

# Deep learning models for distributional regression

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# Outline

- ▶ Probabilistic forecasting and comparative model assessment
- ▶ Motivation: Post-processing ensemble weather simulations
- ▶ Neural networks for distributional regression
- ▶ Advanced machine learning methods for incorporating complex sources of information

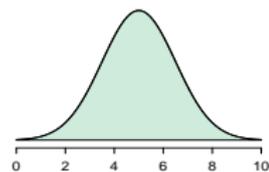
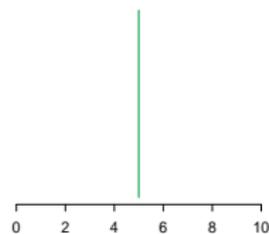
# Probabilistic forecasting

Model predictions should be probabilistic (given as a parametric or simulation-based probability distribution) to

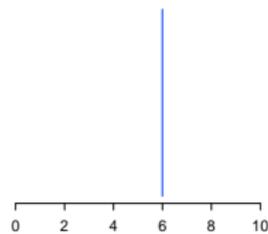
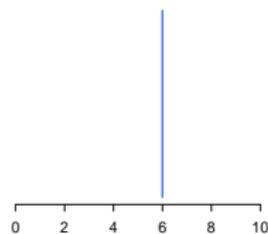
- ▶ quantify inherent uncertainty
- ▶ allow for optimal decision making by obtaining target functionals (mean, quantiles, ...) of the predictive distributions
- ▶ meet increasing popularity and requests across disciplines, in particular in economics and environmental sciences.

# Deterministic and probabilistic predictions

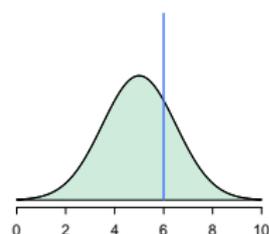
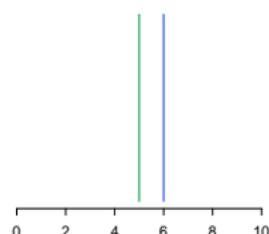
Forecast



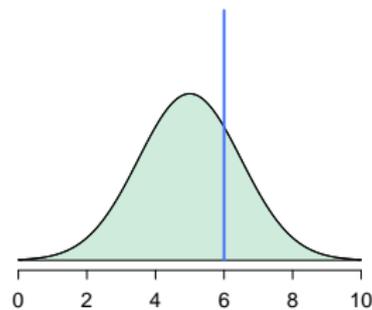
Observation



Comparison



# Evaluation of probabilistic forecasts: Proper scoring rules



A (negatively oriented) **proper scoring rule** is any function

$$S(F, y)$$

such that for all  $F, G$ ,

$$\mathbb{E}_{Y \sim G} S(G, Y) \leq \mathbb{E}_{Y \sim G} S(F, Y).$$

Popular examples include

the **logarithmic score**

$$\text{LogS}(F, y) = -\log(f(y))$$

the **continuous ranked probability score**

$$\text{CRPS}(F, y) = \int_{-\infty}^{\infty} (F(z) - \mathbb{1}\{y \leq z\})^2 dz$$

# Proper scoring rules as tools for model estimation

Proper scoring rules provide useful tools for [parameter estimation](#) in an [M-estimation](#) framework: Determine

$$\hat{\theta} = \arg \min_{\theta \in \Theta} \sum_{i=1}^n S(F_{\theta}, y_i).$$

LogS yields maximum likelihood (ML) estimation, the CRPS provides a robust alternative.

[Computational tools](#): Efficient implementations for parametric and simulation-based predictive models for optimization and large scale evaluation: [R package scoringRules](#).

Jordan, A., Krüger, F. and Lerch, S. (2019)

**Evaluating probabilistic forecasts with scoringRules.**

*Journal of Statistical Software*, 90, 1–37.

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# Numerical weather prediction

Modern weather forecasts rely on physical **numerical weather prediction (NWP)** models of atmospheric processes.

