Systematic Effects among Loss Given Defaults and their Implications on Downturn Estimation*

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January 9, 2018

Abstract

Banks are obliged to provide downturn estimates for loss given defaults (LGDs) in the internal ratings-based approach. While downturn conditions are characterized by systematically higher LGDs, it is unclear which factors may best capture these conditions. As LGDs depend on recovery payments which are collected during varying economic conditions in the resolution process, it is challenging to identify suitable economic variables. Using a Bayesian Finite Mixture Model, we adapt random effects to measure economic conditions and to generate downturn estimates. We find that systematic effects vary among regions, i.e., the US and Europe, and strongly deviate from the economic cycle. Our approach offers straightforward supportive tools for decision makers. Risk managers are enabled to select their individual margin of conservatism based on their portfolios, while regulators might set a lower bound to guarantee conservatism. In comparison to other approaches, our proposal appears to be conservative enough during downturn conditions and inhibits over-conservatism.

Keywords: bank loans; credit risk; random effects

JEL classification: C23, G21, G33

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* The support by the Frankfurt Institute of Risk Management and Regulation is gratefully acknowledged. The authors would like to thank Global Credit Data for granting access to their data base.

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1 Introduction

Banks are obliged to provide conservative estimates for the probability of default (PD) and the loss given default (LGD) in the advanced internal ratings-based (IRB) approach of the Basel regulations (see Basel Committee on Banking Supervision, 2004). While through-the-cycle, i.e., average, PDs are translated into (conservative) conditional PDs by a supervisory mapping function, LGDs are required to reflect conditions of economic downturn. Economic variables might, thus, be a natural choice to identify downturns and to generate consistent downturn estimates. Recently, the EBA published new technical standards (see European Banking Authority, 2017) which emphasize the use of economic or credit risk factors. However, the identification of economic variables seems to be ambiguous in the literature.[1] This might be due to the nature of *workout* LGDs. Recovery payments are collected during the whole resolution process which usually takes multiple years ahead of the default event. Thus, economic or credit risk factors at a specific point in time (e.g., the default time) may not be able to represent the systematic patterns inherent in LGDs.

Although the identification of suitable economic variables seems to be ambiguous, financial institutions are confronted with the need to generate consistent downturn estimates for loss rates and, thus, identify and measure downturns. Following the literature, this may be a challenging task. Economic variables are either not significant or deliver limited explanatory power. To resolve this, we suggest to use time-specific random effects to measure the systematic movement inherent in LGDs over time. Consistent downturn estimates can be generated by considering a conservative quantile of the random effect. This entails a comprehensive supportive tool for decision makers. On the one side, risk managers are enabled to determine the margin of conservatism by selecting an according quantile based on the characteristics of their loan portfolio. On the other side, regulators might set a lower bound to this quantile to guarantee overall conservatism and, thus, the stability of the financial system during crises. Furthermore, our approach bypasses identification issues that may occur to decision makers in terms of selecting variables for LGD modeling among a set of observable variables.

The contribution of this paper is threefold. First, we use access to the unique loss database of Global Credit Data (GCD)[2] to reveal the systematic nature inherent in LGDs. Thereby, we find

[1] See Section 2 for a literature review.

[2] GCD is a non profit initiative which aims to support banks to measure their credit risk by collecting and analyzing historical loss data. See `http://www.globalcreditdata.org/` for further information. Currently, 52 member banks from all around the world share their loss information.

considerable differences among regions, i.e., the US and Europe. Second, we show that random effects strongly deviate from the economic cycle measured by common economic variables. We compare the estimated random effects to macro variables common in the LGD literature. Thereby, not only descriptive differences come to light. The impact of macro variables seems not to be evident or limited regarding its magnitude. Third, we suggest a methodology to generate consistent downturn estimates and compare it to a variety of alternatives.

The remainder of this paper is structured as follows. Section 2 provides a literature overview, while data and methods are explained in Section 3. The empirical results are presented in Section 4. Based on our results, a new method to generate downturn LGDs is introduced in Section 5 and compared to other approaches in this field. Section 6 concludes.

2 Literature Review

The literature regarding loss rates can be divided into two streams. The first one focuses on separate methods, whereby, the LGD is the only dependent variable. The second one applies joint modeling approaches for the PD and the LGD to consider the dependence structure between the two central credit risk parameters.

Table B.1 in Appendix B summarizes the first stream of literature, i.e., separate methods, focusing systematic effects. Typically, macro variables are included to display synchronism in time line. However, some authors completely renounce macro variables in their analysis (see, e.g., Bastos, 2010; Bijak and Thomas, 2015; Calabrese, 2014; Gürtler and Hibbeln, 2013; Matuszyk et al., 2010; Somers and Whittaker, 2007). Others examine univariate significance which (partly) disappears in a multivariate context (see, e.g., Acharya et al., 2007; Brumma et al., 2014; Caselli et al., 2008; Dermine and Neto de Carvalho, 2006; Grunert and Weber, 2009). Acharya et al. (2007) find statistical significance for industry distress dummies but not for continuous variables. They trace this to non linear impacts of macro variables. Krüger and Rösch (2017) adapt quantile regression and report statistical significance of macro variables for the inner quantiles. Again, this can be interpreted as an indication for non linear impacts. In some papers, statistical significance or evidence is not reported (see, e.g., Altman and Kalotay, 2014; Tobback et al., 2014; Yao et al., 2015). Other authors state statistical significance for a variety of macro variables. However, they apply data sets of bonds (see Jankowitsch et al., 2014; Nazemi et al., 2017; Qi and Zhao, 2011), credit cards (see Bellotti and Crook, 2012; Yao et al., 2017), or mortgages (see Leow et al., 2014; Qi and Yang, 2009). Bonds are typically characterized by *market-based* LGDs. This simplifies the identification of significant macro variables as the time a bond spends in default is standardized to 30 days, thus, the final LGD is certain shortly after default and there is no additional uncertainty regarding timing of cash flows. Credit cards and mortgages are among bulk businesses of banks. Resolution processes might be more standardized and related to less uncertainty compared to corporate loans. This also may simplify the identification of significant macro variables.

The second stream of literature models PDs and LGDs jointly and introduces time-specific systematic random effects. The PD an LGD models are linked by joint random variables to measure correlation structures among the risk parameters. Bruche and Gonzalez-Aguado (2010) implements binary variable indicating the state of the cycle. Chava et al. (2011) use a frailty in a hazard-type PD model, however, show that it has no significant impact on the RR. Bade et al. (2011), Rösch and Scheule (2010), and Rösch and Scheule (2014) implement correlated random effects in both models. Thereby, Rösch and Scheule (2014) present a closed form expression for generating downturn LGD estimations which are based on an adverse realization of the random effect using a simple and parsimonious Merton-type model. Keijsersy et al. (2017) adopts random effects that are modeled by a VAR process to link default, loss, and the economic environment. The closest studies to ours are Keijsersy et al. (2017) and Rösch and Scheule (2014). In comparison to Keijsersy et al. (2017), we compare two model specifications for the random effect in order to analyze if cyclical behavior among LGDs is present at all. Due to their model specification, such a distinction is not possible. Furthermore, our analysis enables us to identify regional differences in systematic effects among LGDs. In addition, we provide a downturn LGD method and compare it to others. Their approach is focused on a comprehensive risk framework that models all credit risk components at once. Thus, their insights seem to be more relevant for internal modeling under Pillar II of the Basel regulations, while our approach is strongly driven towards Pillar I which combines credit risk components by a standardized formula. Rösch and Scheule (2014) use a random effect to capture systematic movements that impact PDs and LGDs. As stated by the EBA, downturn LGDs may additionally include systematic effects that are independent of those for PDs. Furthermore, their study is based on *market-based* LGDs. As explained above, their nature strongly differs from *workout* LGDs which is why inferences with respect to the systematic impact are not comparable. Moreover, they do not present results regarding the progress of systematic effects over time which inhibits inferences regarding similarities to the economic cycle.

Summarizing, the identification of systematic effects among LGDs seems to be not trivial in a LGD modeling context and results regarding macro variables are ambiguous in the literature. This applies in a special manner to corporate loan data sets which are characterized by *workout* LGDs and non standardized and, thus, complex resolutions.

3 Data and Methods

3.1 Data

We use access to the unique loss data base of Global Credit Data (GCD) to build the subsample adopted in the paper.The data base includes detailed loss information on transaction basis of 52 member banks all around the world. The LGD is determined by:

$$\mathrm{LGD}_i = 1 - \mathrm{RR}_i, \tag{1}$$

where, LGD_i denotes the LGD of loan i and RR_i the corresponding recovery rate (RR). The RR is calculated as the sum over the present values of relevant transactions divided by the outstanding amount.[3]

To check for the appropriateness of data, an expanded version of the procedure as in Höcht and Zagst (2010) and Höcht et al. (2011) is applied. Two selection criteria are evolved to identify defaulted loans with an extraordinary payment structure pre- and post-resolution. The first criterion (pre-resolution criterion) is calculated as the sum of all relevant transactions including charges-offs divided by the outstanding amount at default. In the second criterion (post-resolution criterion), the sum of post-resolution payments is divided by a fictional outstanding amount at resolution. The barriers are set to $[90\%, 110\%]$ for the pre-resolution and $[-10\%, 110\%]$ for the post-resolution criterion. Loans with realizations outside these intervals are sorted out. Finally, loans with abnormal low and high LGDs ($< 50\%$ and $> 150\%$) are eliminated.[4] A rather homogeneous subsample of defaulted US American and European term loans and lines to small and medium sized enterprises (SMEs) is selected.[5] Some further adjustment mechanisms are applied. We remove loans with an EAD less than 500 USD (5.8%

[3] A more detailed description of the LGD calculation can be found in Betz et al. (2016) and is available from the authors upon request.

[4] Overall, 2.0% of the data is sorted out due to the pre-resolution criterion. The remaining data is reduced by 0.2% based on the post-resolution criterion. Less than 0.1% is eliminated due to abnormal LGD values.

[5] The European sample consists of Germany, Great Britain, Portugal, Ireland, Denmark, Norway, Sweden, Finland, Latvia, Estonia, France, and Poland. In the GCD data base, these European countries are most common. We

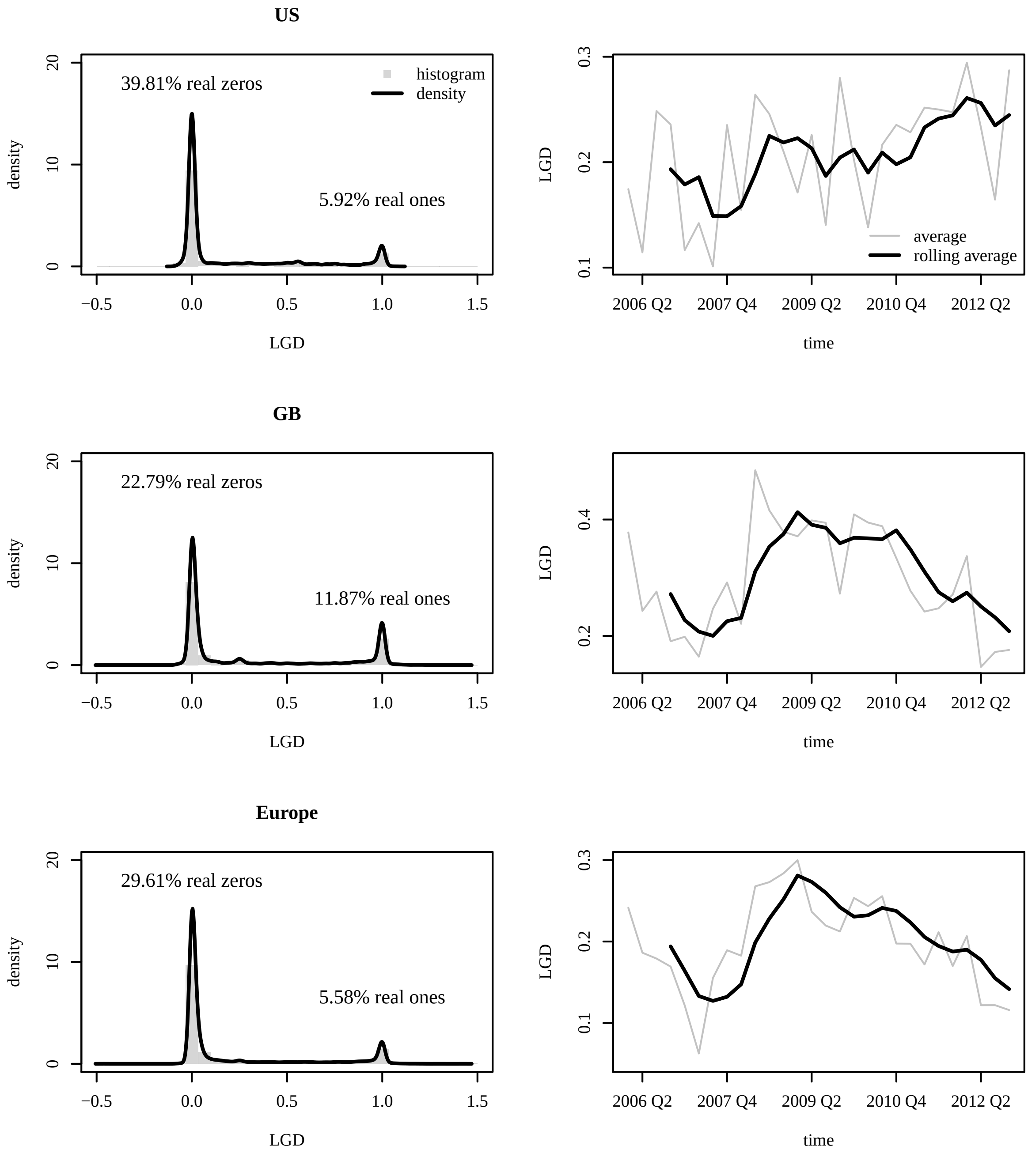
of subsample data) and non resolved loans (13.0% of subsample data). The latter might entail resolution bias as resolved loans in current time periods are often characterized by systematically lower LGDs compared to unresolved loans. Thus, we restrict the time period from 2006 to 2012. We further exclude loans with incomplete observations (7.1% of subsample data). A data set of 2,987 US American and 16,924 European loans including 3,958 British loans remains.

Analyses are performed separately for the three regions – US, GB, and Europe – as fundamental differences might arise regarding the regulatory and economic setting in the US and Europe. Among the European countries, a special status is attributed to GB since it shares similarities with the US, e.g., the origin of legal systems. The left panels of Figure 1 illustrate the distribution of LGDs for the US American, British and European data set. Histograms are presented by gray bars, the black lines show kernel density estimates. In all three regions, LGDs are characterized by a rather extraordinary distributional form. The distribution is extremely bimodal with its modi at values of 0% and 100%. The density mass in between these extremes is rather low. Unlike *market-based* LGDs, the range of *workout* LGDs is not compulsory restricted to [0%, 100%] as workout resolutions can recover more than the EAD (LGD < 0%) due to additional charges or cause costs despite failed workout resolution efforts (LGD > 100%) on the basis of additional expenses. Furthermore, the distribution is shaped by ties. Depending on the data set, we observe between 23% and 40% of LGDs that are exactly zero and between 6% and 11% that are exactly one. These characteristics complicate modeling approaches as values outside the range]0%,100%[hinder common transformations and ties are challenging to estimate by one-stage models. An adequate modeling framework which considers common characteristics of LGDs is crucial for estimation and prediction intentions. The right panels of Figure 1 display the average LGDs in time line. Hereby, considerable deviations between the US and Europe arise. While average LGDs exhibit the maximum during the GFC and slowly decrease to pre-crisis levels afterwards in Europe, average loss rates remain on high levels in the US post-crisis and even further increase in 2011 and 2012. This seems to be a first indication that systematic patterns regarding LGDs differ between the US and Europe, whereas, the systematic patterns of GB seem to be similar to continental Europe.

Table B.2 in Appendix B summarizes descriptive statistics of the dependent and independent variables divided by region. Mean, median, and standard deviation are presented for metric quantities. For variables of categoric nature, proportions of categories are stated. The extraordinary distributional form of LGDs is reflected in its descriptive statistics. While the means

tested an alternative European sample including Great Britain, Portugal, Ireland, Denmark, Norway, and Sweden. Results are similar and may be available upon request.

Figure 1: Histogram and time patterns of LGDs



Note: The left panels of the figure illustrate the distribution of LGDs in the data divided by region. The histograms are represented by gray bars, the black lines show the kernel density estimates. Furthermore, the proportions of values which are exactly zero and one are stated above the modi of the distribution. The right panels of the figure illustrate the time patterns of average LGDs divided by region. The gray lines display the average LGD in the quarters of default, whereas, the black line show the rolling average.

amount to round about 20% to 30% depending on the region, medians are close to 0%. In addition, the standard deviation is relatively large compared to the maximal range $[-50\%, 150\%]$. The exposure at default (EAD) is stated in USD and restricted to values > 0. Thus, its distribution is skewed to the right (median < mean). In the affiliating analysis, we implement the standardized logarithm of the EAD. Facility types are nearly balanced in the data sets, i.e., half of all loans are lines. A majority of loans (70% to 84%, depending on region) is protected by either collateral or guarantee or both. In total, 9% to 15% of loans are granted to corporations of FIRE (finance, insurance, and real estate) affiliation – the rest to other industry sectors.

