A Distributed Lag Approach
to the Generalised Dynamic Factor Model (GDFM)

Research Seminar, Vienna University of Economics and Business

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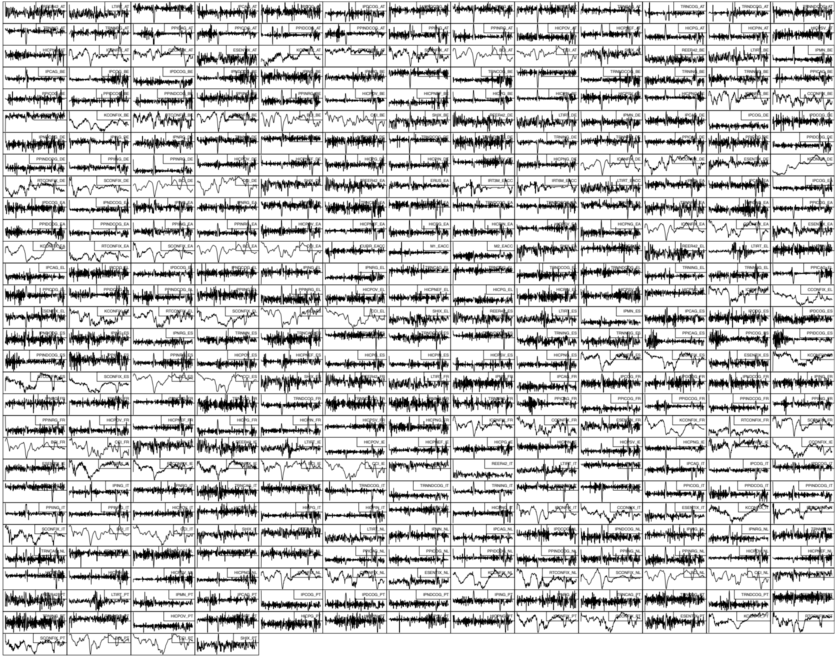


High-Dimensional Time Series

$$(y_{it} : i \in \mathbb{N}, t \in \mathbb{Z}) = (y_{it})$$

$$y_t^n = \begin{pmatrix} y_{1t} \\ \vdots \\ y_{nt} \end{pmatrix} \qquad y_t^{n+1} = \begin{bmatrix} y_t^n \\ y_{n+1,t} \end{bmatrix} \qquad y_t = \begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \end{pmatrix}$$

Assumption A0: Assume (y_t^n) is zero-mean, stationary with existing spectral density for all n, throughout.

The Generalised Dynamic Factor Model

$$y_{it} = \chi_{it} + \xi_{it} = \underline{k}_i(L) \underbrace{\varepsilon_t}_{q \times 1} + \xi_t = \sum_{j=0}^{\infty} K_i(j) \varepsilon_{t-j} + \xi_{it}, \qquad \varepsilon_t \sim WN(I_q)$$

- orthogonality at all leads and lags between the dynamic idiosyncratic an the dynamic common component
- original model by Forni and Lippi (2001) is two-sided in the dynamic factors
- existence of a one-sided representation that is causally subordinated to (y_{it}) has been shown by Forni et al. (2015) for the ARMA case and by Gersing (2024b) for the PND case.
- exploding/bounded eigenvalues in the spectral density of χ_t^n and ξ_t^n respectively.

The Static Approach

$$y_{it} = C_{it} + e_{it} = \Lambda_i F_t + e_{it}$$

- contemporaneous orthogonality between factors and idiosyncratic component
- exploding/bounded eigenvalues of the variance matrix of C_t^n and e_t^n respectively.

A Distributed Lag Approach to the GDFM

$$y_{it} = \underbrace{\beta_{i0} F_t + \ldots + \beta_{ip} F_{t-p}}_{\chi_{it}} + \xi_{it}$$

1. extract factors F_t by static PCA
2. Regress y_{it} on estimated factors + lags

+) simpler to use and understand than so far existing frequency domain based approaches Forni et al. (2000, 2005, 2017); Barigozzi (2022)

+) Incorporate weak factors ⇒ Correct Impulse Responses, Forecasting, etc.

+) Use dynamic structure to achieve better finite sample performance than static PCA: even for pervasive factors

+) canonical decomposition of factor models (Gersing et al., 2023) immediately implied

The Canonical Decomposition of Factor Models

$$y_{it} = \underbrace{C_{it} + \overbrace{e^{\chi}_{it}}_{\chi_{it}} + \xi_{it}}^{e_{it}} = \Lambda_i F_t + \Lambda^w_i F^w_t + \xi_{it}$$

Summary

Static Model/static decomposition

$$y_{it} = C_{it} + e_{it} = \Lambda_i F_t + e_{it}$$

Dynamic Model/dynamic decomposition

$$y_{it} = \chi_{it} + \xi_{it} = \Lambda_i F_t + \Lambda_i^w F_t^w + \xi_{it} = \beta_{i0} F_t + ... + \beta_{ip} F_{t-p} + \xi_{it}$$

Model Assumptions and Identification Results

Dynamic and Static Factor Structure

A1 q-Dynamic Factor Structure

(i) $\sup_n \mu_q\left(f_y^n\right) = \infty$ a.e. on $[-\pi, \pi]$;

(ii) $\operatorname{ess\,sup}_\theta \sup_n \mu_{q+1}(f_y^n) < \infty$.

$\Rightarrow$ implies the dynamic decomposition $y_{it} = \chi_{it} + \xi_{it} = \underline{k}_i(L)\varepsilon_t + \xi_{it}$.

A2 r-Static Factor Structure

(i) $\sup_{n\in\mathbb{N}} \mu_r\left(\Gamma_y^n\right) = \infty$;

(ii) $\sup_{n\in\mathbb{N}} \mu_{r+1}\left(\Gamma_y^n\right) < \infty$.

$\Rightarrow$ implies the static decomposition $y_{it} = C_{it} + e_{it} = \Lambda_i F_t + e_{it}$.

