Empirical Analysis of European Government Yield Spreads

Alois Geyer
University of Economics and Business Administration, Vienna

Stephan Kossmeier
Institute of Advanced Studies, Vienna and Vienna University of Technology

Stefan Pichler[1]
Vienna University of Technology

March 2001

1 Corresponding author: Department of Finance, Vienna University of Technology, Department of Finance, Favoritenstrasse 11, A-1040 Wien, Austria, email: spichler@pop.tuwien.ac.at. This paper was partially supported by the Austrian National Bank‘s Jubilaeumsfonds.

Empirical Analysis of European Government Yield Spreads

Abstract

This paper focuses on the dynamics of yield spreads deducted from government bonds issued by member states of the European Monetary Union (EMU). A descriptive analysis shows that there is a substantial and volatile spread between government zero coupon yields of EMU member countries and German Bund yields. These yield spreads form an important source of additional risk that has to be taken into account by any pricing or risk management model based on or dealing with EMU government bonds. We extract risk factors driving the observed yield spreads by employing a multi-issuer version of the model originally proposed by Duffie and Singleton (1999). We adopt a state-space approach to implement the model that enables us to extract factor series and model parameters simultaneously. We find strong empirical evidence for a global factor that mainly represents the average level of the yield spreads and for a country specific factor for each issuer. Our findings indicate that it may be justified to implement a simplified version of the multi-issuer model that sufficiently captures the main features of the data.

1 Introduction

The formation of the third stage of the European Monetary Union (EMU) in January 1999 based on the Maastricht Treaty changed the structure of the European bond market fundamentally. Exchange rates among member countries were irrevocably fixed, and the Euro was introduced as the new single currency. All EMU member countries have to meet specific economic "convergence" criteria, most notably the public finance criteria restricting the public debt below 60% of GDP and the public deficit below 3% of GDP. As a consequence, the most important source of spreads between interest rates among member states, the existence of exchange rate risk, disappeared. From January 1999 on "riskless" bonds denominated in Euro and traded in a frictionless market are expected to be perfect substitutes and, thus, to trade on the same yield curve.

However, there could be some additional sources of interest spreads remaining even within the EMU. Firstly, there could be country-specific default risk perhaps due to differences in the fiscal policy of different governments or due to external reasons. Secondly, there could be differences in liquidity and taxation among countries and market segments. Finally, there is a small but positive probability of a failure of the EMU that might re-introduce exchange rate risk at some date prior to the redemption of some bonds.

The first two years of EMU experience show that there is a substantial and volatile spread between government bond yields of EMU member countries other than Germany and German Bund yields. These yield spreads form an important source of additional risk that has to be taken into account by any pricing or risk management model. Hence, there is a need for a model that prices EMU government bonds and related derivatives[2] based on explicitely modelling the risk factors underlying the yield spreads. It is the main objective of this paper to propose such a model with special emphasis on the economic explanation of the observed government yield spreads.

The reduced form default risk model proposed by Duffie and Singleton (1999, DS hereafter) seems to be a natural candidate for modelling zero coupon yield spreads. Under the assumption of a loss rate proportional to the market value of a defaultable bond and modelling the default event as a first jump of a Cox process the resulting bond pricing model has a rather simple structure. The product of the intensity and the loss rate, and the riskless short rate are assumed to be driven by affine processes[3]. The pricing model preserves its "affine" structure when the factors driving the intensity/loss rate process are just treated as additional factors in a familiar affine term structure model like Vasicek (1977) or Cox, Ingersoll, and Ross (1985, CIR hereafter). Besides its analytical

[2] There are derivatives traded in the markets, eg spread options, that explicitely refer to the observed yield spreads. Obviously, there is special need for such a pricing model.
[3] See Duffie and Kan (1996).

tractability the use of this model benefits from the experience in implementing and testing affine term structure models in a default-free context.

It has to be emphasized that the structure of this model can also be used to capture liquidity effects. Duffie and Singleton (1997) discuss this issue with respect to the interpretation of endogeneously derived factor series whereas Kempf and Uhrig-Homburg (2000) add an exogeneously specified factor representing liquidity effects to a standard CIR model. However, care must be taken when results concerning liquidity factors are interpreted because of the limited structure of affine models that may not be sufficient to capture the whole variety of possible liquidity effects.

To estimate and test different specifications of the general affine model structure we adopt a state-space approach to implement the model empirically. This approach in a default-free term structure setting first introduced by Chen and Scott (1995) and Geyer and Pichler (1999) enables us to extract factor series and model parameters simultaneously. In contrast to other approaches no additional assumptions about the observation errors[4] are necessary and the error-in-variables problem inherent in two-step approaches is avoided. Duffee (1999) used a similar approach to implement a pricing model for US corporate bonds and Lund (1999) used a non-linear Kalman filter model to analyze the pre-1999 effects of the EMU. Our paper is the first attempt to apply the state-space approach to price EMU sovereign debt and explicitly analyze the extracted factor series.

In a recent paper, Düllmann and Windfuhr (2000) try to test the suitability of one-factor affine models to explain the observed spreads between Italian and German government bond yields. Based on their rather limited data set they ultimately reject all one-factor models tested in their paper. Nevertheless, since their estimation procedure is built upon very strong additional assumptions and since they did not test multi-factor extensions there is no final conclusion about the suitabilty of affine models to model EMU government bond yield spreads in general to be derived from their work.

The results of this paper show that it may be justified to use one global factor that represents the average level of the yield spreads and a country specific factor for each issuer. Our findings indicate that a simplified version of the multi-issuer model can be implemented that fits the data sufficiently well and is computationally much more attractive.

The paper is structured as follows: Section 2 shortly summarizes the main cornerstones of the DS model and describes our implementation of the model in a multi-issuer setup. Section 3 contains a detailed analysis of the problems arising when the DS model is implemented empirically. Special attention is drawn to the state-space approach used in this paper. In section 4 the empirical results are presented. We perform a diagnostic checking in order to assess the quality of the estimated

[4] Some approaches are based on the assumptions that a subset of bonds is priced without any error.

models and analyze the extracted factor series with respect to their economic interpretation. This analysis is based upon the factor loadings and the comparison of the extracted factor series with factors derived from a LISREL model. Section 5 concludes the paper.