To capture the systematic effect among LGDs over time, we use a stochastic latent variable. However, in the literature, it is often controlled for observable macro variables when analyzing systematic effects. This is why we include five macro variables for comparison.[6] We consider the year-on-year (yoy) percentage change of the seasonally adjusted GDP (ΔGDP) to examine the influence of the real economy on LGDs. For Europe, we use the weighted average of country specific GDPs. The quarterly average of yoy log returns of major stock indices (ΔEI) and the level of the volatility index (VIX) are applied to investigate impacts of the financial economy. For the US, we use the S&P 500 and the CBOE Volatility Index. The FTSE and the VSTOXX Volatility Index are applied for GB. Weighted average equity returns and VSTOXX Volatility Index are adopted for Europe. Further common determinants in credit risk are the yoy percentage change of house prices indices (ΔHPI) and the ratio of non-performing loans (NPL ratio). As proxy for house prices, we use real residential property prices. For Europe, the weighted average is adopted. The ratio of non-performing loans is non-performing total loans to total loans for the US and bank non-performing loans to gross loans for GB and Europe (weighted average). The first data type is on quarterly basis, however, not available for European countries. The second one is on yearly basis.[7]

3.2 Modeling Framework

We use a hierarchical model combining a Finite Mixture Model (FMM) with a probabilistic substructure (see Altman and Kalotay, 2014). In FMMs, the dependent variable is assumed to be divided into a finite number of latent classes. In these, the dependent variable follows a

[6] At least one of these macro variables is also considered in Acharya et al. (2007), Altman and Kalotay (2014), Brumma et al. (2014), Caselli et al. (2008), Dermine and Neto de Carvalho (2006), Grunert and Weber (2009), Jankowitsch et al. (2014), Krüger and Rösch (2017), Qi and Yang (2009), Qi and Zhao (2011), and Tobback et al. (2014).

[7] The variables ΔGDP, ΔHPI, and NPL ratio originate from Federal Reserve Economic Data (FRED, see https://fred.stlouisfed.org/), whereas, the variable ΔEI and VIX steams from Thomson Reuter's EIKON.

specific distribution, e.g., Normal distribution with parameters depending on the latent class. We refer to the FMM as the *component model*. The probability of belonging to a latent class is modeled by an Ordered Logit (OL) model. We refer to the probabilistic substructure in form of the OL model as the *probability model*.

Component model | FMM

The LGD as dependent variable Y is assumed to be divided into a finite number of K latent classes. In each class k, the probability density function (PDF) for observation y given k is $f_k(y \mid \theta_k)$, e.g., Normal density functions, with parameters θ_k depending on the latent class k. These constituent PDFs are weighted by $p_1,\ldots,p_K$ to generate the PDF of a finite mixture distribution $g(y \mid \theta_1,\ldots,\theta_K)$:

$$g(y|\theta_1,\ldots,\theta_K) = \sum_{k=1}^{K} p_k \; f_k(y|\theta_k)\,, \tag{2}$$

where, $f_1(y|\theta_1),\ldots,f_K(y|\theta_k)$ are the PDFs with parameters $\theta_1,\ldots,\theta_K$ of the constituent classes $k \in \{1,\ldots,K\}$. To ensure the general properties of a PDF, i.e., $g(y) \geq 0$ for all $y \in \mathbb{R}$ and $\int_{-\infty}^{\infty} g(y) = 1$, $p_k \geq 0$ and $\sum_k p_k = 1$ must hold.

In the following, we assume Gaussian FMMs. The constituent PDFs $f_k(y \mid \theta_k)$ correspond to Normal density functions and the dependent variable Y given a latent class k follows a Normal distribution with parameters μ_k and σ_k. Assuming conditional independence, the likelihood of a Gaussian FMM $\phi(Y_1,\ldots,Y_N \,|\, \mu,\sigma,p)$ is the product of the individual likelihood contributions, which arise from the above densities:

$$\phi(Y_1,\ldots,Y_N \,|\, \mu,\sigma,p) = \frac{1}{(2\pi)^{\frac{N}{2}}} \prod_{i=1}^{N} \left(\sum_{k=1}^{K} \frac{p_k}{\sigma_k} \exp\left[-\frac{(Y_i-\mu_k)^2}{2\sigma_k^2} \right] \right), \tag{3}$$

where, μ is a $(1 \times K)$ vector of component means, σ is a $(1 \times K)$ vector of component standard deviations, and p is a $(1 \times K)$ vector of component weights. N is the number of observations.

The model is estimated via a Markov Chain Monte Carlo (MCMC) sampler. It generates samples from the posterior distribution by constructing reversible Markov-chains. The equilibrium distribution corresponds to the target posterior distribution. The solution via an MCMC sampler is necessary due to the complexity of the model, i.e., the priors are partly non conjugate. Thus, direct sampling from posterior distributions is not possible as they do not have a analytical solution.

To adapt data augmentation in the MCMC sampler, the component weight p_k is replaced with an indicator variable e_{ik} in the likelihood specification of Equation (3):

$$\phi(Y_1,...,Y_N \,|\, \mu,\sigma,e) = \frac{1}{(2\pi)^{\frac{N}{2}}} \prod_{i=1}^{N} \left(\sum_{k=1}^{K} \frac{e_{ik}}{\sigma_k} \exp\left[-\frac{(Y_i-\mu_k)^2}{2\sigma_k^2} \right] \right) . \tag{4}$$

If loan i is a random draw of component k, $e_{ik} = 1$ and zero otherwise. In each step of the MCMC sampler, loan $i \in N$ is assigned to one specific component k and, thus, follows one specific probability density distribution $f(y \mid \mu_k, \sigma_k)$. However, changes of component affiliation remain possible within a chain.

Probability model | OL model

For the probabilistic substructure of the component model we use an OL approach. Hereby, observable covariates are included. To rely on the classical formulation of OL models, we define the component affiliation z_i for each loan i:

$$z_i = k \quad \text{if } e_{ik} = 1 \, , \tag{5}$$

where, e_{ik} is the indicator as of Equation (4). The latent component variable z_i describes the affiliation to individual components k for every loan i in the data. Thus, the variable Z_i follows a categorical distribution with component probabilities p_k:

$$Z_i \sim \text{Cat}(p_i) \, , \tag{6}$$

where, p_i is a $(1 \times K)$ vector of component probabilities.

An underlying variable Z_i^* is defined which represents the true but unobservable dependent variable. This latent variable is determined by a linear model:

$$Z_i^* = x_i\beta + F_{t(i)} + \epsilon_i, \qquad \epsilon_i \sim \text{logistic}, \tag{7}$$

whereby, x_i is a $(1 \times J)$ vector of J standardized independent variables and β the $(J \times 1)$ vector of coefficients. The expression ϵ_i describes the error term. A random effect $F_{t(i)}$ with time stamp $t(i)$ is introduced in the modeling framework to capture systematic effects among LGDs. The time stamp $t(i)$ indicates the default time t in quarters which depends on the loan i as every loan defaulted in a specific quarter, e.g. 2007 Q4. Two loans i and i' which defaulted in the same quarter ($t(i) = t(i') = t$) are characterized by the same realization of the random effect ($f_{t(i)} = f_{t(i')} = f_t$). A positive value of F_t in a specific quarter t leads to a higher value of the latent dependent variable Z_i^* for all loans defaulted in this quarter and vice versa.

We consider two alternative specifications for the random effect. In specification I, the random effect is modeled as an i.i.d. normally distributed random variable and implemented in terms of a random intercept:

$$F_t \sim N(\alpha, \sigma^F), \tag{8}$$

where, α is its mean and σ^F its standard deviation. In other words, the mean α is the intercept in the linear model as of Equation (7). The standard deviation σ^F can be interpreted as the impact of the systematic effect. A higher value of σ^F allows for more extreme realizations of the random effect and, thus, for more extreme time dependent shifts in Z_i^*.

In specification II, the random effect follows an AR(1) process to allow for cyclical movements in the realizations of the random effect:

$$\begin{aligned} F_t &= a + \varphi F_{t-1} + \varepsilon_t \\ \mu_u^F &= \frac{a}{1-\varphi} \\ \sigma_u^F &= \frac{\sigma_c^F}{\sqrt{1-\varphi^2}}, \end{aligned} \tag{9}$$

where, a is the constant and φ the parameter of the AR(1) process. The unconditional mean and standard deviation are denoted by μ_u^F and σ_u^F. The conditional standard deviation σ_c^F corresponds to the standard deviation of the errors ε_t. The stationary condition of an AR(1) process is $|\varphi| < 1$.

The component affiliations Z_i are determined by the location of the latent variable Z_i^* as of Equation (7) to corresponding cut points c_k:

$$Z_i = \begin{cases} 1 & \text{if } Z_i^* \leq c_1 \\ 2 & \text{if } c_1 < Z_i^* \leq c_2 \\ \vdots & \\ K & \text{if } c_{K-1} < Z_i^*. \end{cases} \tag{10}$$

Thus, loan i is assigned to component 1 ($Z_i = 1$) if Z_i^* is smaller or equal than c_1. If Z_i^* lies in between c_1 and c_2, loan i is assigned to component 2 ($Z_i = 2$) and so on. Finally, component K ($Z_i = K$) is assigned if Z_i^* is greater than c_{K-1}. Generally, there are $K - 1$ cut points to estimate within the OL model. Consider loan i and i' which defaulted in the same quarter ($t(i) = t(i') = t$) and, thus, share the same realization of the random effect ($f_{t(i)} = f_{t(i')} = f_t$). For rather high (low) values of F_t the component allocation is shifted towards higher (lower) components for all loans defaulted in the corresponding quarter t.

The corresponding component probabilities as of Equation (6) can be derived by the cumulative distribution function of the Logistic distribution:

$$\begin{aligned}
\mathbb{P}\left(Z_i = 1 \mid x_i, f_{t(i)}\right) &= \mathbb{P}\left(Z_i^* \leq c_1 \mid x_i, f_{t(i)}\right) \\
&= \frac{1}{1 + \exp\left[-\left(c_1 - Z_i^*\right)\right]} \\
\mathbb{P}\left(Z_i = 2 \mid x_i, f_{t(i)}\right) &= \mathbb{P}\left(Z_i^* \leq c_2 \mid x_i, f_{t(i)}\right) - \mathbb{P}\left(Z_i^* \leq c_1 \mid x_i, f_{t(i)}\right) \\
&= \frac{1}{1 + \exp\left[-\left(c_2 - Z_i^*\right)\right]} - \frac{1}{1 + \exp\left[-\left(c_1 - Z_i^*\right)\right]} \\
&\vdots \\
\mathbb{P}\left(Z_i = K \mid x_i, f_{t(i)}\right) &= \mathbb{P}\left(Z_i^* > c_{K-1} \mid x_i, f_{t(i)}\right) \\
&= 1 - \frac{1}{1 + \exp\left[-\left(c_{K-1} - Z_i^*\right)\right]} .
\end{aligned} \tag{11}$$

To guarantee the non negativity of component probabilities p_{ik} ($p_{ik} \geq 0$ for $k \in \{1, \dots, K\}$), $c_1 \leq c_2 \leq \cdots \leq c_{K-1}$ must hold. In analogy to the component allocation in Equation (10), the random effect as of Equation (7), (8), and (9) introduces systematic movement into the component probabilities. Again, consider loan i and i' which defaulted in the same quarter ($t(i) = t(i') = t$) and, thus, share the same realization of the random effect ($f_{t(i)} = f_{t(i')} = f_t$). For high values of F_t ($F_t > \alpha$ in specification I and $F_t > \mu_u^F$ in specification II), the probability of the first component decreases as Z_i^* increases. Simultaneously, the probability of the K-th component increases. The probabilities of the remaining components might be affected to a minor extent. Summarizing, a high realization of the random effect leads to a systematically lower probability for the first component and higher probability for the last component while a low realization of the random effect implies the opposite effect. Loans defaulted in the same quarter are, thus, characterized by systematically lower probabilities of the first and systematically higher probabilities of the last component or systematically higher probabilities of the first and systematically lower probabilities of the last component.

The above OL model suffers from over specification, i.e., an infinite number of solutions exists. Three solution mechanisms are common to solve this problem: (i) fixation of the variance parameter of Z_i^* and fixation of one cut point, (ii) dropping of intercept and fixation of the variance parameter of Z_i^*, and (iii) fixation of two cut points. All three solution mechanisms aim to fix the range of the latent variable Z_i^* and can be transfered into each other by choosing according values. We select (iii) and fix the two outer cut points c_1 and c_{K-1} as the variability of the latent variable itself allows the use of conjugate priors for the random effect in specification I. However, results are reproducible by alternative identification restrictions.

The models are estimated via Bayesian inference.[8] Thus, prior distributions have to be specified for every parameter of the model. Detailed information will be made available in an online appendix.

4 Empirical Results

In this section, we start with the estimation results of the model from the previous section. Subsequently, we focus on systematic effects among LGDs in the US American, British, and European data set by analyzing the implied realizations of the random effects within the modeling approach.

Component model

We begin with the results of the component model, i.e., the Gaussian FMM. Component 1 and 5 are fixed to zero ($\mu_1 = 0$) and one ($\mu_5 = 1$), respectively, in order to capture the number of LGDs which are exactly zero and one. Thus, standard deviations of these components are set to reasonably small values ($\sigma_1 = \sigma_5 = 0.001$). Three further components are estimated within the modeling approach. Posterior means and common Bayesian figures – i.e., highest posterior density intervals (HPDIs), posterior odds ratios[9], naive and time-series standard errors – for the parameters of the component model are shown in Table 1. In specification I, the random effect is normally distributed as in Equation (8), whereas, it follows an AR(1) process as in Equation (9) in specification II. However, the specification of the random effect does not impact the parameter estimates of the component model. We, thus, present only one specification for the US, GB, and Europe.[10]

[8] The MCMC samples are drawn via the Gibbs sampler JAGS.

[9] See results of the probability model for a detailed explanation on how to interpret the posterior odds.

[10] Due to statistical evidence of the AR(1) parameter, we apply specification I for the US and specification II for GB and Europe (see results of the probability model for detailed information). Remaining results are available from the authors upon request.

In the US, components 2 and 4 are centered close to zero and one, respectively. According to their comparably small standard deviations, they cover LGD values around the fixed components 1 and 5. Component 3 exhibits a mean of round about 0.44 and a comparably high standard deviation ($\sigma_3 \approx 0.25$). Thus, it contains LGD values in between the extremes zero and one. Comparing the results of GB with the US, components are slightly shifted. Components 2 and 4 are not as close to the extremes. Component 3 is with a mean of round about 0.24 shifted towards the lower end of the value range. However, it exhibits still the highest standard deviation ($\sigma_3 \approx 0.18$) and, thus, covers large parts of the area in between the extremes. In Europe, results are more similar to GB than to the US as components 2 and 4 are not as close to the extremes as in the US. However, component 4 exhibits the highest standard deviation ($\sigma_4 \approx 0.25$) and, thus, covers the highest proportion of the value range.

Probability model

Loan specific component probabilities are derived based on a probability model, i.e., the underlying OL model as of Equation (7). The results of the probability model are shown in Table 2 and 3. Parameter estimates of the independent variables, i.e., β_{EAD}, β_{Facility}, $\beta_{\text{Protection}}$, and β_{Industry}, should be interpreted in Bayesian terms. In Bayesian inference, posterior distributions of β_j are assumed to be continuous. Thus, specific values in the posterior distributions exhibit a probability of zero. In frequentistic terms, one *true* value is assumed. A null hypothesis for β_j is set up accordingly to reach a yes-no-decision. In Bayesian terms, posterior distributions of parameters β_j are adopted to examine if the results are in favor of a positive or negative impact or if there is no clear one-sided influence. Two concepts might be applied. First, credible intervals, e.g., HPDIs, are intervals in the domain of the posterior distribution. If zero is not included in the credible interval, the domain of the posterior is located in the positive or negative value range – indicating positive or negative impact. Second, Bayes factors might be applied to evaluate statistical evidence and are defined as the relation between posterior and prior odds. Posterior odds are the ratio of the posterior probability mass favoring the sign of the posterior mean to the posterior probability mass of the opposite sign

$$\text{posterior odds}_{\,\text{E}[\beta_j]<0} = \frac{\mathbb{P}(\beta_j < 0 \mid \text{data})}{\mathbb{P}(\beta_j \geq 0 \mid \text{data})}$$
$$\text{posterior odds}_{\,\text{E}[\beta_j]>0} = \frac{\mathbb{P}(\beta_j > 0 \mid \text{data})}{\mathbb{P}(\beta_j \leq 0 \mid \text{data})}\,,$$

Table 1: Results of component model

	posterior mean	HPDI (90%)		posterior odds	naive standard error	time-series standard error
			US | specification I			
μ_1	0.0000			*not estimated*		
μ_2	0.0027	0.0010	0.0045	174.4386	0.0000	0.0000
μ_3	0.4388	0.4199	0.4589	∞	0.0001	0.0001
μ_4	0.9657	0.9589	0.9725	∞	0.0000	0.0000
μ_5	1.0000			*not estimated*		
σ_1	0.0010			*not estimated*		
σ_2	0.0245	0.0229	0.0261	∞	0.0000	0.0000
σ_3	0.2542	0.2405	0.2682	∞	0.0001	0.0001
σ_4	0.0306	0.0244	0.0365	∞	0.0000	0.0000
σ_5	0.0010			*not estimated*		
			GB | specification II			
μ_1	0.0000			*not estimated*		
μ_2	0.0153	0.0144	0.0162	∞	0.0000	0.0000
μ_3	0.2427	0.2164	0.2655	∞	0.0001	0.0002
μ_4	0.9039	0.8867	0.9202	∞	0.0001	0.0001
μ_5	1.0000			*not estimated*		
σ_1	0.0010			*not estimated*		
σ_2	0.0147	0.0138	0.0157	∞	0.0000	0.0000
σ_3	0.2016	0.1839	0.2186	∞	0.0001	0.0001
σ_4	0.1205	0.1066	0.1345	∞	0.0001	0.0001
σ_5	0.0010			*not estimated*		
			Europe | specification II			
μ_1	0.0000			*not estimated*		
μ_2	0.0154	0.0149	0.0159	∞	0.0000	0.0000
μ_3	0.1158	0.1073	0.1244	∞	0.0001	0.0001
μ_4	0.7132	0.6974	0.7292	∞	0.0001	0.0002
μ_5	1.0000			*not estimated*		
σ_1	0.0010			*not estimated*		
σ_2	0.0128	0.0124	0.0133	∞	0.0000	0.0000
σ_3	0.0875	0.0817	0.0931	∞	0.0000	0.0001
σ_4	0.2515	0.2403	0.2621	∞	0.0001	0.0001
σ_5	0.0010			*not estimated*		

Note: The table summarizes the results of the component model. The first column presents the posterior means of the component means μ_k and standard deviations σ_k. The second and third column contain the lower and upper bound of the HPDI to a credibility level of 90%. In the last two columns, the naive and time-series standard error of the chains are presented, whereas, the time-series standard error is calculated based on the effective (N^*_{MCMC}) instead of the real (N_{MCMC}) sample size. Hereby, $N^*_{\mathrm{MCMC}} < N_{\mathrm{MCMC}}$ holds for autocorrelated chains.

