S[x_{t+1}] - x_t = -\left(\frac{B_1(n)}{n} + 1\right)x_t - \frac{B_3(n)}{n}\mathbb{E}_t^S[x_{t+1}]$$

And expected bond excess return is

$$\mathbb{E}_t[q_{t+1}(n-1)^* - q_{t+1}(n-1)] - (q_t(n)^* - q_t(n)) - x_t =$$
$$(B_1(n-1)\lambda - B_1(n) - 1)x_t + (B_3(n-1)\lambda - B_3(n))\mathbb{E}_t^S[x_{t+1}]$$

Note

$$Cov(\mathbb{E}_t[q_{t+1}(n-1)^* - q_{t+1}(n-1)] - (q_t(n)^* - q_t(n)) - x_t, x_t) =$$
$$(B_1(n-1)\lambda - B_1(n) - 1)\mathbb{V}ar(x_{t+1}) + (B_3(n-1)\lambda - B_3(n))Cov(x_{t+1}, \mathbb{E}_t^S[x_{t+1}])$$

When regressing bond excess returns on relative short rates we then obtain a predictability coefficient of

$$\frac{Cov(\mathbb{E}_t[q_{t+1}(n-1)^* - q_{t+1}(n-1)] - (q_t(n)^* - q_t(n)) - x_t, x_t)}{\mathbb{V}ar(x_t)} =$$

$$(B_1(n-1)\lambda - B_1(n) - 1) + (B_3(n-1)\lambda - B_3(n))\frac{\lambda k}{1-(1-k)\lambda^2}$$

Moreover, for the spread we have

$$Cov(\mathbb{E}_t[q_{t+1}(n-1)^* - q_{t+1}(n-1)] - (q_t(n)^* - q_t(n)) - x_t,$$
$$-\left(\frac{B_1(n)}{n}+1\right)x_t - \frac{B_3(n)}{n}\mathbb{E}_t^S[x_{t+1}]) =$$
$$-\left(\frac{B_1(n)}{n}+1\right)(B_1(n-1)\lambda - B_1(n) - 1)\mathbb{V}ar(x_{t+1})$$
$$-\left[\left(\frac{B_1(n)}{n}+1\right)(B_3(n-1)\lambda - B_3(n)) + \frac{B_3(n)}{n}(B_1(n-1)\lambda - B_1(n) - 1)\right]Cov(x_{t+1}, \mathbb{E}_t^S[x_{t+1}])$$
$$-\frac{B_3(n)}{n}(B_3(n-1)\lambda - B_3(n))\mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}])$$

Here

$$\mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}]) = \frac{k^2\lambda^2 + 2k(1-k)\lambda^2\frac{\lambda k}{1-(1-k)\lambda^2}}{1-(1-k)^2\lambda^2}\mathbb{V}ar(x_t)$$

the variance of the spread is

$$\mathbb{V}ar(-\left(\frac{B_1(n)}{n}+1\right)x_t - \frac{B_3(n)}{n}\mathbb{E}_t^S[x_{t+1}]) =$$
$$\left(\frac{B_1(n)}{n}+1\right)^2\mathbb{V}ar(x_t) + 2\left(\frac{B_1(n)}{n}+1\right)\frac{B_3(n)}{n}Cov(\mathbb{E}_t^S[x_{t+1}], x_t) + \frac{B_3(n)^2}{n^2}\mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}])$$

and the predictability coefficient is ratio of the above covariance terms. Similarly the variance of n maturity relative yield is

$$\mathbb{V}ar(-\frac{B_1(n)x_t}{n} - \frac{B_3(n)\mathbb{E}_t^S[x_{t+1}]}{n}) =$$
$$\frac{B_1(n)^2}{n^2}\mathbb{V}ar(x_t) + 2\frac{B_1(n)B_3(n)}{n^2}Cov(\mathbb{E}_t^S[x_{t+1}], x_t) + \frac{B_3(n)^2}{n^2}\mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}])$$

Next consider currencies. Using our previous notation we have

$$\zeta_t^{FX} = -\frac{\bar{\sigma}_\epsilon^2\bar{\varphi}_t^2}{2} - \frac{\sigma_\epsilon^2\varphi_t^2}{2} + \frac{\bar{\sigma}_\epsilon^2\bar{\varphi}_t^{*2}}{2} + \frac{\sigma_\epsilon^2\varphi_t^{*2}}{2} =$$
$$\bar{\sigma}_\epsilon^2(\bar{\varphi}_0\bar{\varphi}_1(z_t - z_t^*) + \bar{\varphi}_1^2(z_t^2 - z_t^{*2}) + 2\bar{\varphi}_1\bar{\varphi}_2\bar{z}_t(z_t - z_t^*)) +$$
$$\sigma_\epsilon^2(\varphi_0\varphi_1(z_t - z_t^*) + \varphi_1^2(z_t^2 - z_t^{*2}) + 2\varphi_1\varphi_2\bar{z}_t(z_t - z_t^*))$$

