A Roller Coaster: Energy Markets, Suboptimal Control and Pensions.

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This talk is based on joint work with David Baños (UiO), Fred Benth (UiO), Carmen Boados-Penas (UoL), Julia Eisenberg (TU Wien), Axel Helmert (msg life).

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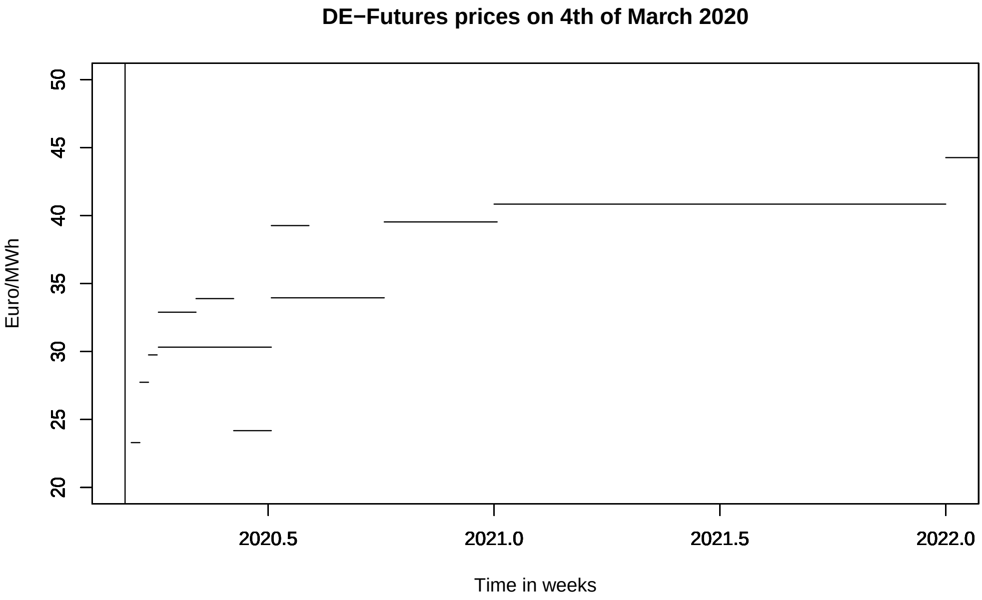
Market structure

1. Reserve markets (delivery in less than 2 hours)
2. Intraday market (delivery in less than 24 hours but more than 2 hours)
3. Spot market (delivery in 1 day)
4. Futures market (delivery in 1 day up to 2 years)

In this talk we are focusing on futures market.

Some market features:

1. Seasonality.
2. High idiosyncratic risk.
3. Flow-commodity.
4. Difficult to store.


DE−Futures prices on 4th of March 2020
Euro/MWh
50
45
40
35
30
25
20
2020.5
2021.0
2021.5
2022.0
Time in weeks

Model wish list

1. Model should incorporate seasonal effects. (Daily, weekly and yearly cycle)
2. Apart from seasonal affects: Markovian structure.
3. Clearly interpretable model factors.
4. Finite dimensionality (smaller dimension is better)
5. New contracts should be readily priceable in the model.
6. Arbitrage free.

Energy futures model

- $F_t(T_1, T_2)$ denotes the time t price of a futures with delivery in the time-interval $[T_1, T_2]$.

$$f_t(x) = f_t(0) + \int_0^t \beta_s(x) ds + \int_0^t \sigma_s(x) dW_s,$$
$$F_t(T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f(t, y - t) dt,$$

1. σ_t, β_t can be chosen to be curves only depending on season and state.
2. New contracts can readily be priced.
3. From (NA) condition: β must have a specific structure:

$$\beta_t(x) = \partial_x f_t(x) + \sigma_t(x) \gamma_t$$

where γ has the same dimension as the driving Brownian motion (with $\gamma = 0$ under EMM).