Table 2: Results of probability model (specification I)

	posterior mean	HPDI (90%)		posterior odds	naive standard error	time-series standard error
				US		
β_{EAD}	-0.0215	-0.0804	0.0408	2.5186	0.0004	0.0005
β_{Facility}	-0.0832	-0.1983	0.0404	6.6864	0.0007	0.0011
$\beta_{\text{Protection}}$	-0.4442	-0.6009	-0.2790	∞	0.0010	0.0019
β_{Industry}	0.2592	0.1002	0.4216	237.0952	0.0010	0.0013
α	2.1402	1.9252	2.3466	∞	0.0013	0.0023
σ^F	0.3522	0.2400	0.4604	∞	0.0007	0.0007
c_1	1.5000			*not estimated*		
c_2	2.4103	2.3529	2.4658	∞	0.0003	0.0003
c_3	3.9466	3.8677	4.0274	∞	0.0005	0.0005
c_4	4.5000			*not estimated*		
				GB		
β_{EAD}	-0.3792	-0.4378	-0.3190	∞	0.0004	0.0004
β_{Facility}	0.2138	0.1112	0.3237	2499.0000	0.0006	0.0009
$\beta_{\text{Protection}}$	-0.4271	-0.5451	-0.3216	∞	0.0007	0.0011
β_{Industry}	-0.1882	-0.3591	-0.0188	26.4725	0.0010	0.0013
α	2.7511	2.5321	2.9818	∞	0.0014	0.0017
σ^F	0.6040	0.4579	0.7499	∞	0.0009	0.0009
c_1	1.5000			*not estimated*		
c_2	2.7395	2.6818	2.7929	∞	0.0003	0.0003
c_3	3.6138	3.5480	3.6771	∞	0.0004	0.0005
c_4	4.5000			*not estimated*		
				Europe		
β_{EAD}	-0.0295	-0.0527	-0.0025	38.0625	0.0002	0.0002
β_{Facility}	0.2686	0.2188	0.3211	∞	0.0003	0.0006
$\beta_{\text{Protection}}$	-0.4176	-0.4715	-0.3655	∞	0.0003	0.0007
β_{Industry}	-0.1811	-0.2462	-0.1180	∞	0.0004	0.0007
α	2.1206	1.9521	2.2807	∞	0.0010	0.0011
σ^F	0.4929	0.3738	0.6038	∞	0.0007	0.0007
c_1	1.5000			*not estimated*		
c_2	2.6594	2.6289	2.6885	∞	0.0002	0.0003
c_3	3.2201	3.1839	3.2555	∞	0.0002	0.0004
c_4	4.5000			*not estimated*		

Note: The table summarizes the results of the probability model with a latent variable specification as of specification I (Equation (7) and (8)). The first column presents the posterior means of the coefficients (β_j), the parameters of the random effect, and the cut points (c_k). The second and third column contain the lower and upper bound of the HPDI to a credibility level of 90%. In the fourth column, posterior odds are displayed. In the last two columns, the naive and time-series standard error of the chains are presented, whereas, the time-series standard error is calculated based on the effective (N^*_{MCMC}) instead of the real (N_{MCMC}) sample size. Hereby, $N^*_{\text{MCMC}} < N_{\text{MCMC}}$ holds for autocorrelated chains.

Table 3: Results of probability model (specification II)

	posterior mean	HPDI (90%)		posterior odds	naive standard error	time-series standard error
			US			
β_{EAD}	-0.0217	-0.0858	0.0353	2.6245	0.0004	0.0005
β_{Facility}	-0.0850	-0.2057	0.0367	6.8555	0.0007	0.0013
$\beta_{\text{Protection}}$	-0.4451	-0.6097	-0.2792	∞	0.0010	0.0022
β_{Industry}	0.2656	0.1030	0.4209	269.2703	0.0010	0.0013
a	2.7054	1.6139	3.8989	∞	0.0070	0.0289
φ	-0.2672	-0.7882	0.2520	0.2694	0.0032	0.0131
σ_c^F	0.3221	0.1934	0.4471	∞	0.0008	0.0016
c_1	1.5000			*not estimated*		
c_2	2.4099	2.3503	2.4639	∞	0.0003	0.0003
c_3	3.9469	3.8657	4.0275	∞	0.0005	0.0005
c_4	4.5000			*not estimated*		
			GB			
β_{EAD}	-0.3825	-0.4439	-0.3232	∞	0.0004	0.0004
β_{Facility}	0.2093	0.1078	0.3176	2499.0000	0.0006	0.0009
$\beta_{\text{Protection}}$	-0.4314	-0.5464	-0.3236	∞	0.0007	0.0011
β_{Industry}	-0.1930	-0.3579	-0.0223	32.1126	0.0010	0.0013
a	0.6779	0.0944	1.2111	999.0000	0.0036	0.0072
φ	0.7504	0.5576	0.9534	∞	0.0013	0.0025
σ_c^F	0.4023	0.2865	0.5188	∞	0.0007	0.0008
c_1	1.5000			*not estimated*		
c_2	2.7397	2.6838	2.7959	∞	0.0003	0.0003
c_3	3.6150	3.5517	3.6807	∞	0.0004	0.0005
c_4	4.5000			*not estimated*		
			Europe			
β_{EAD}	-0.0291	-0.0553	-0.0045	31.7869	0.0002	0.0002
β_{Facility}	0.2685	0.2196	0.3217	∞	0.0003	0.0006
$\beta_{\text{Protection}}$	-0.4184	-0.4685	-0.3634	∞	0.0003	0.0007
β_{Industry}	-0.1840	-0.2525	-0.1210	∞	0.0004	0.0006
a	0.2874	0.0094	0.5458	237.0952	0.0018	0.0044
φ	0.8565	0.7359	0.9862	∞	0.0009	0.0020
σ_c^F	0.2678	0.1982	0.3338	∞	0.0004	0.0005
c_1	1.5000			*not estimated*		
c_2	2.6597	2.6315	2.6907	∞	0.0002	0.0003
c_3	3.2201	3.1841	3.2560	∞	0.0002	0.0004
c_4	4.5000			*not estimated*		

Note: The table summarizes the results of the probability model with a latent variable specification as of specification II (Equation (7) and (9)). The first column presents the posterior means of the coefficients (β_j), the parameters of the random effect, and the cut points (c_k). The second and third column contain the lower and upper bound of the HPDI to a credibility level of 90%. In the fourth column, posterior odds are displayed. In the last two columns, the naive and time-series standard error of the chains are presented, whereas, the time-series standard error is calculated based on the effective (N^*_{MCMC}) instead of the real (N_{MCMC}) sample size. Hereby, $N^*_{\text{MCMC}} < N_{\text{MCMC}}$ holds for autocorrelated chains.

whereas, prior odds are the corresponding ratio of the prior distribution.[11] Thus, the Bayes factor complies with the posterior odds if the prior odds equal to one. This is true for symmetric prior distributions around zero. We set such a prior for the parameters vector β.[12] Hence, the corresponding posterior odds are interpretable in terms of Bayes factors. Following Kass and Raftery (1995), a Bayes factor exceeding 3.2 is deemed as substantial evidence. Values above 10 are assigned with strong evidence, whereas, values above 100 are related to decisive evidence.

The upper panels of Table 2 (specification I) and 3 (specification II) summarize the results of the probability model for the US. The specification of the random effect seems to have no impact on the remaining parameter estimates. Considering the posterior odds, only β_{EAD} exhibits no clear evidence for the sign of the posterior mean (posterior $\text{odds}_{\text{E}[\beta_{\text{EAD}}<0]} \approx \{2.5, 2.6\} < 3.2$). Thus, it can not be stated with conviction whether loans with higher EADs lead to lower or higher LGDs. The remaining variables are of categoric nature. The reference categories are term loan for facility, non protected for protection, and non FIRE for industry. The posterior odds indicate substantial evidence for a negative impact of β_{Facility} (posterior $\text{odds}_{\text{E}[\beta_{\text{Facility}}<0]} \approx \{6.7, 6.9\} > 3.2$), i.e., lines exhibit lower values for Z_i^* and, thus, lower losses, compared to term loans. The evidence for a negative impact of $\beta_{\text{Protection}}$ and a positive impact of β_{Industry} is decisive (posterior $\text{odds}_{\text{E}[\beta_{\text{Protection}}<0]} \rightarrow \infty > 100$ and posterior $\text{odds}_{\text{E}[\beta_{\text{Industry}}>0]} \approx \{237.1, 269.3\} > 100$). Protection is, thus, associated with lower values for Z_i^* and lower losses, whereas, the FIRE industry affiliation seems to be shaped by higher losses. Generally, higher values of Z_i^* imply lower probabilities for lower components and higher probabilities for higher components. Higher values of Z_i^* can, thus, be directly associated with higher losses.

The middle panels of Table 2 (specification I) and 3 (specification II) contain the results of the probability model for GB. Again, the specification of the random effect seems to have no influence on the remaining parameter estimates. Regarding the posterior odds, the evidence for a negative impact of EAD, a positive impact of facility, and a negative impact of protection is decisive (posterior $\text{odds}_{\text{E}[\beta_{\text{EAD}}<0]} \rightarrow \infty > 100$, posterior $\text{odds}_{\text{E}[\beta_{\text{Facility}}>0]} \approx 2499.0 > 100$ and posterior $\text{odds}_{\text{E}[\beta_{\text{Protection}}<0]} \rightarrow \infty > 100$). Higher EADs are associated with lower losses. Lines generate higher losses compared to term loan and protection leads to lower losses. The evidence of a negative sign for industry is strong (posterior $\text{odds}_{\text{E}[\beta_{\text{Industry}}<0]} \approx \{26.5, 32.1\} > 10$). Thus, FIRE affiliation entails lower losses compared to other industries. Comparing these results with the US, two deviations in signs of parameters arise. First, lines are associated with lower

[11] Prior odds equal the ratio of prior probabilities for two states of the world. For instance, the states are $\beta_j < 0$ and $\beta_j \geq 0$.

[12] The prior of the parameter vector β is a Multivariate Normal distribution with mean vector zero.

losses compared to term loans in the US, however, higher losses arise for lines in GB. This might be caused by different business practices, e.g., lines might be closer monitored in the US and, thus, information asymmetries might be reduced to a higher extent. Second, loans granted to corporations of FIRE affiliation are characterized by higher losses in the US, whereas, lower losses occur in GB. Reasons may be found in deviating industry standards. While the financial system in GB is strongly shaped by banks, the US American system is more market-orientated – i.e., corporations might rather fund debt by traded instruments than loans. Thus, corporations depending on bank financing might worse in the first place. The lower panels of Table 2 (specification I) and 3 (specification II) displays the results of the probability model for Europe. The evidence for a negative impact of EAD is strong (posterior odds$_{E[\beta_{\text{EAD}}<0]} \approx \{38.1, 31.8\} > 10$), whereas, the evidence for a positive impact of facility (posterior odds$_{E[\beta_{\text{Facility}}>0]} \rightarrow \infty > 100$), for the impact of protection (posterior odds$_{E[\beta_{\text{Protection}}<0]} \rightarrow \infty > 100$) and industry (posterior odds$_{E[\beta_{\text{Industry}}<0]} \rightarrow \infty > 100$) is decisive. The signs and magnitudes of the posterior means correspond to GB.

Besides the summary of the posterior estimates for β_{EAD}, β_{Facility}, $\beta_{\text{Protection}}$, and β_{Industry}, Table 2 and 3 present the results regarding the parameters of the random effect for specification I (α and σ^F) and specification II (a, φ, and σ_c^F). To evaluate the specifications, we consider the evidence of the AR(1) parameter φ and select specification I if φ is not evident ($0 \in$ HPDI and posterior odds < 3.2) and specification II otherwise.[13] In the US, the AR(1) parameter is not statistically evident as the HPDI includes the zero and the posterior odds amount to round about 0.3. This does not indicate conditional behavior of the random effect and, thus, no cyclical nature of its realizations. Contrary, φ is decisively evident in GB and Europe. Thus, we consider specification I for the US and specification II for GB and Europe. This indicates that the systematic impact on LGDs in GB and Europe is rather of cyclical nature.[14]

Systematic effects among LGDs

In the following, the results regarding the random effect F_t are further analyzed. We introduced F_t to capture systematic effects impacting LGDs of all loans defaulted in the same quarter. In specification I, the random effect is normally distributed with mean α an standard deviation σ^F. Thus, the latent variable Z_i^* is shifted with a constant level of α. Variations in the random effect and, thus, its impact, are captured by the standard deviation σ^F. In specification II, the random

[13] Please note, that the posterior odds of φ are interpretable in terms of a Bayes factor as we set a symmetric prior around zero (normal distribution with mean zero and symmetric truncation $[-1, 1]$).

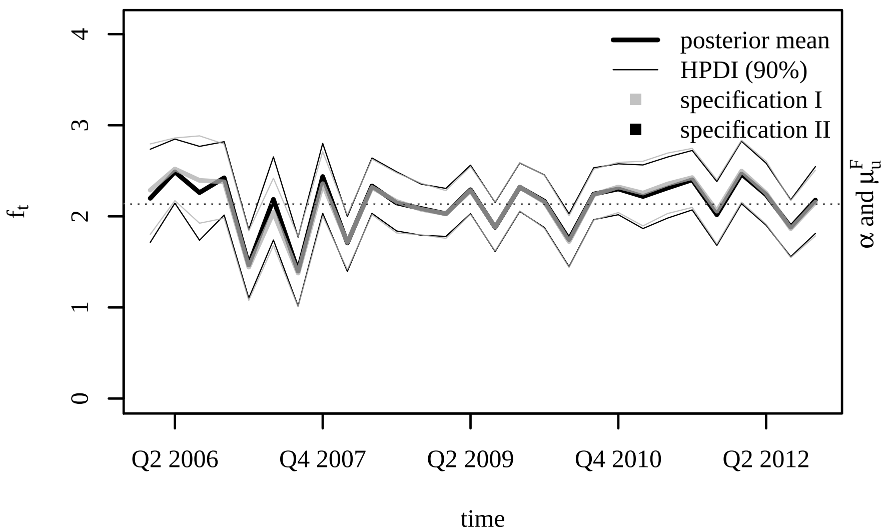
[14] Convergence diagnostics of the selected model specifications will be made available in an online appendix.

effect follows an AR(1) process with intercept a and AR parameter φ. The latent variable Z_i^* is shifted with the unconditional mean of the AR(1) process $\mu_u^F = a/(1-\varphi)$. Its impact is expressed by the unconditional standard deviation $\sigma_u^F = \sigma_c^F / \sqrt{1-\varphi^2}$.

In both specifications, realizations f_t vary through time. For realizations $f_t > \alpha$ (specification I) or $f_t > \mu_u^F$ (specification II), z_i^* is shifted upwards for all loans i defaulted in the same time ($t(i) = t$). Thus, probabilities of low components decrease and probabilities of high components increase. This implies higher average LGDs at time t. If realizations f_t lie below its mean α (specification I) or μ_u^F (specification II), probabilities of low (high) components are increased (decreased) resulting in lower average LGDs at time t. Hence, we expect low realization of F_t in favorable systematic conditions and high realization during adverse systematic conditions.

