Time-varying risk premium in large cross-sectional equity datasets

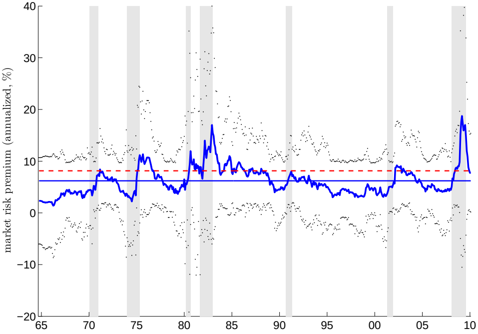
Patrick Gagliardini[a], Elisa Ossola[b] and Olivier Scaillet[c]

[a]University of Lugano,
[b]University of Lugano,
[c]University of Genève and Swiss Finance Institute.

March, 2015

Goal of the paper

- Analysis of time-varying behaviour of risk premia in large equity datasets.



- Test of asset pricing restrictions induced by conditional factor models.

Motivations

"During the period from 1926 to 1999 large stocks earned an annualized average return of 13%, whereas long-term bonds earned only 5.6%. Small stocks earned 18.9% - substantially higher than large stocks."

Jagannathan-Skoulakis-Wang (2009)

- Why do different assets earn different expected rates of return?
  - *Systematic and idiosyncratic risk*
  - *Linear factor models*
- Investors ask for a financial compensation for bearing systematic risk.
- How can we estimate the risk premium of different factors?
  - *Time-varying risk premia*

Two-pass regression methodology

$$R_{i,t} = a_i + b_i' f_t + \varepsilon_{i,t}, \;\; t = 1, ..., T, \;\; i = 1, ..., n$$

$$E\left[R_{i,t}\right] = b_i' \lambda$$

Two-pass methodology
(Black-Jensen-Scholes (1972), Fama-MacBeth (1973)):

1. time series OLS regression to estimate the factor loadings b_i;
2. cross-sectional OLS regression to estimate the vector of risk premia λ.

Usual setting:

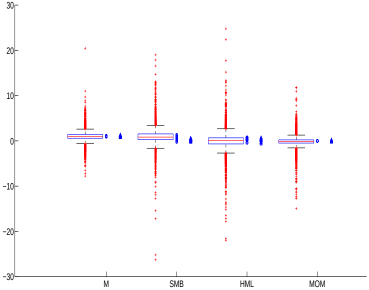
- time-invariant linear factor models of asset returns;
- portfolios with large T and fixed n (balanced panel).

This paper:

- time-varying linear factor models of asset returns;
- individual stocks with large T and large n ($n >> T$ and unbalanced).

Individual stocks versus portfolios

Estimated factor loadings for individual stocks (box-plots), for 25 FF portfolios (circles) and 44 Indu. portfolios (triangles)



Sorting and pooling stocks into portfolios distorts information.

Data-snooping bias (Lo-MacKinlay (1990)).

Ang-Liu-Schwarz (2008), Lewellen-Nagel-Shanken (2010), Berk (2000)

Building blocks of the paper

1. Derivation of no-arbitrage pricing restrictions
   - In a **large economy** (continuum of assets)
     Hansen-Richard (1987), Al-Najjar (1995, 1998)
   - With an **approximate factor structure** for excess returns
     Chamberlain-Rothschild (1983), Al-Najjar (1999)
   - With **conditional** factor models for excess returns
     Ferson-Harvey (1991,1999), Ferson-Schadt (1996), Ghysels (1998), Jagannathan-Wang (1996), and Petkova-Zhang (2005)

Building blocks of the paper

1. Derivation of no-arbitrage pricing restrictions
   - In a **large economy** (continuum of assets)
     Hansen-Richard (1987), Al-Najjar (1995, 1998)
   - With an **approximate factor structure** for excess returns
     Chamberlain-Rothschild (1983), Al-Najjar (1999)
   - With **conditional** factor models for excess returns
     Ferson-Harvey (1991,1999), Ferson-Schadt (1996), Ghysels (1998), Jagannathan-Wang (1996), and Petkova-Zhang (2005)
2. A new two-pass cross-sectional estimator of the risk premia
   - **Large unbalanced panel** of returns
   - Large-sample properties with double asymptotics: $\mathbf{n}, \mathbf{T} \to \infty$
     Bai-Ng (2002, 2006), Stock-Watson (2002), Bai (2003, 2009), Forni-Hallin-Lippi-Reichlin (2000, 2004, 2005), and Pesaran (2006)
   - Comparison with the classical framework:
     balanced panel and $T \to \infty$ with n fixed
     Shanken (1985,1992), Jagannathan-Wang (1998), Kan-Robotti-Shanken (2009), and Shanken-Zhou (2007)

3. Test of the asset pricing restrictions
   - Based on the **cross-sectional SSR**
     Gibbons-Ross-Shanken (1985)
   - Relation to the coefficient of determination R^2 of cross-sectional regression
     Lewellen-Nagel-Shanken (2009), and Kan-Robotti-Shanken (2009)

3. Test of the asset pricing restrictions
   - Based on the **cross-sectional SSR**
     Gibbons-Ross-Shanken (1985)
   - Relation to the coefficient of determination R^2 of cross-sectional regression
     Lewellen-Nagel-Shanken (2009), and Kan-Robotti-Shanken (2009)
4. Empirical analysis comparing results with CRSP **individual stock returns** and Fama-French 25 portfolios
   - Use of individual stocks versus portfolios
     Litzenberger-Ramaswamy (1979), Berk (2000), Ang-Liu-Schwarz (2008), and Avramov-Chordia (2006)
   - Risk premia estimates disagree between individual stocks and portfolios

Outline of the presentation

- Introduction ✓
- Conditional factor model
  - Model setting
  - Functional specification of time-varying coefficients
  - Estimation of betas and risk premia
  - Testing of the asset pricing restrictions
- Empirical results
- Conclusions

Conditional factor model: Model setting

Excess returns generation and asset pricing restrictions:

The excess return $R_t(\gamma)$ of asset $\gamma \in [0,1]$ at date $t = 1, 2, ...,$ satisfies

$$R_t(\gamma) = \beta_t(\gamma)' x_t + \varepsilon_t(\gamma), \tag{1}$$

where:

- $x_t = (1, f_t')'$ and f_t is the $K \times 1$ random vector of observable factors;
- $\beta_t(\gamma) = \left(a_t(\gamma), b_t(\gamma)'\right)'$ contains time-varying coefficients;
- $\varepsilon_t(\gamma)$ is a random vector of error terms s.t. $E[\varepsilon_t(\gamma) | \mathcal{F}_{t-1}] = 0$ and $Cov[\varepsilon_t(\gamma), f_t | \mathcal{F}_{t-1}] = 0$ for any $\gamma \in [0,1]$.