Moreover

$$Cov(\zeta_t^{FX}, x_t) = (-\bar{\sigma}_\epsilon^2\bar{\varphi}_0\bar{\varphi}_1 - \sigma_\epsilon^2\varphi_0\varphi_1)\mathbb{V}ar(x_t)$$

and

$$Cov(\mathbb{E}_t^S\zeta_{t+j}^{FX}, x_t) = (-\bar{\sigma}_\epsilon^2\bar{\varphi}_0\bar{\varphi}_1 - \sigma_\epsilon^2\varphi_0\varphi_1)\mathbb{C}ov(\mathbb{E}_t^S[x_{t+j}], x_t) = \lambda^{j-1}\frac{\lambda k}{1-(1-k)\lambda^2}Var(x_t)$$

and

$$Cov(\zeta_{t+1}^{FX}, x_t) = \lambda(-\bar{\sigma}_\epsilon^2\bar{\varphi}_0\bar{\varphi}_1 - \sigma_\epsilon^2\varphi_0\varphi_1)$$

$$Cov(\mathbb{E}_{t+1}^S\zeta_{t+j+1}^{FX}, x_t) = \lambda^{j-1}(-\bar{\sigma}_\epsilon^2\bar{\varphi}_0\bar{\varphi}_1 - \sigma_\epsilon^2\varphi_0\varphi_1)Cov(\mathbb{E}_{t+1}^S x_{t+2}, x_t) =$$
$$\lambda^{j-1}(-\bar{\sigma}_\epsilon^2\bar{\varphi}_0\bar{\varphi}_1 - \sigma_\epsilon^2\varphi_0\varphi_1)(\lambda k + (1-k)\lambda\frac{\lambda k}{1-(1-k)\lambda^2})$$

$$s_t = \sum_{j=0}^{\infty} \mathbb{E}_t^S[m_{t+j,t+j+1} - m^*_{t+j,t+j+1}] + \underset{j\to\infty}{lim} \mathbb{E}_t^S[s_{t+j}]$$

Hence we obtain a predictability coefficient related to x_t of

$$\beta_{FX,x} = \beta^{IRM} +$$
$$(-\bar{\sigma}_\epsilon^2 \bar{\varphi}_0 \bar{\varphi}_1 - \sigma_\epsilon^2 \varphi_0 \varphi_1)\left(1 - \lambda - \frac{1}{1-\lambda}(\lambda k + (1-k)\lambda \frac{\lambda k}{1-(1-k)\lambda^2}) + \frac{1}{1-\lambda}\frac{\lambda k}{1-(1-k)\lambda^2}\right)$$

We now need to solve for the predictability coefficient related to spread. Here note

$$Cov(\zeta_t^{FX}, \mathbb{E}_t^S[x_{t+1}]) = (-\bar{\sigma}_\epsilon^2 \bar{\varphi}_0 \bar{\varphi}_1 - \sigma_\epsilon^2 \varphi_0 \varphi_1)\frac{\lambda k}{1-(1-k)\lambda^2}$$

$$Cov(\mathbb{E}_t^S \zeta_{t+j}^{FX}, \mathbb{E}_t^S[x_{t+1}]) = (-\bar{\sigma}_\epsilon^2 \bar{\varphi}_0 \bar{\varphi}_1 - \sigma_\epsilon^2 \varphi_0 \varphi_1)\lambda^{j-1} \mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}]) =$$
$$\lambda^{j-1}(-\bar{\sigma}_\epsilon^2 \bar{\varphi}_0 \bar{\varphi}_1 - \sigma_\epsilon^2 \varphi_0 \varphi_1) \mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}])$$

$$Cov(\zeta_{t+1}^{FX}, \mathbb{E}_t^S[x_{t+1}]) = \lambda(-\bar{\sigma}_\epsilon^2 \bar{\varphi}_0 \bar{\varphi}_1 - \sigma_\epsilon^2 \varphi_0 \varphi_1)\frac{\lambda k}{1-(1-k)\lambda^2}\mathbb{V}ar(x_t)$$

$$Cov(\mathbb{E}_{t+1}^S \zeta_{t+j+1}^{FX}, \mathbb{E}_t^S[x_{t+1}]) = \lambda^{j-1} Cov(\mathbb{E}_{t+1}^S \zeta_{t+2}^{FX}, \mathbb{E}_t^S[x_{t+1}]) =$$
$$\lambda^{j-1}(-\bar{\sigma}_\epsilon^2 \bar{\varphi}_0 \bar{\varphi}_1 - \sigma_\epsilon^2 \varphi_0 \varphi_1) Cov(\mathbb{E}_{t+1}^S[x_{t+2}], \mathbb{E}_t^S[x_{t+1}]) =$$
$$\lambda^{j-1}(-\bar{\sigma}_\epsilon^2 \bar{\varphi}_0 \bar{\varphi}_1 - \sigma_\epsilon^2 \varphi_0 \varphi_1)\left(\lambda^2 k \frac{\lambda k}{1-(1-k)\lambda^2}\mathbb{V}ar(x_t) + (1-k)\lambda \mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}])\right)$$

$$Cov(\Theta_t^{FX,RPM}, \mathbb{E}_t^S[x_{t+1}]) = Cov(-\mathbb{E}_t\left[\mathbb{E}_{t+1}^S \sum_{j=0}^{\infty} \zeta_{t+1+j}^{FX} - \mathbb{E}_t^S \sum_{j=0}^{\infty} \zeta_{t+1+j}^{FX}\right], \mathbb{E}_t^S[x_{t+1}]) =$$

$$(\bar{\sigma}_\epsilon^2 \bar{\varphi}_0 \bar{\varphi}_1 + \sigma_\epsilon^2 \varphi_0 \varphi_1)(a_1 + a_2 - a_3)$$

Here

$$a_1 = \frac{\lambda^2 k}{1-(1-k)\lambda^2} \mathbb{V}ar(x_t)$$

$$a_2 = \frac{1}{1-\lambda}\left(\lambda^2 k \frac{\lambda k}{1-(1-k)\lambda^2} \mathbb{V}ar(x_t) + (1-k)\lambda \mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}])\right)$$

$$a_3 = \frac{1}{1-\lambda} \mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}])$$

