# Efficient Simulation and Other Numeric Methods for Stochastic Disease Spread Models

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# Outline

- **Introduction**
- **Model Definition and Examples**
- $R_0$ **Calculation for Overlapping Mixing Groups**
- **Practical Applications of** $R_0$

# Disease Spread Models

- **Deterministic Disease Spread Models (S-I-R)**
  Kermack-McKendrick Models
- **Stochastic Disease Spread Models (S-I-R)**
  Greenwood and Red Frost Models
  1. Graph Based Models
  2. Agent Based Models with Mixing Groups

# Agent Based Models with Mixing Groups

- **Advantages**
  1. Allow to model complex systems
  2. Incorporate census data in the models
- **Disdvantages**
  1. No closed form solution characterization of important epidemiological quantities
  2. Require simulation for analysis

# Single Mixing Group

- **Discrete Time Stochastic Model**
- **States:** Susceptible(S) - Infected(I) - Recovered(R)
- **Change in States:** Newly Infected (NI)
- **Discrete Time Duration:** Random disease time with pmf
- **Binomial Chain Assumption:** The number of newly infected per day has binomial distribution

# Single Mixing Group

- **The number of newly infected generated on time interval t**

$$NI_t \sim Bin(n = S_{t-1}, 1 - (1 - p_c)^{I_{t-1}}) \quad (1)$$

# Basic reproduction Number $R_0$

- **Basic Reproduction Number $R_0$:** expected number of secondary cases that one case would produce in an entirely susceptible population
- $R_0 > \mathbf{1}$ $\rightarrow$ Positive probability of an outbreak
- $R_0 < \mathbf{1}$ $\rightarrow$ An outbreak not possible

# $R_0$ for Single Mixing Group

- To calculate $R_0$ there is a single infected case and (n-1) susceptible
- **The probability of being infected**

$$p_{inf} = \sum_d (1 - (1 - p_c)^d) P_D[d] \quad (2)$$

- **Basic Reproduction Number $R_0$ Formula**

$$R_0 = \sum_d (1 - (1 - p_c)^d) P_D[d](n - 1). \quad (3)$$

# Overlapping Mixing Groups

- **Possible Mixing Groups**
  - Community
  - Neighborhood
  - School
  - Work
  - Family

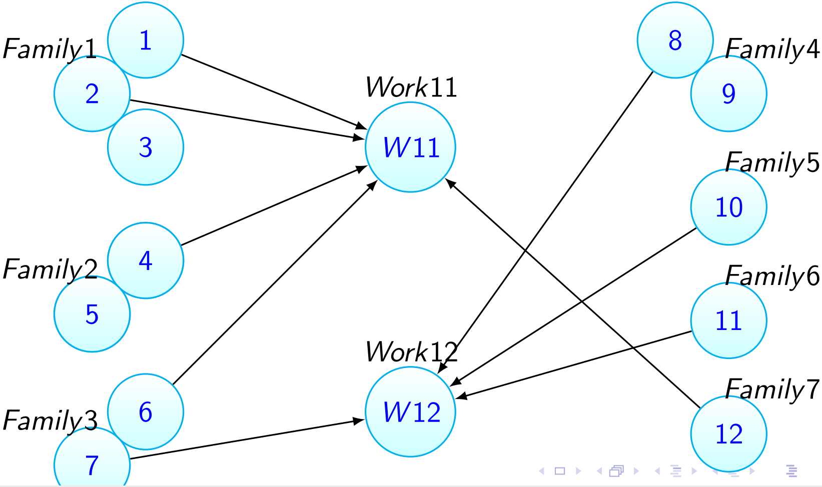
# Overlapping Mixing Groups

- The infection probabilities depend on the number of infected individuals in the respective mixing groups
- The probability of being infected of an individual with $I_c$, $I_n$, $I_w$, and $I_f$ infected individuals in his community, neighbor, work and family

$$p_{inf} = 1 - (1 - p_c)^{I_c}(1 - p_n)^{I_n}(1 - p_w)^{I_w}(1 - p_f)^{I_f} \tag{4}$$

- The total number of newly infected is sum of independent not identically distributed Bernoulli variates

# Population Graph for Small Population Model



# Population Matrix for Small Population Model

| Individual ID | Family Number | Work Number | Community |
|---|---|---|---|
| 1 | 1 | 11 | 100 |
| 2 | 1 | 11 | 100 |
| 3 | 1 | - | 100 |
| 4 | 2 | 11 | 100 |
| 5 | 2 | - | 100 |
| 6 | 3 | 11 | 100 |
| 7 | 3 | 12 | 100 |
| 8 | 4 | 12 | 100 |
| 9 | 4 | - | 100 |
| 10 | 5 | 12 | 100 |
| 11 | 6 | 12 | 100 |
| 12 | 7 | 11 | 100 |