Together they imply the canonical decomposition $y_{it} = \Lambda_i F_t + \Lambda_i^w F_t^w + \xi_{it}$

Fundamentalness of the Static Factors

It is quite natural to assume that the same common shocks which drive the whole economy, also drive the static factors (F_t) and are fundamental for them. Formally, we require that there is representation of the form:

$$F_t = \sum_{j=0}^{\infty} B_F(j)\varepsilon_{t-j} = \underline{b}_F(L)\varepsilon_t,$$

where $\underline{b}_F(L)$ is a causal transferfunction that has a causal left inverse, say $\underline{c}_F(L)$, of dimension $q \times r$ such that

$$\varepsilon_t = \sum_{j=0}^{\infty} C_F(j)F_{t-j} = \underline{c}_F(L)F_t.$$

Identification from Static Factors

$$\chi_{it} = \text{proj}\left(y_{it} \mid \mathbb{H}_t(\varepsilon)\right).$$

A3 Identification from Static Factors

(i) (Fundamentalness) The common shocks are fundamental for the statically pervasive factors, i.e.

$$\mathbb{H}_t(F) = \mathbb{H}_t(\varepsilon)$$

(ii) (Finite lag order) For every i the projection of y_{it} on $\mathbb{H}_t(F)$ can be expressed by at most p lags, i.e.

$$\text{proj}\left(y_{it} \mid \mathbb{H}_t(F)\right) = \chi_{it} = \beta_{i0}F_t + \cdots + \beta_{ip}F_{t-p}$$

Further Justification

Assume the well known model

$$y_{it} = \underline{\lambda}_i(L) f_t + \xi_{it} = \underbrace{\lambda_{i0} f_t + ... + \lambda_{ip} f_{t-p}}_{\chi_{it}} + \xi_{it}$$

$$= \left[\lambda_{i0} \cdots \lambda_{ip}\right] \begin{pmatrix} f_t \\ \vdots \\ f_{t-p} \end{pmatrix} + \xi_{it} = \bar{\Lambda}_i \bar{F}_t + \xi_{it}, \text{ say.}$$

$$\underline{a}(L) f_t = (1 - A_1 L - \cdots - A_{p_f} L^{p_f}) f_t = b\varepsilon_t, \qquad \varepsilon_t \sim WN(0, I_q), \text{ and } \operatorname{rk} b = q,$$

And assume (f_t) is following a VAR(p_f) process.

Obviously

$$\chi_{it} = \underline{\lambda}_i(L) \underline{a}^{-1}(L) \varepsilon_t = \underline{k}_i(L) \varepsilon_t. \qquad \text{(This is a GDFM see proof in Gersing et al. (2023)).}$$

Further Justification

Now in this representation $\chi_t^n = \bar{\Lambda}^n \bar{F}_t$, say only $r < r_\chi = q(p+1)$ eigenvalues diverge, i.e. lagged factors are less pervasive. Then

$$F_t = T\bar{F}_t \qquad T \in \mathbb{R}^{r \times r_\chi}$$

In particular this means that not all r_χ eigenvalues in $\bar{\Lambda}^{n\prime}\Lambda^n$ diverge.

State Space Representation

We can represent this as a state space system:

$$F_t = T\bar{F}_t \quad \text{(observation equation)},$$

$$\underbrace{\begin{pmatrix} f_{t+1} \\ f_t \\ \vdots \\ f_{t-p+2} \end{pmatrix}}_{\bar{F}_{t+1}} = \underbrace{\begin{bmatrix} A_1 & A_2 & \cdots & A_p \\ I_q & & & 0 \\ & \ddots & & \\ & & I_q & 0 \end{bmatrix}}_{\mathcal{A}} \underbrace{\begin{pmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-p+1} \end{pmatrix}}_{\bar{F}_t} + \underbrace{\begin{bmatrix} b \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{G} \varepsilon_{t+1}$$

$$\bar{F}_{t+1} = \mathcal{A}\bar{F}_t + G\varepsilon_{t+1}, \quad \text{(transition equation)}.$$

Further Justification

Theorem

Consider the set of minimal stable systems $(A_1, ..., A_p, b, T) \equiv (A, G, T)$, then (ε_t) is fundamental for (F_t) in the following cases:

1. For $r = q$ if $\det\left[\underline{t}(z)\right] \neq 0$ for all $|z| < 1$, where $\underline{t}(z) = T\underline{S}(z)$ with $\underline{S}(z) := \left(I_q, I_q z, ..., I_q z^{p-1}\right)'$.
2. For $r > q$ generically.

Asymptotic Theory

Motivation

- The procedure resembles a "Factor-Augmented-Regression" Bai and Ng (2006); Bernanke et al. (2005)
- The theory from Bai and Ng (2006) cannot be applied in our set up because they assume independence of (F_t) and (e_{it})
- Here: $e_{it} = \Lambda^{w,n} F_t^w + \xi_{it}$ and F_t^w is a linear transformation of $F_t, F_{t-1}, ..., F_{t-p}$ $\Rightarrow$ we must allow correlation at lags.
- In the proofs I need to use the canonical decomposition of factor models from Gersing et al. (2023) to allow for correlation at lags.
- I provide a different proof for the factor asymptotics trying to make the assumptions imposed more transparent
- First asymptotic results with heteroscedasticity over time for the GDFM

Asymptotic Theory

High level description of the Assumptions for **consistency**: **A4** and **A5**

- Global boundedness of the loadings $\|\Lambda_i\| < \mathcal{B}_\Lambda < \infty$, $\|\Lambda_i^w\| < \mathcal{B}_\Lambda$, distinct eigenvalues, $n^{-1}\Lambda^{n'}\Lambda^n \to \Gamma_\Lambda > 0$
- Global bound of the idiosyncratic variances: $\mathbb{E}\left[\xi_{it}^2\right] < \mathcal{B}_\xi < \infty$, global bound for the first eigenvalue of the variance matrix $\sup_t \sup_n \mu_1 \left(\mathbb{E}\left[\xi_t^n \xi_t^{n'}\right]\right) < \infty$, $\sup_{n\in\mathbb{N}} \mu_1 \left(\Lambda^{w,n'}\Lambda^{w,n}\right) < \mathcal{B}_{\Lambda^w}$.
- **Moment estimation**: $\hat{\Gamma}_F(h), \hat{\gamma}_\xi^{ij}(h), \hat{\Gamma}_{F\xi_i}(h)$ converge in mean square to the population second moments at the usual rates.