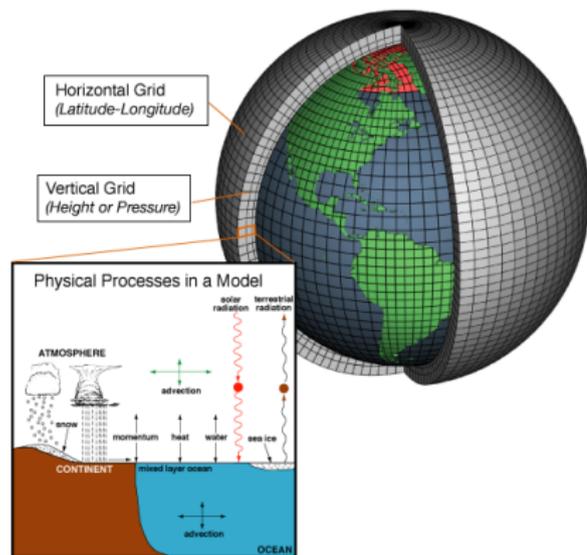


Image source: NOAA<sup>1</sup>

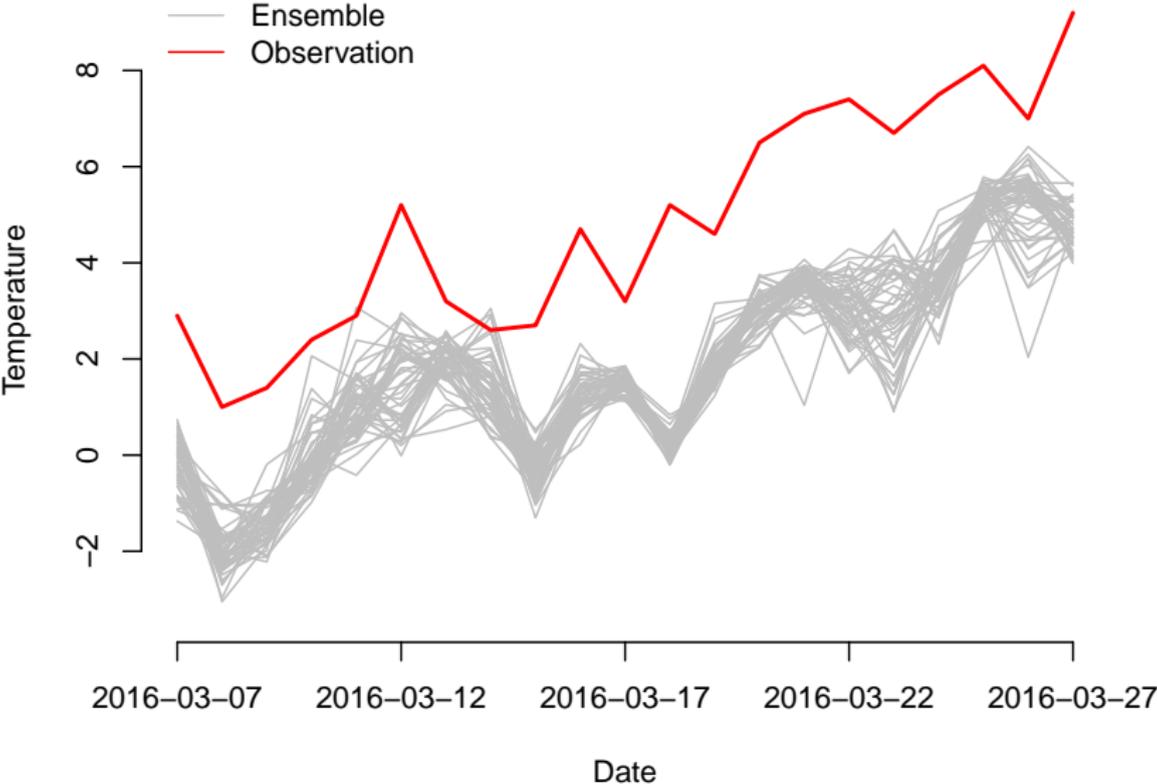
However, there are major sources of **uncertainty** (**initial conditions**, **physical models**).

**Ensemble** simulations seek to quantify uncertainty and provide probabilistic forecasts.

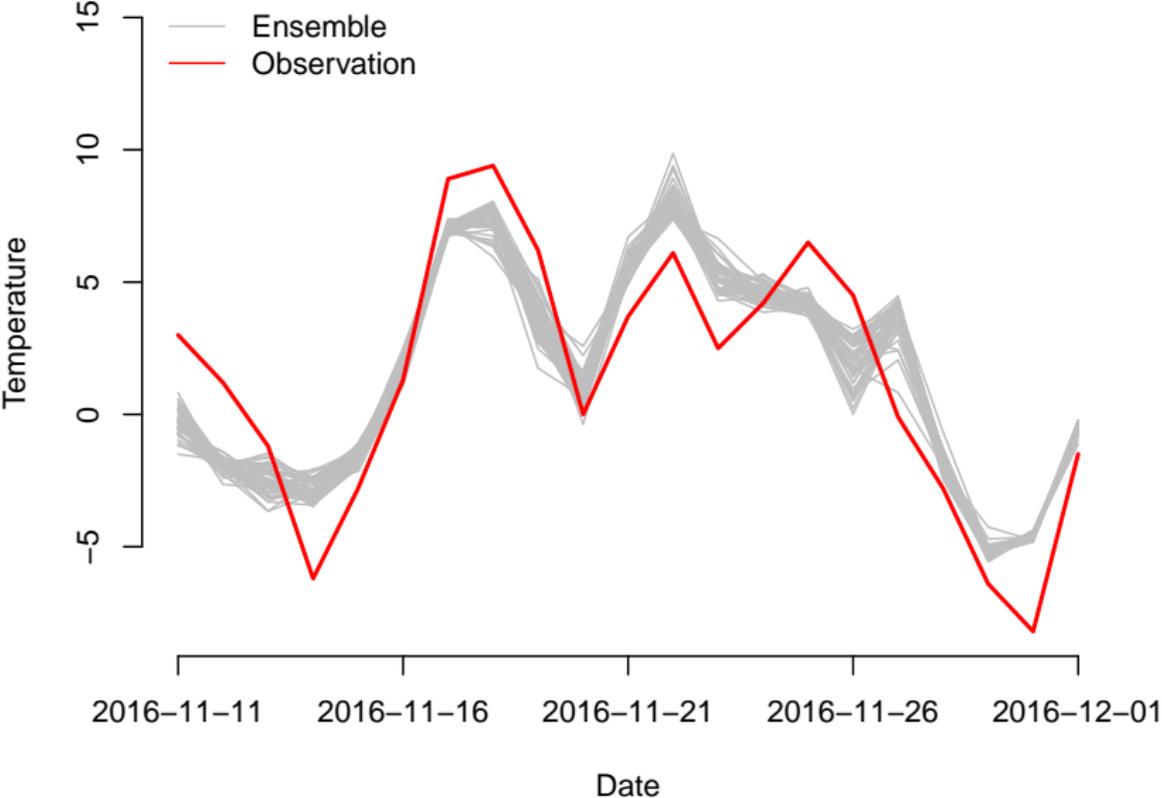
Despite continued improvements, ensemble forecasts are subject to **model biases** and **lack calibration**.