(Hansen-Richard (1987))

Assumption 1:

Approximate factor structure: (Chamberlain-Rothschild (1983)) conditional var-cov matrix $\Sigma_{\varepsilon,t,n} = \left[Cov\left[\varepsilon_t(\gamma_i), \varepsilon_t(\gamma_j) | \mathcal{F}_{t-1}\right]\right]_{i,j}$ for $i,j = 1,...,n$ is s.t. $n^{-1} eig_{max}\left(\Sigma_{\varepsilon,t,n}\right) \stackrel{L^2}{\to} 0$ as $n \to \infty$, for a.e. sequences (γ_i) in $[0,1]^\infty$;

No asymptotic arbitrage opportunities: there are no portfolios that approximate arbitrage opportunities when the number of assets increases.

Proposition 1: Asset pricing restriction

There exists a unique vector $\nu_t \in \mathbb{R}^K$ such that

$$a_t(\gamma) = b_t(\gamma)' \nu_t \qquad \left(\text{i.e., } E\left[R_t(\gamma) | \mathcal{F}_{t-1}\right] = b_t(\gamma)' \lambda_t\right) \qquad (2)$$

for almost all $\gamma \in [0,1]$, where $\lambda_t = \nu_t + E\left[f_t | \mathcal{F}_{t-1}\right]$ is the vector of time-varying risk premia.

Large economy with a continuum of assets:

⇒ derivation of an **empirically testable exact pricing restriction**.

⇒ **robustness** of factor structures to asset **repackaging** (Al-Najjar (1999)).

Unbalanced nature of the panel:

$I_t(\gamma)$ admits value 1 if the return of asset γ is observable at date t, and 0 otherwise (Connor-Korajczyk (1987)).

The sampling scheme:

A sample of n assets is obtained by drawing i.i.d. indices γ_i according to a probability distribution G on $[0,1]$.

⇒ **cross-sectional limits exist and are invariant to reordering of assets.**

⇒ sample of n assets and T observations of excess returns $R_{i,t} = R_t(\gamma_i)$, $I_{i,t} = I_t(\gamma_i)$, $\varepsilon_{i,t} = \varepsilon_t(\gamma_i)$ and $\sigma_{ij,t} = E\left[\varepsilon_{i,t}\varepsilon_{j,t} | \mathcal{F}_{\underline{t}}, \gamma_i, \gamma_j\right]$ for $i = 1, ..., n$ and $t = 1, ..., T$.

⇒ **random coefficient panel** model with $\beta_{i,t} = \beta_t(\gamma_i)$.

Functional specification of time-varying coefficients

Information set $\mathcal{F}_{t-1}$ contains lagged observations of:

- $Z_t \in \mathbb{R}^p$, vector of common instruments:
  - the constant and the observable factors f_t,
  - additional observable variables Z_t^*.
- $Z_{i,t} \in \mathbb{R}^q$, vector of asset-specific instruments:
  - firm characteristics,
  - stocks returns.

Assumption 2:

Factor loadings: $b_{i,t} = B_i Z_{t-1} + C_i Z_{i,t-1}$, where $B_i \in \mathbb{R}^{K \times p}$ and $C_i \in \mathbb{R}^{K \times q}$, for any asset i and $t = 1, 2, \ldots$;

Risk premia: $\lambda_t = \Lambda Z_{t-1}$, where $\Lambda \in \mathbb{R}^{K \times p}$, for any t;

Factors: $E\left[f_t | \mathcal{F}_{t-1}\right] = F Z_{t-1}$, where $F \in \mathbb{R}^{K \times p}$, for any t.

Assumption 2 and Proposition 1 imply:

$$a_{i,t} = Z'_{t-1} B_i \left(\Lambda - F\right) Z_{t-1} + Z_{i,t-1}{}' C_i{}' \left(\Lambda - F\right) Z_{t-1}.$$

- The **conditional factor model** (1), for the sample observations, becomes

$$R_{i,t} = \beta_i' x_{i,t} + \varepsilon_{i,t}, \tag{3}$$

  where:
  - regressor $x_{i,t}$ involves cross-terms of instruments Z_{t-1}, $Z_{i,t-1}$ and f_t;
  - time-invariant parameters $\beta_i = \left(\beta'_{1,i}, \beta'_{2,i}\right)'$ are (unconditional) transformations of matrices B_i, C_i, Λ and F.
- The **asset pricing restriction** (2) implies the parameter restriction

$$\beta_{1,i} = \beta_{3,i}\nu, \tag{4}$$

  where:
  - $\beta_{3,i}$ is a trasformation of matrices B_i and C_i;
  - $\nu = vec\left[\Lambda' - F'\right]$.

Estimation of betas and risk premia

1. ***Time series OLS regression** for the first pass:*

$$\hat{\beta}_i = \left(\sum_t I_{i,t} x_{i,t} x'_{i,t} \right)^{-1} \sum_t I_{i,t} x_{i,t} R_{i,t}, \; i = 1, ..., n.$$

Problem: If $T_i = \sum_t I_{i,t}$ is small, the inversion of $\hat{Q}_{x,i} = \frac{1}{T_i} \sum_t I_{i,t} x_{i,t} x'_{i,t}$ can be unstable.