Figure 2 illustrates the course of the random effect over time in the US. Specification I is plotted in gray, specification II in black. Posterior means are displayed by thick lines, HPDIs at credibility levels of 90% by thin lines, α (specification I, gray dotted line) and μ_u^F (specification II, black dotted line) are nearly identical. However, the specification of the random effect rarely

Figure 2: Random effect in time line (US)

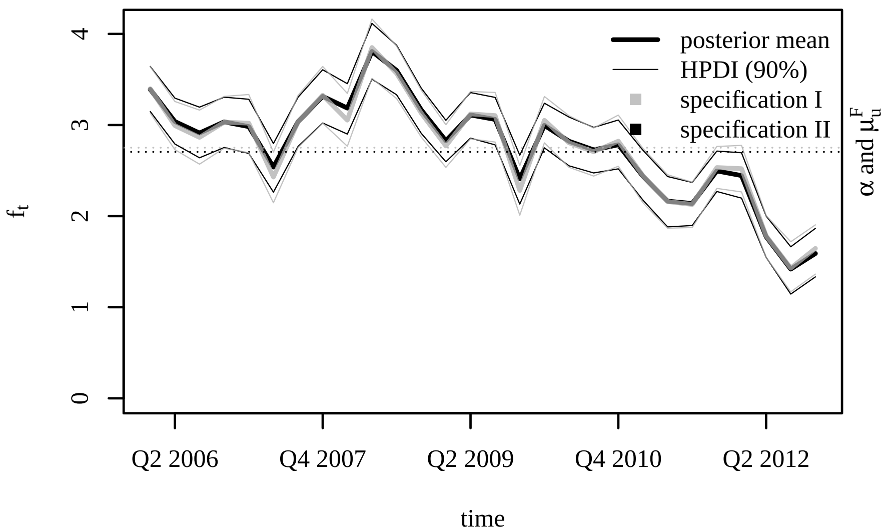


Note: The figure illustrates the course of the random effect over time. The thick gray line represents the posterior means for specification I, whereas, the HPDI to a credibility of 90% is expressed by thin gray lines. The thick black line shows the posterior means for specification II. The corresponding HPDI is displayed by thin black lines. The horizontal dotted lines represent the mean of the random effect (α for specification I and μ_u^F for specification II).

influences its individual realizations. Realizations f_t seem to independently proceed around the means α (specification I) and μ_u^F (specification II). This supports the evidence of an i.i.d. random effect. Despite its rather random process, the systematic effect is characterized by low realizations pre crisis indicating lower average LGDs. An upward shift during the GFC is less pronounced. After a slight drop in 2010 Q2, the random effect remains on a rather high level indicating high average LGDs in the recent time periods. Thus, the random effect does not seem to display the time patterns of common macro variables. However, it simulates the time series of average LGDs in the data. The upper panels of Figure A.1 in Appendix A contrasts average LGDs (black lines) with the random effect (gray lines). In the left panel, average LGDs and the realizations f_t for specification I per quarter are displayed.[15] The rolling averages of both time series are plotted in the right panel.

Figure 3 illustrates the course of the random effect over time in GB. The presentation corresponds to Figure 2. The GFC is clearly identifiable in the random effect of GB. The highest

Figure 3: Random effect in time line (GB)

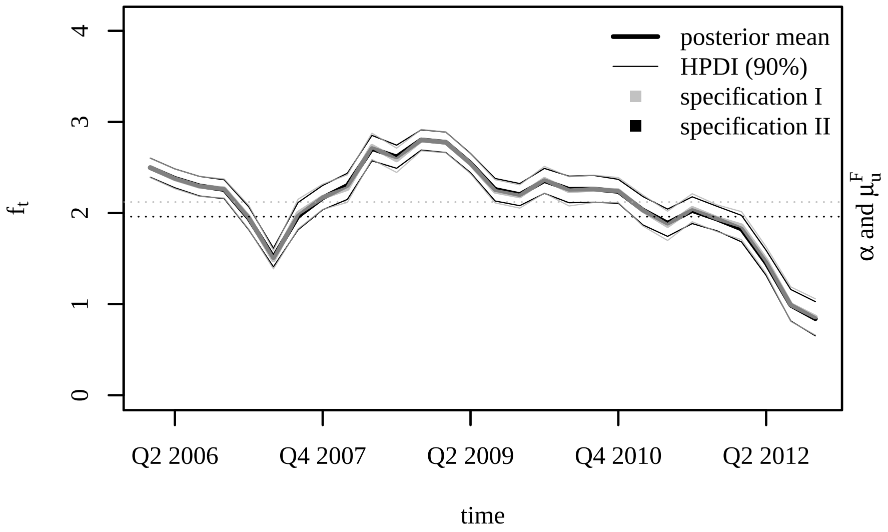


Note: The figure illustrates the course of the random effect over time. The thick gray line represents the posterior means for specification I, whereas, the HPDI to a credibility of 90% is expressed by thin gray lines. The thick black line shows the posterior means for specification II. The corresponding HPDI is displayed by thin black lines. The horizontal dotted lines represent the mean of the random effect (α for specification I and μ_u^F for specification II).

realization f_t is in 2008 Q2 during the summit of the crisis. Afterwards, the realizations of the random effect constantly decline until the minimum is reached in recent time periods. In analogy to the US, the random effect reproduces the time patterns of average LGDs. The middle panels of Figure A.1 in Appendix A contrast the two time series. However, the British random effect seems to exhibit a rather cyclical behavior compared to the US. This is already indicated by the evidence of φ in Table 3. Figure 4 illustrates the course of the random effect over time in Europe. The presentation corresponds to Figure 2. The European random effect shows strong

[15] Specification II is skipped for presentational purposes. However, the realizations f_t are similar.

Figure 4: Random effect in time line (Europe)



Note: The figure illustrates the course of the random effect over time. The thick gray line represents the posterior means for specification I, whereas, the HPDI to a credibility of 90% is expressed by thin gray lines. The thick black line shows the posterior means for specification II. The corresponding HPDI is displayed by thin black lines. The horizontal dotted lines represent the mean of the random effect (α for specification I and μ_u^F for specification II).

similarity to GB. The GFC is clearly observable, however, the crisis seems prolonged compared to GB as f_t is still near its maximum in 2009 Q1. Following the GFC, the random effect slowly declines. Again, the random effect simulates the time series behavior of average LGDs in the data. The lower panels of Figure A.1 in Appendix A contrasts average LGDs and f_t.

Systematic effects vs. macro variables

The time patterns of the random effects (see Figure 2 for the US, Figure 3 for GB, and Figure 4 for Europe) might be a first indication that macro variables might not be suitable to capture the intrinsic systematic effects among LGDs. To examine this in more detail, we consider macro variables.

In this context, Figure A.2, A.3, and A.4 in Appendix A contrast the course of the considered macro variables – ΔGDP, ΔEI, VIX, ΔHPI, and NPL ratio – to the course of the random effects. The y-axis is reversed for ΔGDP, ΔEI, and ΔHPI to ensure an intuitive interpretation. If macro variables imply the same information as the random effects, their time patterns should be congruent with the random effect. However, this does not seem to be the case in the US (see Figure A.2). While stronger deteriorations are indicated by the macro variables in the GFC (upward movement), macro variables return to pre-crisis levels at the end of 2009. Contrary, the random effect further increases after the GFC. Just the course of the NPL ratio seems to capture parts of this movement as a rather slow recovery post crisis is indicated by this variable. In GB and Europe, the time series patterns of macros are more similar to the random effects (see Figure A.3 and A.4). However, the random effect lies above macros variables post crisis. While sharp rebound pushes the economic indicators back to their pre crisis levels, the easing is slower in terms of the random effects and, thus, average LGDs. In addition, the NPL ratio seems incapable of capturing this behavior as it remains on its crisis levels until the end of 2012, whereas, slow recovery is indicated by the random effects. Table B.3 in Appendix B summarizes the pairwise correlations of the displayed time series. Perfect multicollinearity would be indicated by a correlation coefficient of 100%. However, the correlation is negative for most of the macro variables (ΔGDP, ΔEI, VIX, and ΔHPI) in the US. Just the course of the NPL ratio exhibits a certain collinearity to the random effect. In GB and Europe, more macro variables exhibit positive correlations to the random effect (ΔGDP and ΔEI in GB and ΔGDP, ΔEI, VIX, and ΔHPI in Europe).

In the light of the above, macro variables might not be suited to capture the true systematic effects among LGDs. However, we estimate the models with macro variables instead of random effects to analyze their impact. Variable selection is not trivial considering highly correlated time series such as macro variables as multicollinearity might arise. This endangers model stability. Small changes in model specifications or on data side might provoke huge alterations in parameter estimates. Furthermore, parameter estimates tend to be less precise and standard errors large. We, thus, decide to include just one of the considered macro variables at a time. Table 4 summarizes the results of the models with macro variables. The presentation of the remaining parameters (β_{EAD}, β_{Facility}, $\beta_{\text{Protection}}$, and β_{Industry}) is skipped as no changes in the signs and magnitudes arise.[16] We assume favorable economic conditions to be associated with lower LGDs, thus, negative impacts of ΔGDP, ΔEI, and ΔHPI and positive signs for VIX and NPL ratio. Comparing the signs of the posterior means to the expected signs of the macro variables, discrepancies arise. In the US, only the posterior mean of the NPL ratio exhibits the expected sign. In GB and Europe, all considered macro variables except the NPL ratio show intuitive signs. However, there is no statistical evidence for the impact of VIX and ΔHPI in GB (posterior $\text{odds}_{E[\beta_{\text{VIX}}>0]} \approx 1.4 < 3.2$ and posterior $\text{odds}_{E[\beta_{\text{HPI}}<0]} \approx 2.0 < 3.2$).[17] Counterintuitive signs and the lack of statistical impact cast doubt on the use of macro variables for LGD modeling. These results do not claim to universal validity. However, the identification problem of macro variables is emphasized.

[16] The results are available from the authors upon request.

[17] Please note, that the posterior odds of β_{GDP}, β_{EI}, β_{VIX}, β_{HPI}, and $\beta_{\text{NPL ratio}}$ are interpretable in terms of a Bayes factor as we set symmetric priors around zero, i.e., normal distributed priors with means zero.

Table 4: Results of macro models

		posterior mean	HPDI (90%)		posterior odds	naive standard error	time-series standard error
				US			
β_{GDP}	−	0.0266	-0.0291	0.0821	3.4703	0.0003	0.0003
β_{EI}	−	0.0062	-0.0529	0.0617	1.3175	0.0003	0.0003
β_{VIX}	+	-0.0319	-0.0880	0.0255	4.5866	0.0003	0.0004
β_{HPI}	−	0.0483	-0.0065	0.1069	11.8041	0.0003	0.0003
$\beta_{\text{NPL ratio}}$	+	0.0661	0.0078	0.1218	33.6021	0.0003	0.0004
				GB			
β_{GDP}	−	-0.0908	-0.1381	-0.0429	768.2308	0.0003	0.0003
β_{EI}	−	-0.1012	-0.1501	-0.0548	2499.0000	0.0003	0.0003
β_{VIX}	+	0.0063	-0.0397	0.0550	1.4213	0.0003	0.0003
β_{HPI}	−	-0.0134	-0.0603	0.0348	2.0331	0.0003	0.0003
$\beta_{\text{NPL ratio}}$	+	-0.3264	0.0078	0.1218	∞	0.0003	0.0003
				Europe			
β_{GDP}	−	-0.1781	-0.2020	-0.1556	∞	0.0001	0.0001
β_{EI}	−	-0.1845	-0.2083	-0.1621	∞	0.0001	0.0001
β_{VIX}	+	0.1694	0.1459	0.1918	∞	0.0001	0.0001
β_{HPI}	−	-0.2314	-0.2535	-0.2073	∞	0.0001	0.0001
$\beta_{\text{NPL ratio}}$	+	-0.0853	-0.1088	-0.0619	∞	0.0001	0.0001

Note: The table summarizes the results of the macro models. The presentation is reduced to the results regarding the macro variables itself. Every macro variable was separately included in a model without random effect. The first column includes the expected signs of the posterior means as bad macro economic environment should entail higher LGDs. The second column presents the posterior means of the coefficients of the macro variables (β_{GDP}, β_{EI}, β_{VIX}, β_{HPI}, $\beta_{\text{NPL ratio}}$). The third and fourth column contain the lower and upper bound of the HPDI to a credibility level of 90%. The fifth column includes posterior odds, while, in the last two columns, the naive and time-series standard error of the chains are presented, whereas, the time-series standard error is calculated based on the effective (N^*_{MCMC}) instead of the real (N_{MCMC}) sample size. Hereby, $N^*_{\text{MCMC}} < N_{\text{MCMC}}$ holds for autocorrelated chains.

To check the robustness of these findings, we reestimate the models considering macro variables and random effects. Among the macro variables with intuitive signs, we select those offering the highest statistical evidence.[18] Thus, the NPL ratio is selected for the US as it is the only macro variable with an intuitive sign, ΔEI for GB due to its high posterior odds ratio, and ΔHPI for Europe as its HPDI is furthest from zero. Table B.4 in Appendix B summarizes the results of the combined models.[19] Statistical evidence vanishes for all considered macro variables. First, posterior odds ratios are smaller than 3.2. Second, the corresponding HPDIs to a credibility level of 90% include zero. In contrast, the parameters of the random effect – α and σ^F for specification I and a, φ, and σ_c^F for specification II – remain nearly unchanged compared to the original model specification without the inclusion of macro variables (see Table 2 and 3).

Summarizing the above results, the identification of appropriate macro variables seems chal-

[18] Results for the remaining macro variables are available from the authors upon request.

[19] The presentation of the remaining parameters (β_{EAD}, β_{Facility}, $\beta_{\text{Protection}}$, and β_{Industry}) is skipped. Results are available from the authors upon request.

lenging in an LGD modeling context. Macro variables in general seem to be not entirely suitable to capture the true systematic effects deriving LGDs. This might be due to three reasons. First, LGD observations are treated as they arise at the default time t. However, LGD realizations are not available until the defaulted loan is completely resolved. In our data sets, default resolution takes typically between one and five years. During the resolution process, recovery payments are processed. Thus, the realized LGD in t does not only depend on the economic condition in t but on the conditions during the whole resolution process ($t + \Delta t$). As consequence, estimated random effects with time stamp t are rather an aggregated proxy of the economic conditions during $t + \Delta t$, where, Δt corresponds to the resolution time. Second, financial institutions have to deal with high stocks of non-performing loans post crises. This may enforce fast and potentially cost-intensive resolutions as institutions want to settle open claims. Slower recovery as indicated by economic proxies such as macro variables might be the consequence. Third, systematic effects on LGDs might not be purely of economic nature. In the US, rather high realizations of the random effect occur since 2010 Q2. Even through the time period from 2010 Q2 to 2013 Q2 is considered as crisis by the OECD, it is surprising that LGDs are averagely higher compared to the GFC. Thus, it is possible that something beyond economic conditions systematically increasing LGDs. As the implementation of Basel II into US law proceeds post crisis, regulatory effects on LGDs could be conceivable. Besides, regulations might lead to adjustments in banking practice which also could influence loss rates.

Posterior predictive distribution

In the following, we briefly analyze the posterior predictive distributions of LGDs as generated by the models. We focus on its ability to capture the patterns of the empirical LGD distribution (see, e.g., left panels of Figure 1 in Section 3.1). As we do not compare different models, we concentrate on graphical tools.

Figure A.5 in Appendix A illustrate the characteristics of the posterior predictive distribution for the US, GB, and Europe. The left panels contrast kernel density estimates of the posterior predictive distribution (gray line) to the empirical LGD distribution (dotted line). As the band width is fixed to 0.015 for both kernel density estimates, their height is comparable despite ties in the empirical data. As the particular shape of LGD distributions is challenging to illustrate via density estimates due to the extreme modi at zero and one, the right panels present the quantile-quantile (qq) plot contrasting the quantiles of the empirical distribution (x-axis) to the quantiles of the posterior predictive distribution (y-axis). The bisector (black line) represents optimality as it indicates that the quantiles of both distributions correspond. The models seem to be characterized by a good fit regarding the distributional form of the posterior predictive distribution in-sample in all three considered regions (see left panels of Figure A.5). Thus, the assumption of a Gaussian FMM, i.e., five normally distributed components, as component model seems adequate. The right panels of Figure A.5 support this impression as the dots in the qq plot almost perfectly lie on the optimality line indicating that the quantiles of the posterior predictive distribution comply with the quantiles of the empirical distribution.

5 A Downturn LGD Model

Systematic effects as revealed in Section 4 directly affect downturn LGD estimation. Following Basel Committee on Banking Supervision (2004) and Basel Committee on Banking Supervision (2005), financial institutions have to provide estimates, that "[...] reflect economic downturn conditions where necessary to capture the relevant risks [...]". First, this ensures conservativeness and, thus, prevents LGD predictions being systematically too low. Second, it seems to be driven by the need for counter-cyclical safety buffers. These buffers are accumulated in favorable economic conditions and absorb unexpected economic losses during downturns. Hence, downturn LGDs should provide safety buffers that are (too) conservative during good and normal conditions, but are sufficient once critical conditions emerge. However, consistent approaches do not exist so far. Different suggestions are even presented in more recent publications (see, e.g. Tobback et al., 2014; Calabrese, 2014; Bijak and Thomas, 2015).