<sup>1</sup>[https://celebrating200years.noaa.gov/breakthroughs/climate\\_model/AtmosphericModelSchematic.png](https://celebrating200years.noaa.gov/breakthroughs/climate_model/AtmosphericModelSchematic.png)

# Example: Ensemble forecasts of temperature



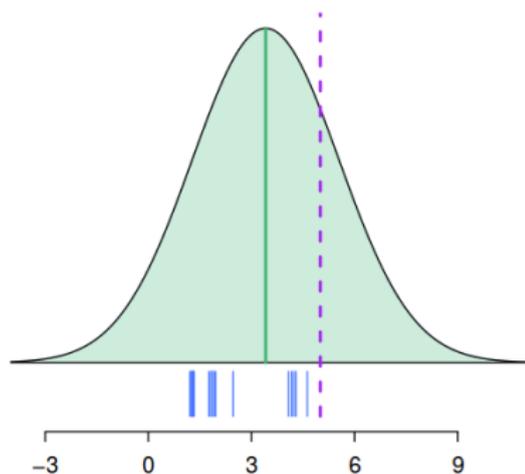
# Example: Ensemble forecasts of temperature



# Statistical post-processing of ensemble forecasts

Ensemble simulations typically fail to accurately quantify model uncertainty and require **calibration** via **statistical post-processing**.

**Example: Temperature:** Using ensemble predictions of temperature as input, the **post-processed** forecast takes the form of a Gaussian distribution.



$$y|\mathbf{X}^{t2m} \sim \mathcal{N}(\mu, \sigma),$$

$$\mu = a + b \cdot \text{mean}(\mathbf{X}^{t2m})$$

$$\sigma = c + d \cdot \text{sd}(\mathbf{X}^{t2m})$$

# Distributional regression models for post-processing

Model probability distribution of target variable  $y$  given ensemble model output  $\mathbf{X}$  by a parametric distribution  $F_{\theta}$ ,

$$y|\mathbf{X} \sim F_{\theta}, \quad \text{where} \quad \theta = g(\mathbf{X}).$$

Limitations of fully parametric approaches:

- ▶ requires choice of **link function**  $g$  connecting predictors  $\mathbf{X}$  and distribution parameters  $\theta$ 
  - ▶ difficult to specify **functional form** of dependencies if many possible predictors are available
- ▶ requires **estimation** of parameters of  $g$ 
  - ▶ **global** (using all training data) or **local** (location-specific) models?
- ▶ requires choice of **parametric model**  $F_{\theta}$

## Advanced benchmark models

Including additional predictors is not straightforward. To avoid overfitting, predictor selection strategies are required.

- ▶ **Gradient boosting** approach (EMOS-loc-bst model) proposed by [Messner et al. \(2017, MWR\)](#): Assume

$$(\mu, \sigma) = \left( \mathbf{X}^T \boldsymbol{\beta}, \exp(\mathbf{X}^T \boldsymbol{\gamma}) \right),$$

and iteratively update coefficient vector entries improving the current model fit most.

- ▶ **Quantile regression forest (QRF)** model proposed by [Taillardat et al. \(2016, MWR\)](#): Nonparametric quantile regression based on random forests. Quantile estimates are obtained from an ensemble of decision trees.

Have to be implemented as **local** models to achieve good forecasts.

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# Neural networks for post-processing ensemble forecasts

**Novel semi-parametric approach:** Estimate distribution parameters  $\theta$  directly by training a **neural network** to

- ▶ learn arbitrary nonlinear relations in an automated, data-driven manner,
- ▶ generate local adaptivity in globally estimated models,
- ▶ gain meteorological insight from trained models.

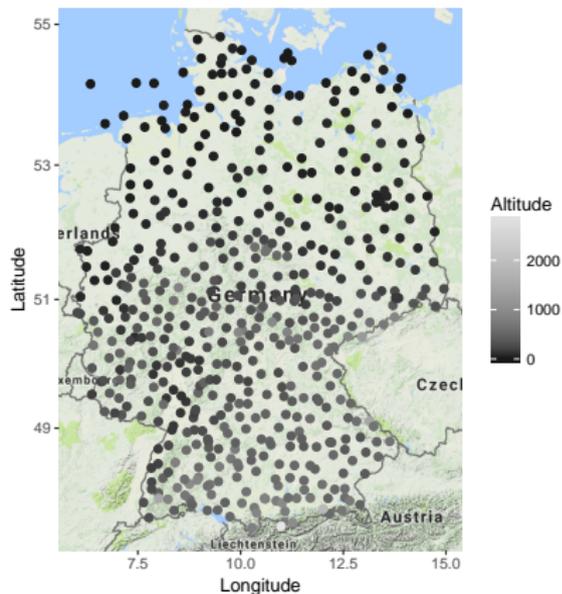
Rasp, S. and Lerch, S. (2018)