# Probabilities for Small Population Model

| Probability in Family | Probability in Work | Probability in Community | Disease Duration |
|---|---|---|---|
| 0.2 | 0.1 | 0.05 | 1 |

$$p_{inf} = 1 - (1 - p_c)^{I_c}(1 - p_n)^{I_n}(1 - p_w)^{I_w}(1 - p_f)^{I_f} \tag{5}$$

- Only Individual 1 is initially infected: probability that Individual 2 is infected

$$p_{inf} = 1 - (1 - 0.05)^1(1 - 0.1)^1(1 - 0.2)^1 = 0.316 \tag{6}$$

# Probabilities for Individual One is Initially Infected

| ID | Family Number | Work Number | Community | Mixing Groups | Infection Probability |
|---|---|---|---|---|---|
| 2 | 1 | 11 | 100 | F-W-C | 0.316 |
| 3 | 1 | - | 100 | F-C | 0.240 |
| 4 | 2 | 11 | 100 | W-C | 0.145 |
| 5 | 2 | - | 100 | C | 0.05 |
| 6 | 3 | 11 | 100 | W-C | 0.145 |
| 7 | 3 | 12 | 100 | C | 0.05 |
| 8 | 4 | 12 | 100 | C | 0.05 |
| 9 | 4 | - | 100 | C | 0.05 |
| 10 | 5 | 12 | 100 | C | 0.05 |
| 11 | 6 | 12 | 100 | C | 0.05 |
| 12 | 7 | 11 | 100 | W-C | 0.145 |
| | | | | | $\sum = 1.291$ |

- Four different intersection four different probabilities

# Individual $R_0$

| ID | Family Number | Work Number | Community | $R_0$ |
|---|---|---|---|---|
| 1 | 1 | 11 | 100 | 1.291 |
| 2 | 1 | 11 | 100 | 1.291 |
| 3 | 1 | - | 100 | 0.93 |
| 4 | 2 | 11 | 100 | 1.12 |
| 5 | 2 | - | 100 | 0.74 |
| 6 | 3 | 11 | 100 | 1.12 |
| 7 | 3 | 12 | 100 | 1.025 |
| 8 | 4 | 12 | 100 | 1.025 |
| 9 | 4 | - | 100 | 0.74 |
| 10 | 5 | 12 | 100 | 0.835 |
| 11 | 6 | 12 | 100 | 0.835 |
| 12 | 7 | 11 | 100 | 0.93 |

- Four different intersection four different probabilities

# Population $R_0$

- Randomly selected initially infected individual with equal probabilities
- Different individual $R_0$ values
- Not possible to observe probability of an outbreak
- The outbreak probability depends on individual $R_0$ of starting infected individual

# $R_0$ for Intervention Strategies

- Evaluation of intervention strategies
- The maximum number of infectious cases, the total number of infectious cases, the average attack rates etc.
- $R_0$ gives more information than descriptive statistics
- Whether to decrease maximum individual $R_0$ below 1

# Possible Intervention Strategies

The use of $R_0$ allows only to evaluate intervention methods implemented from the very beginning of the infection

- Vaccination
- Stay Home
- Use of Antiviral Drugs

# Model for Scenario Analysis

- Similar to the model of Longini et al. (2004)
- 100 single living
- 50 families with size two
- 34 families with size three
- 37 families with size four
- 5 families with size five
- 3 families with size six
- Single family with size seven
- Disease length2, 3, 4, 5, and 6 with probabilities 0.21, 0.19, 0.18, 0.22, and 0.2.

# Vaccination Strategies

Vaccination takes place before the infection starts & Individuals develop immunity
It is possible to generate vaccination strategy based on individual $R_0$

- $R_0$ without vaccination
- $R_0$ with 50% random vaccination
- $R_0$ with 50% optimal vaccination

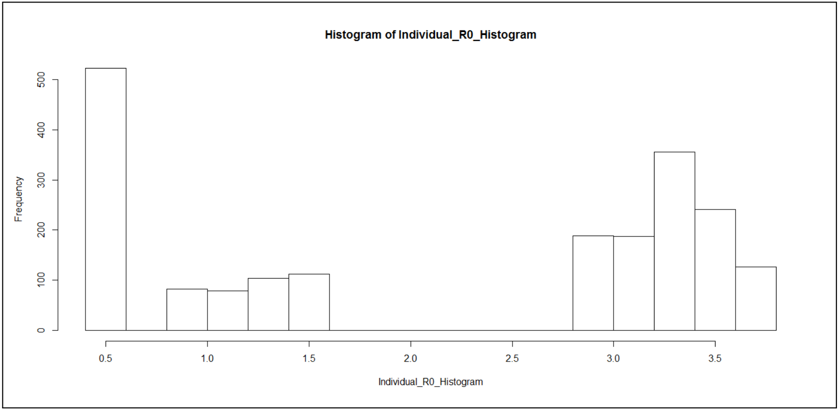



Figure: Individual $R_0$ histogram without vaccination.