Idea: Apply a **trimming approach**:

$$\mathbf{1}_i^{\chi} = \mathbf{1}\left\{ CN\left(\hat{Q}_{x,i}\right) \leq \chi_{1,T}, \tau_{i,T} \leq \chi_{2,T} \right\},$$

with $\chi_{1,T} > 0$ and $\chi_{2,T} > 0$ and where $CN\left(\hat{Q}_{x,i}\right) = \sqrt{\frac{eig_{max}\left(\hat{Q}_{x,i}\right)}{eig_{min}\left(\hat{Q}_{x,i}\right)}}$ is the condition number of $\hat{Q}_{x,i}$ (Greene (2008)), and $\tau_{i,T} = T/T_i$.

2. ***Cross-sectional WLS regression*** *for the second pass:*

$$\hat{\nu} = \left(\sum_i \hat{\beta}'_{3,i} \hat{w}_i \hat{\beta}_{3,i} \right)^{-1} \sum_i \hat{\beta}'_{3,i} \hat{w}_i \hat{\beta}_{1,i},$$

where $\hat{w}_i = \mathbf{1}_i^{\chi} \left(diag\left[\hat{v}_i \right] \right)^{-1}$ and $\hat{v}_i$ is a consistent estimator of $v_i = AsVar\left[\sqrt{T} \left(\hat{\beta}_{1,i} - \hat{\beta}_{3,i}\nu \right) \right]$.

The **estimator of time-varying risk premia** is

$$\hat{\lambda}_t = \hat{\Lambda} Z_{t-1},$$

where $\hat{\Lambda}$ is deduced by

$$vec\left[\hat{\Lambda}' \right] = \hat{\nu} + vec\left[\hat{F}' \right],$$

and $\hat{F}$ is the estimator of F in the SUR regression: $f_t = F Z_{t-1} + u_t$.

Large sample properties

Asymptotic scheme: ***simultaneous double asymptotic***

$n, T \to \infty$ such that $n = T^{\bar{\gamma}}$ with $\bar{\gamma} > 0$.

Assumption 3: Heteroschedasticity and cross-sectional dependence

a) $E\left[\varepsilon_{i,t} \middle| \left\{\varepsilon_{j,\underline{t-1}}, \gamma_j, j = 1, ..., n\right\}, \mathcal{F}_{\underline{t}}\right] = 0$, *with* $\varepsilon_{j,\underline{t-1}} = \{\varepsilon_{j,t-1}, \varepsilon_{j,t-2}, \cdots\}$;

b) $M^{-1} \leq E\left[\varepsilon^2_{i,t} | \mathcal{F}_t, \gamma_i\right] = \sigma_{ii,t} \leq M, \quad i = 1, ..., n$ *for a constant* $M < \infty$;

c) $E\left[\frac{1}{n}\sum_{i,j} E\left[|\sigma_{ij,t}|^2 \, |\gamma_i, \gamma_j\right]\right]^{1/2} \leq M$, *with* $\sigma_{ij,t} = E\left[\varepsilon_{i,t}\varepsilon_{j,t} | \mathcal{F}_t, \gamma_i, \gamma_j\right]$.

Assumption 3 accommodates **non Gaussian**, **conditionally heteroschedastic**, weakly **serially and cross-sectionally dependent** error terms.

Proposition 2: Asymptotic distribution

As $n, T \to \infty$ such that $n = o(T^3)$, estimators $\hat{\nu}$, $\hat{\Lambda}$ and $\hat{\lambda}_t$ are consistent and asymptotically normal:

a) $\sqrt{nT}\left(\hat{\nu} - \nu - \frac{1}{T}\hat{B}_\nu\right) \Rightarrow N\left(0, \Sigma_\nu\right)$, *where* $\hat{B}_\nu / T$ *is a bias term;*

b) $\sqrt{T} vec\left[\hat{\Lambda}' - \Lambda\right] \Rightarrow N\left(0, \Sigma_\Lambda\right)$, *where*

$$\Sigma_\Lambda = \left(I_K \otimes Q_z^{-1}\right) \Sigma_u \left(I_K \otimes Q_z^{-1}\right),$$

with $Q_z = E\left[Z_t Z_t'\right]$ *and* $\Sigma_u = E\left[u_t u_t' \otimes Z_{t-1} Z_{t-1}'\right]$;

c) $\sqrt{T}\left(\hat{\lambda}_t - \lambda_t\right) \Rightarrow N\left(0, H_{t-1}\Sigma_\Lambda H_{t-1}'\right)$, *where* H_{t-1} *is a trasformation of* Z_{t-1}.

Estimation of ν does not affect accuracy of risk premia estimates.

Properties:

- Estimators $\hat{\nu}$, $\hat{\Lambda}$ and $\hat{\lambda}_t$ feature **different convergence rates** $\sqrt{nT}$ and $\sqrt{T}$.
- Bias term $\hat{B}_\nu / T$ is induced by the Error-in-Variable (EIV) problem.

Time-invariant case ($Z_t = 1$ and $Z_{i,t} = 0$):

- $R_{i,t} = a_i + b_i' f_t + \varepsilon_{i,t}$ and $a_i = b_i' \nu$;
- $\hat{\lambda} = \hat{\nu} + \frac{1}{T} \sum_t f_t$ and $\hat{\nu} = \left(\sum_i \hat{w}_i \hat{b}_i \hat{b}_i' \right)^{-1} \sum_i \hat{w}_i \hat{b}_i \hat{a}_i$ with $\hat{w}_i = \hat{v}_i^{-1}$;
- for $n, T \to \infty$, $\sqrt{T} \left(\hat{\lambda} - \lambda \right) \Rightarrow N \left(0, \Sigma_f \right)$;
- for **fixed** n, $T \to \infty$, $\sqrt{T} \left(\hat{\lambda} - \lambda \right) \Rightarrow N \left(0, \Sigma_f + \frac{1}{n} \Sigma_\nu \right)$ (Shanken (1992), Jagannathan-Wang (1998)).