Structural implication (FDR on vector space + NA)

1. If realised on a finite dimensional space V of curves, then one must have $\sigma_t, \beta_t \in V$ and V must be invariant under the derivative.
2. Spaces of functions which are invariant under the derivative have a basis of the form

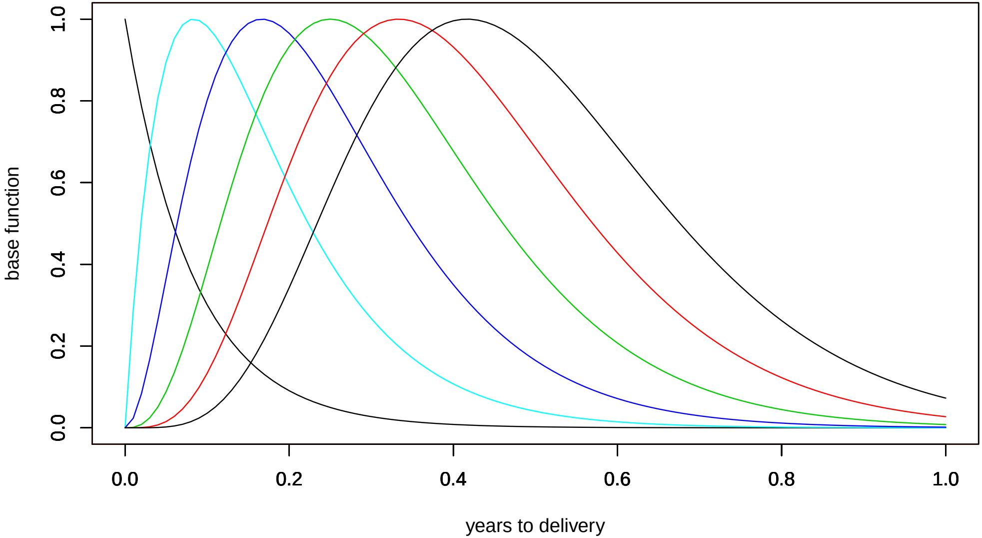
$$x \mapsto \mathrm{Re}(p(x) \cdot e^{\alpha x})$$

   where p is a complex polynomial and $\alpha \in \mathbb{C}$.

Our choice of generating curves:

$$f_{n,\alpha} : x \mapsto \frac{x^n}{n!} e^{-\alpha x}$$

for some $\alpha \geq 0$ and $n = 0, 1, \ldots, N$.
Allowed α: 0 or $\frac{1}{\text{contract length}}$.
With the second choice $f_{n,\alpha}$ has its maximum in n/α.


base function
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0
years to delivery

Statistical result

1. We use the curves $f_{n,\alpha}$ with $n = 0, 1$, $\alpha = 0, \frac{1}{\text{week}}, \frac{1}{\text{month}}, \frac{1}{\text{quarter}}, \frac{1}{\text{year}}$ (dimension$= 10$)
2. Estimated volatility is 99% on the curves:

$$f_{0,\frac{1}{\text{week}}}, \quad f_{1,\frac{1}{\text{week}}}, \quad f_{1,\frac{1}{\text{month}}}, \quad f_{1,\frac{1}{\text{quarter}}}, \quad f_{1,\frac{1}{\text{year}}}$$

   with estimated correlations less than 10%.
3. The model does still involve all curves to capture the initial state and for dynamic reasons.
4. Possible seasonality in the volatility has been ignored so far.

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Stochastic optimal control

1. The goal of stochastic optimal control is to maximise some quantity in expectation over a class of processes (controlled process) which is parametrised by some other stochastic processes (the control).
2. A typical setup: c is prog. measurable process chosen with values in some interval $[a, b]$ and

$$dX_t^c = \beta(t, c_t, X_t^c)dt + \sigma(t, c_t, X_t^c)dW_t,$$

$$\mathrm{E}\Big[\int_0^T g(s, c_s, X_s^c)ds + f(X_T^c)\Big] \to \text{Maximise over } c$$

g is called running gain and f is called terminal gain. A maximiser c^* of the above quantity is called an optimal control.