We also have

$$Cov(\mathbb{E}_t\left[\mathbb{E}_{t+1}^S \sum_{j=0}^{\infty} x_{t+1+j} - \mathbb{E}_t^S \sum_{j=0}^{\infty} x_{t+1+j}, \mathbb{E}_t^S[x_{t+1}]\right] =$$

$$\left[1 + \frac{\lambda k}{1-\lambda}\right]\left[\mathbb{E}_t x_{t+1} - \mathbb{E}_t^S x_{t+1}\right] Cov(\mathbb{E}_t x_{t+1} - \mathbb{E}_t^S x_{t+1}, \mathbb{E}_t^S x_{t+1}) =$$

$$\left[1 + \frac{\lambda k}{1-\lambda}\right](\lambda Cov(x_t, \mathbb{E}_t^S[x_{t+1}]) - \mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}]))$$

Hence for the spread we obtain

$$
\begin{gathered}
Cov(\mathbb{E}_t^S[s_{t+1}] - s_t + x_t, -\left(\frac{B_1(n)}{n} + 1\right)x_t - \frac{B_3(n)}{n}\mathbb{E}_t^S[x_{t+1}]) = \\
-\left(\frac{B_1(n)}{n} + 1\right)\beta_{FX,x}\mathbb{V}ar(x_t) + \\
-\frac{B_3(n)}{n}\left(\left[1 + \frac{\lambda k}{1-\lambda}\right](\lambda Cov(x_t, \mathbb{E}_t^S[x_{t+1}]) - \mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}]))\right) + \\
-\frac{B_3(n)}{n}\left(Cov(\zeta_{t+1}^{FX}, \mathbb{E}_t^S[x_{t+1}]) + Cov(\Theta_t^{FX,RPM}, \mathbb{E}_t^S[x_{t+1}])\right)
\end{gathered}
$$

Again the variance of the spread is

$$
\begin{gathered}
\mathbb{V}ar(-\left(\frac{B_1(n)}{n} + 1\right)x_t - \frac{B_3(n)}{n}\mathbb{E}_t^S[x_{t+1}]) = \\
\left(\frac{B_1(n)}{n} + 1\right)^2 \mathbb{V}ar(x_t) + 2\left(\frac{B_1(n)}{n} + 1\right)\frac{B_3(n)}{n}Cov(\mathbb{E}_t^S[x_{t+1}], x_t) + \frac{B_3(n)^2}{n^2}\mathbb{V}ar(\mathbb{E}_t^S[x_{t+1}])
\end{gathered}
$$

Predictability Coefficients for the Rational Model

The predictability coefficients for the rational model are obtained as a special case of the above coefficients. Here

$$
q_t(n)^* - q_t(n) = B_{1,r}(n)x_t
$$

The spread is

$$
q_t(n)^* - q_t(n) = -B_{1,r}(n)x_t/n - x_t = -(B_{1,r}(n)/n + 1)x_t
$$

And expected bond excess return is

$$
\mathbb{E}_t[q_{t+1}(n-1)^* - q_{t+1}(n-1)] - (q_t(n)^* - q_t(n)) - x_t = (B_{1,r}(n-1)\lambda - B_{1,r}(n) - 1)x_t
$$

Hence the predictability coefficient is

$$\frac{Cov(\mathbb{E}_t[q_{t+1}(n-1)^* - q_{t+1}(n-1)] - (q_t(n)^* - q_t(n)) - x_t, x_t)}{Var(x_t)} =$$
$$(B_{1,r}(n-1)\lambda - B_{1,r}(n) - 1)$$

And for the spread:

$$\frac{Cov(\mathbb{E}_t[q_{t+1}(n-1)^* - q_{t+1}(n-1)] - (q_t(n)^* - q_t(n)) - x_t, -(B_{1,r}(n)/n+1)x_t)}{Var(-(B_{1,r}(n)/n+1)x_t)} =$$
$$-\frac{B_{1,r}(n-1)\lambda - B_{1,r}(n) - 1}{B_{1,r}(n)/n+1}$$

In the case of rational expectations we have

$$\mathbb{E}_t[s_{t+1}] - s_t + x_t = \frac{\bar{\sigma}_\epsilon^2 \bar{\varphi}_t^2}{2} + \frac{\sigma_\epsilon^2 \varphi_t^2}{2} - \frac{\bar{\sigma}_\epsilon^2 \bar{\varphi}_t^{*2}}{2} - \frac{\sigma_\epsilon^2 \varphi_t^{*2}}{2} =$$
$$\bar{\sigma}_\epsilon^2(\bar{\varphi}_0\bar{\varphi}_1(z_t - z_t^*) + \bar{\varphi}_1^2(z_t^2 - z_t^{*2}) + 2\bar{\varphi}_1\bar{\varphi}_2\bar{z}_t(z_t - z_t^*)) +$$
$$\sigma_\epsilon^2(\varphi_0\varphi_1(z_t - z_t^*) + \varphi_1^2(z_t^2 - z_t^{*2}) + 2\varphi_1\varphi_2\bar{z}_t(z_t - z_t^*))$$

The the predictability coefficients for currencies are:

$$\frac{Cov(\mathbb{E}_t[s_{t+1}] - s_t + x_t, x_t)}{Var(x_t)} = -\bar{\sigma}_\epsilon^2\bar{\varphi}_0\bar{\varphi}_1 - \sigma_\epsilon^2\varphi_0\varphi_1$$

and for the spread:

$$\frac{Cov(\mathbb{E}_t[s_{t+1}] - s_t + x_t, -(B_{1,r}(n)/n+1)x_t)}{Var(-(B_{1,r}(n)/n+1)x_t)} =$$
$$\frac{\bar{\sigma}_\epsilon^2\bar{\varphi}_0\bar{\varphi}_1 + \sigma_\epsilon^2\varphi_0\varphi_1}{B_{1,r}(n)/n+1}$$

Moreover, the variance of n maturity relative yield is $\frac{B_{1,r}(n)^2}{n^2}$.