Estimation of asymptotic variance Σ_ν

Problem: Σ_ν involves the double sum

$$S_{v_3} = \lim_{n\to\infty} E\left[\frac{1}{n}\sum_{i,j}\frac{\tau_i\tau_j}{\tau_{ij}}\left(Q_{x,i}^{-1}S_{ij}Q_{x,j}^{-1}\right)\otimes v_{3,i}v_{3,j}'\right],$$

over $S_{ij} = E[\varepsilon_{i,t}\varepsilon_{j,t}x_{i,t}x_{j,t}'|\gamma_i,\gamma_j]$, where $v_{3,i} = vec[\beta_{3,i}'w_i]$.

Plugging-in $\hat{S}_{ij} = \frac{1}{T_{ij}}\sum_t I_{i,t}I_{j,t}\hat{\varepsilon}_{i,t}\hat{\varepsilon}_{j,t}x_{i,t}x_{j,t}'$ leads to divergent accumulation of statistical errors.

Idea:

Assume a sparsity structure for the S_{ij} and use a thresholded estimator (Bickel-Levina (2008), Fan-Liao-Mincheva (2011))

$$\tilde{S}_{ij} = \hat{S}_{ij}\mathbf{1}_{\|\hat{S}_{ij}\|\geq\kappa}.$$

Sparsity condition is applied on the error terms and *not* on the excess returns!

Testing of the asset pricing restriction

$$\mathcal{H}_0\text{: there exists } \nu \in \mathbb{R}^{pK} \text{ such that } \beta_1(\gamma) = \beta_3(\gamma)\nu, \\ \text{for almost all } \gamma \in [0,1].$$

- The statistic is $\hat{\xi}_{nT} = T\sqrt{n}\left(\hat{Q}_e - \frac{1}{T}\hat{B}_\xi\right)$, where
  - $\hat{Q}_e = \frac{1}{n}\sum_i \hat{e}_i' \hat{w}_i \hat{e}_i$, with $\hat{e}_i = \hat{\beta}_{1,i} - \hat{\beta}_{3,i}\hat{\nu}$, is the cross-sectional weighted SSR (Gibbons-Ross-Shanken (1989));
  - $\hat{B}_\xi = 0.5p(p+1) + pq$ is the recentering term.

Proposition 3: Asymptotic distribution of the test statistic under $\mathcal{H}_0$

Under $\mathcal{H}_0$, we have $\tilde{\Sigma}_\xi^{-1/2}\hat{\xi}_{nT} \Rightarrow N(0,1)$, as n, $T \to \infty$ such that $n = o(T^2)$, where $\tilde{\Sigma}_\xi$ is an estimator of the asymptotic variance that involves the thresholded estimator $\tilde{S}_{ij}$.

- More restrictive condition on the relative rate of n and T wrt Prop. 2.

Data description

Base assets:

- 9,936 stocks with monthly returns from Jul1964 to Dec2009 after merging CRSP and Compustat databases;
- 25 Fama-French (FF) and 44 industry (Indu.) monthly portfolios returns.

Factors:

- $f_t = (r_{m,t}, r_{smb,t}, r_{hml,t}, r_{mom,t}) = (\text{market, size, value, momentum})$.

Instrumental variables:

- common variables $Z_t = (1, Z_t^*)'$:
  - term spread: difference between yields on 10-year Treasurys and 3-month T-bills;
  - default spread: yield difference between Moody's Baa and Aaa-rated corporate bonds.
- firm characteristics $Z_{i,t}$:
  - book-to-market equity.

Estimated risk premia and ν for the time-invariant models

$$\hat{\lambda} = \hat{\nu} + \frac{1}{T}\sum_t f_t$$

Four-factor model				
	Stocks ($n = 9,936$, $n^{\chi} = 9,902$)		Portfolios ($n = n^{\chi} = 25$)	
	bias corrected estimate (%)	95% conf. interval	point estimate (%)	95% conf. interval
λ_m	8.14	(3.26, 13.02)	5.70	(0.73, 10.67)
λ_{smb}	2.86	(-0.50, 6.22)	3.02	(-0.48, 6.51)
λ_{hml}	-4.60	(-8.06, -1.14)	4.81	(1.21, 8.41)
λ_{mom}	7.16	(2.56, 11.75)	34.03	(9.98, 58.07)
ν_m	3.29	(2.88, 3.69)	0.85	(-0.10, 1.79)
ν_{smb}	-0.41	(-0.95, 0.13)	-0.26	(-1.24, 0.72)
ν_{hml}	-9.38	(-10.12, -8.64)	0.03	(-0.95, 1.01)
ν_{mom}	-1.47	(-2.86, -0.08)	25.40	(1.80, 49.00)

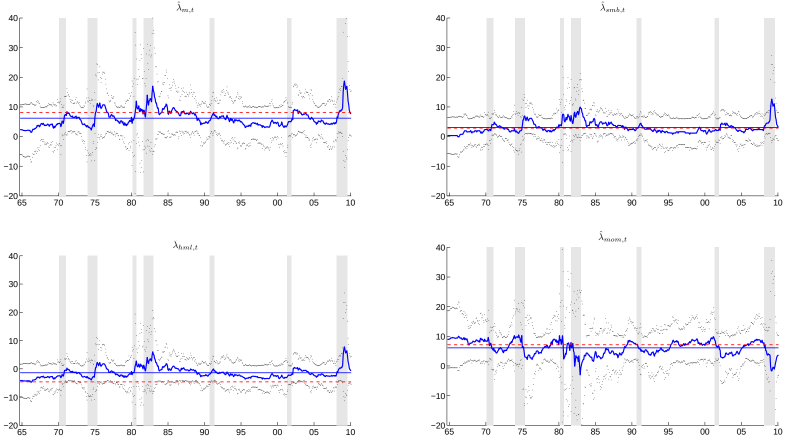
Fama-French model

	Stocks ($n = 9,936$, $n^{\chi} = 9,902$)		Portfolios ($n = n^{\chi} = 25$)	
	bias corrected estimate (%)	95% conf. interval	point estimate (%)	95% conf. interval
λ_m	7.77	(2.89, 12.65)	5.04	(0.11, 9.97)
λ_{smb}	2.64	(-0.72, 5.99)	3.00	(-0.42, 6.42)
λ_{hml}	-5.18	(-8.65, -1.72)	5.20	(1.66, 8.74)
ν_m	2.92	(2.48, 3.35)	0.18	(-0.51, 0.87)
ν_{smb}	-0.63	(-1.11, -0.15)	-0.27	(-0.93, 0.40)
ν_{hml}	-9.96	(-10.62, -9.31)	0.41	(-0.32, 1.15)

