Realized Networks

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Introduction

High Frequency Data Based Volatility Estimation

- Over the last decade, the availability of intra-daily high frequency trade, quote and order book data has boosted research on the construction of efficient ex-post measure of daily return variability

- These estimator are typically called realized volatility estimators

- Extensive literature on the topic:
  Andersen, Bollerslev, Diebold and Labys (2003); Ait-Sahalia, Mykland and Zhang (2005); Bandi and Russell (2006); Barndorff-Nielsen, Hansen, Lunde, and Shephard (2009);

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Two Challenges

- The multivariate generalizations of these estimators, aka realized covariance, have not not been as widely applied as their univariate counterparts
- Besides numerical challenges, realized covariance estimation suffers from two challenges which are inherently linked to covariance estimation for large number of assets:
  (cf Ledoit and Wolf, 2004; Hautsch, Kyj and Oomen, 2012)
  1. Precise estimation of the covariance
  2. Interpretation of the dependence structure of the assets

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Regularizing the Realized Covariance

- In this work we introduce a regularization approach inspired by the network literature
- The approach consist of shrinking the inverse covariance matrix. This turns out to have a natural interpretation in terms of a partial correlation dependence structure among variables.
- This regularization approach attempts to provide precise estimates and enhance interpretation of the realized covariance

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Roadmap

1. We propose a LASSO–based regularization procedure for realized covariance estimation.
   - It shrinks the off diagonal elements of the inverse realized covariance to zero
   - Regularized estimator can be interpreted as a partial correlation network

   We call our estimator the Realized Network

2. We establish the large sample properties of the estimator.
   - Establish conditions of consistent covariance estimation and network selection
   - We consider the vanilla Realized Covariance estimators as well as extensions that take into account for factor structure and market microstructure frictions

3. Advantages of the methodology are illustrated by means of a simulation study and an empirical illustration

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Related Literature

- ## Covariance Regularization

  Ledoit and Wolf (2004), Fan, Liao, Mincheva (2011), Ledoit and Wolf (2012), ...

  - ### Realized Covariance Regularization:

    Hautsch, Kyj and Oomen (2012); Corsi, Peluso, and Audrino (2015); Malec, Hautsch, Kyj (2015); Wang and Zhou (2010); Tao, Wang and Zhou (2013);

- ## Network Estimation in Econometrics and Statistics:

  Meinshausen & Buhlmann (2006); Brownlees & Barigozzi (2013); Diebold & Yilmaz (2013); Billio, Getmansky, Lo and Pellizzon (2012); Hautsch, Schaumburg and Schienle (2014); Medeiros & Mendes (2015);

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Framework

Covariance of the Efficient Price

- $y(t)$ is the efficient log–price of n assets at time t ($y(0) = 0$)
- The dynamics of $y(t)$ are given by

$$y(t) = \int_0^t b(u)du + \int_0^t \Theta(u)dB(u) \ ,$$

where $B(u)$ Brownian motion and $\Theta(u)$ is the spot covolatility

- Integrated Covariance: the covariance matrix of daily return $y = y(1)$

$$\text{Var}(y) = \int_0^1 \Sigma(t)dt = \Sigma^\star$$

where $\Sigma(t) = \Theta(t)\Theta(t)'$.

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Partial Correlation Network

- We are interested in the partial correlation network structure of the daily return y
- The network associated with the system is an undirected graph

1. the components of y denote vertices
2. the presence of an edge between i and j denotes that i and j are partially correlated and the value of the partial correlation measures the strength of the link

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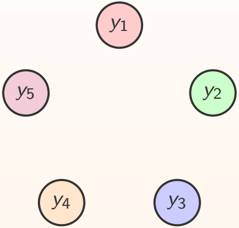

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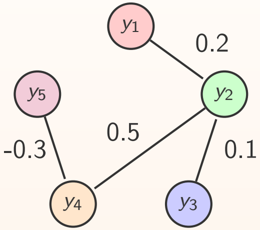

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Refresher on Partial Correlation

- Partial Correlation measures (cross-sect.) linear conditional dependence between y_i and y_j given on all other variables:

$$\rho^{ij} = \text{Cor}(y_i, y_j | \{y_k : k \neq i, j\}).$$

- Partial Correlation is related to Linear Regression:
For instance, consider the model

$$y_1 = c + \beta_{1\,2} y_2 + \beta_{1\,3} y_3 + \beta_{1\,4} y_4 + \beta_{1\,5} y_5 + u_1$$

β_{13} is different from 0 $\Leftrightarrow$ 1 and 3 are partially correlated

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If there is exist a partial correlation path between nodes i and j, then i and j are correlated (and viceversa).