**Neural networks for post-processing ensemble weather forecasts,**  
*Monthly Weather Review*, 146, 3885–3900.

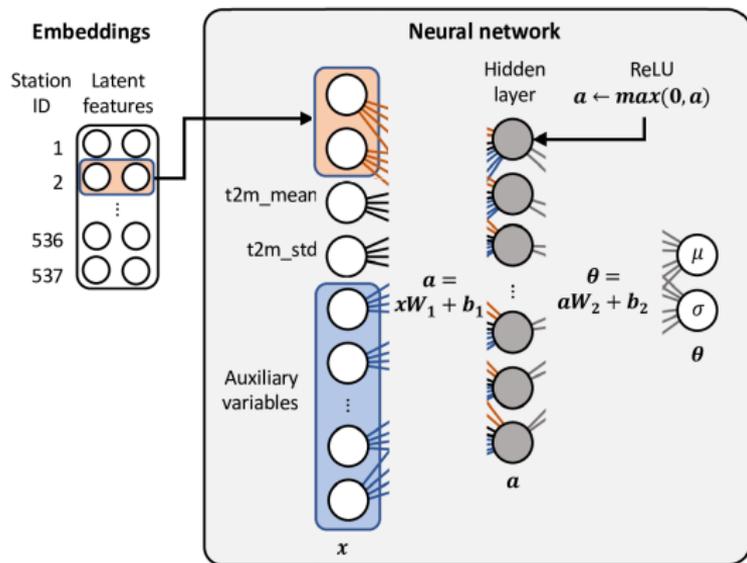
Python/R code available at <https://github.com/slerch/ppnn>.

# Data

- ▶ data from 2007–2016
- ▶ 48 hours-ahead ECMWF 50-member ensemble forecasts of temperature (and 17 other variables)
- ▶ DWD station observations at 537 locations
- ▶ data from 2016 used as evaluation set
- ▶ two training datasets: 2015 and 2007–2015



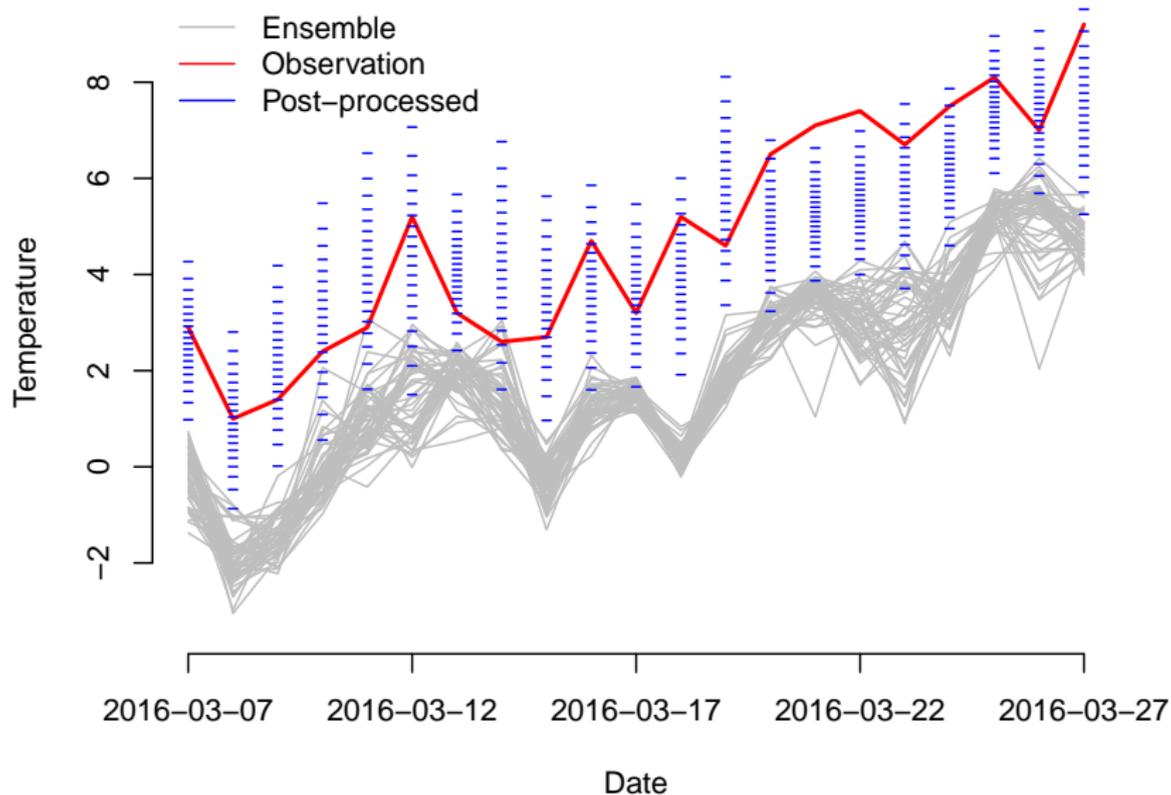
# Neural networks for distributional regression