A Trick with Davis-Kahan

Using the variant of the Davis-Kahan-Theorem from Yu et al. (2015) used also in Barigozzi (2022).

Lemma: Population Eigenvectors and Eigenvalues

Under Assumption A2 and A4, we have

(i) $\left\|p_j\left(\Gamma_y^n\right) - p_j(\Gamma_C^n)\right\| = \mathcal{O}(n^{-1})$, for $1 \leq j \leq r$;

(ii) $|\mu_j\left(\Gamma_y^n\right) - \mu_j(\Gamma_C^n)| = \mathcal{O}(1)$ for $1 \leq j \leq r$.

With this we can show that $n^{1/2}\left\|\hat{\mathcal{K}}_j(\hat{\Gamma}_y^n) - \mathcal{K}_j(\Gamma_C^n)\right\| = \mathcal{O}_P(\max(T^{-1/2}, n^{-1}))$, while $\mathcal{K} = M^{-1/2}(\Gamma_y^n)P(\Gamma_y^n)$ and $\hat{\mathcal{K}} = \mathcal{K}(\hat{\Gamma}_y^n)$. Then it's all about killing idiosyncratic terms with $\mathcal{K}, \hat{\mathcal{K}}$...

Consistency of Factors and Loadings Space and Common Component

Let $\hat{W}_t^{y,n} := M^{-1/2}(\hat{\Gamma}_y^n)\underbrace{P(\hat{\Gamma}_y^n)}_{r\times n} y_t^n$, (sample normalised principal components)

$\hat{x}_t := \left(\hat{W}_t^{y,n'}, ..., \hat{W}_{t-p}^{y,n'}\right)'$ (stacked version)

Theorem

Under Assumptions A1-A3, with $\hat{H} = \hat{W}'F(F'F)^{-1}$, $\hat{\mathcal{H}} = \hat{x}'x(x'x)^{-1}$, $\hat{H}_\Lambda = \left(\Lambda^{n'}\Lambda^n\right)^{-1}\Lambda^{n'}\hat{\Lambda}^n$

(i) $\left\|\hat{W}_t^{y,n} - \hat{H}F_t\right\| = \mathcal{O}_P(\max(n^{-1/2}, T^{-1/2}))$ and $\left\|\hat{x}_t - \hat{\mathcal{H}}x_t\right\| = \mathcal{O}_P(\max(n^{-1/2}, T^{-1/2}))$

(ii) $\left\|\hat{\Lambda}_i - \Lambda_i\hat{H}_\Lambda\right\| = \mathcal{O}_P(\max(n^{-1/2}, T^{-1/2}))$ and $\left\|\hat{\beta}_i - \beta_i\hat{\mathcal{H}}^{-1}\right\| = \mathcal{O}_P(\max(n^{-1/2}, T^{-1/2}))$

(iii) $\left\|\hat{C}_{it} - C_{it}\right\| = \mathcal{O}_P(\max(n^{-1/2}, T^{-1/2}))$, $\|\hat{\chi}_{it} - \chi_{it}\| = \mathcal{O}_P(\max(n^{-1/2}, T^{-1/2}))$ and $\|\hat{e}_{it}^{\chi} - e_{it}^{\chi}\| = \mathcal{O}_P(\max(n^{-1/2}, T^{-1/2}))$

Asymptotic Theory

Assumptions for **Asymptotic Normality**: **A6** and **A7**

- CLTs for $T^{-1/2}\sum_{t=1}^{T} x_t\xi_{it}$ and $\left(I_{p+1}\otimes n^{-1/2}\Lambda^{n\prime}\right)(\xi_t^{n\prime},...,\xi_{t-p}^{n\prime})'$
- Stronger conditions limiting the serial correlation in the idiosyncratic component.

Asymptotic Normality of the Loadings

Theorem

Under Assumptions A0-A7, as $\sqrt{T}/n \to 0$, with $\widehat{\mathcal{H}} = T^{-1}\sum_{t=1}^{T} \hat{x}_t x_t' \left(T^{-1}\sum_{t=1}^{T} x_t x_t'\right)^{-1}$, we have

$$\sqrt{T}\left(\hat{\beta}_i - \beta_i \widehat{\mathcal{H}}^{-1}\right) \Rightarrow \mathcal{N}\left(0, asy\Gamma_{\hat{\beta}_i}\right)$$

The asymptotic variance $asy\Gamma_{\hat{\beta}_i}$ is given by

$$asy\Gamma_{\hat{\beta}_i} := \Gamma_x^{-1}\left(I_{p+1} \otimes P_\Lambda\right)\Omega_{x\xi}(i)\left(I_{p+1} \otimes P_\Lambda'\right)\Gamma_x^{-1}. \tag{1}$$

Estimation Following Bai (2003); Bai and Ng (2006); Barigozzi (2022) for estimating the middle term, robust to heteroskedasticity, we may use either one of the following:

$$\left(I_{p+1} \otimes P_\Lambda\right)\widehat{\Omega_{x\xi}(i)}\left(I_{p+1} \otimes P_\Lambda\right)' = \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T} \hat{x}_t \hat{\xi}_{it} \hat{\xi}_{is} \widehat{x}_s' \kappa(t,s) \tag{2}$$

where $\kappa(t,s)$ is a suitable kernel with bandwidth M_T.
The final estimator for the asymptotic variance is then given by

$$asy\hat{\Gamma}_{\hat{\beta}_i} = \left(\frac{1}{T}\sum_{t=1}^{T} \hat{x}_t \hat{x}_t'\right)^{-1} \left(I_{p+1} \otimes P_\Lambda\right)\widehat{\Omega_{x\xi}(i)}\left(I_{p+1} \otimes P_\Lambda\right)' \left(\frac{1}{T}\sum_{t=1}^{T} \hat{x}_t \hat{x}_t'\right)^{-1}.$$