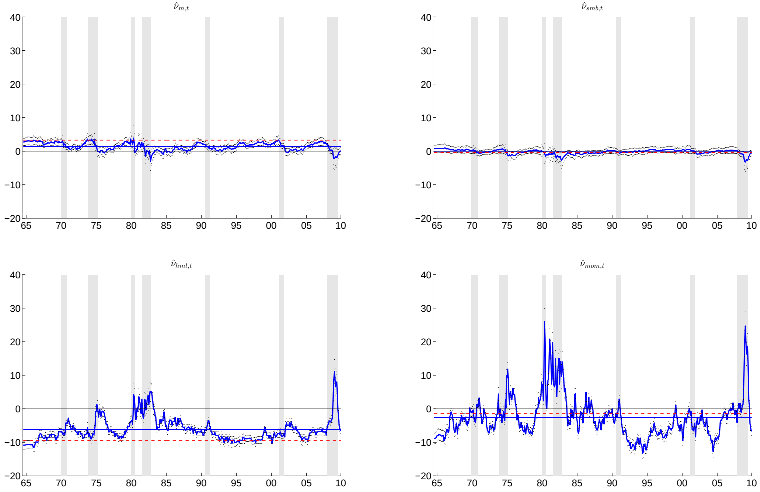
CAPM

	Stocks ($n = 9,936$, $n^{\chi} = 9,904$)		Portfolios ($n = n^{\chi} = 25$)	
	bias corrected estimate (%)	95% conf. interval	point estimate (%)	95% conf. interval
λ_m	7.42	(2.54, 12.31)	6.98	(1.93, 12.02)
ν_m	2.57	(2.17, 2.97)	2.12	(0.85, 3.40)

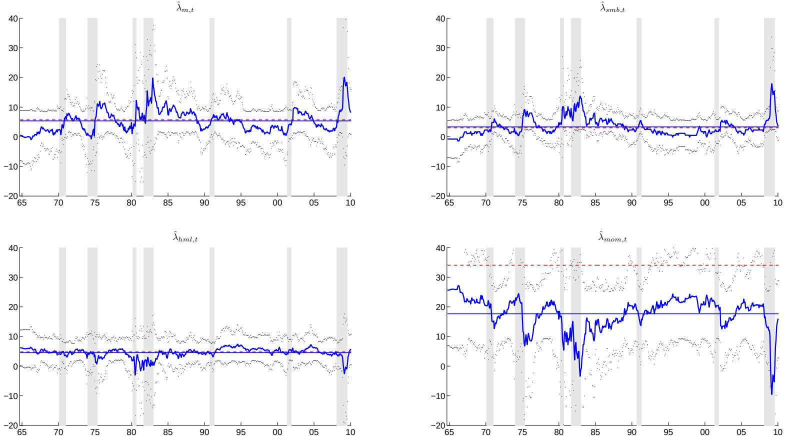
Paths of estimated risk premia $\hat{\lambda}_t = \hat{\Lambda} Z_{t-1}$ on individual stocks ($n = 9,936$, $n^{\chi} = 3,900$)



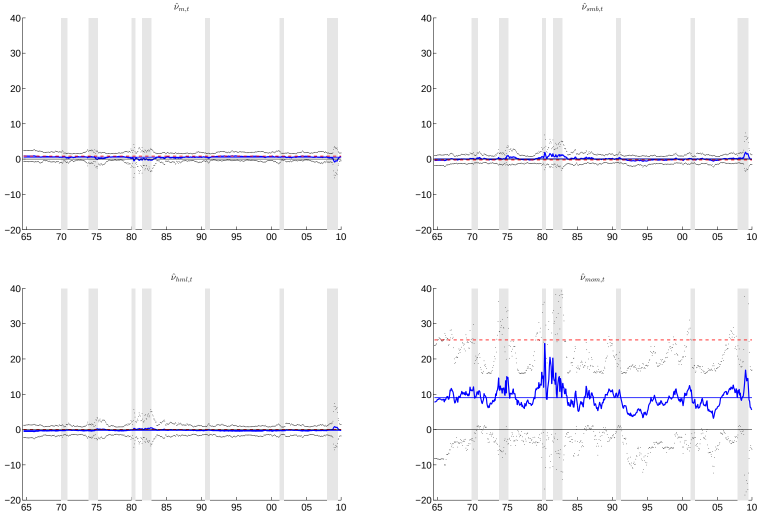
Paths of estimated $\hat{\nu}_t$ on individual stocks ($n = 9,936,\ n^{\chi} = 3,900$)



Paths of estimated risk premia with $n = 25$ portfolios



Paths of estimated $\hat{\nu}_t$ with $n = 25$ portfolios



Effects of $vec[F']$ and ν on time-varying risk premia

		$vec[F']$	ν $(n = 9,936)$	ν $(n = 25)$
m	*const*	4.8322 (0.2653, 9.3990)	1.3744 (0.7069, 2.0419)	0.5251 (−0.4713, 1.5216)
	ds_{t-1}	3.0353 (−2.6803, 8.7509)	−0.6032 (−1.2688, 0.0623)	−0.2916 (−1.1622, 0.5790)
	ts_{t-1}	1.8677 (−2.8399, 6.5754)	−0.9254 (−1.5626, −0.2881)	0.0828 (−0.6666, 0.8323)
smb	*const*	3.2739 (0.0410, 6.5067)	−0.2130 (−0.8680, 0.4421)	0.0607 (−0.9808, 1.1122)
	ds_{t-1}	2.5468 (−0.5998, 5.6934)	−0.5948 (−1.1499, −0.0396)	0.4134 (−0.6139, 1.4407)
	ts_{t-1}	0.2855 (−2.6271, 3.1982)	−0.2157 (−0.7443, 0.3128)	−0.1966 (−0.9686, 0.5753)
hml	*const*	4.7772 (1.7905, 7.7639)	−6.1642 (−6.8543, −5.4741)	−0.2267 (−1.3144, 0.8611)
	ds_{t-1}	−1.7898 (−5.5963, 2.0167)	3.5981 (2.8995, 4.2967)	0.2187 (−1.0365, 1.4740)
	ts_{t-1}	0.8933 (−2.2598, 4.0465)	−0.4292 (−1.0043, 0.1458)	−0.0073 (−0.8766, 0.8620)
mom	*const*	8.6543 (−4.2482, 13.0605)	−2.5592 (−3.4153, −1.7031)	9.0179 (0.4294, 17.6064)
	ds_{t-1}	−7.3714 (−14.6656, −0.0771)	6.0148 (5.1168, 6.9131)	1.9403 (−6.0003, 9.8808)
	ts_{t-1}	1.5804 (−2.8226, 5.9833)	−3.2960 (−4.0246, −2.5673)	−2.5080 (−9.9869, 4.9710)