2 A pricing model for EMU government bonds

This section briefly summarizes the cornerstones of the DS model and discusses the extension of the model to a multi-issuer context.

We take as given an arbitrage free setting, where all contingent claims are priced in terms of a numeraire and an equivalent martingale measure. The dynamics of the numeraire asset are described as

$$d\Pi_t = r_t \Pi_t dt \tag{1}$$

where $r_t = r(X_t)$ denotes the risk free short rate that itself depends on a $M \times 1$ vector of state variables or factors driven by Ito processes.

Consider two classes of claims in this economy: First, default-free zero bonds, which pay one unit of currency at time to maturity *t+T*, $P(t,T)$. The prices of these bonds are given as the expectation of the stochastic discount factor under the equivalent martingale measure.

$$P(t,T) = E_t^Q \left[\exp\left(-\int_t^{t+T} r_u du \right) \right] \tag{2}$$

Equation 2 characterizes the default free term structure in the economy. Throughout this paper, we assume that this default free term structure is given by the observed prices of German sovereign bonds.[5]

Second, there is a set of defaultable zero bonds, $V_C(t,T)$, promising to pay one unit of currency at time to *t+T*. The defaultable term structures are defined by the sovereign debt of other EMU countries. Here C is an index indicating the particular defaultable term structure under analysis. Now, for each issuer default can occur at a random point of time, τ. Along the lines of Lando (1998) DS assume that the default event is the first jump of a Cox process (ie a Poisson counting

[5] From a purely theoretical point of view this assumption seems rather strong. However, this assumption is natural from a practicioner's point of view and comparable empirical studies are also based on this assumption (eg, Düllmann and Windfuhr (2000).

process with stochastic parameter) with intensity $\eta_C(Y_t)$, where Y_t denotes a $K \times 1$ vector of state variables or factors driven by Ito processes. When one issuer country defaults, the values of all bonds issued by this particular country are reduced to a fraction $L_{C,t}$ of their pre-default market value. This assumption is crucial for the derivation of the following key result. DS show that under this assumption of *fractional recovery of market value* defaultable claims can be priced in the same way as their default-free counterparts just with a risk-adjusted short rate. Therefore the prices of the defaultable bonds are given by

$$V_C(t,T) = E_t^Q\left[\exp\left(-\int_t^{t+T}(r_u + s_{C,u})du\right)\right] \tag{3}$$

where $s_{C,t} = \eta_{C,t}L_{C,t}$ is usually called the short spread and can be interpreted as the mean instantaneous loss rate due to default under the risk neutral measure. Thus, the risk-adjusted short rate is simply the sum of the risk-free short rate and the short spread for the issuing country. Equation (3) can be seen as a multi-issuer version of the DS model.

Comparison of equations 2 and 3 sheds light on the convenience of the DS model. There is a clear analogy between conventional models of the risk free term structure and models of defaultable term structures. By parameterizing the sum in the integral in equation 3 or its components, the technical toolbox that has been developed for risk free interest rate modelling becomes available. However, due to the fact that only the product of the two relevant quantities is considered, default probabilities and recovery rates are not separately identifiable from observable prices of risky bonds.

There is an alternative interpretation of s_t in equation 3 in terms of a fractional carrying cost of the defaultable bond due to illiquidity. Alternatively, if s_t is negative, it can be interpreted as a fractional convenience yield due to liquidity. Grinblatt (1995) introduces this setup and fits a Vasicek model of liquidity effects on bond prices. Ultimately, it is an empirical question if yield spreads are predominantly caused by default risk or illiquidity. One of the advantages of latent factor models is that the economic meaning of factors need not be specified in advance. It is most likely that in the market that we analyse both effects play a role. In that sense s_t can be interpreted as a sum of a credit risk component and an illiquidity component. We do not parameterize each component separately but resulting factor time series will be analysed in order to determine which of the possible explanations is more appropriate.

Assuming that both r and s are linear combinations of 'affine' factors (for a characterisation of affine term structure models see Duffie and Kan, 1996) the solution to equation 3 has the following structure:

$$V_C(t,T) = \left(\prod_{k=1}^{M} A_k(T) \prod_{k=1}^{K} A_{C,k}(T) \right) \cdot \exp\left[-\sum_{k=1}^{M} B_k(T) X_t^k - \sum_{k=1}^{K} B_{C,k}(T) \delta_C Y_t^k \right] \tag{4}$$

$$r_t = \sum_{k=1}^{M} X_t^k, \quad s_t^C = \sum_{k=1}^{K} \delta_{C,k} Y_t^k$$

where the expressions $A_k(T)$ and $B_k(T)$ denote maturity dependent functions related to the factors driving the risk free term structure (i.e. X_t^k), whereas the expressions $A_{C,k}(T)$ and $B_{C,k}(T)$ denote maturity dependent functions related to the factors driving the yield spreads (i.e. Y_t^k). The exact functional form of A and B depends on the specification of the factor processes. The country specific weights, $\delta_{C,k}$ enable us to incorporate arbitrary linear dependence structures between the short spreads of different issuers into the model. Note that for $\delta_{C,k} = 0$ the function $A_{C,k}(T) = 1, \forall T$ which allows us to restrict the dependence of a certain issuer's spread to a subset of the driving factors. Furthermore, linear dependence between the short rate and the short spread can be introduced by some Y_t^k being linear combinations of some X_t^k.