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- Network is entirely characterized by the integrated concentration matrix $\mathbf{K}^\star = (\Sigma^*)^{-1} = (k^\star_{ij})$:

$$\rho^{ij} = \frac{-k^\star_{ij}}{\sqrt{k^\star_{ii} k^\star_{jj}}}$$

  In particular, the nonzero entries of $\mathbf{K}^*$ correspond to the linkages of the network.

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Estimation Strategy

- Assumption:
  1. We assume that the underlying (idiosyncratic) partial correlation network is sparse
- Objectives:
  1. Estimate the integrated covariance
  2. Detect the nonzero linkages of the network, equivalent to detecting the nonzero entries of the integrated concentration matrix.
- Strategy: We are going to tackle both objectives simultaneously by introducing a sparse integrated concentration matrix estimator
  1. We are going to introduce an appropriate estimator of the integrated covariance
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Realized Covariance

- Assume the log prices $y_i(t)$ of all assets are observed at the same grid $t_0, t_1, t_2, t_3, \ldots, t_M$
- The RC estimator is denoted by $\overline{\boldsymbol{\Sigma}}_{\text{RC}} = (\overline{\sigma}_{\text{RC},ij})$,

$$\overline{\sigma}_{\text{RC},ij} = \sum_{k=1}^{M} (y_{i\,k} - y_{i\,k-1})(y_{j\,k} - y_{j\,k-1})$$

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Realized Network Estimator

- The Realized Network Estimator is defined as

$$\widehat{\mathbf{K}} = \arg\min_{\mathbf{K}\in\mathcal{S}^n} \left\{ \mathrm{tr}(\overline{\Sigma}\mathbf{K}) - \log\det(\mathbf{K}) + \lambda \sum_{i\neq j} |k_{ij}| \right\}$$

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Theory

Preliminaries

Key ingredient to establish results is a concentration inequality of the realized volatility estimator.

- Let M denote the number of intra–daily returns used to compute the realized covariance

- Assume that the realized covariance estimator satisfies

$$\mathrm{P}\left(\left|\overline{\sigma}_{ij} - \sigma^{\star}_{ij}\right| > x\right) \leq a_1 M^{\alpha} \exp\left\{-a_2 \left(M^{\beta} x\right)^{\gamma}\right\} .$$

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Remarks

- In particular, (in the absence of microstructure noise) the classic realized volatility estimator satisfies

$$\mathrm{P}\left(\left|\overline{\sigma}_{ij} - \sigma^{\star}_{ij}\right| > x\right) \leq a_1 \exp\left\{-a_2 M x^2\right\}$$

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Realized Network Estimator Properties

- There are two main results that is interesting to establish for the Realized Network estimator:
  1. Consistent Estimation
  2. Consistent Selection
- Theory builds up on general results established by Ravikumar *et al.* (2012)

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Consistent Estimation

Theorem: Consistent Concentration Estimation

Let $\lambda = \frac{8}{\alpha} M^{-\beta} \left(\frac{\log(a_1 n^{\tau})}{a_2} \right)^{\frac{1}{\gamma}}$ *for some* $\tau > 2$.

Let

$$M > C \left(\log \left(a_2 (a_1^{\frac{1}{\beta\gamma}} C_0(d)^{\frac{1}{\beta}}) n^{\tau} \right) \right)^{\frac{1}{\beta\gamma}} C_0(d)^{\frac{1}{\beta}},$$

where C_0 *is a function of the max vertex degree* d

Then, for n *sufficiently large*

$$\mathrm{P} \left(||\widehat{\mathbf{K}} - \mathbf{K}^{\star}||_{\infty} \leq 2 C_{\Gamma^*} \left(1 + \frac{8}{\alpha} \right) M^{-\beta} \left[\frac{\log \left(a_1 M^{\alpha} n^{\tau} \right)}{a_2} \right]^{\frac{1}{a_0}} \right) \geq 1 - \frac{1}{n^{\tau - 2}}$$

where $C_{\Gamma^{\star}}$ *is a constant that depends on* Σ.