6.6 On Estimating k

This section derives the slope coefficient in the regression where forecast errors are explained by forecast revisions. Similarly to Coibion and Gorodnichenko (2012) we have

$$\mathbb{E}_t^S[x_{t+1}] = (1-k)\lambda\mathbb{E}_{t-1}^s[x_t] + k\lambda x_t$$

$$\mathbb{E}_t[x_{t+1}] = \lambda x_t.$$

Multiplying the first expression by λ^{j-1}:

$$\mathbb{E}_t^S[x_{t+j}] = (1-k)\mathbb{E}_{t-1}^s[x_{t+j}] + k\mathbb{E}_t[x_{t+j}],$$

where we used the property $\mathbb{E}_t^S[x_{t+j}] = \lambda^{j-1}\mathbb{E}_t^S[x_{t+1}]$. From this it follows that

$$k(\mathbb{E}_t[x_{t+j}] - \mathbb{E}_t^S[x_{t+j}]) = (1-k)(\mathbb{E}_t^S[x_{t+j}] - \mathbb{E}_{t-1}^S[x_{t+j}]).$$

Hence

$$x_{t+j} - \mathbb{E}^S[x_{t+j}] = \frac{1-k}{k}(\mathbb{E}_t^S[x_{t+j}] - \mathbb{E}_{t-1}^S[x_{t+j}]) + u_{t+j},$$

where u_{t+j} is zero mean and orthogonal to time t information. Hence $\beta_1^{FR} = \frac{1-k}{k}$ and $\beta_0^{FR} = 0$.

6.7 Bond Dollar Return Differences

Table shows IX the regression results for bond dollar return differences. The coefficients are mechanically the sums of the corresponding coefficients for bond local currency return differences and excess currency returns shown in table II. However, this does not hold exactly due to missing observations.

Bond Dollar Return Differences										
	3 month rate					yield spread				
	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1(3)$	$s.e$	R^2	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1$	$s.e$	R^2
Panel			0.15	0.67	0.0002			1.06	0.80	0.0041
AUS	-0.001	0.007	0.60	0.83	0.0025	0.008	0.008	0.59	1.24	-0.0009
CAN	0.007	0.004	-0.04	0.69	-0.0025	0.012***	0.005	1.22	0.80	0.0085
GER	0.013***	0.005	-1.12	0.89	0.0099	0.029***	0.009	3.61**	1.62	0.0348
JAP	0.007	0.006	1.98**	0.95	0.0241	-0.007	0.013	-1.23	2.46	-0.0006
NZL	0.011	0.014	-0.70	1.98	-0.0010	0.006	0.006	-0.47	1.99	-0.0031
SWE	-0.004	0.005	-1.08	1.31	0.0147	-0.007	0.005	1.77	1.23	0.0135
CH	0.003	0.005	0.99	0.87	0.0063	0.005	0.009	0.94	1.69	0.0008
UK	0.001	0.005	0.42	1.07	-0.0009	0.006	0.007	0.46	1.21	-0.0014

Table IX shows the results from regressing the relative bond (foreign minus US) returns measured in dollars on short rate and yield spread differences. The standard errors of the panel regression (country fixed effects) are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

	$\hat{\lambda}_0$	$s.e$	$\hat{\lambda}_1$	$s.e$	R^2
panel			0.982***	0.009	0.978
CAN	0.024	0.034	0.956***	0.019	0.934
GER	-0.005	0.032	0.992***	0.015	0.991
JAP	-0.078	0.029	0.979***	0.008	0.990
NOR	0.049	0.072	0.986***	0.021	0.974
SWE	-0.008	0.043	0.986***	0.016	0.981
CH	-0.004	0.046	0.986***	0.016	0.962
UK	0.045	0.046	0.972***	0.021	0.970

Table X shows the results from regressing the monthly short rate differential (foreign minus US) on its first lag excluding the sample period after 2008. The standard errors of the panel regression are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

6.8 Robustness Checks for Empirical Analysis

We conduct several robustness checks for our results. First, some authors such as Engel (2016) voluntarily leave the period after the financial crisis out from the sample due to possible changes in the driving forces of currencies. Similarly this period might be extraordinary for the bond market due to low interest rates and unconventional monetary policies. Tables X, XI and XII replicate tables IV, V and II but now excluding the period after 2008. Excluding this period does not alter the key results: rather many of results become stronger. The results in the after 2008 subsample are somewhat weaker and mostly not statistically significant. However, the sample period is fairly short. Many of our results also become stronger if we omit Japan, where interest rates have been very low during most of the sample period.