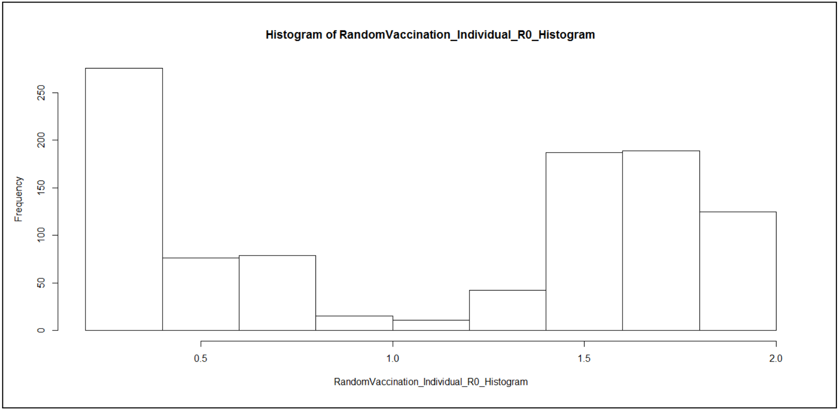



Figure: Individual $R_0$ histogram with 50% random vaccination strategy.

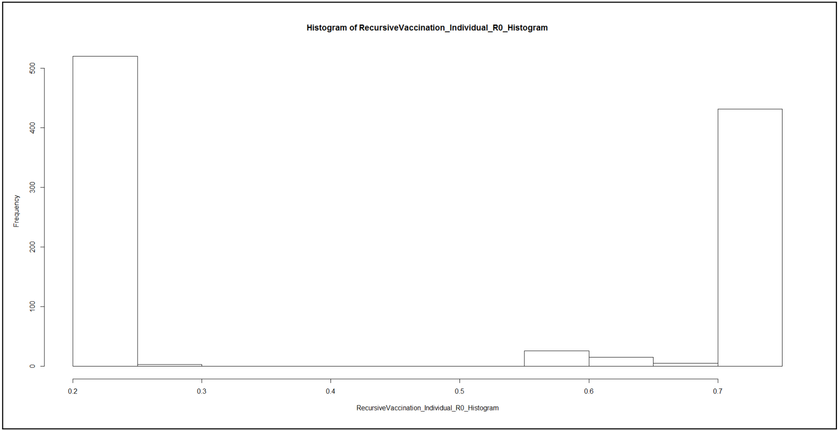



Figure: Individual $R_0$ histogram with 50% optimal vaccination strategy.

## Household Quarantine

- Infected individuals after some days of infection do not have contacts with other individuals outside of their households
- Not effective because disease spread before indicating symptoms

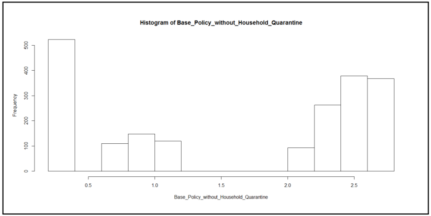



Figure: Base policy with household quarantine.

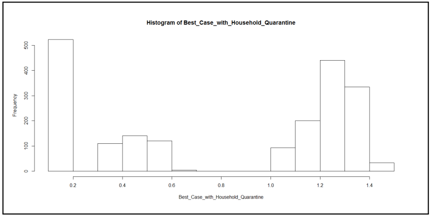



Figure: Best case with household quarantine.

# Use of Antiviral Drugs

- The use of antiviral drugs evaluated here prevents infection given exposure
- Reduce the probability of transmission to others given infection and the probability of being infected given exposure
- Not very effective due to shortage of supply and expensive

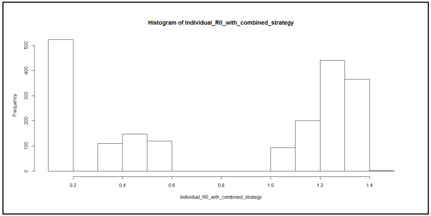



Figure: $R_0$ distribution with combined strategy.

THANK YOU..