Consistent Selection

Theorem: Consistent Network Selection

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where C_1 *is a function of the max vertex degree* d.
Then, for n *sufficiently large*

$$\mathrm{P}\left(\mathrm{sign}(\widehat{k}_{ij}) = \mathrm{sign}(k^\star_{ij}), \forall i, j \in \{1, \ldots, n\} \right) \geq 1 - \frac{1}{n^{\tau - 2}}.$$

Estimator Precision and Sparsity

- It is useful to analyse how the expression simplify depending on the degree of sparsity of the networks for the realized volatility estimator without noise and asynchronicity

  1. If the max degree d is zero, then the sample size M has to be at least $O((\log n))$
  2. If the max degree d is $O(n)$, then the sample size M has to be at least $O((\log n)n^2)$

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Extensions

Factors

- In finance, it is customary to assume that returns have a factor structure. In practice, it is more interesting to analyse the partial correlation structure of assets conditional on the factors
- To this extent we assume factors to be observed and we augment our system $y(t)$ with their corresponding efficient price processes
- The covariance matrix of the augmented system can be expressed as

$$\Sigma^{\star} = \begin{bmatrix} \Sigma^{\star}_{AA} & \Sigma^{\star}_{FA} \\ \Sigma^{\star}_{AF} & \Sigma^{\star}_{FF} \end{bmatrix},$$

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Factors

- Then the covariance of the assets can be expressed as

$$\boldsymbol{\Sigma}^{\star}_{AA} = \mathbf{B}\boldsymbol{\Sigma}^{\star}_{FF}\mathbf{B}' + \boldsymbol{\Sigma}^{\star}_{I},$$

where

$$\mathbf{B} = \boldsymbol{\Sigma}^{\star}_{AF}\left[\boldsymbol{\Sigma}^{\star}_{FF}\right]^{-1} \text{ and } \boldsymbol{\Sigma}^{\star}_{I} = \boldsymbol{\Sigma}^{\star}_{AA} - \boldsymbol{\Sigma}^{\star}_{AF}\left[\boldsymbol{\Sigma}^{\star}_{FF}\right]^{-1}\boldsymbol{\Sigma}^{\star}_{FA}.$$

- Notice, that if the factor is pervasive (i.e. $\mathbf{B}$ is not sparse), then the concentration matrix of the assets is not sparse

- In this case it is natural to define the network on the basis of the idiosyncratic covariance matrix $\boldsymbol{\Sigma}^{\star}_{I}$. We call this the idiosyncratic partial correlation network.

Factors

- Then the covariance of the assets can be expressed as

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Factors: Estimation Strategy

- We estimate the idiosyncratic realized network applying GLASSO to

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(We show that if $\overline{\Sigma}$ satisfies our concentration assumption, then $\overline{\Sigma}_I$ also does.)

- We estimate the covariance of the assets by

$$\widehat{\Sigma}_{AA} = \overline{\mathbf{B}}\,\overline{\Sigma}_{FF}\,\overline{\mathbf{B}}' + \widehat{\Sigma}_{I\,\lambda},$$

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Microstructure Noise & Asynchronous Trading

- It is customary to assume that the econometrician does not observe the efficient price y but a contaminated version x defined as

$$x_i(t_{i\,k}) = y_i(t_{i\,k}) + u_i(t_{i\,k})$$

  where
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Microstructure Robust Covariance Estimators

- Our estimation strategy in case of microstructure noise consist of regularizing a Robust RC estimator.

- Many estimators are available in the setting we are working on. In this work we focus on the Two Scales Realized Covariance (TSRC) and Multivariate Realized Kernel (MRK) estimators based on Pairwise–Refresh Time.

- Fan et al. (2012) establish a concentration inequality for the TSRC estimator that allow us to use our theorem for this estimator. For the MRK estimator we develop a novel concentration inequality which allows us to apply the theory.