Note that our assumptions imply that under correct beliefs $k = 1$. However, in theory some underweighting might be statistically optimal e.g. due to noisy observations. We

	$\hat{\beta}_0$	*s.e*	$\hat{\beta}_1$	*s.e*	R^2	*implied k*
panel			1.648**	0.500	0.061	0.38
CAN	0.333	0.263	0.049	0.362	0.001	0.95
GER	0.538**	0.258	2.826***	0.681	0.152	0.26
JAP	0.432	0.311	1.936***	0.656	0.081	0.34
NOR	0.664	0.511	3.315***	0.979	0.172	0.23
SWE	-0.150	0.396	2.103***	0.800	0.092	0.32
CH	0.593*	0.339	1.766**	0.724	0.073	0.36
UK	0.412	0.266	0.580	0.480	0.010	0.63

Table XI shows the results from regressing the difference in forecast error (foreign minus US) when forecasting short rates 12 months ahead on the difference in short rate forecast revisions excluding the sample period after 2008. The standard errors of the panel regression are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

now test this assumption of the model. Using the actual short rate process we obtain a panel estimate $k \approx 0.983$.[27] Moreover, $k = 1$ clearly cannot be rejected. Therefore assuming $k = 1$ under correct beliefs seems empirically reasonable.

The baseline model assumes an AR(1)-process for the short rate differential under the objective measure. We now estimate a sticky expectations AR(2)-version of the model. Table XIII replicates VI but now under the assumption that the true short rate differential process is AR(2). These are obtained using simulations as the AR(2)-version does not allow for simple closed-form expressions. One can see that the sticky expectations AR(2) -model gives somewhat more accurate results for the predictability coefficients.

Finally, in theory variables other than forecast revisions might be important to predicting forecast errors concerning short rate differences. We have not found evidence for this. Table XIV shows the results when we modify the regression to include unemployment

[27] This can be estimated either using the above regression procedure or maximum likelihood.

PANEL A: Bond Local Currency Return Differences Before 2008										
	3 month rate					yield spread				
	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1(3)$	$s.e$	R^2	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1$	$s.e$	R^2
panel			-1.322***	0.310	0.022			1.421**	0.530	0.012
AUS	0.002**	0.002	-1.382* *	0.600	0.030	0.001	0.001	2.333**	1.074	0.026
CAN	0.002*	0.001	-1.989***	0.562	0.051	0.001*	0.001	2.805***	0.777	0.050
GER	-0.002*	0.001	-0.806**	0.400	0.007	-0.001	0.001	0.764	0.529	0.005
JAP	-0.003**	0.002	-1.367***	0.512	0.089	-0.000	0.001	0.621	0.508	0.002
NZL	0.002	0.002	-1.675***	0.650	0.013	-0.001	0.001	0.975	0.873	0.004
SWE	0.002	0.001	-1.195**	0.616	0.025	0.001	0.001	1.579	1.070	0.022
CH	-0.003***	0.001	-1.163**	0.553	0.011	-0.001	0.001	1.198*	0.655	0.011
UK	0.002	0.002	-1.544***	0.667	0.017	-0.001	0.001	0.892	0.815	0.003

PANEL B: Currency Excess Returns Before 2008										
	3 month rate					yield spread				
	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1(3)$	$s.e$	R^2	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1$	$s.e$	R^2
panel			1.758**	0.515	0.026			-2.834**	0948	0.020
AUS	-0.001	0.003	1.527**	0.621	0.027	-0.000	0.002	-2.999***	1.200	0.033
CAN	0.002	0.001	1.424**	0.642	0.024	0.003**	0.001	-2.112***	0.840	0.025
GER	0.008***	0.003	-.285	1.323	0.000	0.007**	0.003	-1.088	1.748	0.003
JAP	0.001**	0.003	4.253***	1.115	0.066	-0.003*	0.002	-4.617***	1.133	0.045
NZL	-0.004	0.003	2.386***	0.370	0.100	-0.005*	0.003	-4.926***	0.602	0.125
SWE	-0.002	0.002	0.942	1.107	0.011	-0.001	0.002	-0.409	1.649	0.001
CH	0.004*	0.002	1.958*	1.206	0.016	-0.000	0.002	-2.781*	1.518	0.022
UK	0.001	0.002	1.648*	0.997	0.014	0.001	0.002	-2.507	1.596	0.016

Table XII shows the results from regressing the relative bond (foreign minus US) and currency returns on short rate and yield spread differences excluding the sample after 2008. The standard errors of the panel regression are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

Regression	β (Data)	β^{IRM}	β^{RP}	$\beta^{RPM}+\beta^{PCM}+$error
LHS: Currency Excess Return , RHS: short rate difference (x_t)	1.489	0.948	NA	NA
LHS: Currency Excess Return , RHS: yield spread difference	-1.943	-1.35	NA	NA
LHS: Local Currency Bond Return Difference, RHS: short rate difference (x_t)	-1.259	-0.99 (79 %)	NA	NA
LHS: Local Currency Bond Return Difference, RHS: yield spread difference	1.250	1.40	NA	NA
LHS: Dollar Bond Return Difference, RHS: short rate difference (x_t)	0.23	0.04	NA	NA
LHS: Dollar Bond Return Difference, RHS: yield spread difference	-0.69	0.05	NA	NA

Table XIII shows key statistics measured from the data (panel regressions) as well as those predicted by the model, AR(2) -version of the model, 1 month horizon.

or inflation rate differences. Here these additional variables are insignificant and the coefficient on forecast revisions of similar magnitude than before.