Does it work well?

1. Optimal controls are in many examples impossible to find.
2. Numerics for stochastic optimal control is an active topic and typically yields convergent schemes with convergence rates but implicit error constants.
3. $\longrightarrow$ If we choose for a numerical approach: When is a chosen control good? Or, if we simply take a reasonable appearing control, is it actually good?
4. $\longrightarrow$ We actually need explicit error bounds for the error between a chosen control and the unknown optimal control.

Density and occupation estimates

Say

$$dX_t = \beta_t dt + \sigma dW_t$$

($\sigma > 0$ a constant) and $|\beta_t|$ bounded by C. Then X_t has density ρ_t and (a version of ρ satisfies)

$$\rho_t(x) \leq \frac{\varphi(a)}{\sqrt{\sigma^2 t}} + C\Phi(a)$$

where φ and Φ are the density and distribution function of the standard normal law and

$$a := C\sqrt{\sigma^2 t} - \frac{|x - X_0|}{\sqrt{\sigma^2 t}}$$

The expected local time $\eta_t(x)$ of X at level x has similar explicit bounds and these bounds do not require constant diffusion coefficient.

Estimating the error to unknown optimal control

1. Control problem with T deterministic, σ constant and $\beta(t,c,x) \in [-1,1]$.

$$dX_t^c = \beta(t, c_t, X_t^c)dt + \sigma dW_t, \quad \mathrm{E}\Big[f(X_T^c)\Big] \to \text{Maximise over } c$$

2. Say $c_t = \gamma(t, X_t^c)$ is the chosen control and its performance function is given by

$$U(t,x) := \mathrm{E}_{(t,x)}[f(X_T^c)], \quad t \in [0,T].$$

3. One has $U(T,x) = f(x)$ and $U(t, X_t^c)$ is a martingale.

$$\begin{aligned}
\mathrm{E}[f(X_T^{c*})] &= U(0,X_0) + \int_0^T \mathrm{E}\left[\dot{U}(t, X_t^{c^*}) + U'(t, X_t^{c^*})\beta_t^* + U''(t, X_t^{c^*})\frac{\sigma^2}{2}\right] dt \\
&= U(0,X_0) + \int_0^T \mathrm{E}\left[U'(t, X_t^{c^*})(\beta_t^* - \beta(t, c_t, X_t^{c^*}))\right] dt \\
&\leq \mathrm{E}[f(X_T^c)] + \int_0^T \int_{\mathbb{R}} \max_{b \in \{1,-1\}} \left[U'(t,x)(b - \beta(t,x))\right] \alpha_t(x) dx dt
\end{aligned}$$

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Risk factors impacting pension schemes

A pension scheme is a financial contract between a pension provider and the member(s) of the plan; established for the purpose of providing an income in retirement for the member(s).

Problems for insurance companies:

1. Longevity risk (Creates stress to some pension schemes)
2. Low interest rate environment. (Problem for guaranteed interest rate. Creates huge stress to private pension provider)

Some classical products

1. PAYG (Typical state-pension. Defined benefits, sometimes defined contribution)
2. Unit-linked (Contract between single person and insurance company)
3. Annuity pools (Only for retirement phase)