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Two–Scale Realized Covariance (TSRC)

- Let $(x^r_{i\,k}, x^r_{j\,k})$ denote the "pairwise refresh time" adjusted observed prices for stock i and j

- The TSRC estimator is denoted by $\overline{\mathbf{\Sigma}}_{\text{TS}} = \left(\overline{\sigma}_{\text{TS},ij}\right)$,

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where $m_K = \frac{m-K+1}{K}$ and $m_J = \frac{m-J+1}{J}$.

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Simulation Study

Simulation Study

- Simulation study used to analyse the finite sample properties of the procedure agains a number of benchmarks using different specifications for the covariance matrix.

- We simulate a $n = 50$ dimensional system with the following features:
  1. Price process follows a diffusion with constant covariance Σ
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- Estimators are assessed on the basis of the RMSE (Frobenius) and Stein's Kullback–Leibler (KL) Loss

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Simulation Study

In particular

- Three simulation settings:
  1. Design 1: Covariance Matrix with Network Structure
  2. Design 2: Covariance Matrix with Factor Structure
  3. Design 3: Covariance Matrix with Spatial Structure
- Three estimators
  1. Realized Covariance
  2. Two Scales Realized Covariance
  3. Multivariate Realized Kernel
- Four regularization procedures
  1. No Regularization
  2. Shrinkage Regularization (Ledoit & Wolf, 2004)
  3. Factor Regularization (POET, Fan, Liao and Mincheva, 2012)
  4. Network Regularization

(Notice that eigenvalue cleaning has to be applied in some instances too)

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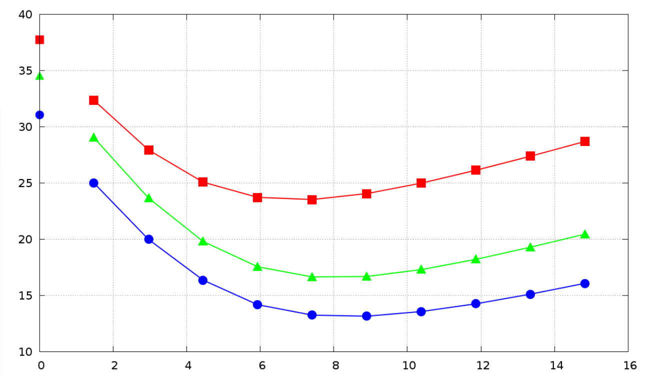
Simulation Study

		No regular.	Shrinkage	Factor	Network
		Design 1			
RC	KL	65.89	55.13	48.04	46.06
	RMSE	55.77	44.98	57.15	40.43
TSRC	KL	51.95	48.72	30.32	28.32
	RMSE	29.97	29.56	29.13	19.35
MRK	KL	53.44	48.64	36.95	30.93
	RMSE	32.50	29.56	29.38	16.94
		Design 2			
RC	KL	73.54	52.07	27.85	46.65
	RMSE	68.65	54.59	51.98	45.55
TSRC	KL	52.30	10.99	3.94	36.67
	RMSE	30.09	20.84	19.02	20.11
MRK	KL	60.67	46.32	6.19	38.45
	RMSE	31.03	22.33	23.43	26.69
		Design 3			
RC	KL	36.29	15.53	17.61	31.16
	RMSE	39.46	19.28	37.69	37.28
TSRC	KL	40.26	4.64	3.35	9.36
	RMSE	15.95	10.65	12.01	12.09
MRK	KL	28.34	4.59	5.43	7.10
	RMSE	19.38	10.52	19.49	18.38

RMSE vs Shrinkage Parameter



RMSE as a function of the tuning parameter λ for the realized volatility (square), realized kernel (triangle) and two scales (circle)

Empirical Application

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- We consider a panel of 96 NYSE Bluechips
  (≈ constituents of the S&P 100)

- We estimate realized covariance for each week of 2009
  using the the last weekday of data available

- Realized covariance is estimated using the
  Realized Network estimator based on TSRC.
  (tuning parameter λ chosen via the BIC)

- Estimators are computed using trade prices from the NYSE–TAQ
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- We focus on the idiosyncratic covariance matrix
  We analyse interdependence conditional on the market factor.