6.9 A Noisy Information Model

This section notes, similarly to Coibion and Gorodnichenko (2015), that a noisy information model implies sticky expectations. Assume each agent observes

$$s_t = x_t + v_t$$

Here the noise is given by $v_t \sim N(0, \sigma_v^2)$. The inference problem can be solved using the standard recursion formulas for the Kalman filter (see e.g. Hamilton, 1994). The solution for conditional expectation converges to a sticky expectations specification, where the underreaction coefficient is given by the Kalman gain

$$k = \frac{1+\Delta-\eta(1+\lambda^2)}{1+\Delta+\eta(1+\lambda^2)}.$$

Here $\Delta^2 = [\eta(1-\lambda^2)+1]^2 + 4\eta\lambda^2$ and $\eta = \frac{\sigma_v^2}{\sigma_\epsilon^2}$. Moreover, the conditional volatility converges to

	$\hat{\beta}_0$	$s.e$	$\hat{\beta}_1$	$s.e$	$\hat{\beta}_2$	$s.e$	R^2
			PANEL A: Unemployment				
panel			1.086***	0.389	0.074	0.051	0.051
CAN	0.217	0.153	-0.209	0.217	0.083	0.083	0.007
GER	0.149	0.211	1.605**	0.682	0.057	0.332	0.100
JAP	0.849***	0.325	1.687***	0.433	0.256**	0.101	0.240
NOR	0.890	0.556	2.050***	0.706	0.174	0.110	0.132
SWE	-0.283	0.283	1.438***	0.573	-0.021	0.106	0.056
CH	0.671	0.384	1.089*	0.537	0.127	0.077	0.081
UK	0.175	0.203	0.561	0.375	-0.017	0.058	0.012
			PANEL B: Inflation				
panel			1.069***	0.393	-0.069	0.086	0.004
CAN	0.271	0.158	-0.222	0.213	-0.025	0.101	0.004
GER	0.289	0.195	1.628**	0.682	-0.029	0.136	0.075
JAP	0.023	0.214	1.748***	0.484	-0.154	0.116	0.122
NOR	0.355	0.271	2.000***	0.711	0.038	0.152	0.091
SWE	-0.353	0.275	1.490**	0.585	-0.036	0.145	0.056
CH	0.166	0.163	0.954*	0.517	-0.083	0.128	0.037
UK	0.195	0.163	0.545	0.349	-0.158	0.114	0.036

Table XIV shows the results from regressing the difference in forecast error (foreign minus US) when forecasting short rates 12 months ahead on the difference in short rate forecast revisions and on an additional regressor, either the unemployment differences or the inflation differences. The standard errors of the panel regression are calculated using the (Driscoll and Kraay, 1998) methodology with 13 lags, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation. The standard errors for individual regressions are corrected for heteroskedasticity and autocorrelation (Newey and West, 1987). *, ** and *** denote significance at 10%, 5% and 1% levels respectively.

$$Var_t^S(x_{t+1}) = \frac{1-k}{1-(1-k)\lambda^2}\sigma_\epsilon^2.$$

However, this is not the only way to arrive at a sticky expectations specification (see e.g., Coibion and Gorodnichenko, 2015).

6.10 Individual vs Aggregate Expectational Errors

The main analysis effectively assumes that all agents share a common probability measure S. However, we can also define this measure as a weighted average of some individual measures $\{S_i\}_{i=1}^N$. We next discuss this observation more formally.

In the case of heterogeneous expectations, we can always decompose the currency risk premium as

$$\underbrace{\Theta_t^{FX}}_{\text{Currency premium}} = \underbrace{\zeta_{t,i}^{FX}}_{\text{Individual risk premium differential}} \underbrace{+\mathbb{E}_t\left[\mathbb{E}_{t+1,i}^S\sum_{j=0}^{\infty}x_{t+1+j} - \mathbb{E}_{t,i}^S\sum_{j=0}^{\infty}x_{t+1+j}\right]}_{\text{Individual interest rate misperception effect}}$$
$$\underbrace{-\mathbb{E}_t\left[\mathbb{E}_{t+1,i}^S\sum_{j=0}^{\infty}\zeta_{t+1+j,i}^{FX} - \mathbb{E}_{t,i}^S\sum_{j=0}^{\infty}\zeta_{t+1+j,i}^{FX}\right]}_{\text{Individual risk premium misperception effect}} + \underbrace{\mathbb{E}_t[\lim_{j\to\infty}\mathbb{E}_{t+1,i}^S[s_{t+j}] - \lim_{j\to\infty}\mathbb{E}_{t,i}^S[s_{t+j}]]}_{\text{Individual permanent component misperception effect}}$$
$$\equiv \zeta_{t,i}^{FX} + \Theta_{t,i}^{IRM} + \Theta_{t,i}^{RPM} + \Theta_{t,i}^{PCM}.$$

Here the components are as before but subjective expectations are taken under agent i's measure S_i. Because this holds for any agent, we can then use a weighted sum of these expectations to obtain:

$$
\begin{aligned}
\underbrace{\Theta_t^{FX}}_{\text{Currency premium}} = \underbrace{\bar{\zeta}_t^{FX}}_{\text{Aggregate risk premium differential}} &+ \underbrace{\mathbb{E}_t\left[\bar{\mathbb{E}}_{t+1}^S \sum_{j=0}^{\infty} x_{t+1+j} - \bar{\mathbb{E}}_t^S \sum_{j=0}^{\infty} x_{t+1+j}\right]}_{\text{Aggregate interest rate misperception effect}} \\
\underbrace{-\mathbb{E}_t\left[\bar{\mathbb{E}}_{t+1}^S \sum_{j=0}^{\infty} \zeta_{t+1+j,i}^{FX} - \bar{\mathbb{E}}_t^S \sum_{j=0}^{\infty} \zeta_{t+1+j,i}^{FX}\right]}_{\text{Aggregate risk premium misperception effect}} &+ \underbrace{\mathbb{E}_t[\underset{j\to\infty}{lim} \bar{\mathbb{E}}_{t+1}^S[s_{t+j}] - \underset{j\to\infty}{lim} \bar{\mathbb{E}}_{t,i}^S[s_{t+j}]]}_{\text{Aggregate permanent component misperception effect}} \\
&\equiv \bar{\zeta}_t^{FX} + \bar{\Theta}_t^{IRM} + \bar{\Theta}_t^{RPM} + \bar{\Theta}_t^{PCM}.
\end{aligned}
$$