Asymptotic Normality of the Factors

Theorem

Under Assumptions A0-A7, as $\sqrt{n}/T \to 0$, we have

$$\sqrt{n}\left(\hat{x}_t - \widehat{\mathcal{H}} x_t\right) \Rightarrow \mathcal{N}(0, asy\Gamma_{\hat{x}_t})$$

The asymptotic variance $asy\Gamma_{\hat{x}_t}$ is given by

$$asy\Gamma_{\hat{x}_t} := \left(I_{p+1} \otimes D_{\Lambda}^{-1} P_{\Lambda}\right) \Theta_{\Lambda\Xi}(t) \left(I_{p+1} \otimes P_{\Lambda}' D_{\Lambda}^{-1}\right).$$

Asymptotic Normality of the Dynamic Common Component

Theorem
Under Assumptions A0-A7, as $\sqrt{n}/T \to 0$ and $\sqrt{T}/n \to 0$, we have

$$\frac{\hat{\chi}_{it} - \chi_{it}}{\sqrt{\frac{1}{T} U_{it} + \frac{1}{n} V_{it}}} \Rightarrow \mathcal{N}(0,1),$$

where $U_{it} := x_t' \Gamma_x^{-1} \Omega_{x\xi}(i) \Gamma_x^{-1} x_t$ and $V_{it} = \beta_i \left(I_{p+1} \otimes \Gamma_\Lambda^{-1} \right) \Theta_{\Lambda\Xi}(t) \left(I_{p+1} \otimes \Gamma_\Lambda^{-1} \right) \beta_i'$.

Asymptotic Normality of the Weak Common Component

Theorem

Under Assumptions A0-A7, as $\sqrt{n}/T \to 0$ and $\sqrt{T}/n \to 0$, we have

$$\frac{\hat{e}_{it}^{\chi} - e_{it}^{\chi}}{\sqrt{\frac{1}{T}U_{it} + \frac{1}{n}V_{it}}} \Rightarrow \mathcal{N}(0,1),$$

where

$$\begin{aligned} U_{it} &:= x_t'\Gamma_x^{-1}\Omega_{x\xi}(i)\Gamma_x^{-1}x_t + F_t'\Omega_{Fe}(i)F_t - 2x_t'\Gamma_x^{-1}\Omega_{x\xi,Fe}(i)F_t \\ V_{it} &:= \beta_i\left(I_{p+1}\otimes\Gamma_\Lambda^{-1}\right)\Theta_{\Lambda\Xi}(t)\left(I_{p+1}\otimes\Gamma_\Lambda^{-1}\right)\beta_i' + \Lambda_i\Gamma_\Lambda^{-1}\Theta_{\Lambda e}(t)\Gamma_\Lambda^{-1}\Lambda_i' \\ &\quad - 2\beta_i\left(I_{p+1}\otimes\Gamma_\Lambda^{-1}\right)\Theta_{\Lambda\Xi,\Lambda e}(t)\Gamma_\Lambda^{-1}\Lambda_i'. \end{aligned}$$

Simulations

Simulation

$$y_{it} = \underbrace{C_{it} + \overbrace{e^{\chi}_{it}}}_{\chi_{it}}\overbrace{\quad + \xi_{it}}^{e_{it}} = \Lambda_i F_t + \Lambda^w_i F^w_t + \xi_{it}$$

$$y_{it} = \chi_{it} + \xi_{it} = \underbrace{\lambda_{i1} f_t + \lambda_{i2} f_{t-1}}_{\chi_{it}} + \xi_{it}$$

$$f_t = a f_{t-1} + \varepsilon_t \qquad \text{with } \varepsilon_t \sim iidN(0, \underbrace{\sqrt{1-a^2}}_{\sigma_\varepsilon}), \quad \text{so } \mathbb{V}\, f_t = 1$$

Suppose

- $\sum_{i=1}^{n} \lambda_{i1}^2 \to \infty$ for $n \to \infty$ (pervasive)
- $\sup_n \sum_{i=1}^{n} \lambda_{i2}^2 < \infty$ (non-pervasive)

DGPs for the Dynamic Idiosyncratic Component

We consider different DGPs for the dynamic idiosyncratic component, based on the specification

$$\xi_{it} = \alpha_i \xi_{i,t-1} + \varepsilon_{it}^{\xi}$$

with $\varepsilon_{it}^{\xi} \sim iidN(0, \sigma_{\xi i}^2)$ and independent of (ε_t) the shocks of the factor process, with $\mathbb{E}(\varepsilon_{it}^{\xi}\varepsilon_{jt}^{\xi}) = \tau^{i-j}, i, j = 1, ..., n$, with $\tau \in \{0, 0.5\}$ if $i - j \leq 10$ and $\mathbb{E}(\varepsilon_{it}^{\xi}\varepsilon_{jt}^{\xi}) = 0$ otherwise; last $\alpha_i = \{0, \delta_i\}$ with $\delta_i \sim iidU(0, \delta)$ and $\delta \in \{0, 0.5\}$. The parameters τ and δ are crucial to control the cross-sectional and serial correlation in the dynamic idiosyncratic component, respectively.