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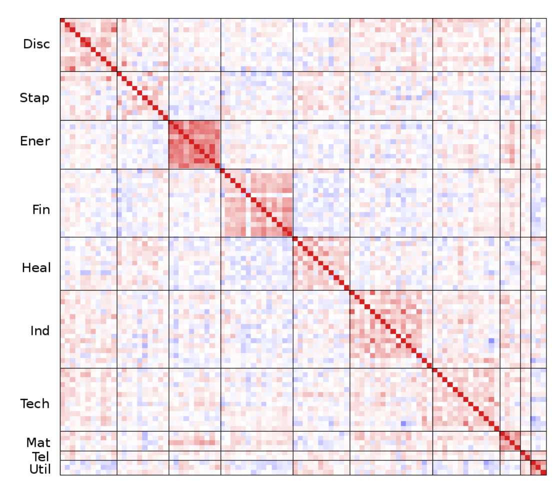
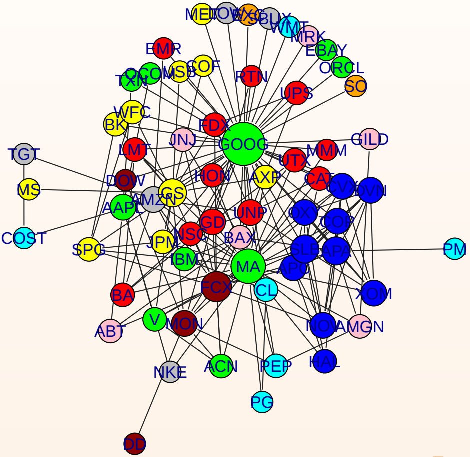
Realized Network Estimates

Realized Correlation Heatmap on 2009-07-02



Realized Network Estimates

Realized Network on 2009-07-02



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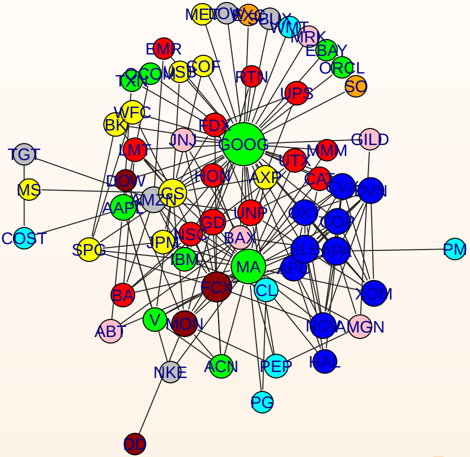
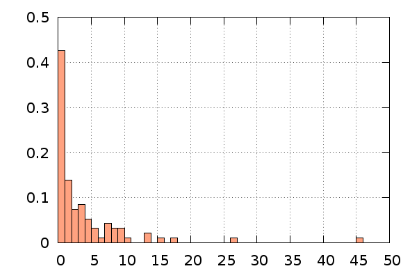


Degree and Partial Correlation Distribution



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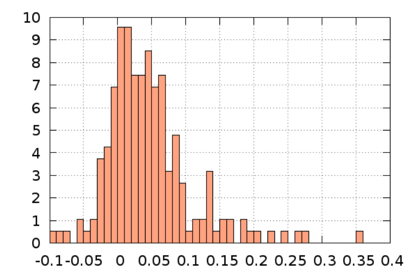

Partial Correlation

Predictive Analysis

- GMV portfolio prediction exercise:
  1. Construct the GMV portfolio weights using the MRK

     Competitors: Unconstrained, Constrained, Shrinkage and Realized Network
  2. Use the weights to construct daily GMV portfolio for the following week.
  3. Compute the variance of the daily portfolios over the full year

- More precise covariance estimators deliver GMV portfolio weights that generate smaller out-of–sample portfolio variances

  (cf Engle and Colacito, 2006)

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Predictive Analysis: GMV Comparison

	No regular	Diagonal	Network	Shrinkange	Factor	Block-Factor
RC	39.10	40.53	26.16	31.86	31.38	32.68
TSRC	37.22	41.41	26.22	29.38	30.60	31.58
MRK	32.83	35.51	24.52	27.81	28.49	28.02

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- Propose a LASSO regularization procedure for realized covariance estimators. We call the regularized estimator Realized Network.

- Highlights:
    1. The procedure delivers more precise estimates of the covariance when the partial correlation structure of the assets is sparse.
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Questions?

Thanks!