Time variation tests

$\mathcal{H}_0^F$: $Avec\left[F'\right] = 0$	$\mathcal{H}_0^\nu$: $A\nu = 0$	
	Stocks ($n = 9,936$)	Portfolios ($n = 25$)
11.8765 (0.1570)	389.27 (0.0000)	1.5566 (0.9920)

- Matrix A is a selection matrix for the components of $vec\left[F'\right]$ and ν corresponding to the effects of the instruments.
- For individual stocks, we reject time-invariance of risk premia implied by the rejection of $\mathcal{H}_0^\nu$.
- The aggregation in the 25 FF portfolios completely masks the time variation of the risk premia.

$\mathcal{H}_0^\nu$: $\nu = 0$	
Stocks ($n = 9,936$)	Portfolios ($n = 25$)
785.93 (0.0000)	9.0885 (0.9650)

- For the 25 FF portfolios, we do not reject the nullity of vector ν ($\Rightarrow$ the nullity of ν_t for all t).

- 44 Indu. portfolios: the empirical results look different from the estimates on the 25 FF portfolios, and similar to those of individual stocks.

To explain the differences between individual stocks and portfolios:

- Long-only factors: the time-invariant estimates of ν are different from zero for individual stocks and equal to zero for the 25 FF portfolios.
- Time variation of $b_{i,t}$: the FF portfolios betas are more stable than the individual stocks and 44 Indu. portfolios betas.
- Pseudo-true values: the pseudo-true values for value factor are different from the individual stocks and the portfolios.

The time-invariant models for the individual stocks are misspecified.

- Limits-to-arbitrage and missing factor impact: a comparison of idiosyncratic risk between individual stocks and portfolios.

- Robustness checks:
  - estimation of Fama-French factor model and CAPM

    The estimates are similar to those for the four-factor model with individual stocks and the 25 FF portfolios.

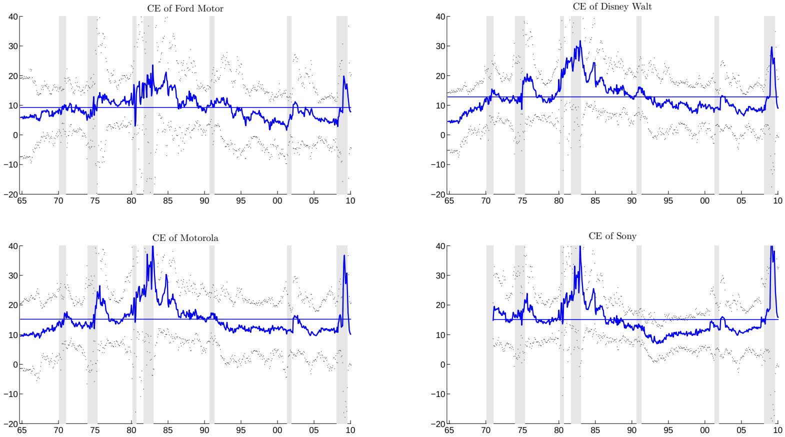
  - estimation of the four-factor model by using several sets of asset-specific and common instruments
  - estimation of the four-factor model by assuming that $b_{i,t} = C_i Z_{i,t-1}$

    The paths of risk premia $\hat{\lambda}_t$ feature similar patterns for the four-factor models.

- Value-weighted estimates for individual stocks:
  - qualitatively unchanged results
  - wider confidence intervals than WLS estimation.

Paths of estimated cost of equity

Cost of equity: $CE_{i,t} = r_{f,t} + b'_{i,t}\lambda_t$



Test results for asset pricing restriction in the time-invariant model

	$\mathcal{H}_0$: $a(\gamma) = b(\gamma)' \nu$		$\mathcal{H}_0$: $a(\gamma) = 0$	
	$n^{\chi} = 1,400$ $N(0,1)$	$n = 25$ χ^2_{n-K}	$n^{\chi} = 1,400$ $N(0,1)$	$n = 25$ χ^2_n
		Four-factor model		
Test statistic	2.0088	35.2231	19.1803	74.9100
p-value	0.0223	0.0267	0.0000	0.0000
		Fama-French model		
Test statistic	2.9593	83.6846	28.0328	87.3767
p-value	0.0015	0.0000	0.0000	0.0000
		CAPM		
Test statistic	8.2576	110.8368	11.5882	111.6735
p-value	0.0000	0.0267	0.0000	0.0000

Test results for asset pricing restriction in the time-varying model

	$\mathcal{H}_0: \beta_1(\gamma) = \beta_3(\gamma)\nu$		$\mathcal{H}_0: \beta_1(\gamma) = 0$	
	$n^{\chi} = 1,373$	$n = 25$	$n^{\chi} = 1,373$	$n = 25$
	$N(0,1)$	$\frac{1}{n}\sum_j eig_j \chi_j^2$	$N(0,1)$	$\frac{1}{n}\sum_j eig_j \chi_j^2$
		Four-factor model		
Test statistic	3.2514	13.4815	3.8683	14.3080
p-value	0.0000	0.0000	0.0000	0.0000
		Fama-French model		
Test statistic	3.1253	15.7895	3.8136	15.9038
p-value	0.0000	0.0000	0.0000	0.0000
		CAPM		
Test statistic	1.7322	9.2934	1.7381	9.6680
p-value	0.0416	0.2076	0.0411	0.0000

Conclusions

Finance Theory:

- We derive empirically testable no-arbitrage restrictions in a multi-period conditional economy with a continuum of assets and an approximate factor structure.