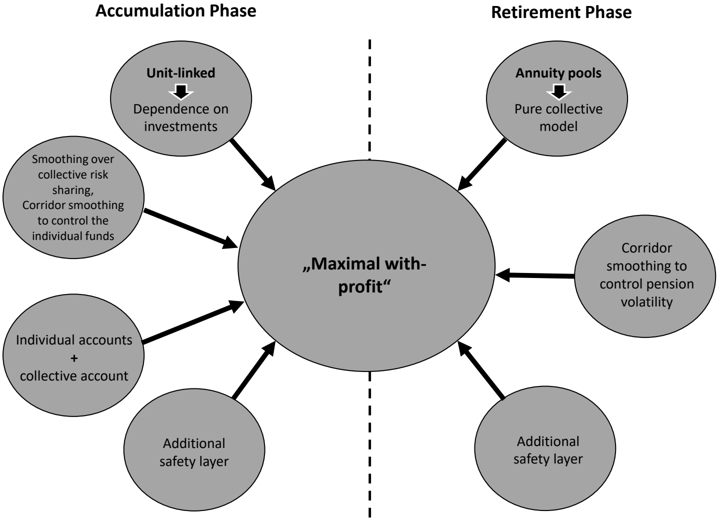
Our approach: Maximal with-profit

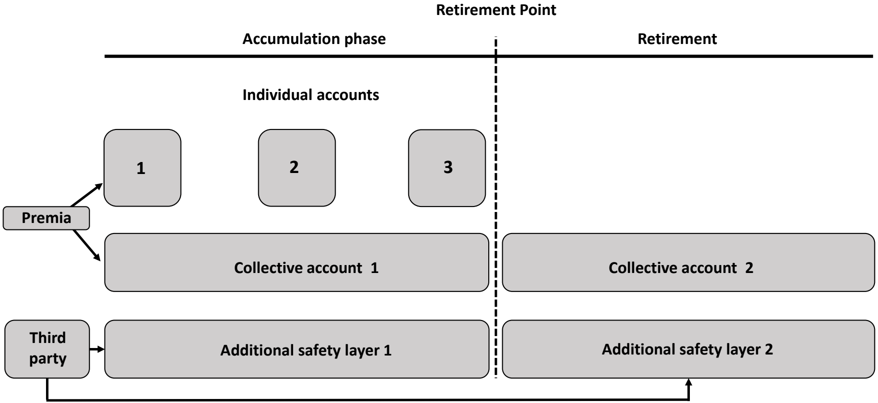
Features in the accumulation phase:

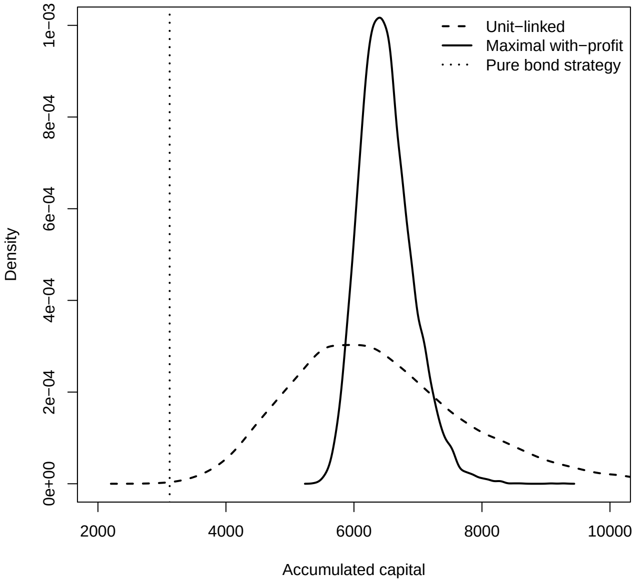
1. Unit-linked type account
2. Collective account
3. Exchange via a volatility smoothing mechanism

Features in the retirement phase:

1. Annuity pool
2. Smoothing mechnism


Accumulation Phase
Retirement Phase
Unit-linked
Dependence on investments
Annuity pools
Pure collective model
Smoothing over collective risk sharing,
Corridor smoothing to control the individual funds
„Maximal with-profit“
Corridor smoothing to control pension volatility
Individual accounts
+
collective account
Additional safety layer
Additional safety layer


Retirement Point
Accumulation phase
Retirement
Individual accounts
1
2
3
Premia
Collective account 1
Collective account 2
Third party
Additional safety layer 1
Additional safety layer 2


Unit−linked
Maximal with−profit
Pure bond strategy
Density
0e+00
2e−04
4e−04
6e−04
8e−04
1e−03
2000
4000
6000
8000
10000
Accumulated capital

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Thank you for your attention!