Here, given some weights $\{w\}_{i=1}^N$ the aggregate subjective risk premium is given by

$$
\bar{\zeta}_t^{FX} = \sum_{i=1}^{N} w_i \zeta_{t,i}^{FX}
$$

and for any variable y we define the aggregate expectation as

$$
\bar{\mathbb{E}}_t^S[y] = \sum_{i=1}^{N} w_i \mathbb{E}_{t,i}[y].
$$

The individual bond risk premium decompositions can be aggregated in a similar way.

Given heterogenous expectations, each component of our decompositions can be interpreted as an average of the components defined under the individual probability measures. The interest rate misperception component can still be used to measure the contribution of interest rate forecast errors to variation in bond and currency premia. However, this component reflects errors in average expectations and might be different under the subjective probability measure of some agent i.

Recently, Bordalo et al. (2019) argue that the underreaction result discussed for example by Coibion and Gorodnichenko (2015) is partly driven by aggregation. For some variables, individual forecasts are rather prone to overreaction. How would these arguments affect the results in this paper?

First, it is important to note that the overreaction result in Bordalo et al. (2019) does not apply to short term interest rates. Rather the authors find that individual short rate forecasts underreact though the amount of this underreaction is somewhat smaller than that suggested by aggregate level data.[28]

Second, Bordalo et al. (2019) agree that the sticky expectations process provides a good description of aggregate level survey expectations. As explained using decompositions above, this aggregate short rate expectations process is naturally used to define the overall contribution of short rate forecast errors to variation in bond and currency premia.

The findings of Bordalo et al. (2019) are still relevant for providing a microfounded theory of expectations formation. However, for example Juodis and Kucinskas (2019) argue that the empirical findings concerning overreaction in some individual level survey expectations can be caused by noise similar to measurement error, which is reduced by aggregation.

Heterogeneity can have interesting effects on the determination of the aggregate subjective risk premium $\bar{\zeta}_t$. For example disagreement can induce speculative trading between agents, which can create a link between the level of disagreement and asset returns. Because we focus on the effects of short rate forecast errors, a formal study of such effects is beyond the scope of this paper.[29]

Finally, heterogeneity can imply that some agents are more important in determining asset prices than others. It might be reasonable, for example, to overweight forecasts

[28] Interestingly, Gabaix (2019) notes that the variables for which the underreaction result is most robust tend to be highly persistent variables. Short rates are indeed very persistent. For additional discussion see also Bouchaud et al. (2018).

[29] For the effects of disagreement on bond markets see e.g. Xiong and Yan (2010) and Giacoletti et al. (2018), for FX markets see Bacchetta and Wincoop (2006), Buraschi et al. (2010) and Molavi et al. (2021).

provided by institutions with larger bond and currency positions. However, because such exposure is difficult to measure, we follow Coibion and Gorodnichenko (2015) and simply average over the individual forecasts provided by different institutions. Note that the use of professional forecasts might provide a conservative estimate of the biases reflected in asset prices.

6.11 On Exchange Rate Disconnect

This paper has argued that sticky short rate expectations can go a long way in explaining bond and currency dynamics. However, this section notes that one puzzle still remains: the exchange rate disconnect puzzle.

The classic puzzle of Fama (1984) pertains to a regression slope coefficient. While UIP predicts that high interest rate currencies should depreciate, the data suggests that on average these currencies rather appreciate. However, the explanatory power of such regressions is still small and the forecast error variance large.

Perhaps even more surprisingly, macroeconomic variables have trouble explaining contemporaneous movements in exchange rates. While exchange rates are correlated with such aggregate variables, these variables cannot explain the bulk of FX rate movements. This finding, attributed to Meese and Rogoff (1983), is often dubbed the *exchange rate disconnect puzzle*.

We next describe the possible sources of FX volatility. Using the decomposition derived earlier but for FX rate changes we obtain:

$$\Delta s_t = \mathbb{E}_t^S \sum_{j=0}^{\infty} x_{t+j} - \mathbb{E}_{t-1}^S \sum_{j=0}^{\infty} x_{t+j-1} - \mathbb{E}_t^S \sum_{j=0}^{\infty} \zeta_{t+j}^{FX} + \mathbb{E}_{t-1}^S \sum_{j=0}^{\infty} \zeta_{t+j-1}^{FX} + \lim_{j\to\infty} \mathbb{E}_t^S[s_{t+j}] - \lim_{j\to\infty} \mathbb{E}_{t-1}^S[s_{t+j-1}] \equiv \Gamma_t^{IR} + \Gamma_t^{RP} + \Gamma_t^{PC}.$$

That is exchange rate movements depend on changes in the expected path of interest rates (Γ_t^{IR}), expected path of risk premia (Γ_t^{RP}) and the permanent component of the FX rate (Γ_t^{PC}). Then for FX rate variance we obtain[30]

$$Var(\Delta s_t) = Cov(\Delta s_t, \Delta s_t) =$$
$$Cov(\Gamma_t^{IR}, \Delta s_t) + Cov(\Gamma_t^{RP}, \Delta s_t) + Cov(\Gamma_t^{PC}, \Delta s_t)$$

and hence

$$1 = \frac{Cov(\Gamma_t^{IR}, \Delta s_t)}{Var(\Delta s_t)} + \frac{Cov(\Gamma_t^{RP}, \Delta s_t)}{Var(\Delta s_t)} + \frac{Cov(\Gamma_t^{PC}, \Delta s_t)}{\Delta s_t}$$

How much of the contemporaneous FX rate changes can the short rates channel explain? Does allowing for sticky expectations increase this number?