Given the limited availability of data and also the convenience offered by analytical pricing equations we choose to implement one of the specifications with known analytical solutions. The two most widely used specifications that satisfy this condition are the Vasicek model, where the evolution of the factors is governed by Ornstein-Uhlenbeck processes and the CIR model, where the factor dynamics are given by square root processes. The following expression reproduces the factor process specification of the CIR model under the empirical (or 'true') measure.

$$dY_t^k = \kappa_k \left(\theta_k - Y_t^k \right) dt + \sigma_k \sqrt{Y_t^k} dW_t^k \qquad k = 1, \ldots, K$$

Here W_t denotes a $K \times 1$ vector of independent standard Brownian motions, κ_k denotes the speed of mean reversion, θ_k is the long term mean of the process, and σ_k is the volatility parameter, respectively. In default-free interest rate modelling, usually the CIR model is considered to be superior to Gaussian specifications, because first of all, under certain parameter restrictions non-negativity of interest rates is assured and second, the volatility structure of the process allows for some degree of heteroscedasticity, which matches empirical evidence for time series of short term interest rates. In the Vasicek model the short rate is normally distributed and homoscedastic. However, for spread modelling it is not yet clear which model to prefer. From a pure credit spread perspective, non-negativity of spreads seems to be a desirable property. Also, almost all empirical work to date has employed the CIR specification. It should be noted that if spreads are not only due to credit risk but also to liquidity differences, then non-negativity of factors is not necessarily an advantage. It is conceivable that in some cases or in some time periods the market for defaultable instruments is more liquid and that therefore liquidity effects have a negative influence on yields. If the liquidity effect can dominate the default effect then spreads can become negative and a Vasicek-type model should be preferred.

Another issue that has to be considered in model specification is the identification of parameters. Here lies the main disadvantage of the Vasicek model. Dai and Singleton (1999) show that in a multi-factor Vasicek model it is not possible to separately identify market prices of risk parameters from zero bond data. This is due to the fact that the market prices of risk only appear in the pricing function for the Vasicek model as a linear combination with the mean of the process, θ. Identification is only possible by using time series information. However, the time series of the short rate is usually highly persistent and therefore time series estimates of θ are unstable and highly imprecise. This identification problem in Gaussian multi-factor specifications is also discussed in De Jong (2000) and Ang and Piazzesi (2000). Ball and Torous (1996) describe the unit root problem in time series estimation of term structure parameters.

Given the considerations above, we choose to apply the CIR model as a first step. In the CIR model the expressions for the functions $A(T)$ and $B(T)$ for a specific factor are given by[6]

$$A(T) = \left[\frac{2\phi_1 \exp(\phi_2 T / 2)}{\phi_4} \right]^{\phi_3} \tag{5}$$

$$B(T) = \frac{2[\exp(\phi_2 T) - 1]}{\phi_4}, \tag{6}$$

[6] For notational convenience we suppress the factor dependent indices in equations (5) and (6). Further, we do not differentiate between functions referring to the risk free short rate and the country dependent functions referring to the short spreads.

where $\phi_1 = \sqrt{(\kappa+\lambda)^2 + 2\sigma^2\delta}$, $\phi_2 = \kappa + \lambda + \phi_1$, $\phi_3 = 2\kappa\theta/\sigma^2$, and $\phi_4 = 2\phi_1 + \phi_2[\exp(\phi_1 T) - 1]$. The parameter λ denotes the market price of risk associated with the factor. Note that for functions related to the factors driving the risk free short rate, $\delta = 1$ in this notation.

The main question in a multi-issuer context is what dependence structure to impose on the stochastic evolution of the short spreads. The empirical results (see section 4) show that there is common variation in empirical spreads of different issuers. Therefore in principle, a specification with unrestricted correlation structure would be desirable. However, estimating such a rich model seems technically highly challenging, if at all feasible. First of all, it is well known that estimation of dynamic term structure models requires long time series of bond data due to the persistent nature of bond yields. Since the EMU started in 1999, the time series available for this analysis is very limited. Second, even if a time series of adequate length would be available, joint estimation of the parameters of about 10 correlated stochastic processes seems a formidable task because of the high dimension of the parameter space. Consequently, as a first approach, we assume independence among the spread processes of different issuers. Still, this exercise is a natural first step towards a slightly more elaborate specification.

Formally, we set $\delta_{C,k} = \mathbf{1}_{\{k \in M^C\}}$, where the sets M^C are disjoint subsets of the index set $\{1,...,K\}$. This structure allows us to estimate the spread related parameters for each issuer separately.

3 Model Estimation

The DS model has been estimated for various specifications and markets. Duffie and Singleton (1997) estimate a two-factor model with independent CIR factors for interest rate swap yields. Duffee (1999) specifies and estimates a three-factor CIR model for corporate bond yields. The short rate is driven by two factors and the short spread by the same two factors plus an idiosyncratic factor. Thus using common factors the model can capture correlation between the short rate and the short spread. However, as Duffie and Singleton (1999) point out, if the correlation in this setting is unrestricted in sign (i.e. negative correlation is allowed) then the short spread looses its non-negativity property. In the most general case affine processes can capture rich dependence structures amongst factors while still retaining non-negativity. Dai and Singleton (2000) analyse the specification of 3-factor affine models and characterize the trade off between dependence and volatility modelling. However, for this richer specifications the maturity dependent constants are only available quasi-analytically, i.e. the corresponding ordinary differential equations have to be solved numerically. Also, the parameter space is expanded considerably when rich dependence structures are taken into account.

Duffie, Pedersen and Singleton (2000) use a specification for the combined effect of liquidity and credit risk to model Russian sovereign debt. In their model yields of different issues of Russian debt depend on a set of common factors and additional idiosyncratic risk factors. Düllmann and Windfuhr (2000) analyse both specifications for the spread between German and Italian government bonds in a univariate setting and conclude that no specification performs significantly better than the other in terms of pricing errors.

Econometric estimation of parameters of continuous time processes is a non-trivial issue. One of the advantages of using affine models is that there is a rich and well-developed literature on parameter estimation for such processes.