Downturn estimation via random effects

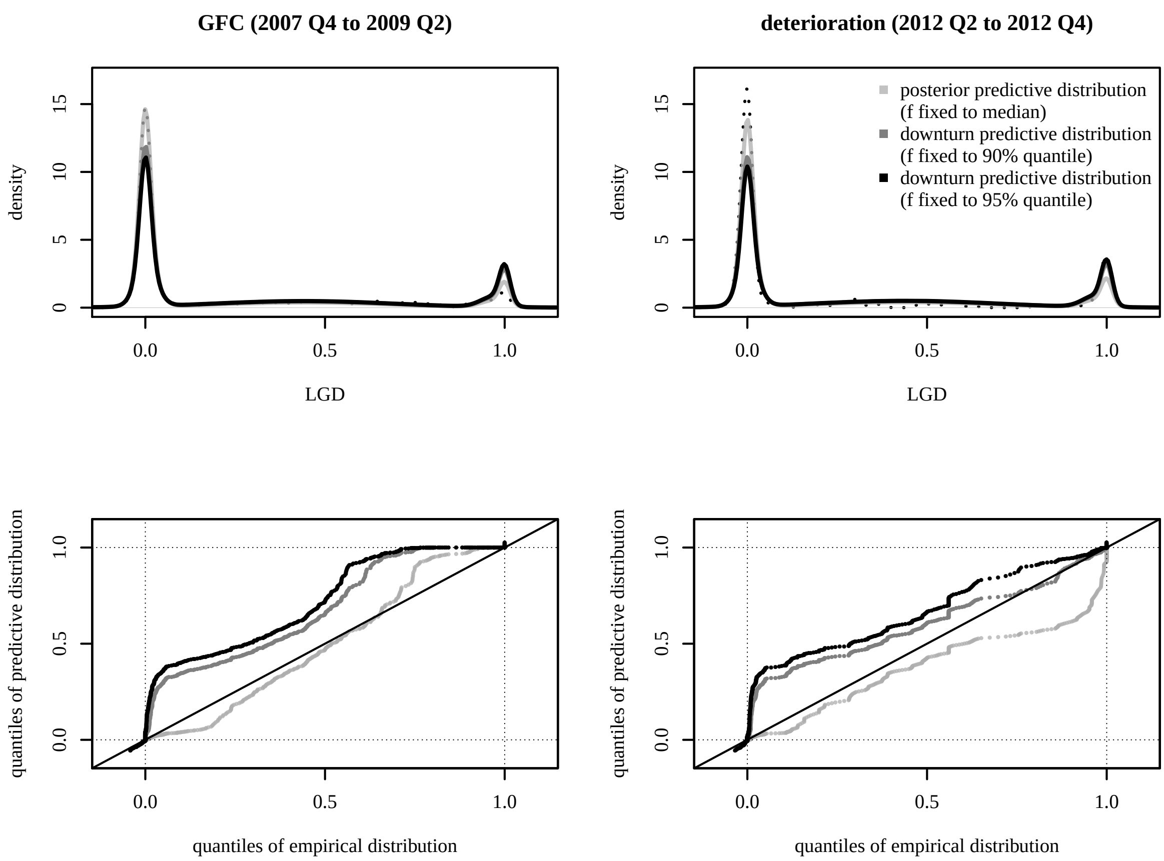
Random effects capture systematic patterns in terms of comovement. Hence, these unobservable variables might be an appropriate control factor for downturn estimates. Downturn LGDs are based on negative systematic conditions, i.e., on assuming a high realization f_t ($>> \alpha$ in specification I, $>> \mu_u^f$ in specification II). Hence, we do not sample the random effect in the MCMC chains as in the posterior predictive distribution, but set it to a conservative quantile for all iterations instead.[20] By doing so, we generate LGD distributions reflecting unfavorable systematic conditions. We analyze downturn LGD distributions for two time periods in each region. In all regions, we consider the GFC. According to the OECD, the financial crisis is terminated in the period from 2007 Q4 to 2009 Q2 in the US, whereas, it is shifted by one quarter in GB and Europe (2008 Q1 to 2009 Q3). Considering the time patterns of average

[20] In the following, we apply the 90% quantile and 95% quantile.

LGDs as in Figure 1, we evaluate additional time periods which are characterized by high average losses. We will refer to this periods as deterioration periods. In the US, we select the time span from 2012 Q2 to 2012 Q4.[21] The year right after the GFC is applied in GB and Europe (2009 Q4 to 2010 Q3).

Figure 5 contrast the posterior predictive and downturn predictive distributions to the empirical LGD distribution in the GFC and deterioration period for the US. The upper panels display

Figure 5: Posterior and downturn distribution for the GFC and a deterioration period (US)



Note: The figure contrasts the empirical LGD distribution to the posterior (light gray) and downturn (dark gray and black) predictive distribution for the GFC (2007 Q4 to 2009 Q2, left panels) and a deterioration period (2012 Q2 to 2012 Q4, right panels). The upper panels display the kernel density estimates, the lower panels the quantile-quantile (qq) plots.

the kernel density estimates of the data (dotted lines), the posterior predictive distribution (light gray lines), and two downturn predictive distributions, whereby, different quantiles, i.e., 90% (dark gray lines) and 95% (black lines), of the random effect are applied. The lower panels show the corresponding qq plots. The posterior predictive distribution is reflected by light gray dots, the downturn predictive distribution based on the 90% quantile of the random

[21] Under the terms of the OECD, the time period from 2012 Q2 to 2013 Q2 is classified as recession period in the US.

effect by dark gray dots and the posterior predictive distribution based on the 95% quantile of the random effect by black dots. During the GFC, the posterior predictive distribution seems to be sufficiently conservative as it fits the empirical LGD distribution quite well. However, this time period is characterized by rather low probability mass at total loss compared to the US American data set as a whole. Thus, the posterior predictive distribution implying average systematic conditions already overestimates high LGD realizations up to a certain degree. Accordingly, quantiles of the downturn predictive distributions lie always higher than empirical quantiles. In the deterioration period, the posterior predictive distribution underestimates the probability mass of high and total losses. However, the downturn predictive is capable of capturing this systematic higher fraction. Figure A.6 and A.7 in Appendix A illustrate the corresponding analytics for GB and Europe. The presentation corresponds to Figure 5. Results are almost similar. However, the posterior predictive distribution does not seem to be sufficiently conservative during the GFC as the empirical LGD distribution is characterized by higher probability masses at total loss in GB and Europe regarding this time period.[22]

Generally, these downturn LGD distributions may be considered too conservative. However, the distance between the downturn predictive distribution and the optimality line is directly impacted by the selected conservative quantile. Confidence levels smaller than 90% result in a lower gap. Hence, downturn estimates are adjustable according to the needs of the risk manager or regulator. The main advantage of the presented model and the implied downturn approach via random effects is the option to generate conservative estimates in accordance with the characteristic nature of LGDs.

Downturn estimation via macro variables

In the literature, several suggestions to generate downturn estimations exist. The most common one refers to the inclusion of macro variables in the modeling context. Hence, we derive posterior and downturn predictive distributions based on the model including a macro variable instead of the random effect. In each region, we select the macro variable with the highest statistical evidence. Thus, we apply the NPL ratio for the US, ΔEI for GB, and ΔHPI for Europe.[23] In analogy to the downturn generation based on random effects, a conservative

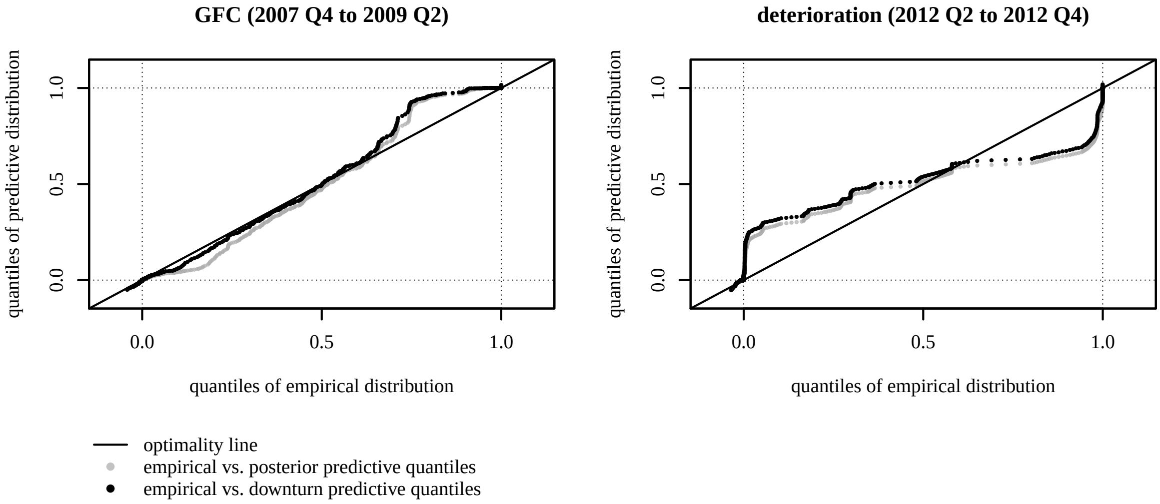
[22] To examine robustness, we evaluate the downturn distributions on an out-of-time basis. The training set contains the time period from 2006 Q1 to 2010 Q1. The test set consists of the time period from 2010 Q2 to 2012 Q4. Results are presented in Figure A.8 in Appendix A. The downturn distributions are still conservative if fitted on a out-of-time basis.

[23] Results regarding the remaining macro variables are available form the authors upon request.

quantile of the macro variable is selected to generate the downturn predictive distribution.[24]

Figure 6 contrasts the quantiles of the empirical distribution in the GFC and the deterioration period to the corresponding posterior and downturn predictive distributions for the US. The

Figure 6: Posterior and downturn distribution for the GFC and a deterioration period based on the macro model containing the NPL ratio (US)



Note: The figure contrasts the empirical LGD distribution to the posterior (light gray) and downturn (dark gray and black) predictive distribution of the macro model containing the NPL ratio instead of a random effect for the GFC (2007 Q4 to 2009 Q2, left panel) and a deterioration period (2012 Q2 to 2012 Q4, left panel).

course of quantiles regarding the posterior predictive distributions based on the macro model is rather similar compared to the random effect model (see Figure 5). However, the corresponding downturn distribution does not sufficiently capture the quantiles of the empirical LGD distribution in the time period from 2012 Q2 to 2012 Q4. Comparing the downturn distributions of the macro model with the one of the random effect model in Figure 5, the gap to the corresponding posterior predictive distributions seems undersized. This might be due to a rather small posterior mean of the coefficient ($\beta_{\text{NPL ratio}} \approx 0.07$, see Table 4) and a HPDI (90%) which nearly reaches zero (HPDI (90%) = $[0.0078, 0.1218]$, see Table 4). As macro variables do not seem to be sufficient to capture the true systematic effect impacting LGDs, generating downturn distributions via macro variables might be less effective. Figures A.6 and A.7 in Appendix A illustrate the corresponding analytics for GB and Europe. As the posterior means of the macro variables are of higher magnitude ($\beta_{\text{EI}} \approx -0.10$ for GB and $\beta_{\text{HPI}} \approx -0.23$ for Europe), the downturn predictive distribution differs clearer from the posterior predictive distribution. In Europe, the resulting downturn distribution is sufficiently conservative in both considered periods. However, the downturn distribution underestimates high-loss components in GB

[24] We select a conservative quantile of 99%.

during the GFC. Furthermore, even though reasonable downturn LGD estimates can be derived by the inclusion of selected macro variables, the identification issue which macro variable is most appropriate to use is avoided by the use of the random effect model.

Alternative downturn concepts

Besides the consideration of stressed economic conditions to derive downturn estimates, further approaches in literature exist. Bijak and Thomas (2015), who also adapt Bayesian inference, suggest to use a conservative quantile of the posterior predictive distribution, whereas, Calabrese (2014) proposes an upper, i.e., rather conservative, component based on a frequentistic mixture model to reproduce a downturn distribution. Besides, Board of Governors of the Federal Reserve System (2006) proposed a linear mapping function to generate downturn estimates ($\text{LGD}_{\text{downturn}}$) based on through-the-cycle estimates (LGD_{TTC}). We will refer to the mapping function as FED proposal:

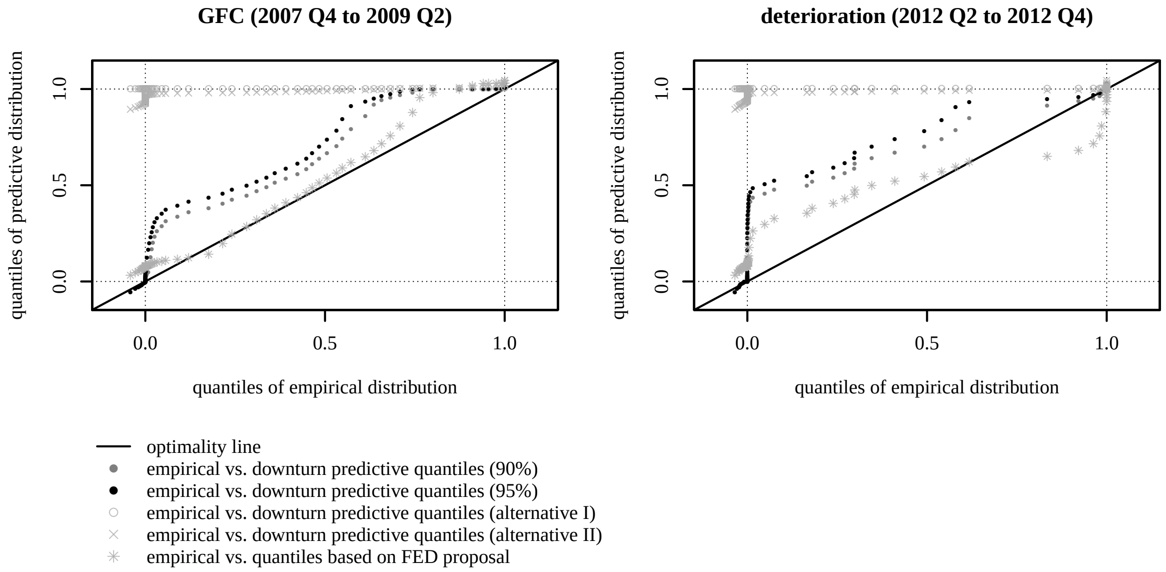
$$\text{LGD}_{\text{downturn}} = 0.08 + 0.92 \cdot \text{LGD}_{\text{TTC}} . \tag{12}$$

To adapt the approach of Bijak and Thomas (2015) we use a conservative quantile of the individual posterior predictive distributions of each loan.[25] To incorporate the suggestion of Calabrese (2014) in the adapted modeling framework, we employ component 4 as downturn distribution. As Calabrese (2014) excludes real zeros and ones and estimates a mixture of Beta distributions on the remaining data, we neglect component 5. This might most likely reproduce her approach.[26] To adopt the FED proposal, we generate posterior predictive distributions based on median realizations of random effects on which we apply Equation (12).

Figure 7 contrasts the downturn predictive distribution via a random effect (gray and black dots) to the approaches of Bijak and Thomas (2015) (alternative I, gray cycles), Calabrese (2014) (alternative II, gray crosses), and the FED proposal (gray stars) for the US. Both alternative approaches generate far more conservative downturn distributions compared to applying random effects as the resulting downturn distributions do not capture the whole range of LGDs $[-50\%, 150\%]$. The probability mass is highly centered around one, i.e., total loss. However, the empirical LGD distribution is shaped by high probability masses around zero, i.e., no loss, even in quarter with high average LGDs. This pattern seems to be neglected by the approaches suggested by Bijak and Thomas (2015) and Calabrese (2014). In the GFC, the FED proposal

[25] We select a conservative quantile of 99%, however, alternative conservative quantiles can be applied.
[26] As an alternative, a weighted mixture of several components can be adopted, e.g., component 3 and 4.

Figure 7: Downturn distribution for the GFC and a deterioration period based on alternative concepts (US)



Note: The figure contrasts the empirical LGD distribution to downturn predictive distributions. The black and gray dots represent the downturn approach via a random effect, whereas, the suggestion of Bijak and Thomas (2015) (alternative I) is displayed by gray cycles and the proposal of Calabrese (2014) (alternative II) by gray crosses. The FED approach is displayed by gray stars. The figure refers to the GFC (2007 Q4 to 2009 Q2, left panel) and a deterioration period (2012 Q2 to 2012 Q4, right panel).

seems to fit the empirical LGD distribution quite well. However, the posterior predictive distribution almost succeeds as well in this time period (see Figure 5). The FED proposal does not produce sufficiently conservative estimates in the deterioration period. Figure A.11 and A.12 in Appendix A confirm these findings for GB and Europe.

In general, fundamental deviations among the approaches arise. While downturn distributions generated based on a random effect still cover the whole LGD range, the alternatives result in rather constant conservative values (Bijak and Thomas, 2015) or in restricted and conservative single component distributions (Calabrese, 2014). The FED proposal does not seem to be able to constantly generate sufficiently conservative estimates. The approach suggested in this paper is based on a shift in component probabilities in critical systematic conditions. Fixing the random effect to a conservative quantile decreases probabilities of low, i.e., *good*, components and increases the probability of high, i.e., *bad*, components. Although the deteriorated systematic surrounding is reflected in the downturn distribution, the whole range of LGDs remains covered. This functionality is based on the empirical observations of LGDs as the bi-modality remains during crises periods.