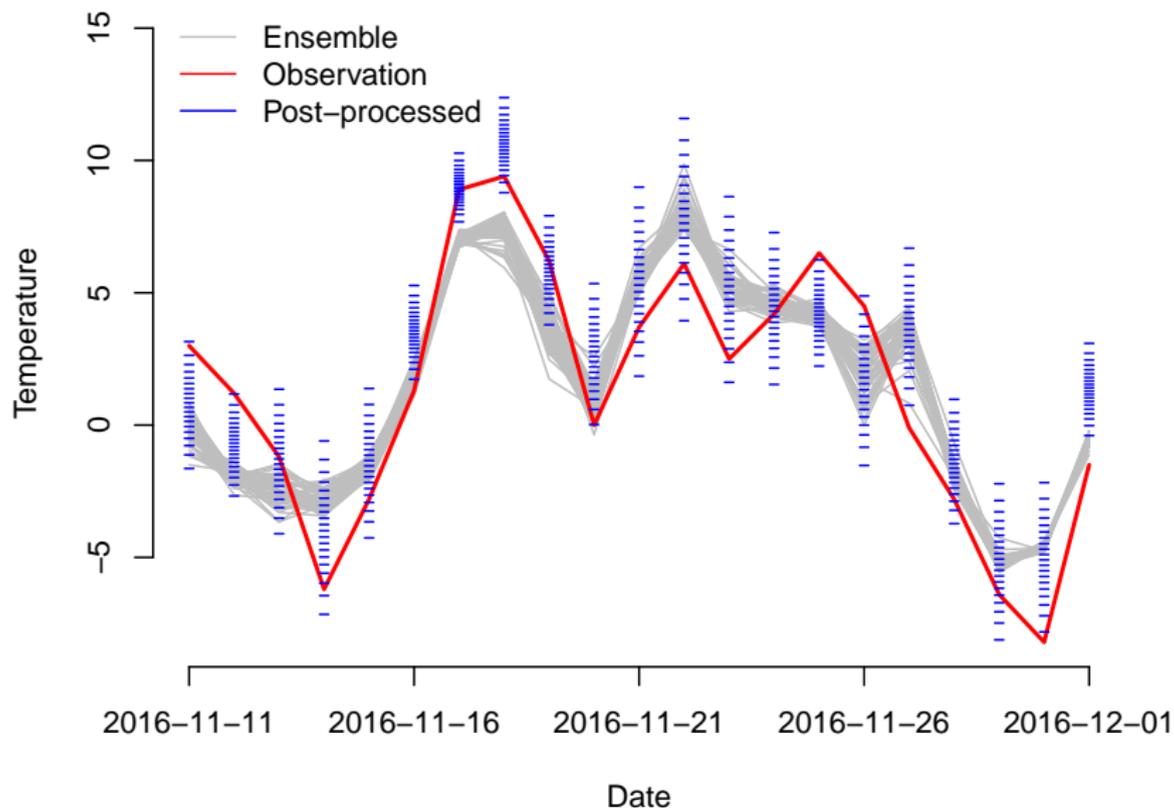
- ▶ **Input:** Predictor variables (NWP quantities, station characteristics).
- ▶ **Output:** Distribution parameters  $\theta$
- ▶ **Embeddings** generate local adaptivity.

Training via CRPS minimization (mathematically principled non-standard choice).

# Example: Ensemble forecasts of temperature



## Example: Ensemble forecasts of temperature

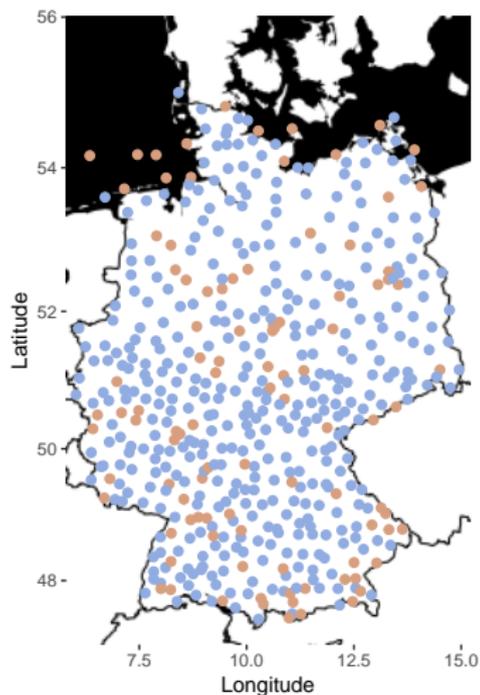


# Overview of results

CRPS: Continuous ranked probability score, [lower is better](#)

Model	Description	Mean CRPS for training period	
		2015	2007–2015
Raw ensemble		1.16	1.16
<i>Benchmark post-processing methods</i>			
EMOS-gl	Global EMOS	1.01	1.00
EMOS-loc	Local EMOS	0.90	0.90
EMOS-loc-bst	Local EMOS with boosting	0.85	0.80
QRF	Local quantile regression forest	0.95	0.81
<i>Neural network models</i>			
NN-aux-emb	Neural network with auxiliary predictors and station embeddings	<b>0.82</b>	<b>0.78</b>