Side note: Impulse Responses

$$y_{it} = \underbrace{C_{it} + \overbrace{e^{\chi}_{it}}^{e_{it}}}_{\chi_{it}} + \xi_{it} = \Lambda_i F_t + \Lambda_i^w F_t^w + \xi_{it}$$

$$y_{it} = \chi_{it} + \xi_{it} = \underbrace{\lambda_{i1} f_t + \lambda_{i2} f_{t-1}}_{\chi_{it}} + \xi_{it} \qquad \text{with } \xi_{it} \sim iidN(0,1)$$

$$f_t = a f_{t-1} + \varepsilon_t \qquad \text{with } \varepsilon_t \sim iidN(0, \sqrt{1-a^2}), \quad \text{so } \mathbb{V} f_t = 1$$

- Transfer-function of the static common component:

$$\underline{k}_i^C(L) = (\lambda_{i1} + \lambda_{i2} a)(1 - aL)^{-1}$$

- Transfer-function of the dynamic common component:

$$\underline{k}_i^{\chi}(L) = (\lambda_{i1} + \lambda_{i2} L)(1 - aL)^{-1}$$

Simulation

DGP 1: including weak factors

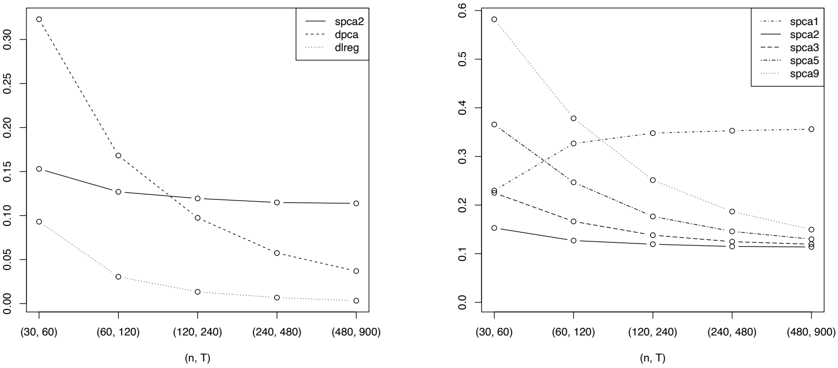
$$y_t^n = \begin{bmatrix} 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \end{bmatrix} \begin{pmatrix} f_{1t} \\ f_{2t} \end{pmatrix} + \xi_t^n$$

which yields

$$C_{it} = af_t, \quad e_{it}^{\chi} = f_{t-1} - af_t \quad \text{for } 1 \leq i \leq 10$$

$$C_{it} = f_t, \quad e_{it}^{\chi} = 0 \quad \text{else.}$$

Simulation: Consistency and Finite Sample Performance

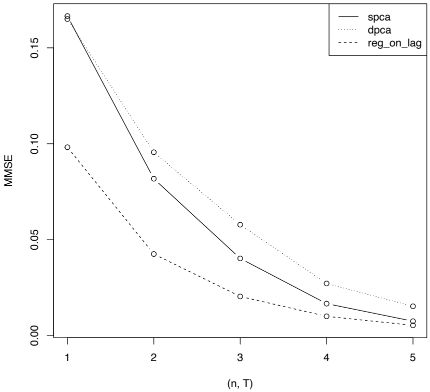


DGP1: Simulation Mean Squared Error of $\chi_{1,t}$: 500 replications,
$(n, T) = (30, 60), (60, 120), (120, 240), (240, 480), (480, 900)$,
spca: estimation with $r = 2$ principal components, **reg_on_lag**: regression on first pc and lag, **dpca**: estimation with

$q = 1$ dynamic principal components. PC with non-perv. factors not consistent, see Onatski (2012).

Simulation - Reconsider Efficiency

DGP2: No weak factors, Gersing et al. (2023)



DGP2: $\tilde{\lambda}_{i1} \sim iidU(1,1), \tilde{\lambda}_{i2} \sim iidU(0.1, 0.1)$. Simulation Mean Squared Error of avg($\chi_{i,t}$): 500 replications , $(n,T) = (30,60), (60,120), (120,240), (240,480), (480,900)$

spca: $r = 2$ PCs, **reg_on_lag**: regression on first PC and lag, **dpca**: $q = 1$ dynamic PCs

Coverage Rates for Confidence Intervals

(n, T)	(30,60)	(60,120)	(120,240)	(240,480)	(480,900)
e_{1t}^{χ}, $\tau = 0$, $\delta = 0$	0.653	0.904	0.932	0.944	0.950
χ_{1t}, $\tau = 0$, $\delta = 0$	0.826	0.930	0.948	0.948	0.934
C_{1t}, $\tau = 0$, $\delta = 0$	0.846	0.932	0.924	0.942	0.934
e_{1t}^{χ}, $\tau = 0.5$, $\delta = 0$	0.544	0.854	0.892	0.916	0.918
χ_{1t}, $\tau = 0.5$, $\delta = 0$	0.768	0.892	0.910	0.920	0.946
C_{1t}, $\tau = 0.5$, $\delta = 0$	0.802	0.896	0.922	0.914	0.926
e_{1t}^{χ}, $\tau = 0$, $\delta = 0.5$	0.602	0.892	0.922	0.932	0.956
χ_{1t}, $\tau = 0$, $\delta = 0.5$	0.844	0.918	0.942	0.932	0.936
C_{1t}, $\tau = 0$, $\delta = 0.5$	0.806	0.950	0.934	0.956	0.948
e_{1t}^{χ}, $\tau = 0.5$, $\delta = 0.5$	0.560	0.874	0.894	0.926	0.928
χ_{1t}, $\tau = 0.5$, $\delta = 0.5$	0.764	0.882	0.910	0.926	0.932
C_{1t}, $\tau = 0.5$, $\delta = 0.5$	0.812	0.896	0.938	0.946	0.948

Empirical Application

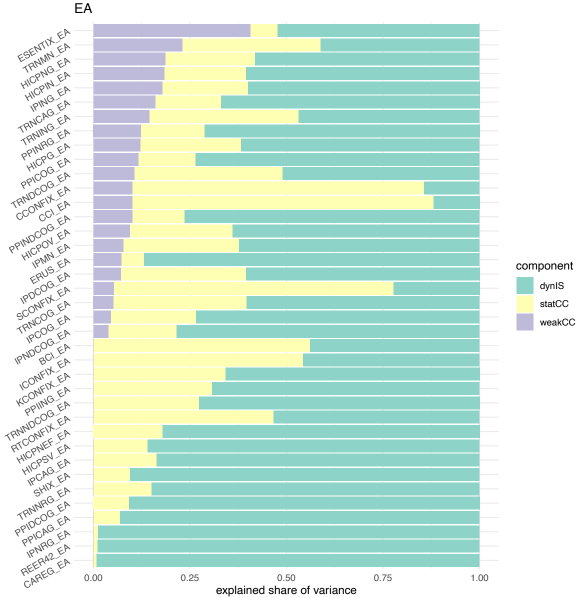
Monthly Euro Area Macroeconomic Data from all Major Economies