The general problem is that the state variables, in our case the short rate and the short spread are not observable. Simple approaches usually define proxies for the short rate (e.g., a short term money market rate) and the short spread and use standard time-series methods to estimate the parameters under the empirical measure and in a second step calibrate the model to market prices to obtain the market price of risk parameters.[7] However there are serious drawbacks associated with this procedure. Firstly, reliable price series of liquid instruments are usually not available for very short maturities. Additionally, even prices of instruments of very short maturity contain risk premia which may lead to substantially biased estimation results. Secondly, this approach neglects the information which is available in the cross section of bonds prices with different maturities.[8]

There are more advanced methods to use both time series and cross sectional information. The first approach which we call 'inversion approach' is due to Pearson and Sun (1992) and Chen and Scott (1993). In this setting, the K unobservable factors are measured by assuming that K bonds in the cross section are priced without error and then inverting the pricing relationship of the model.[9] The results, of course, depend strongly on the choice of the 'input' bonds. As other two-stage estimation procedures this approach suffers from a error-in-variables problem

A different and technically more involved approach uses a state space framework and Kalman filtering to jointly estimate the parameters of the process and the factor series. This approach has first been used by Chen and Scott (1994) and Geyer and Pichler (1999). For a technical description

[7] see Kempf and Uhrig-Homburg (2000).
[8] For a detailed discussion of this matter see Geyer and Pichler (1999).
[9] Düllmann and Windfuhr adopt such an approach in their analysis.

of the method see section 4.1. The advantage of this approach is that it is consistent with the underlying theoretical model since no additional restrictions on the structure of observation errors are necessary. The same set of parameters that govern the dynamics of the factor processes also determines the term structure at any given point in time. The factors need not be specified beforehand, but are extracted from the data.

4 Empirical Analysis

4.1 Data

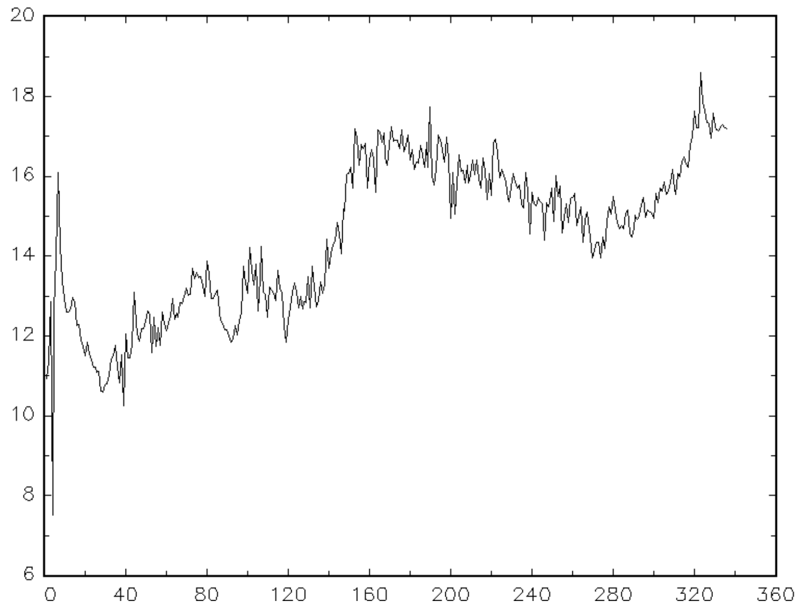
For the empirical analysis presented in this paper we use daily time-series of zerobond yields of all EMU member states for maturities of 2, 3, …, 10 years. These zerobond yields are provided by Reuters Ltd. and are based on the quoted prices of liquid, non-callable government bonds. The use of a set of standardized maturities across all issuers allows us to obtain time-series of yield spreads by simply subtracting the yields observed for Germany from the yields observed for the relevant issuer. Due to the lack of reliable zero bond yield data for some countries, in this version of the paper the empirical analysis is restricted to Austria , Belgium, Italy, The Netherlands and Spain.

Figure 1 shows the yield spread averaged across maturities and countries. Two distinct features can be seen: an extremely high volatility during the first days of January 1999 and a general tendency of increasing spreads. Looking at the country level (while still averaging across maturity) shows that this trend is prevalent in each country, though on different levels (see Table 1). While Italy's spread is clearly above the average, the spread for The Netherlands is far below the average. The overall trend in yield spreads indicates that at least two factors are necessary for modelling the data. One factor has to account for the long-term behavior whereas the second factor may account for country and/or maturity specific features.

Table 1: Mean and standard deviation of credit spreads (averaged across maturities) in basis points.

	Austria	Belgium	Italy	The Netherlands	Spain
Mean	13.7	15.4	21.6	7.6	14.1
std.dev.	2.0	2.8	2.3	1.4	2.4

Figure 1: Observed yield spread between each country and Germany averaged across maturities and countries (January 4, 1999 – April 26, 2000).



4.2 The state-space approach

In order to estimate parameters and to extract the unobservable state variables from yield spreads observed at discrete time intervals we use a state-space formulation of the CIR model. The exact state-space formulation for a K-factor yield spread model with state-vector

$$y_t = (Y_{t,1},...,Y_{t,K})'$$

is based on the assumption that $y_0, y_1,..., y_t$ is a Markov process with $y_0 \sim p(y_0)$ and $y_t \mid y_{t-1} \sim p(y_t \mid y_{t-1})$. $p(y_0)$ is the density of the initial state and $p(y_t \mid y_{t-1})$ is the transition density.