# Station-specific comparison of NN and benchmark models

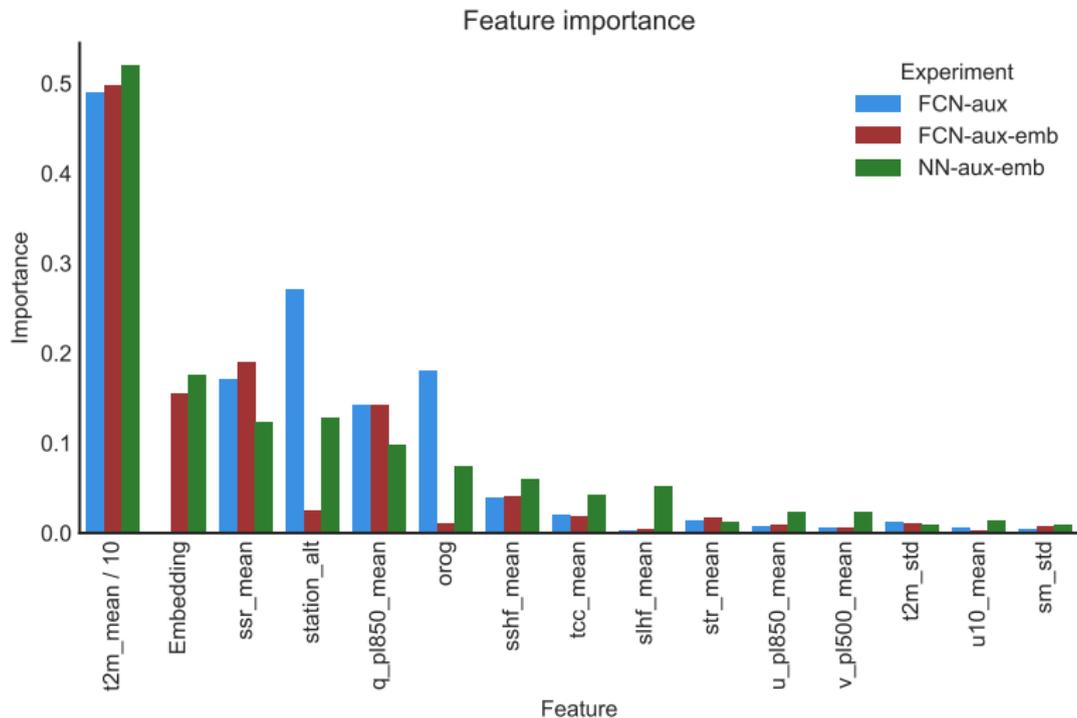


Station-specific best model is a  
NN model / benchmark model

NN models perform best at more  
than 80% of the stations.

Differences are statistically signifi-  
cant at a large fraction of stations.

# Peeking into the black box of neural network models



Change in mean CRPS after permuting a single input variable according to a **random permutation** across stations and dates.

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# Prospects of modern ML for distributional regression

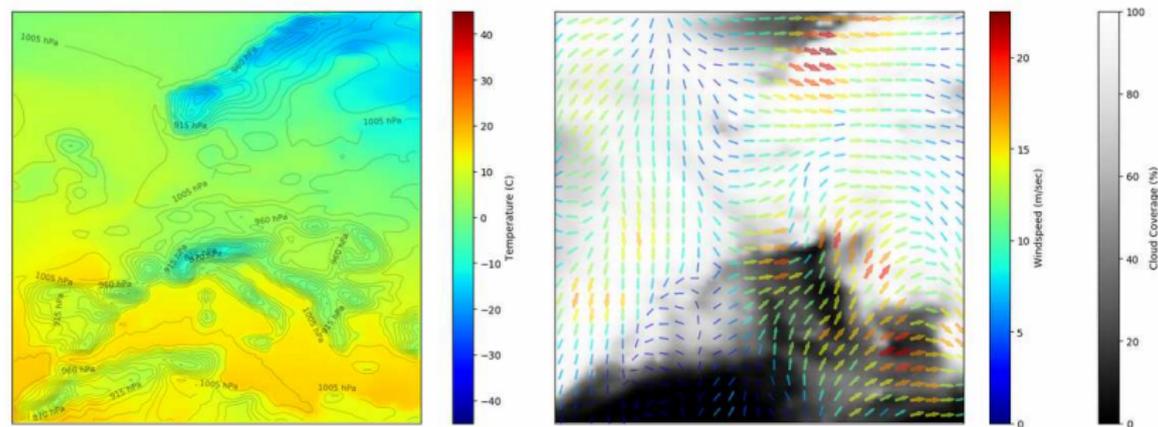
Modern AI methods provide unprecedented tools for **data analysis and prediction**.

In particular, machine learning can be useful for

- ▶ incorporating **spatial**, **temporal** and **inter-variable** information into model building and estimation,
- ▶ incorporating **prior knowledge** about underlying (e.g. physical) processes into models,
- ▶ flexible modelling of complex response distributions.

# Spatial information

Ensemble forecasts are **gridded 2D fields** of forecasts of weather variables. Thus far, those were **interpolated** to station locations.

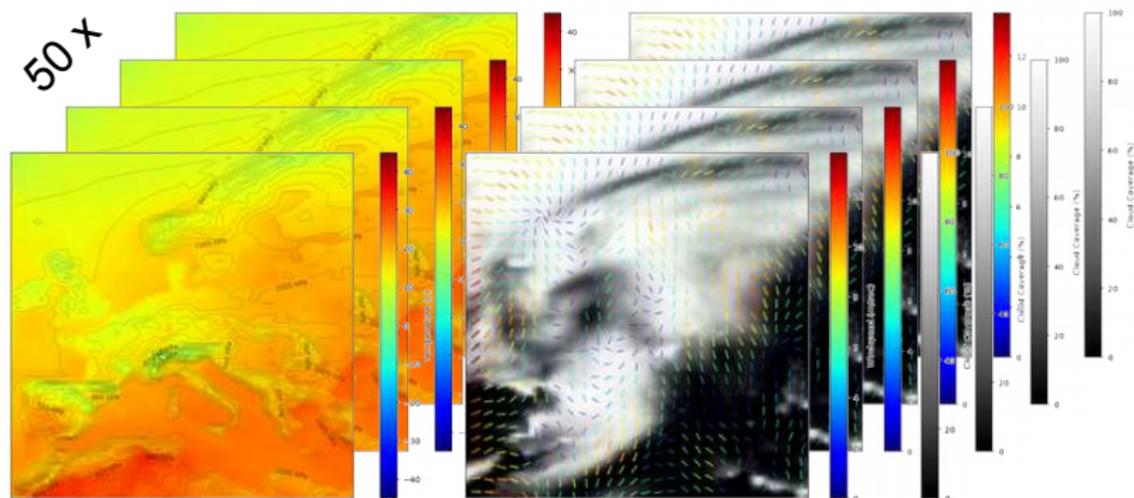


Gridded ECMWF forecasts over Europe ( $0.5^\circ$  resolution,  $81 \times 81$  pixels)

However, large-scale **spatial structure** and **predictability information** (e.g., 'weather regimes') get lost in the interpolation step.

## Ensemble information

Ensemble members provide 50 **physically coherent** forecasts of weather variables. Thus far, only **mean and standard deviation** of (interpolated) ensemble forecasts were used.