Barigozzi et al. (2024)

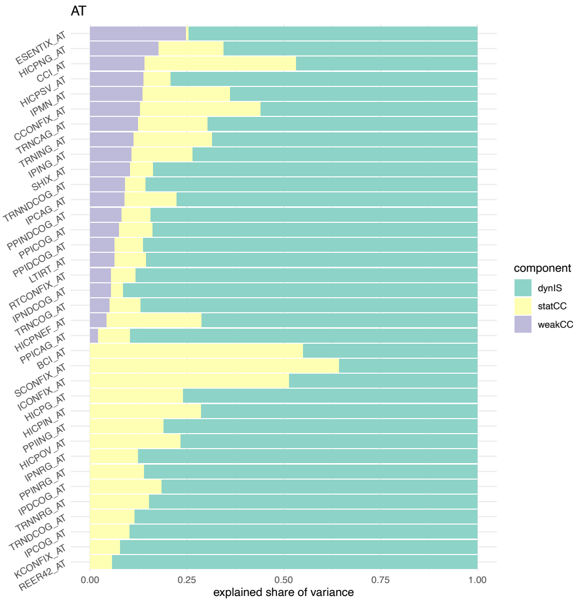
Data contains COVID-period, preprocessing: stationarity transformation, removal of univariate + multivariate outliers for parameter estimation. Compute Factors and Components with original raw data.

1) Estimate F_t, i.e. x_t
2) Select with BIC lag order for each i with $p_{max} = 12$, then regress $y_{it} \to \hat{W}_t, \hat{W}_{t-1}, ..., \hat{W}_{t-p}$.
3) Estimate standard deviations and compute confidence intervals.

Whole Euro Area



Austria



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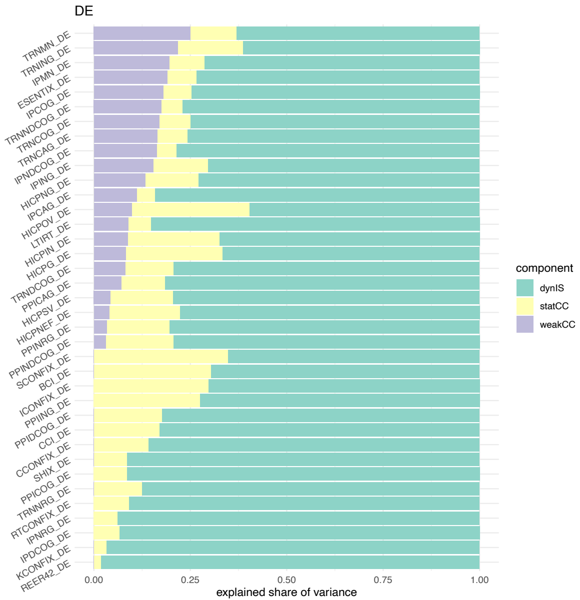

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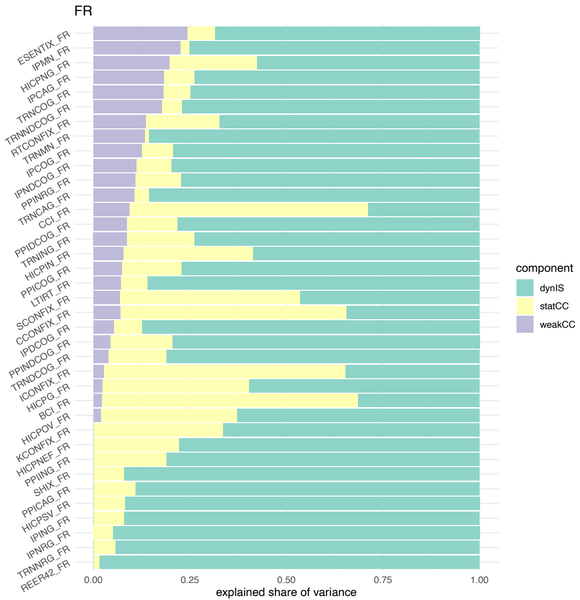
France



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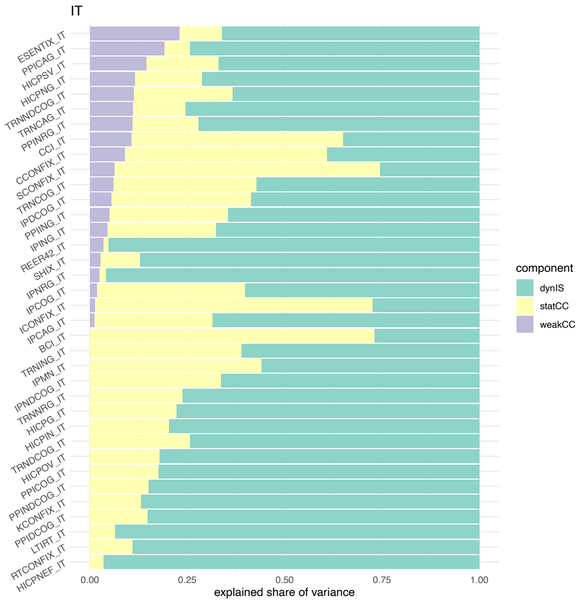
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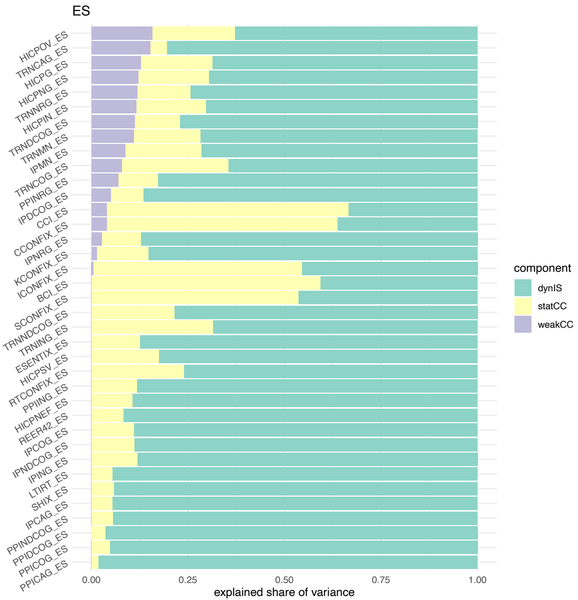