It is known that the exact transition density for the CIR-model is the product of K non-central χ^2 densities (see CIR 1985). Estimation of the unobservable state variables with an approximate Kalman filter in combination with quasi-maximum-likelihood (QML) estimation of the model parameters can be carried out by substituting the exact transition density by a normal density:

$$y_t \mid y_{t-1} \sim N(\mu_t, Q_t).$$

μ_t and Q_t are determined in such a way that the first two moments of the approximate normal and the exact transition density are equal. The elements of the *K*-dimensional vector μ_t are defined as

$$\mu_{t,k} = \theta_k [1 - \exp(-\kappa_k \Delta t)] + \exp(-\kappa_k \Delta t) Y_{t-1,k},$$

where Δt is a discrete time interval. Q_t is a $K \times K$ diagonal matrix with elements

$$Q_{t,k} = \sigma_k^2 \frac{1 - \exp(-\kappa_k \Delta t)}{\kappa_k} \left(\frac{\theta_k}{2} [1 - \exp(-\kappa_j \Delta t)] + \exp(-\kappa_k \Delta t) Y_{t-1,k} \right),$$

Let $z_t = (Z_{t,1}, ..., Z_{t,n})'$ be the *n*-dimensional vector of yield spreads observed at time *t*. $Z_t(T)$ is related to equation 4 as follows:

$$Z_t(T) = -\frac{1}{T} \sum_{k=1}^{K} \log[A_{C,k}(T)] + \frac{1}{T} \sum_{k=1}^{K} B_{C,k}(T) Y_t^k . \qquad (7)$$

We assume that z_t are conditionally independent given y_{t-1} and z_t is independent of y_s, $s \neq t$ given y_t with $z_t \mid y_t \sim p(z_t \mid y_t)$. The observation density $p(z_t \mid y_t)$ is based on the linear relation between observed yield spreads and the state variables. The measurement equation for observed yield spreads is:

$$z_t = a_t + b_t y_t + \varepsilon_t \qquad \varepsilon \sim NID(0, H) \qquad (t = 1, ..., N),$$

where *N* is the number of observations. a_t is a $n \times 1$ vector with elements (see equations 4, 5 and 7):

$$a_t(T_i) = -\sum_{k=1}^{K} \frac{\phi_{k,3}}{T_i} \log\left(\frac{2\phi_{k,1} \exp(\phi_{k,2} T_i / 2)}{\phi_{k,4}} \right) \qquad (i = 1, ..., n),$$

b_t is a $n \times K$ matrix defined as (see equations 4, 6 and 7):

$$b_t(T_i) = \frac{2[\exp(\phi_{k,1}T_i) - 1]}{\phi_{k,4}T_i} \qquad (i = 1,...,n).$$

H is the variance-covariance matrix of ε_t. In our empirical application the number of observed bonds and the associated maturities do not change over time. Therefore H has constant dimension $n\times n$ and is assumed to be a diagonal matrix. In the empirical analysis we assume that the errors for each maturity have the same standard deviation (ie., all diagonal elements are identical $h_i = h$).

This model is directly applicable to the Kalman filter recursion which can be briefly described as follows. For a given set of parameter values a prediction of the factors y_t (and their variance-covariance matrix Σ_t) is made at the beginning of time t based on the factor estimate from time t–1:

$$Y_{k,t|t-1} = \theta_k[1 - \exp(-\kappa_k \Delta t)] + \exp(-\kappa_k \Delta t) Y_{k,t-1|t-1}.$$

The vector of expected yield spreads conditional on time t–1 information is given by

$$\hat{z}_t = a_t + b_t y_{t|t-1} \qquad (8)$$

When z_t is observed an updated state estimate $y_{t|t}$ is computed such that the fit to observed yield spreads for the current date is optimal in the mean squared error sense. The prediction and updating sequence is repeated for each $t=K+1,...,N$.

The Kalman filter provides all the necessary information to calculate the quasi log-likelihood (see Harvey 1989, p.126):

$$\log L = -0.5 \log 2\pi (N-K)n - 0.5 \sum_{t=K+1}^{N} \log|F_t| - 0.5 \sum_{t=K+1}^{N} v_t F_t^{-1} v_t ,$$

where v_t are $n\times 1$ vectors of errors $z_t - \hat{z}_t$, $\hat{z}_t$ is the vector of predicted yield spreads based on equation (8) using the factor estimate before updating ($y_{t|t-1}$), and $F = b_t \Sigma_t b_t^{'} + H$.

The CIR model differs from standard Kalman filter applications because of the non-negativity restriction on state variables. We modify the standard Kalman filter by simply replacing any negative element of the state estimate $y_{t|t}$ with zero. Therefore, in general, the Kalman filter is not strictly a linear estimator for the state variables, but it is linear for $y_t > 0$.

4.3 Empirical Results

4.3.1 Factor analysis with LISREL

In order obtain some insight into the covariance-structure of the data we use a constrained factor analysis. This technique is also known as confirmatory factor analysis, structural equation or LISREL modeling (see Bollen 1989). It deviates from standard factor analysis in that it allows to impose constraints on the factor loadings. A large variety of different assumptions can be tested in this framework. While any assumptions that may be imposed on the covariance among variables (yield spreads across countries and maturities) may rely on economic considerations, the methodology itself is purely data driven. It is not possible to impose constraints on dynamic properties of the extracted factors, e.g., in order to make them comparable to a particular stochastic process.

We found strong support for using one global factor and a country-specific factor for each country (i.e. n+1 factors for n countries in the sample). The global factor is assumed to affect all maturities in all countries, whereas the country-specific factors are only affecting the spreads in that specific country. Factor loadings are different for each maturity. Reframed in terms of a regression equation the model may be written as

$$Z_{ij} = g_j F^g + c_{ij} F_i^c + \varepsilon_{ij},$$

where Z_{ij} is the observed yield spread for country i and maturity j, F^g is the global factor and g_j the maturity-specific loading for this factor. F_i^c is the country-specific factor for country i and c_{ij} the country- and maturity-specific loading for this factor.

The global factor is assumed to be uncorrelated with the country factors. Relaxing the assumption of zero correlation with the global factor has almost no effect, however. Correlations with the global factor are below 0.05. The country factors, however, are correlated with each other as shown in Table 2.