Econometric Theory:

- Simple two-pass cross-sectional regressions allow us to estimate the time-varying risk premia implied by conditional linear asset pricing models using the returns of individual stocks.
- The risk premia estimator is consistent and asymptotically normal when $n, T \to \infty$.

Empirics:

- We observe a disagreement between the empirical results derived by sorting and pooling stocks into portfolios and by extracting the information directly from the individual stocks.

Work in progress...

- Define a simple diagnostic criterion for approximate factor structure in large cross-sectional equity datasets.

- *Main idea:* If the set of observable factors is correctly specified, the errors are weakly cross-sectionally correlated.

1. A new diagnostic criterion for approximate factor structure in large cross-sectional datasets.
2. The simple criterion is based on three steps:
   1. compute the largest eigenvalue of a variance-covariance matrix;
   2. substract a penalty;
   3. conclude on the validity of the approximate factor structure if criterion value is negative.
3. Empirical results:
   1. we cannot select a model with zero common factors in the errors for the time-invariant specifications;
   2. we provide penalised scree plots that show the cutoff point for each model;
   3. we conclude on the validity of the approximate factor structure for time-varying specifications.

Link with the well-known incidental parameters problem in the fixed effects nonlinear panel literature

Write the time-invariant factor model, with asset pricing restriction $a_i = b_i'\nu$, as:

$$R_{i,t} = b_i'(f_t + \nu) + \varepsilon_{i,t},$$

where the b_i are the individual effects and ν is the common parameter.

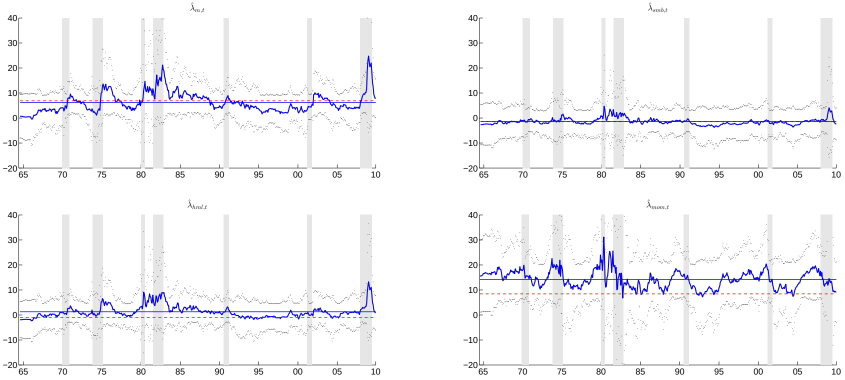
(Hahn-Kuersteiner (2002), Hahn-Newey (2004)): $y_{i,t} \sim h(\cdot; b_i, \nu)$

- Similar type of analytical bias correction for the estimator of ν.
- Same condition $n = o(T^3)$ for the asymptotic analysis.
- However, our setting is semi-parametric and accommodates cross-sectional dependence.

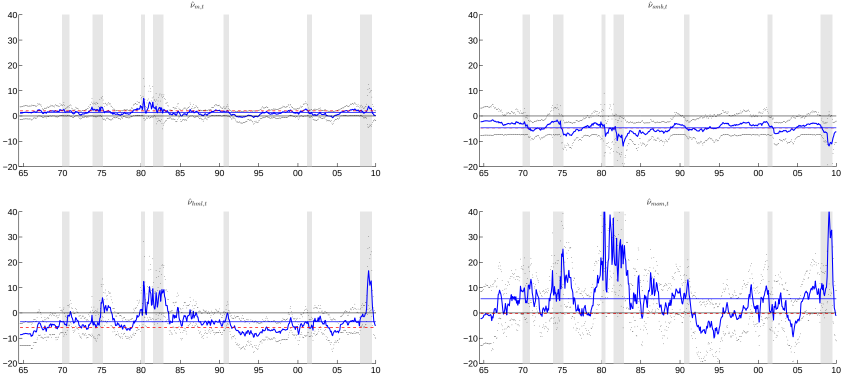
Estimated risk premia and ν for the time-invariant models with $n = 44$

	point estimate (%)	95% conf. interval		point estimate (%)	95% conf. interval
Four-factor model					
λ_m	6.87	(1.86, 11.88)	ν_m	2.02	(0.90, 3.13)
λ_{smb}	-1.46	(-5.57, 2.84)	ν_{smb}	-4.72	(-7.40, -2.05)
λ_{hml}	-0.97	(-5.49, 3.57)	ν_{hml}	-5.75	(-8.66, -2.84)
λ_{mom}	8.42	(-3.11, 19.96)	ν_{mom}	-0.20	(-10.78, 10.37)
Fama-French model					
λ_m	6.58	(1.60, 11.56)	ν_m	1.74	(0.73, 2.72)
λ_{smb}	-2.24	(-6.46, 1.98)	ν_{smb}	-5.51	(-8.07, -2.95)
λ_{hml}	-1.40	(-5.57, 2.95)	ν_{hml}	-6.19	(-8.82, -3.56)
CAPM					
λ_m	5.95	(0.98, 10.99)	ν_m	1.09	(-0.15, 2.35)