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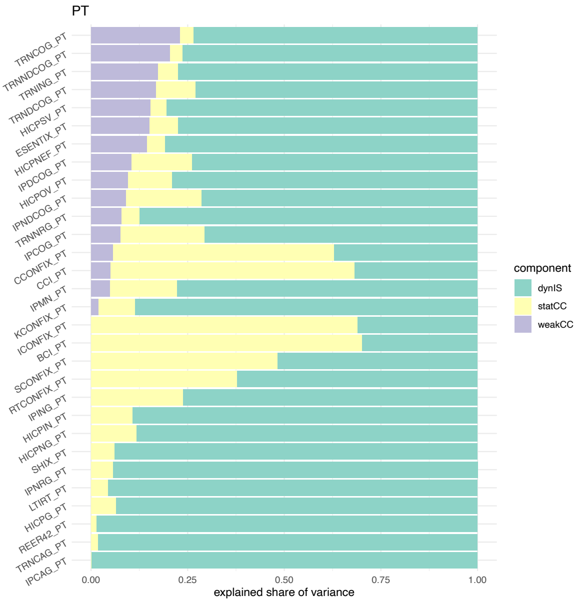

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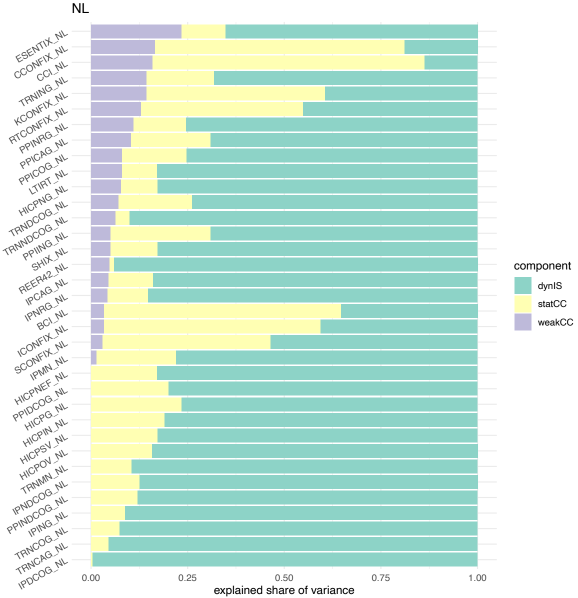
Netherlands



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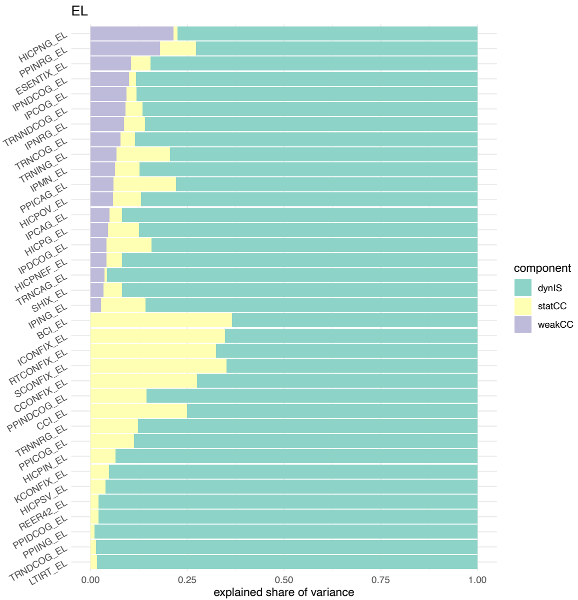

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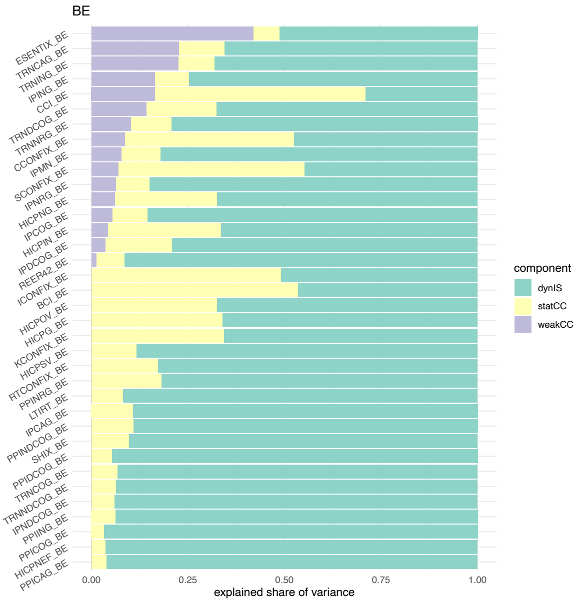
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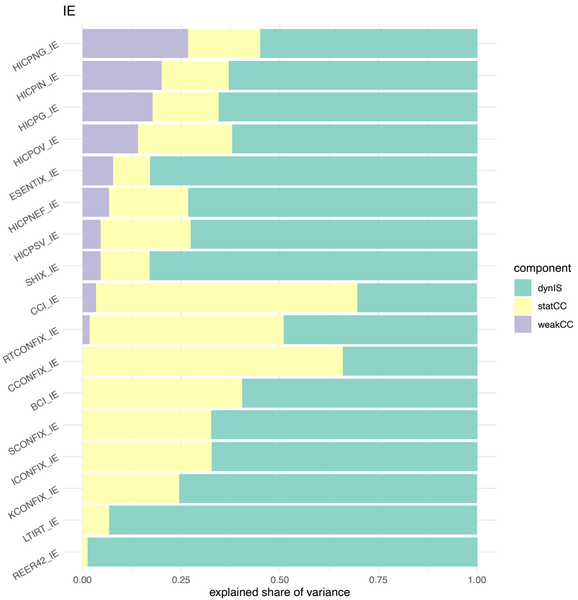
Ireland