Dependent on the intention, the presented approach may be more or less favorable compared to the alternative suggestions by Bijak and Thomas (2015) and Calabrese (2014). Restricting downturn distributions to a certain value range might result in overestimation of risk and, thus, in extreme safety buffers. While this is in line with the conservativity principle, holding too much capital creates new risk and burdens the solvency of banks as opportunity cost increase and operating business is hampered. The approach presented in this paper generates reasonable safety buffers and is adoptable to the needs of risk managers and regulators. By illustrating this, we aim to contribute to the ongoing discussion on how to define and estimate downturn LGDs.

6 Conclusion

This paper examines systematic effects among LGDs in the US and Europe. While we observe cyclical patterns of these effects among LGDs in Europe, this is not true for the US. Either way, systematic impacts on LGDs strongly deviate from the economic cycle measured by common macro variables. A reason for this finding might be found in the collection process of recovery payments. Collection processes might take multiple years, thus, varying economic conditions impact final LGDs. This might hamper the use of economic variables with a specific time stamp. Given these considerations, the random effect in our model may be interpreted as the average systematic impact of LGDs during the resolution process.

Our results contribute to the discussion how to provide consistent downturn estimates for LGDs. The presented approach offers a straightforward regulation mechanism for decision makers. If economic variables are supposed to be used, we believe it is indispensable to include multiple time leads or lags at once to stimulate the impact of the economic environment. This might complicate the regulation mechanism for risk managers and regulators as further issues rise. Conservative quantiles have to be selected for every lead and lag, which offers additional decision-making scope. Regulation, e.g., in the sense of a lower bound, might get more complex as modeling alternatives multiply.

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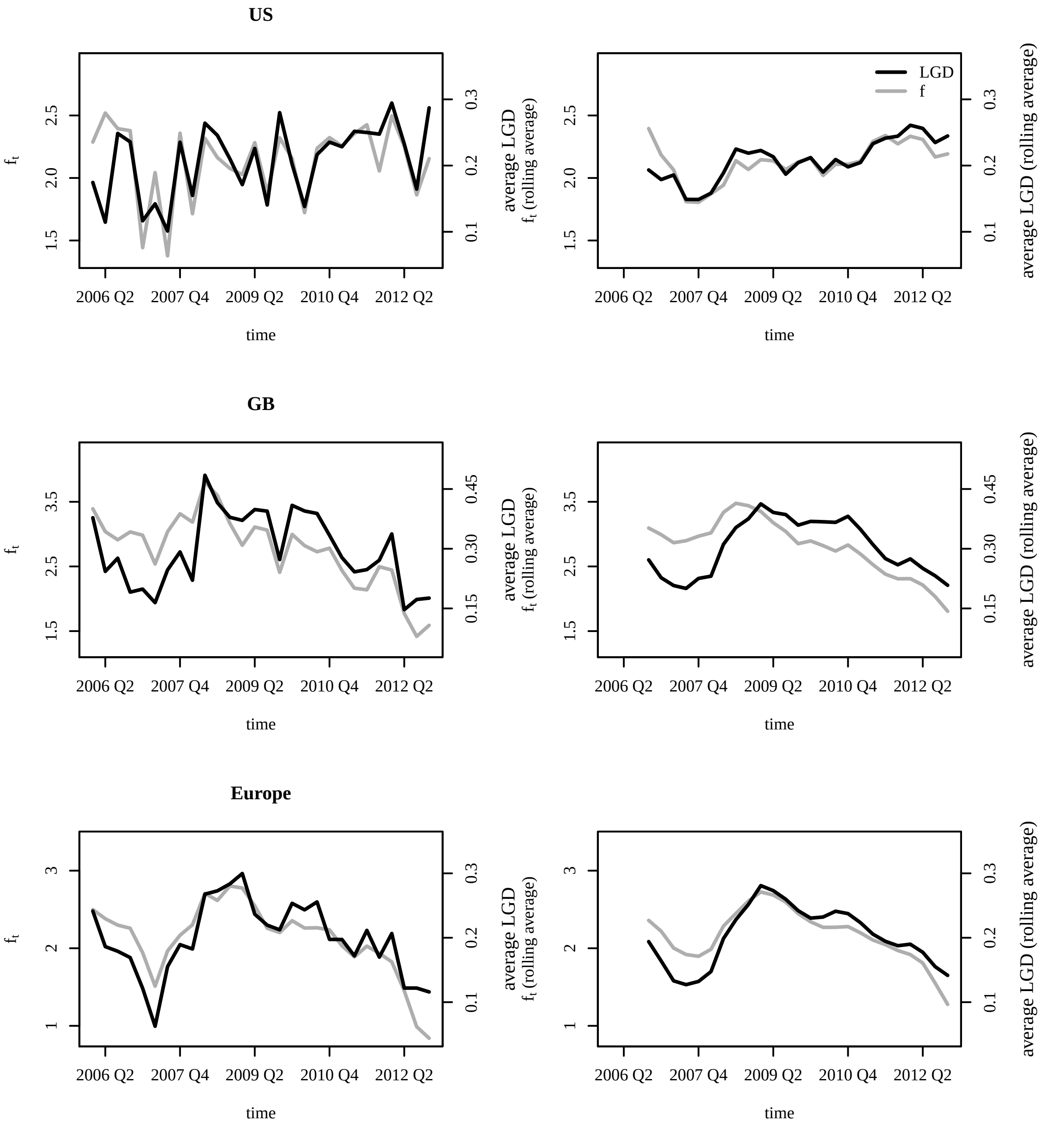
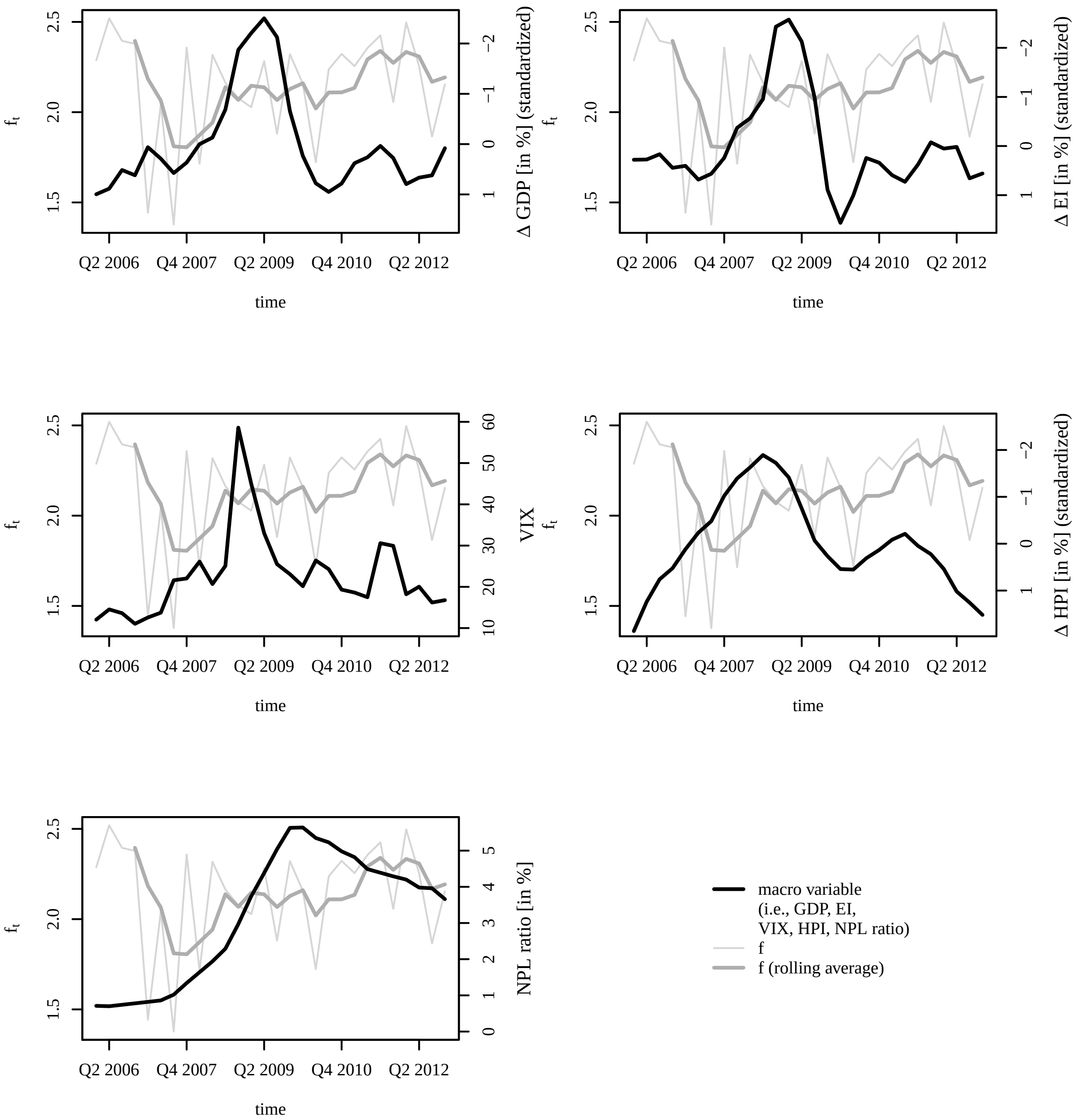
A Further Figures

Figure A.1: Random effect vs. average LGD



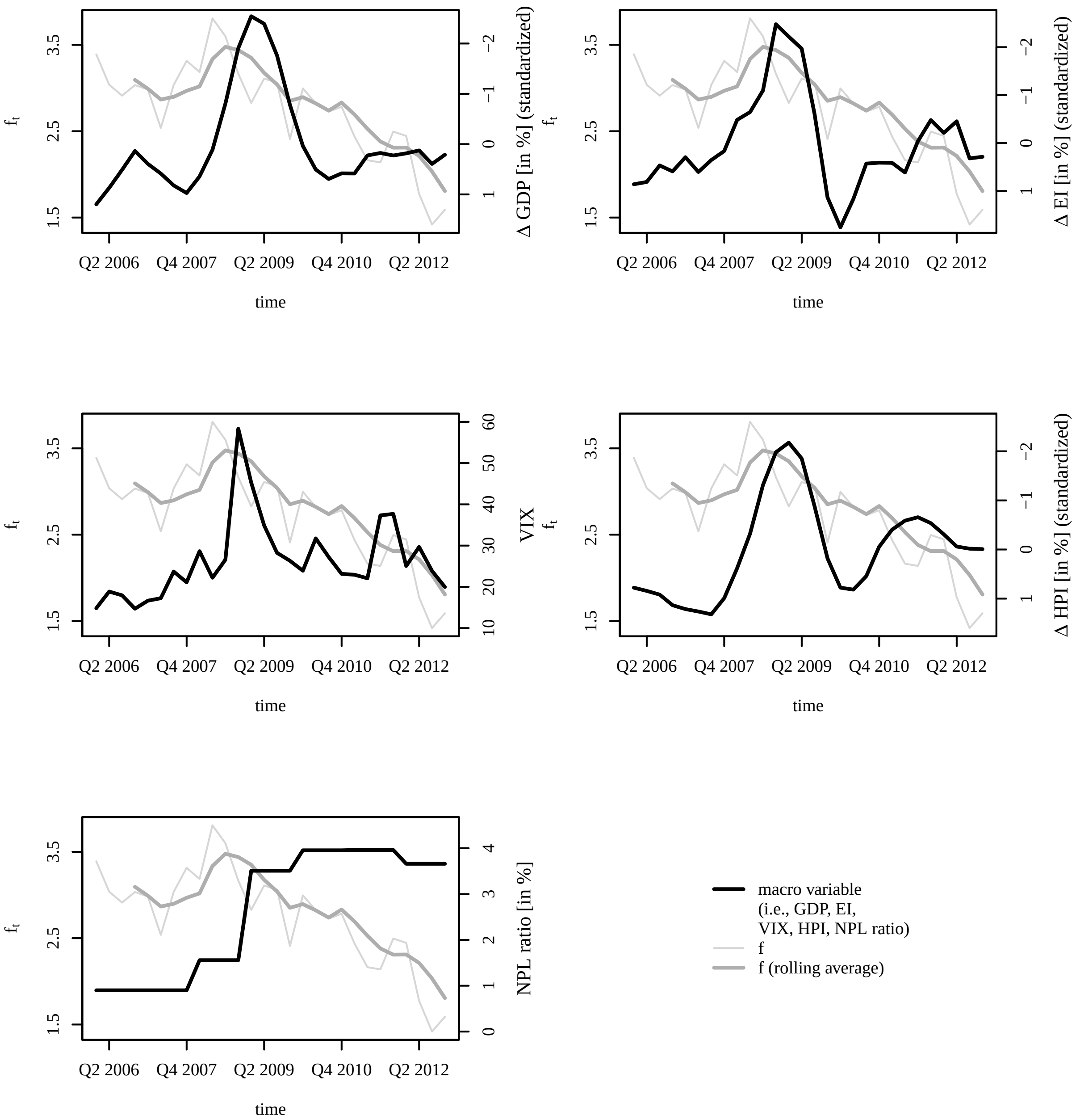
Note: The figure illustrates the synchronism of the estimated posterior means of the random effect (specification I for the US and specification II for GB and Europe) and the average LGD in time line. In the left panels of the figure, the course of the random effect (gray line) and the quarterly average LGD (black line) is displayed. For representational purpose the rolling average of the random effect (gray line) and the rolling quarterly average of the LGD (black line) is plotted in the right panels.

Figure A.2: Random effect vs. macro variables (US)



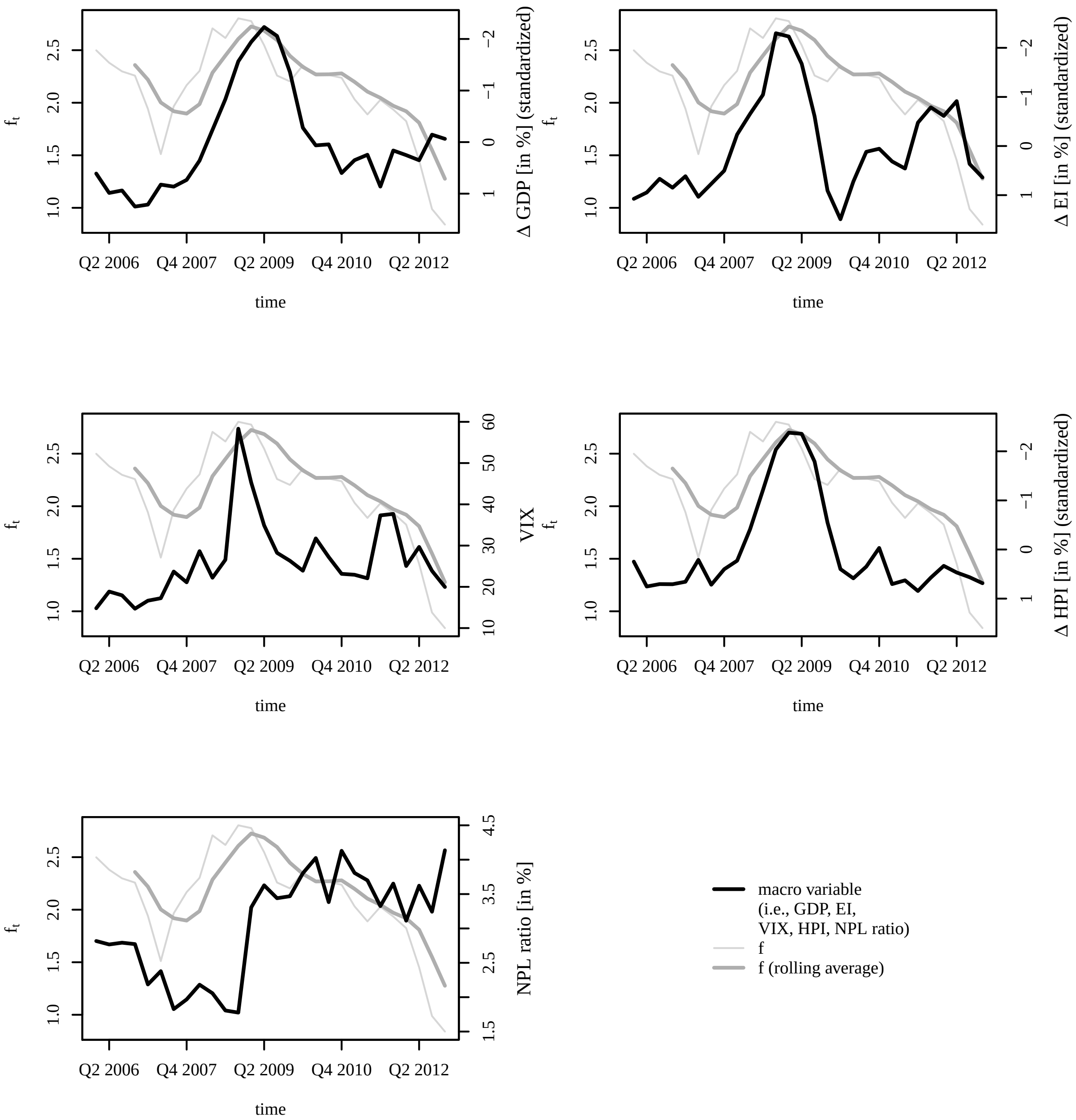
Note: The figure contrasts the course of the macro variables and the random effect over time. The macro variables (i.e., GDP, S&P 500, VIX for the US, HPI, and NPL ratio) are represented by the black lines. The thin gray lines map the posterior means of the random effect (specification I). For representation purpose, the rolling average of the posterior means is illustrated by thick gray lines.