Using the main calibration of the model and survey data, we find a value of 4 per cent for the sticky expectations model and 3 per cent for the rational expectations model. News to the subjectively expected path of short rates appears to explain slightly more of the FX rate variation than news to the rationally expected path of short rates.

These values can be sensitive to model assumptions. We do not observe the path of subjectively expected path of future short rates. Rather we apply our calibrated model to estimate this component from short term expectations. Changing the value of the persistence parameter to 0.995 from 0.987 would increase the share explained by subjective expectations to roughly 11 per cent and changing it to 0.999 would already mean that the expectations account for most of the variation in exchange rates. This would be without similar large effects for the FX predictability coefficients. Note that a version of the sticky expectations model where agents also perceive the persistence of the short rate process as higher than actual has been applied e.g. by Brooks et al. (2019).

[30] This decomposition is related to that discussed by Stavrakeva and Tang (2020).

In addition, this model is based on aggregated survey data. Because aggregation reduces volatility, a decomposition obtained with individual survey data might show a higher share of variance explained by the path of short rate forecasts.

It is still likely that the path of subjectively expected short rates does not explain 100 per cent of FX volatility. As such, this is not a problem for the results of this paper. We have argued that sticky short rate forecasts can explain most of the predictability of currency and bond returns due to variation in short rate and yield spread differences. We have not claimed that they explain most of FX volatility. Moreover, even our strongest conditions only imply

$$\frac{Cov(\Gamma_t^{RP}, \Delta s_t)}{Var(\Delta s_t)} = 0$$

but generally

$$\frac{Cov(\Gamma_t^{PC}, \Delta s_t)}{Var(\Delta s_t)} \neq 0.$$

In particular the assumption of constant risk premia implies that they do not contribute to FX volatility. But to satisfy condition NLRM, it is sufficient to merely assume that agents have rational beliefs concerning the permanent component of the FX rate.

But what then explains FX volatility? While a comprehensive answer is beyond the scope of this paper, we next explore some possibilities. First, consider the following version of the model:

$$s_t = \mathbb{E}_t^S \sum_{j=0}^{\infty} x_{t+j} + \lim_{j \to \infty} \mathbb{E}_t[s_{t+j}].$$

Here, a sticky expectations process determines the expected path of short rates but agents have rational beliefs concerning the long run component of the FX rate. This model satisfies the strongest conditions considered in the paper: SE, CRP and NLRM. Now the sticky expectations mechanism can explain return predictability. On the other hand shocks to the permanent component also contribute to FX volatility but not to return predictability. Moreover, even though changes in this component are unpredictable, it can be contemporeneously correlated with short rate shocks.

One interpretation of our results is that subjective risk premia contribute less to standard bond and currency predictability patterns than sticky short rate forecast errors. However, risk premium shocks might still explain most of FX volatility. In particular a volatile subjective risk premium that is weakly correlated with (previous period) short rates and yield spreads could explain most of FX volatility yet contribute weakly to bond and currency predictability.

But what would explain such shocks to risk premia or the permanent component of the FX rate? Note that the disconnect puzzle suggests that these shocks are weakly correlated with macroeconomic fundamentals.[31] Hence they might represent type of sentiment shocks that alters agents' views about currency return potentials or long-run exchange rate values.[32] They could also represent noise trader shocks similar to those in Itskhoki and Mukhin (2021), be related to convenience yields analyzed by Jiang et al. (2018) or time-varying parameters discussed by Fratzscher et al. (2015).

For the main purposes of this paper, condition CRP could be replaced with the following much weaker condition:

Condition UCRP

The risk premium components are uncorrelated with short rate differentials: $Cov(\Theta_t^{IRM}, x_t) = Cov(\Theta_t^{IRM,B}(n), x_t) = 0$

[31] Alternatively, these shocks might provide information about future short rates or risk premia but be weakly correlated with current macroeconomic variables. Also these shocks might be connected with macroeconomic fundamentals but this connection is offset for exchange rates due to interractions with the different channels.

[32] Because FX rates have a fairly strong cross-sectional factor structure (Korsaye et al., 2020), generally these shocks might have to be correlated across currencies.

Now imposing SE, UCRP and NLRM instead of SE, CRP and NLRM would still attribute all of the bond and currency predictability due to short rate variation to forecast errors concerning short rates. However, here subjective risk premium shocks could help solve the exchange rate disconnect puzzle. An example of a model satisfying Condition UCRP would be one with i.i.d. subjective bond and currency premia.

Finally, note that the inability to fully explain the exchange rate disconnect puzzle is shared by key competing risk-based explanations. Macrofinance models like the long run risk model or the habit model imply that the economy can be described by a small set of aggregate level variables. These variables should then fully explain all FX rate movements. For example in the standard habit model (Verdelhan, 2010) the economy is described by a single state variable: consumption habit that is a simple function of current and past consumption values. Movements in the habit variable should explain all of FX volatility. But empirically FX rate changes are weakly correlated with aggregate variables like consumption growth.