Paths of estimated risk premia with $n = 44$ portfolios



Paths of estimated $\hat{\nu}_t$ with $n = 44$ portfolios



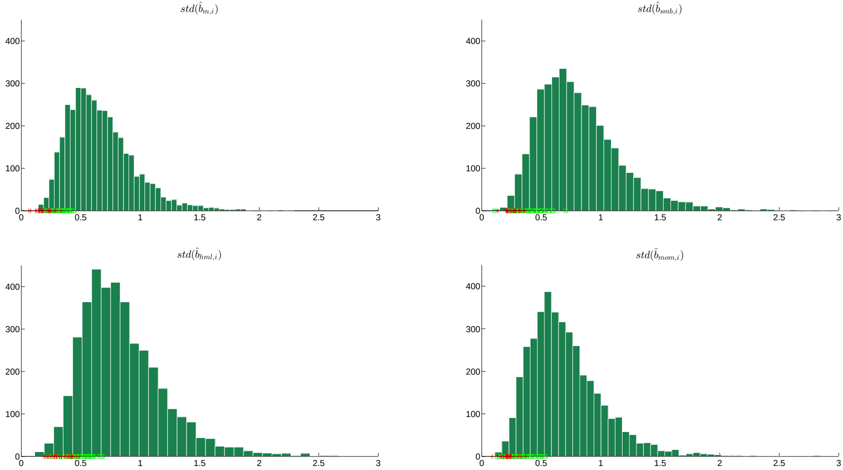
↩

Estimated risk premia and ν for the time-invariant three factor model with long-only factors

	Stocks ($n = 9,936$, $n^{\chi} = 9,846$)	FF Portfolios ($n = 25$)	Indu. Portfolios ($n = 44$)
	bias corrected estimate (%) (95 % conf.interval)	bias corrected estimate (%) (95 % conf.interval)	bias corrected estimate (%) (95 % conf.interval)
λ_m	7.49 (2.61, 12.37)	4.72 (−0.22, 9.66)	6.57 (1.60, 11.54)
λ_s	9.24 (2.66, 15.82)	9.12 (2.54, 15.71)	4.69 (−2.27, 11.65)
λ_h	5.46 (−0.09, 11.02)	10.30 (4.70, 15.90)	5.16 (−0.92, 11.23)
ν_m	2.64 (2.14, 3.13)	−0.14 (−0.90, 0.62)	1.72 (0.79, 2.65)
ν_s	0.30 (−0.27, 0.88)	0.19 (−0.08, 0.45)	−4.25 (−6.25, −1.96)
ν_h	−4.06 (−4.50, −3.63)	0.77 (0.04, 1.51)	−4.37 (−6.83, −1.91)

↩

Cross-sectional distributions of the standard deviations of $\hat{b}_{k,i,t}$, over time



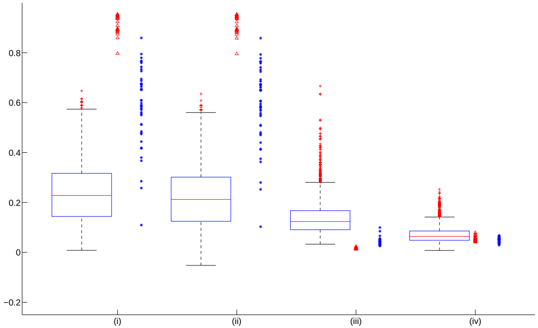
Estimated pseudo-true values of parameter ν

		$n = 9,936$	$n = 25$		$n = 44$	
			CW	TVW	CW	TVW
$\nu_t = \bar{\nu}$, $b_{i,t}$ constant	ν^*_m	1.3772	1.3772	0.4453	1.3772	1.0312
	ν^*_{smb}	-0.2122	-0.2122	0.4779	-0.2122	0.0657
	ν^*_{hml}	-6.1636	-6.1636	-3.0085	-6.1636	-5.8395
	ν^*_{mom}	-2.5507	-2.5507	-0.7216	-2.5507	-4.5657
$\nu_t = \bar{\nu}$, $b_{i,t}$ time-varying	ν^*_m	1.3406	2.6374	0.6123	1.6079	0.9199
	ν^*_{smb}	0.1490	0.1940	0.7492	0.1824	0.8432
	ν^*_{hml}	-6.5468	-9.8461	-3.4016	-6.1935	-6.4573
	ν^*_{mom}	-6.6899	-3.5831	-2.6132	-5.4675	-8.0675

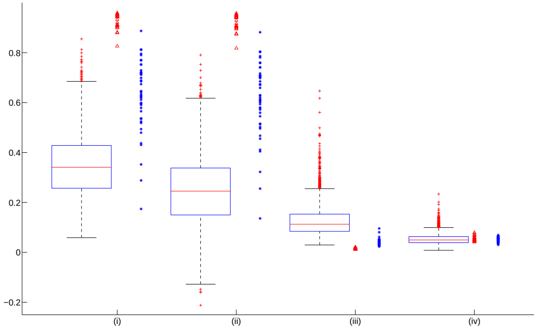
		$n = 9,936$	$n = 25$		$n = 44$	
			CW	TVW	CW	TVW
$\nu_t = \hat{\nu}_t$, $b_{i,t}$ constant	ν^*_m	1.3788	1.3788	0.8521	1.3788	1.0816
	ν^*_{smb}	-0.2158	-0.2158	0.4970	-0.2158	0.1172
	ν^*_{hml}	-6.1291	-6.1291	-3.9565	-6.1291	-5.9395
	ν^*_{mom}	-2.4741	-2.4741	-0.9824	-2.4741	-4.2506
$\nu_t = \hat{\nu}_t$, $b_{i,t}$ time-varying	ν^*_m	1.0201	1.5269	-0.0080	1.4433	0.6526
	ν^*_{smb}	0.1678	0.1870	0.8511	-0.3721	0.6996
	ν^*_{hml}	-6.0848	-8.1776	-2.6871	-6.6668	-6.5043
	ν^*_{mom}	-4.8815	-3.9304	-1.6555	-6.0449	-7.4999

$\hookleftarrow$

Cross-sectional distributions of $\hat{\rho}_i^2$, $\hat{\rho}_{ad,i}^2$, $IdiVol_i$, and $SysRisk_i$, for the time-invariant four-factor model



Cross-sectional distributions of $\hat{\rho}_i^2$, $\hat{\rho}_{ad,i}^2$, $IdiVol_i$, and $SysRisk_i$, for the time-varying four-factor model



Cross-sectional distributions of $\hat{\beta}_{1,i}'\hat{\beta}_{1,i}$