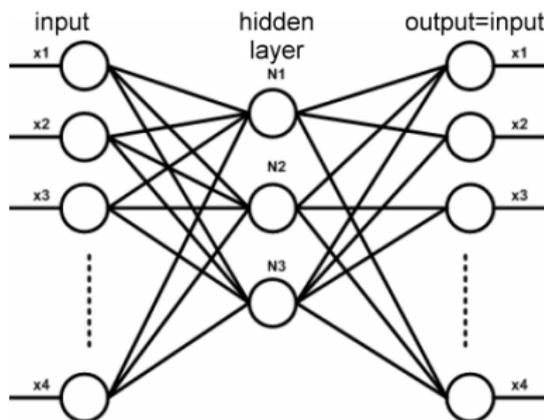


Possibly important **uncertainty information** might get lost by the use of summary statistics.

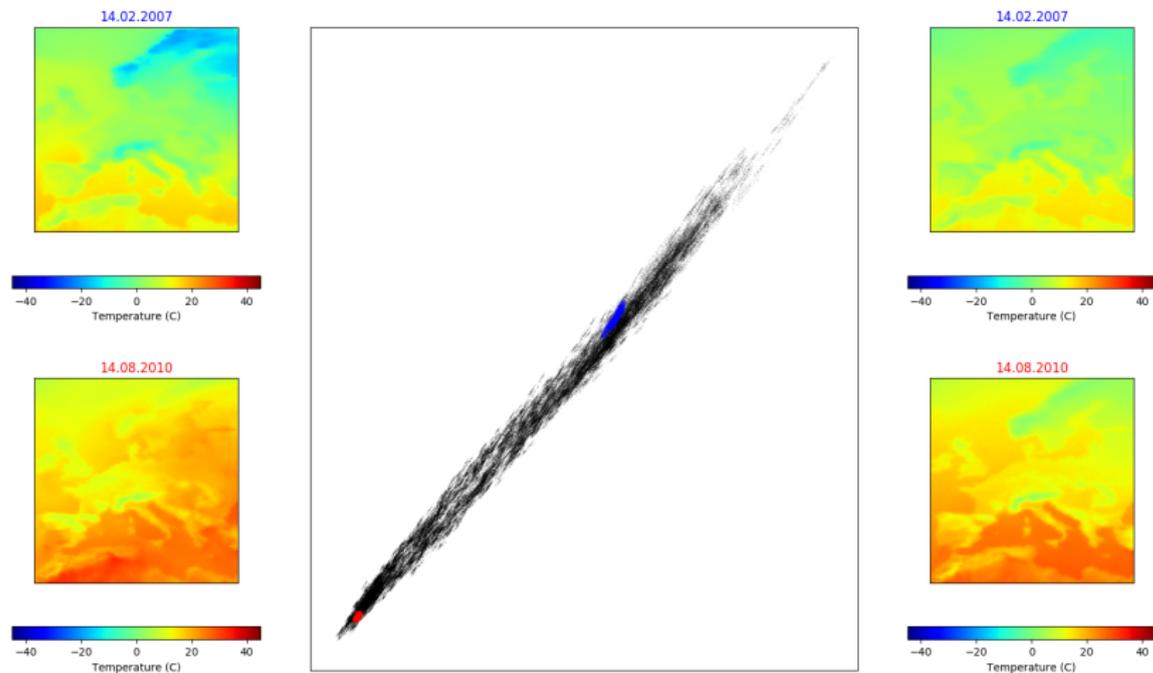
# Deep autoencoders for dimensionality reduction

Specific NN architectures to find compact representation of inputs (**unsupervised**) by

- ▶ training the network to re-create its own inputs
- ▶ creating a bottleneck by using fewer hidden units than inputs



# Projections of ensemble forecasts (temperature)

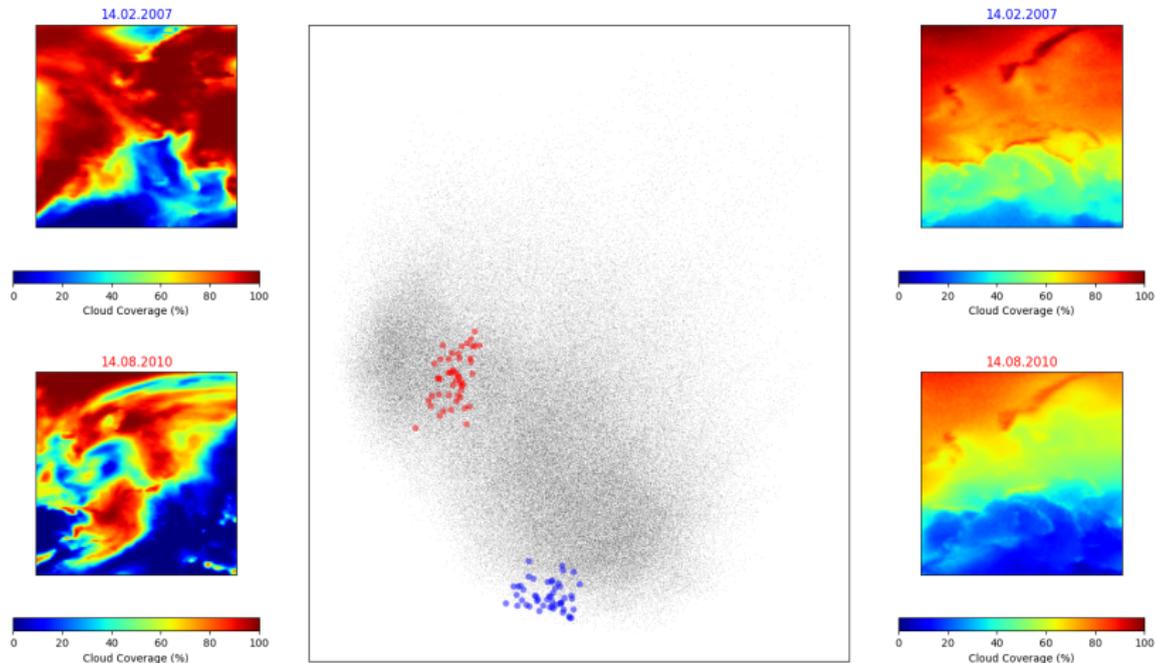


Left: Example input forecast fields from two days.