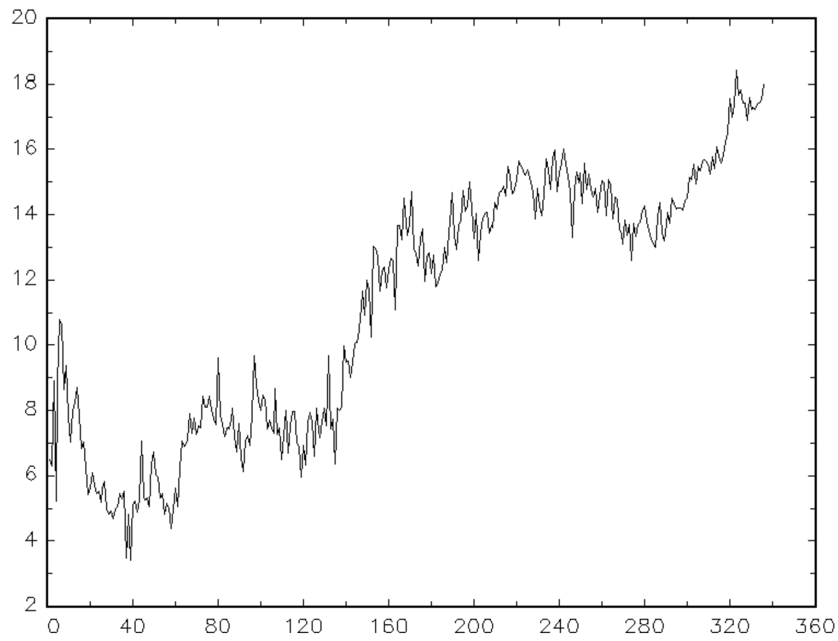
Table 2: Correlation among factors from LISREL analysis.

	global factor	country factor Austria	country factor Belgium	country factor Italy	country factor Netherlands	avg. squared loadings with global factor	avg. squared loadings with country factor
Austria	0.0					0.371	0.272
Belgium	0.0	-0.83				0.462	0.181
Italy	0.0	-0.04	0.04			0.434	0.198
The Netherlands	0.0	0.68	-0.77	-0.10		0.305	0.168
Spain	0.0	-0.80	0.78	0.32	-0.78	0.488	0.253

In order to investigate the strength of association to the global and country-specific factors we compute the average of squared loadings for each country and factor (averages are across maturities and countries for g_j and across maturities for c_{ij}). Table 2 shows Belgium, Italy and Spain are more strongly related to the global factor than Austria and The Netherlands.

Figure 2 shows the scores of the global factor from the LISREL analysis which is very similar to the overall trend in credit spreads already observed in Figure 1. The high level of volatility at the beginning of the sample is not captured by this factor, however.

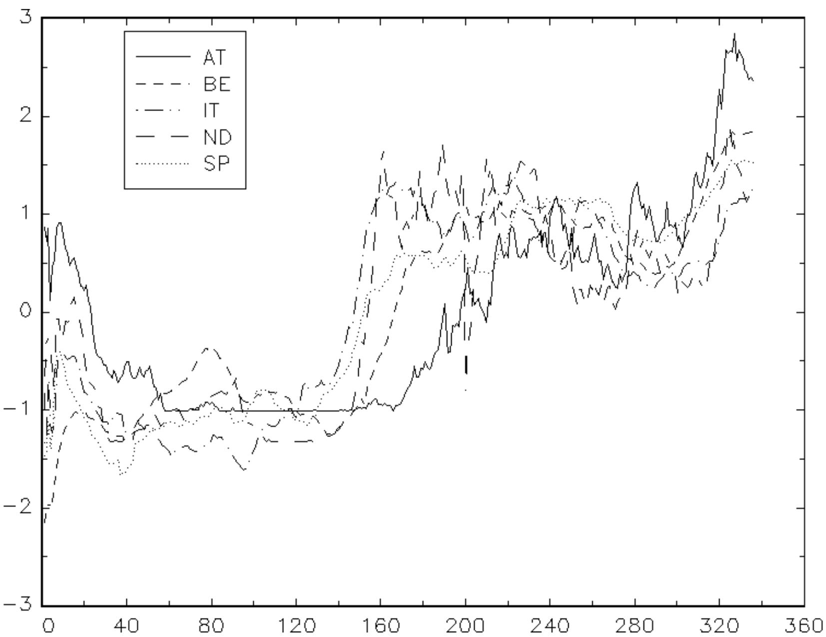
Figure 2: The global factor in credit spreads obtained from the LISREL analysis.



4.3.2 Extracted factors from the CIR model

Figure 3 shows the global factor from the CIR model for each country. The Figure shows a high degree of correspondence of the factors, both among each other and compared to the LISREL factor in Figure 2. The global factors from the CIR model pick up the general trend in the yield spreads, already shown in Figure 1. Note, however, that in the LISREL analysis data for all countries and maturities has been used simultaneously, whereas in the state-space approach a CIR two-factor model has been estimated for each country separately neglecting the corresponding information contained in the data from the remaining countries.

Figure 3: The global factors from the CIR model for each country.



Comparing country-specific factors from LISREL to the second factor from the CIR models for each country provides a mixed picture. Whereas the LISREL and the CIR factor for Austria, Belgium and Spain are rather strongly correlated (0.91, 0.86 and 0.79, respectively), the two factors for Italy and The Netherlands are only weakly correlated (0.17 and 0.14). Figures 4 and 5, however, show that the factor series are not as different as the low correlations suggest. Only in the second half of the sample the two factors (start to) deviate from each other.

In general, however, these comparisons show a remarkable similarity between factors derived from two distinctly different approaches. This supports a simplified structure of our multi-issuer model. It does not appear to be necessary to model all countries jointly in a large state-space model with restrictions on the correlations among country-specific or other factors. Note, that this fact might be essential for the applicability of this model for practical purposes.

Figure 4: Country-specific factors from LISREL and CIR model for Italy.