Figure A.3: Random effect vs. macro variables (GB)



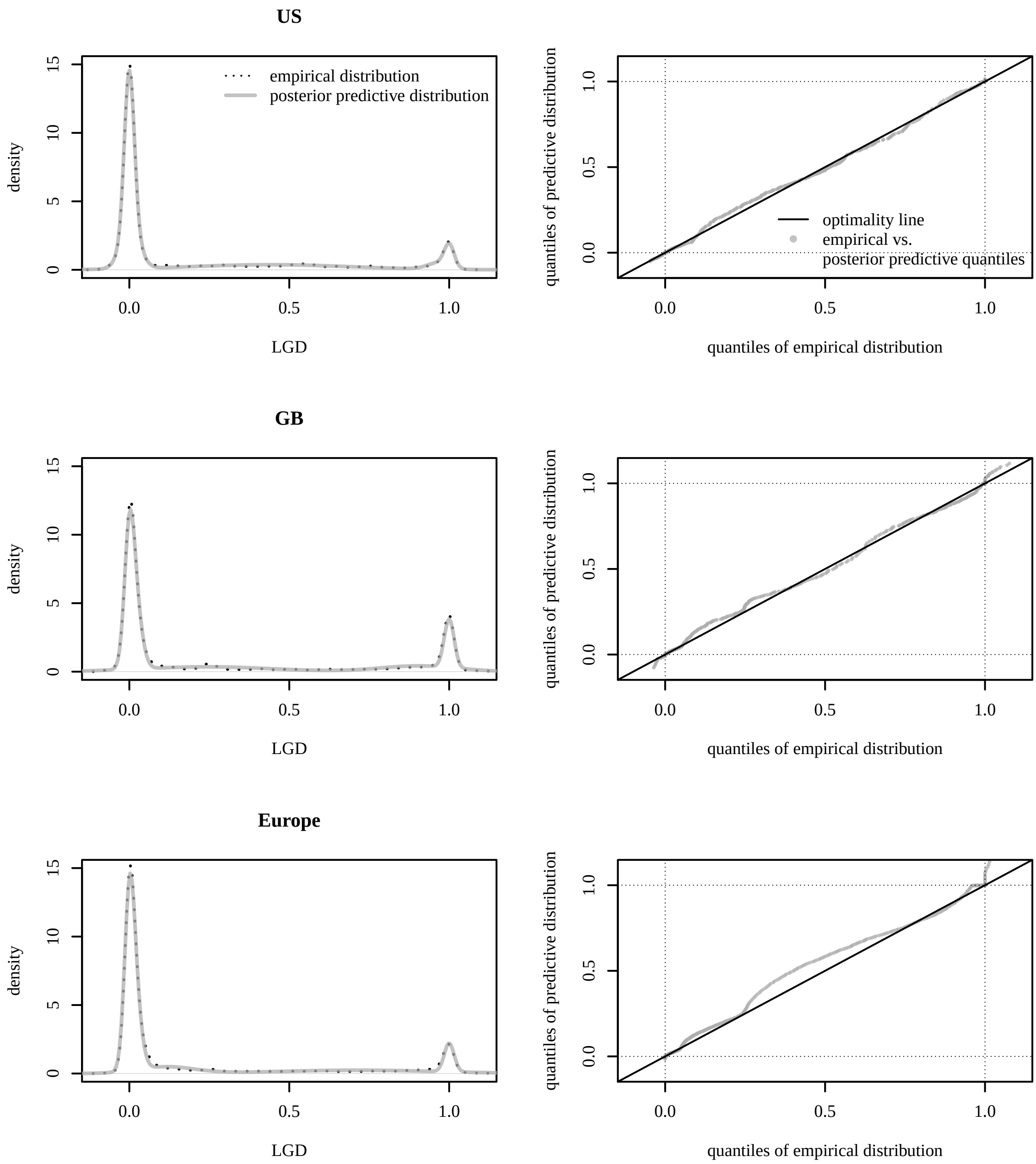
Note: The figure contrasts the course of the macro variables and the random effect over time. The macro variables (i.e., GDP, FTSE, VIX for Europe, HPI, and NPL ratio) are represented by the black lines. The thin gray lines map the posterior means of the random effect (specification II). For representation purpose, the rolling average of the posterior means is illustrated by thick gray lines.

Figure A.4: Random effect vs. macro variables (Europe)



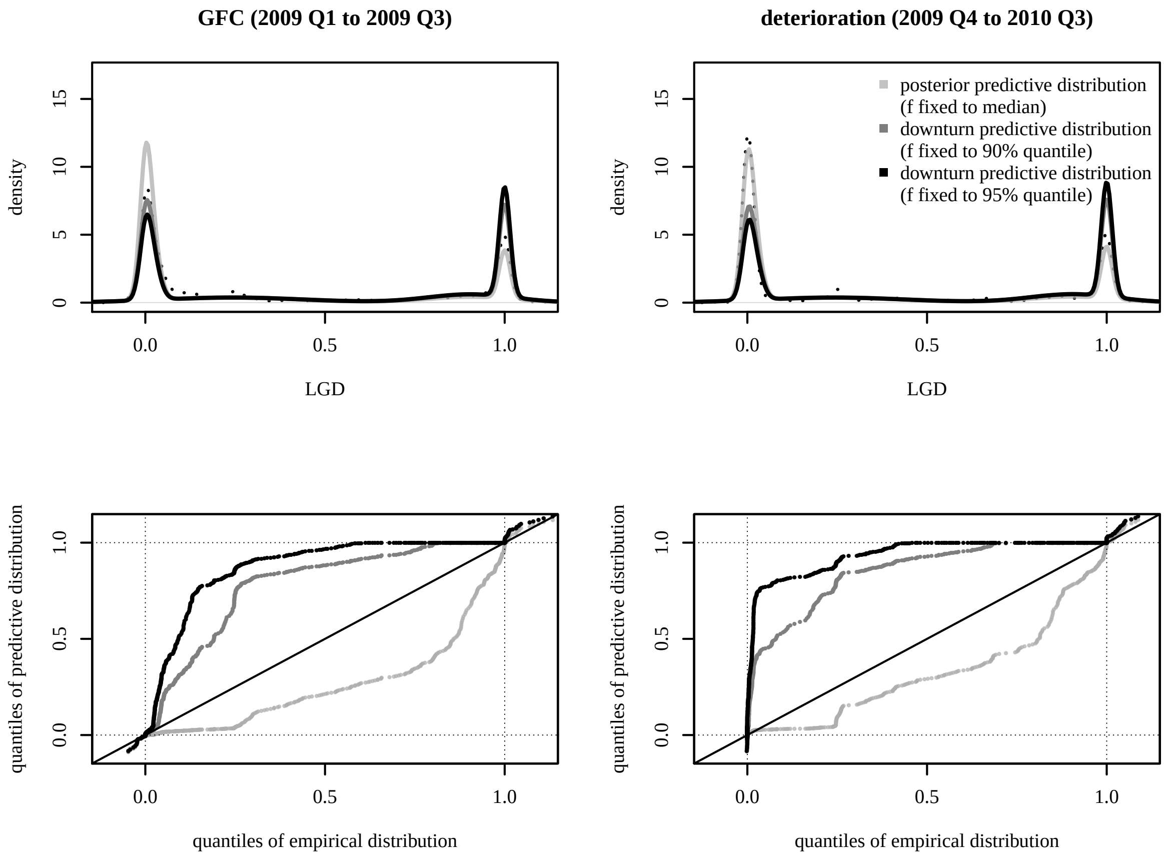
Note: The figure contrasts the course of the macro variables and the random effect over time. The macro variables (i.e., the weighted average of GDP, EIs, HPIs, and NPL ratio as well as the VIX for Europe) are represented by the black lines. The thin gray lines map the posterior means of the random effect (specification II). For representation purpose, the rolling average of the posterior means is illustrated by thick gray lines.

Figure A.5: Posterior predictive distribution



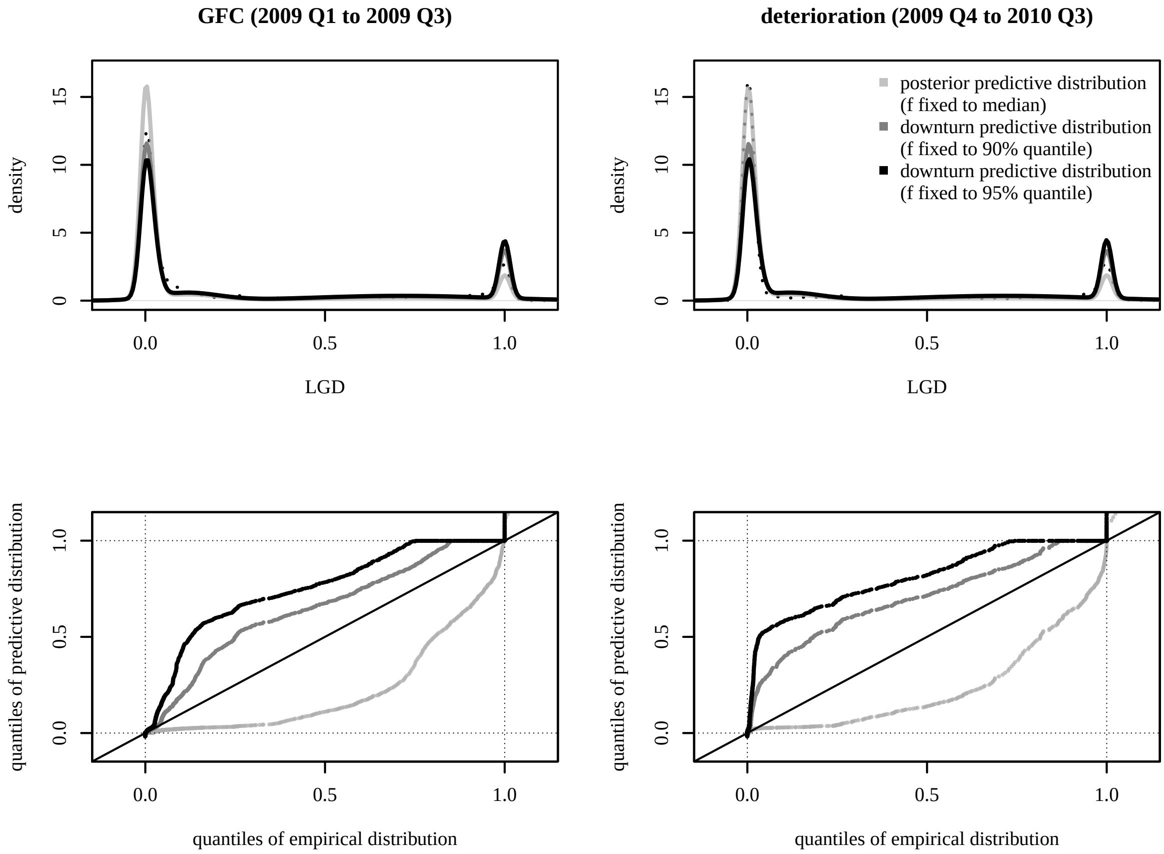
Note: The figure illustrates the posterior predicted distribution for the US American, British, and European data set. In the left panels, the kernel density estimates of the posterior predicted (gray line) is contrasted to the kernel density estimate of the empirical data (dotted line). The band widths are fixed to 0.015 for both kernel density estimates to ensure comparability regarding the height of the density. Thus, the curves are comparable in spite of ties. As differences are hard to identify due to the high probability masses at the two modi, quantile-quantile (qq) plots are presented in the right panels. The gray dots represent the quantiles of the empirical data plotted against the quantiles of the posterior predicted distribution. The bisector (black line) represents optimality.

Figure A.6: Posterior and downturn distribution for the GFC and a deterioration period (GB)



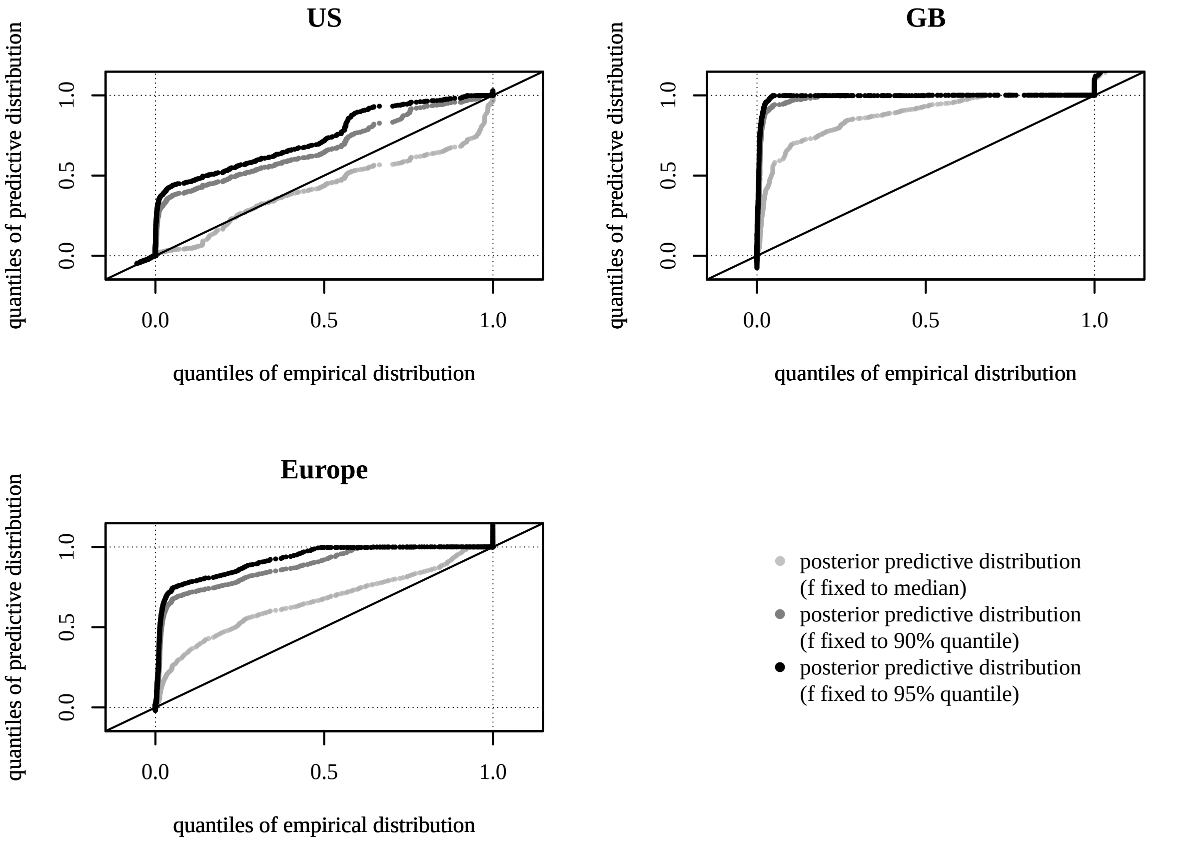
Note: The figure contrasts the empirical LGD distribution to the posterior (light gray) and downturn (dark gray and black) predictive distribution for the GFC (2009 Q1 to 2009 Q3, left panels) and a deterioration period (2009 Q4 to 2010 Q3, right panels). The upper panels display the kernel density estimates, the lower panels the quantile-quantile (qq) plots.

Figure A.7: Posterior and downturn distribution for the GFC and a deterioration period (Europe)



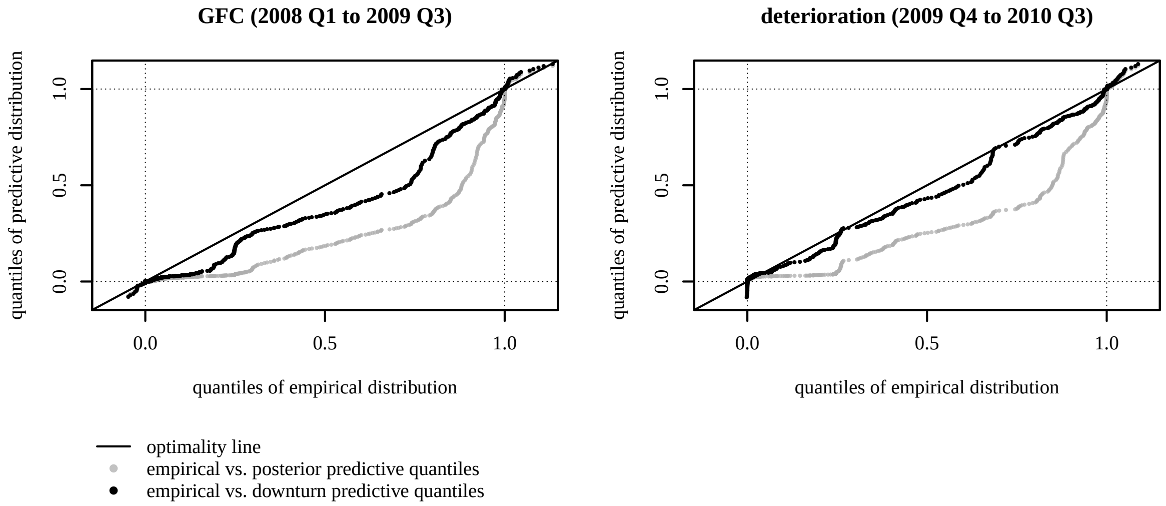
Note: The figure contrasts the empirical LGD distribution to the posterior (light gray) and downturn (dark gray and black) predictive distribution for the GFC (2009 Q1 to 2009 Q3, left panels) and a deterioration period (2009 Q4 to 2010 Q3, right panels). The upper panels display the kernel density estimates, the lower panels the quantile-quantile (qq) plots.