Middle: Ensemble members in projected space (blue: top, red: bottom).

Right: Reconstructed fields.

# Projections of ensemble forecasts (cloud cover)

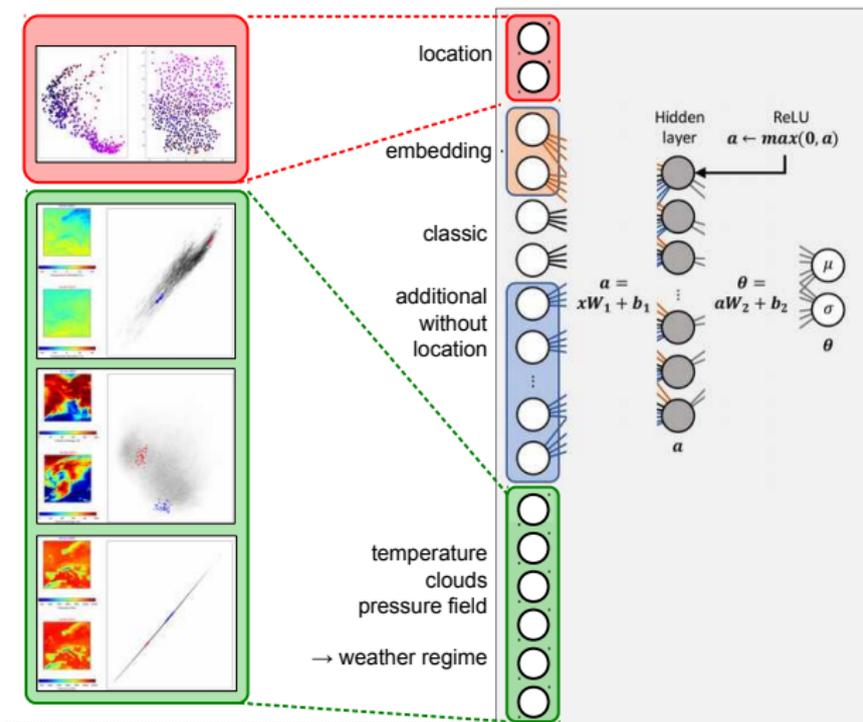


Left: Example input forecast fields from two days.

Middle: Ensemble members in projected space (blue: top, red: bottom).

Right: Reconstructed fields.

# Autoencoder representations as additional NN-input



**Preliminary** results suggest improvements in mean CRPS (0.78  $\rightarrow$  0.76).  
Ongoing joint work with Kai Polsterer and Antonio D'Isanto.

# Incorporating physical knowledge into ML models

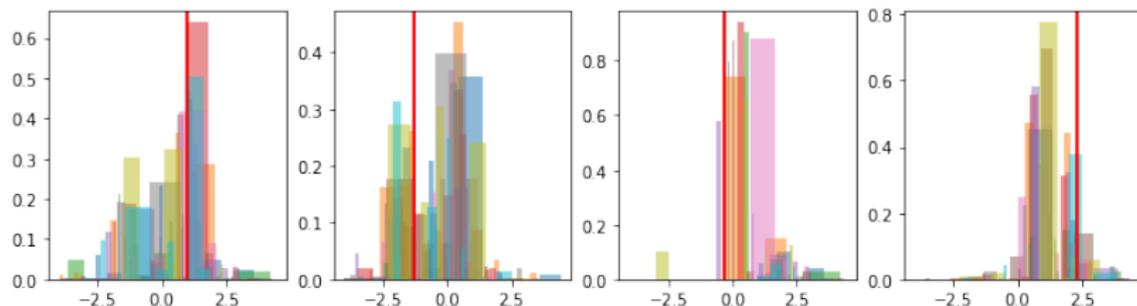
Modern AI methods provide new approaches to better understand and utilize **interactions of domain knowledge and statistics**, which is key to improving forecasting systems and optimize predictions.

In the context of ensemble post-processing, examples include

- ▶ incorporating predictability information contained in large-scale weather patterns ('weather regimes'), e.g. utilizing indicators **as predictors**
- ▶ **stratified model estimation** by objectively identified and meteorologically meaningful dynamic subregions of storms (PhD project of Benedikt Schulz)

# Neural networks for nonparametric distributional regression

The choice of a suitable parametric forecast distribution  $F_\theta$  remains a challenge for parametric approaches.



NN-based **nonparametric** distributional regression methods may allow to flexibly model complex response distributions.

Ongoing joint work with Stephan Rasp, M.Sc. thesis by Marvin Bischoff on electric load forecasting.

# Summary

- ▶ semi-parametric distributional regression models based on neural networks
- ▶ flexible, automated and data-driven modelling of **nonlinear relations** between predictors and distribution parameters
- ▶ perform better than **state of the art approaches** and allow to gain **meteorological insight** from trained models
- ▶ compressing complex spatial data might **improve performance and add to interpretability**

Rasp, S. and Lerch, S. (2018)

**Neural networks for post-processing ensemble weather forecasts,**  
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