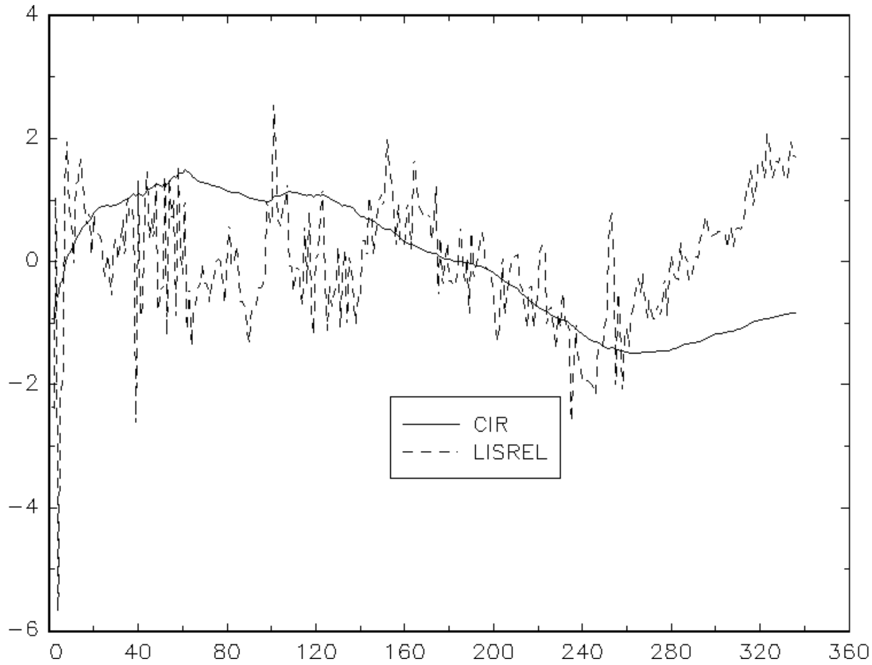
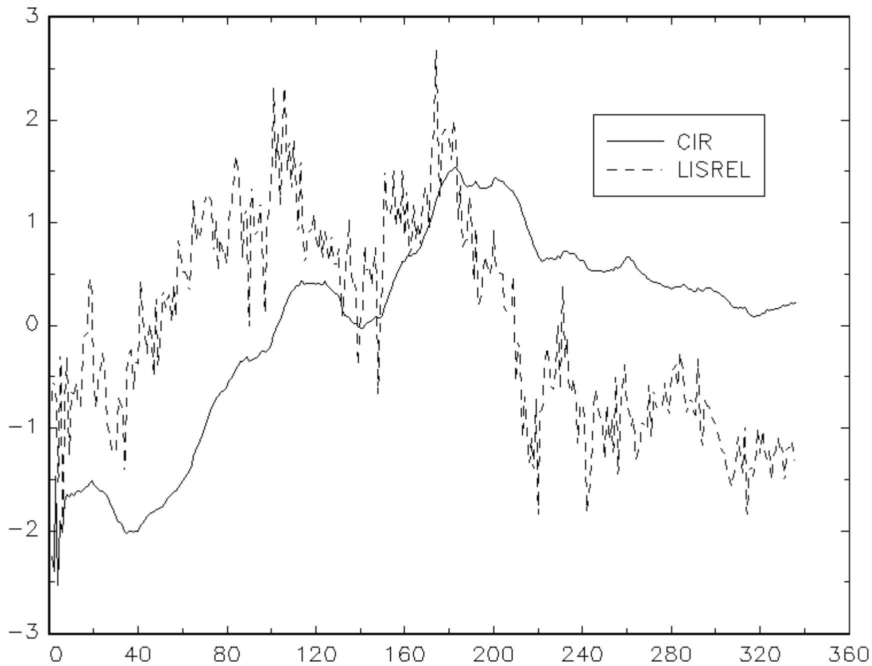


Figure 5: Country-specific factors from LISREL and CIR model for The Netherlands.



4.3.3 One-step ahead prediction of credit spreads from the CIR model

Whereas the LISREL analysis mainly attempts to explain the covariance in the data, the state-space approach can be viewed as an optimal one-step ahead prediction of the observations. In order to illustrate the flexibility implied by the CIR model consider Figure 6, which shows four selected days from Italy's sample. The state-space approach yields two predictions, which are both shown in these diagrams: one is made ex-ante, before the current observation ($z_{t|t-1}$) and the second is made ex-post, after the current observation has been taken into account ($z_{t|t}$). The peculiar, s-shaped behavior of the observed term structure is very well captured by the two-factor CIR model. In general, the predicted term structure reacts in terms of level shifts, curvature and steepness/flatness to changes in the observed credit spreads.

Figure 6: Observed spread and fit obtained from the CIR model for Italy.