Figure A.8: Posterior and downturn distribution based on out-of-time estimation



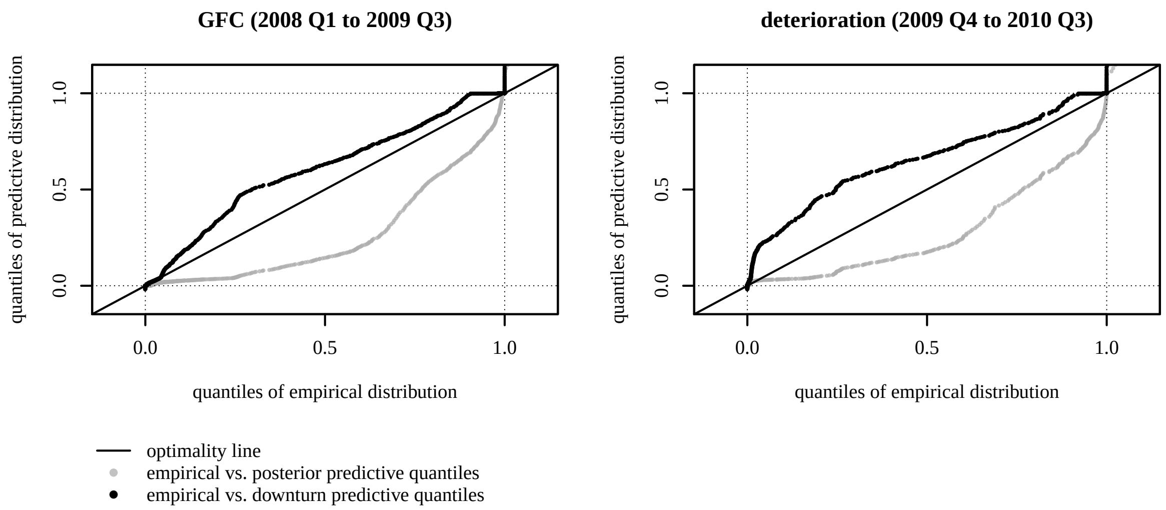
Note: The figure contrasts the empirical LGD distribution to the posterior (light gray) and downturn (dark gray and black) predictive distribution based on out-of-time estimation. The training set contains the time period from 2006 Q1 to 2010 Q1. The test set consists of the time period from 2010 Q2 to 2012 Q4.

Figure A.9: Posterior and downturn distribution for the GFC and a deterioration period based on the macro model containing the EI (GB)



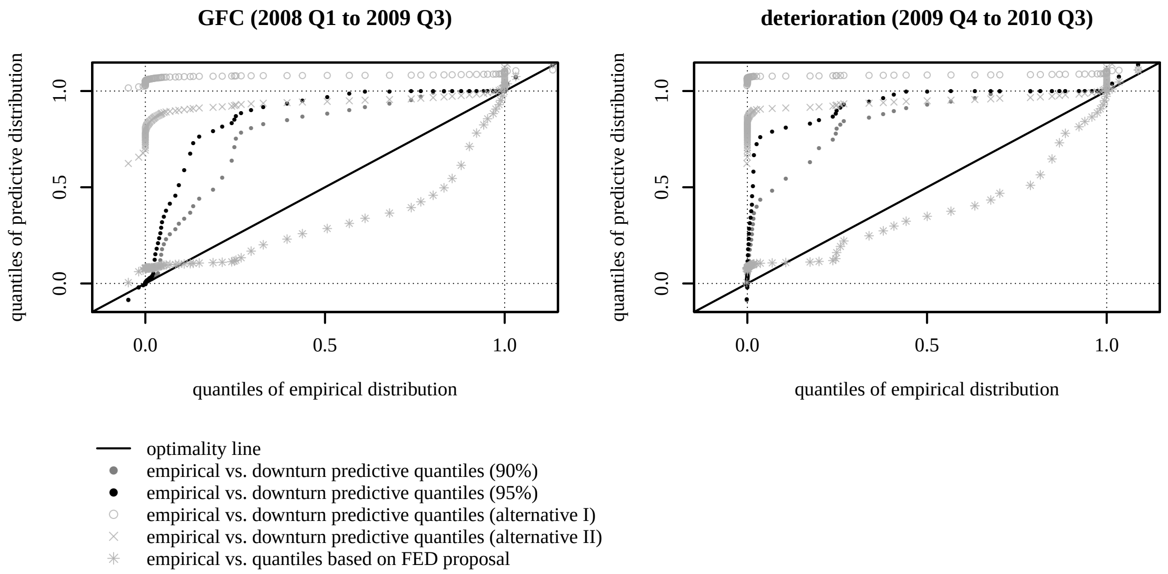
Note: The figure contrasts the empirical LGD distribution to the posterior (light gray) and downturn (dark gray and black) predictive distribution of the macro model containing the EI instead of a random effect for the GFC (2008 Q1 to 2009 Q3, left panel) and a deterioration period (2009 Q4 to 2010 Q3, left panel).

Figure A.10: Posterior and downturn distribution for the GFC and a deterioration period based on the macro model containing the HPI (Europe)



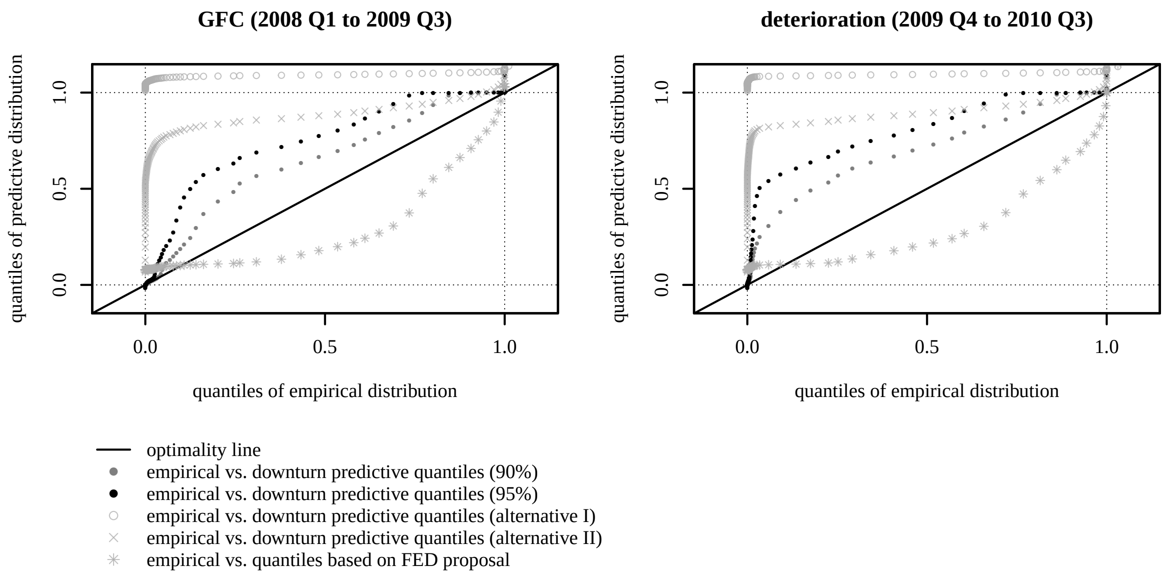
Note: The figure contrasts the empirical LGD distribution to the posterior (light gray) and downturn (dark gray and black) predictive distribution of the macro model containing the HPI instead of a random effect for the GFC (2008 Q1 to 2009 Q3, left panel) and a deterioration period (2009 Q4 to 2010 Q3, left panel).

Figure A.11: Downturn distribution for the GFC and a deterioration period based on alternative concepts (GB)



Note: The figure contrasts the empirical LGD distribution to downturn predictive distributions. The black and gray dots represent the downturn approach via a random effect, whereas, the suggestion of Bijak and Thomas (2015) (alternative I) is displayed by gray cycles and the proposal of Calabrese (2014) (alternative II) by gray crosses. The FED approach is displayed by gray stars. The figure refers to the GFC (2008 Q1 to 2009 Q3, left panel) and a deterioration period (2009 Q4 to 2010 Q3, right panel).

Figure A.12: Downturn distribution for the GFC and a deterioration period based on alternative concepts (Europe)



Note: The figure contrasts the empirical LGD distribution to downturn predictive distributions. The black and gray dots represent the downturn approach via a random effect, whereas, the suggestion of Bijak and Thomas (2015) (alternative I) is displayed by gray cycles and the proposal of Calabrese (2014) (alternative II) by gray crosses. The FED approach is displayed by gray stars. The figure refers to the GFC (2008 Q1 to 2009 Q3, left panel) and a deterioration period (2009 Q4 to 2010 Q3, right panel).

B Further Tables

Table B.1: Literature review

	Data	Method	Country	Time	Macro variables	Result
Acharya et al. (2007)	Bonds and loans (market-based RRs)	Regression	US	1982–1999	• Industry macro variables • Aggregates default rates	• Industry distress dummies (–***) • Aggregated macro variables lose significance
Altman and Kalotay (2014)	Bonds and loans (workout RRs)	Bayesian FMM	US	1987–2011	Industry specific default likelihood	Statistical evidence not reported
Bastos (2010)	Loans (workout RRs)	• Regression • Regression trees	Portugal	1995–2000	No	
Bijak and Thomas (2015)	Loans (workout LGDs)	Bayesian FMM	UK	1987–1998	No	
Brumma et al. (2014)	Loans (workout LGDs)	Regression	International	2000–2010	Varity of macro variables	Significance only for cash flow weighted LGDs
Calabrese (2014)	Loans (workout RRs)	FMM	Italy	1975–1999	No	
Caselli et al. (2008)	Loans (workout LGDs)	Regression	Italy	1990–2004	Varity of macro variables	SMEs: Macro variables statistically or economically non significant
Bellotti and Crook (2012)	Personal credit cards (workout RRs)	• Regression (Tobit) • Regression trees	UK	1999–2005	• Interest rate • Unemployment • Earnings growth	• Interest rate (–***) • Unemployment (–***) Not economically significant • Earnings growth (+*)
Dermine and Neto de Carvalho (2006)	Loans (workout RRs)	Regression	Portugal	1995–2000	• GDP • Default rate	No statistical significance
Grunert and Weber (2009)	Loans (workout RRs)	Regression	Germany	1992-2003	• GDP • Loss provisions • Unemployment	No statistical significance
Gürtler and Hibbeln (2013)	Loans (workout RRs)	Regression (model improvements)	Germany	2006–2008	No	
Jankowitsch et al. (2014)	Bonds (market-based RRs)	Regression	US	2002–2010	• Market default rate • Industry default rate • Federal funds rate • Slope of term structure	• Market default rate (–***) • Industry default rate (–***) • Federal funds rate (+***) • Slope of term structure (+***)
Krüger and Rösch (2017)	Loans (workout LGDs)	Quantile regression	US	2000–2014	• S&P 500 • TED spread • Term spread • VIX	• S&P 500 (significant for inner quantiles) • TED spread (significant for inner quantiles)
Leow et al. (2014)	Mortgages and loans (workout LGDs)	• Two-stage model • Regression	UK	1990–2002 (mortgages) 1989–1999 (loans)	Varity of macro variables (with different time stamps)	Mixed results regarding significance (might be due to non linear impacts)
Matuszyk et al. (2010)	Loans (workout LGDs)	Two-stage model	UK	1989–2004	No	
Nazemi et al. (2017)	Bonds (market-based LGDs)	Fuzzy decision fusion	US	2002–2012	Principal components of 104 macro variables	Principal components have high contribution to R^2
Qi and Yang (2009)	Mortgages (workout LGDs)	Regression	US	1990–2003	• HPI • Stress dummy • Current LTV	• HPI (not included) • Stress dummy (+***) • Current LTV (+***)
Qi and Zhao (2011)	Bonds (market-based and workout LGDs)	Varity of methods	US	1985–2008	• (Industry) distance to default • (Industry) default rate • Market return • Interest rate	• Industry distance to default (–***) • Default rate (+***) • Market return (–***) • Interest rate (+***)
Somers and Whittaker (2007)	Mortgages (workout LGDs)	Quantile regression	EU	since 1990	No	
Tobback et al. (2014)	Loans (workout LGDs)	• Regression • Regression trees • Non linear models	US	1984–2011	Varity of macro variables	Statistical significance not reported
Yao et al. (2015)	Bonds (workout LGDs)	Support vector regression	US	1985–2012	• GDP • Unemployment • S&P 500 • Interest rate	Statistical significance not reported
Yao et al. (2017)	Credit cards (workout LGDs)	Support vector machines	UK	2009–2010	• Unemployment • CPI • HPI	• Unemployment (+***) • CPI (–***) • HPI (–***)

Note: The table summarizes LGD related literature with focus on systematic effects (i.e., impact of macro variables).

Table B.2: Descriptive statistics

		US	GB	Europe
		Dependent		
LGD	mean	0.2057	0.2949	0.1958
	median	0.0000	0.0201	0.0080
	standard deviation	0.3438	0.4121	0.3466
		Loan specific| metric		
EAD	mean	25,553,822.04	1,080,731,810.36	1,303,325,643.26
	median	509,213.80	144,311,828.61	168,649,302.14
	standard deviation	587,085,828.35	7,963,230,966.87	9,103,356,249.39
		Loan specific | categoric		
Facility	term loan	43.63%	46.34%	60.22%
	line	56.37%	53.66%	39.78%
Protection	no	16.13%	28.88%	30.45%
	yes	83.87%	71.12%	69.55%
Industry	non FIRE	85.88%	91.26%	84.62%
	FIRE	14.12%	8.74%	15.38%
		Macro variables		
Δ GDP	mean	1.11%	0.70%	1.24%
	median	1.69%	1.31%	1.92%
	standard deviation	2.07%	2.67%	2.75%
Δ EI	mean	1.91%	1.49%	2.35%
	median	7.60%	6.74%	11.86%
	standard deviation	20.54%	17.11%	26.22%
VIX	mean	22.7187	26.4292	26.4292
	median	20.4113	23.8916	23.8916
	standard deviation	10.1888	9.5692	9.5692
Δ HPI	mean	-5.70%	-1.56%	-1.88%
	median	-4.74%	-1.65%	0.42%
	standard deviation	7.68%	7.52%	5.75%
NPL ratio	mean	3.20%	2.63%	3.04%
	median	3.85%	3.51%	3.27%
	standard deviation	1.79%	1.36%	0.76%

Note: The table summarizes descriptive statistics of dependent and independent variables. For metric variables the mean, median, and standard deviation is presented. Proportions are given for variables of categoric nature. FIRE is an abbreviation for corporations which are active in the finance, insurance or reals estate industry.

Table B.3: Pairwise correlations of macro variables and random effect

	US	GB	Europe
−Δ[GDP (standardized)]	-11.64%	12.39%	34.44%
−Δ[EI (standardized)]	-0.50%	20.91%	38.41%
VIX	-11.17%	-1.04%	33.84%
−Δ[HPI (standardized)]	-17.46%	-2.03%	54.41%
NPL ratio	12.26%	-58.74%	-32.32%

Note: The table summarizes the pairwise correlations of the macro variables (i.e., GDP, EI, VIX, HPI, and NPL ratio) with the random effect posterior means.

Table B.4: Results of combined models

	posterior mean	HPDI (90%)		posterior odds	naive standard error	time-series standard error
				US		
$\beta_{\text{NPL ratio}}$	0.0480	-0.0736	0.1701	3.0306	0.0007	0.0011
α	2.1408	1.9280	2.3576	∞	0.0013	0.0024
σ^F	0.3559	0.2344	0.4642	∞	0.0007	0.0007
				GB		
β_{EI}	-0.0641	-0.2917	0.1627	0.4637	0.0014	0.0044
a	0.6977	0.0973	1.2440	768.2308	0.0037	0.0073
φ	0.7439	0.5412	0.9515	∞	0.0013	0.0026
σ_c^F	0.4120	0.2901	0.5282	∞	0.0008	0.0009
				Europe		
β_{HPI}	-0.1909	-0.3989	0.0232	0.0769	0.0013	0.0082
a	0.3435	0.0166	0.6503	343.8276	0.0022	0.0045
φ	0.8347	0.6924	0.9851	∞	0.0010	0.0021
σ_c^F	0.2644	0.1946	0.3319	∞	0.0004	0.0007

Note: The table summarizes the results of the combined models where a macro variable and the random effect are included. The presentation is reduced to the results regarding the macro variables itself and the parameters of the random effect. The first column presents the posterior means of the parameters. The second and third column contain the lower and upper bound of the HPDI to a credibility level of 90%. The fourth column includes posterior odds, while, in the last two columns, the naive and time-series standard error of the chains are presented, whereas, the time-series standard error is calculated based on the effective (N^*_{MCMC}) instead of the real (N_{MCMC}) sample size. Hereby, $N^*_{\text{MCMC}} < N_{\text{MCMC}}$ holds for autocorrelated chains.