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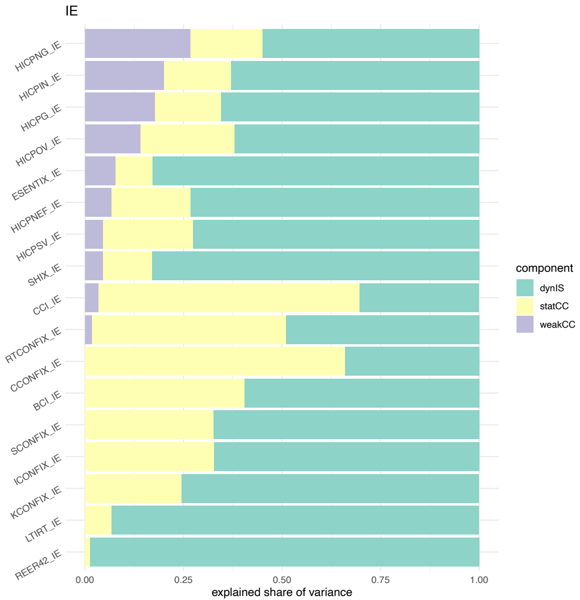
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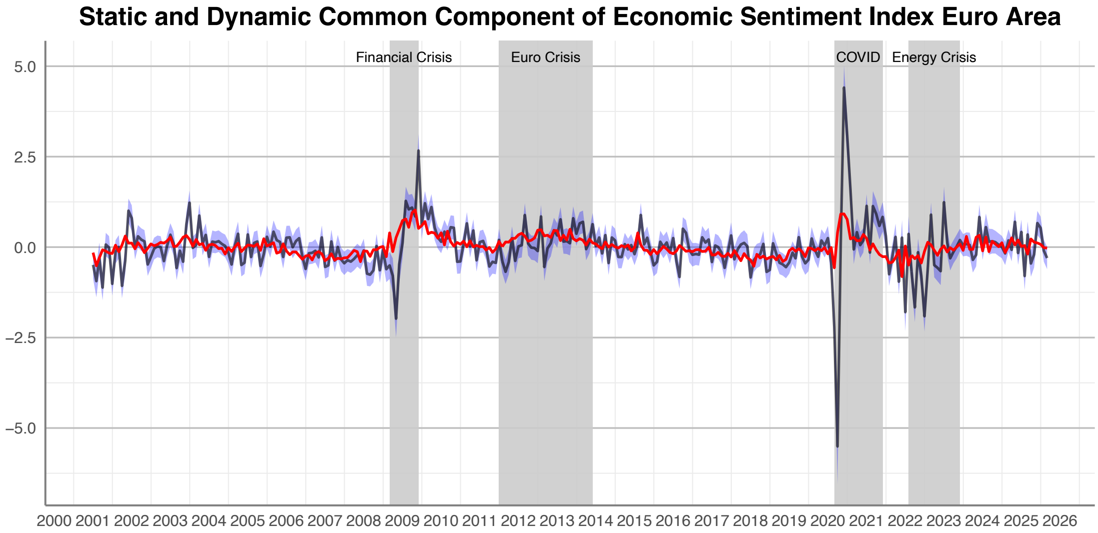
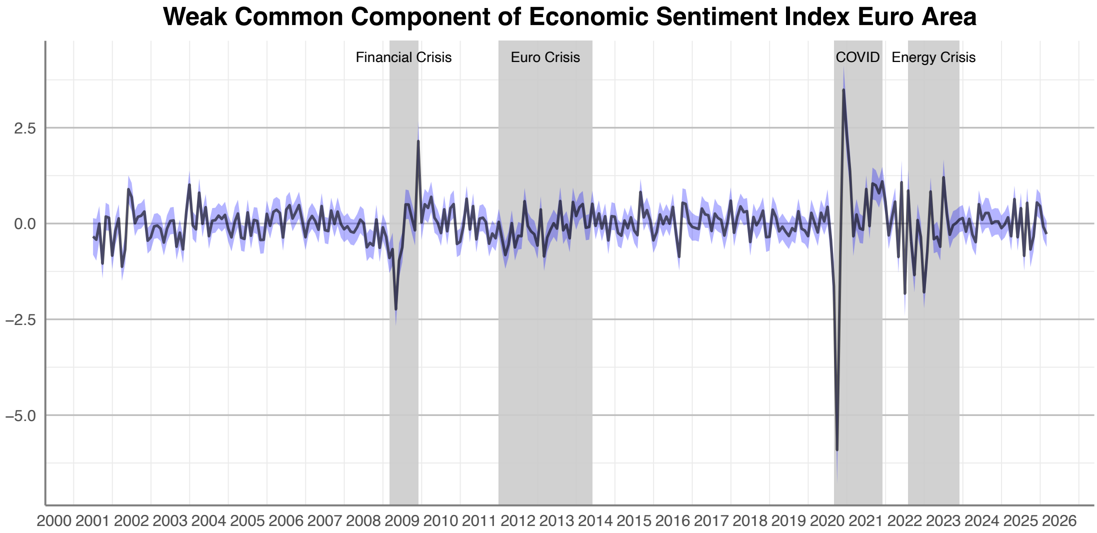
Confidence Intervals for the Dyn. and the Stat. Common Component



Confidence Intervals for the Weak Common Component



Conclusion

$$y_{it} = \underbrace{C_{it} + \overbrace{e^{\chi}_{it}}_{\chi_{it}} + \xi_{it}}^{e_{it}} = \Lambda_i F_t + \Lambda_i^w F_t^w + \xi_{it}$$

- Simple way to estimate the GDFM and the canonical decomposition above
- Even if there are no weak factors: efficiency gains from regressing on lags
- Lots of empirical evidence for the presence of weak factors and consequently a difference between the GDFM and the static approach

Thank You

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