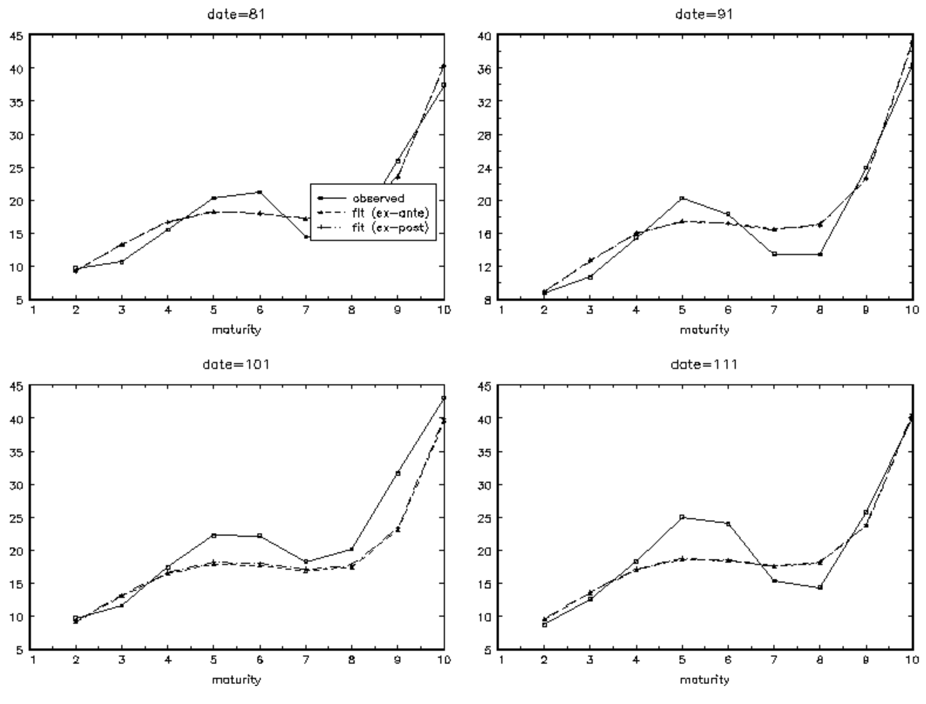


Table 3 contains the average one-step ahead prediction error (not standardized) for the 2-factor CIR model in basis points. Except for Belgium the average error is more or less unbiased. Looking across maturities, however, we see that errors follow a distinct +/– pattern. That is mainly introduced by the peculiar s-shaped behavior of the observed term structure. Although the CIR model implies a high degree of flexibility, we find that it is not possible to capture the observed curvature sufficiently. This deficiency can be explained by the constraints imposed on the factor processes. The same set of parameters affects both the dynamic behavior of the factors as well as the cross-sectional fit implied by the CIR model.

Table 3: Average one-step ahead prediction errors from the CIR model in basis points.

Maturity	Austria	Belgium	Italy	The Netherlands	Spain
2	0.027	3.351	3.331	1.284	0.324
3	1.754	5.591	-2.234	0.908	0.013
4	1.942	5.694	-2.292	1.029	0.013
5	2.446	6.495	2.658	2.111	0.458
6	0.752	5.88	3.503	1.571	0.784
7	-1.973	4.432	-1.543	-0.498	1.01
8	-1.74	4.422	-3.379	-0.678	0.549
9	0.681	6.469	0.85	0.757	0.609
10	-2.848	6.679	-1.292	-0.328	1.068
average	0.116	5.446	-0.044	0.684	0.537

It also turns out that the estimated parameters of the two-factors may strongly differ among countries. We only find a strong similarity for the risk parameter λ for the country-specific factor, which is around –1.3 (see Table 4). The large differences between theta do not imply, however, that the average levels of the yield spreads differ in a similar way. Since the extracted factors correspond to maturity zero the estimated level of theta depends on the shape of the yield spread structure at maturities of two to four years.

Table 4: Estimated parameters of the CIR model for each country.

country factor	κ	σ	λ	θ	h
Austria	0.1044	0.0067	-1.1511	0.0162	3.5499
Belgium	0.4916	0.0040	-1.8644	0.0009	2.5532
Italy	0.0004	0.0011	-1.4649	0.0120	3.5993
The Netherlands	0.0016	0.0028	-1.3257	0.0046	1.7239
Spain	0.2488	0.0094	-1.4348	0.0056	1.9784
global factor					
Austria	2.2192	18.5462	-67.9991	7.8591	
Belgium	1.8321	0.2529	-3.1263	0.1067	
Italy	0.0000	0.2362	-0.8083	3.8568	
The Netherlands	5.3730	13.5362	-5.4722	7.1446	
Spain	0.0506	0.1739	-0.9021	0.3580	

A way to summarize and to compare the individual parameter values is to compute the probabilities of default for each country for a bond with maturity of one year. This may be interesting from a regulatory point of view since for the recent Basle proposal the one year default probability is of critical importance. For that purpose we compute the predicted yield spreads for maturity one as in equation (8) and to obtain the default probabilities under the empirical measure we set $\lambda = 0$ and use the factor estimates after updating ($y_{t|t}$).This default probability can be computed for every day in the sample. The mean probability (across time) for each country is given in Table 5. Note, however, that it is only valid to interpret these probabilities as pure default probabilities if other effects such as liquidity are not taken into account.

Table 5: Average default probability (in %) under the empirical measure for a maturity of one year implied by the CIR model for each country.

	one year default probability (in %)
Austria	0.02414
Belgium	0.00426
Italy	0.04853
The Netherlands	0.05895
Spain	0.01383

5 Summary

This paper investigates some major issues involved in modeling the dynamics of yield spreads between government bonds issued by member states of the European Monetary Union (EMU) and Germany. We formulate a multi-issuer version of the model originally proposed by Duffie and Singleton (1999). The model enables us to incorporate arbitrary linear dependence structures between the yield spreads of different issuers. We use a state-space approach to extract series of factors driving the observed yield spreads assuming the CIR-type factor specification. In addition, we use a LISREL approach to investigate the question of an appropriate factor structure, that does not impose any constraints from an economic model. We find strong empirical evidence for a global factor that mainly represents the average level of the yield spreads and for a country specific factor for each issuer. This is supported by two entirely different estimation approaches. Our findings indicate that it may be justified to implement a simplified version of the multi-issuer model that sufficiently captures the main features of the data.

References

Ang, M., and M. Piazzesi, 2000, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent factors, working paper

Ball, C., and W. Torous, 1996, "Unit roots and the estimation of interest rate dynamics", Journal of Empirical Finance, 3, 215-238

Bollen, K.A., 1989, Structural equations with latent variables, New York:Wiley.

Chen, R., and L. Scott, 1993, "Maximum likelihood estimation for a multifactor equilibrium model of the term structure of interest rates", Journal of Fixed Income, 3, 14-31

Chen, R., and L. Scott, 1995, "Multifactor Cox Ingersoll Ross Models of the term structure: Estimates and test from a Kalman filter model", working paper, University of Georgia

Cox, J.C., J.E. Ingersoll, and S.A. Ross, 1985, "A theory of the term structure of interest rates", Econometrica, 53, 385-407

Dai, Q., and K. Singleton, 2000, "Specification Analysis of Affine Term Structure Models", Journal of Finance, forthcoming

De Jong, F., 2000, "Time Series and Cross-section Information in Affine Term-Structure Models", Journal of Business and Economic Statistics, 18, 300-314

Duffee, G., 1999, Estimating the Price of Default Risk", Review of Financial Studies, 12, 197-226

Duffie, D. , and R. Kan, 1996, A yield-factor model of interest rates, Mathematical Finance, 6, 379-406

Duffie, D., and K. Singleton, 1997, "An Econometric Model of the Term Structure of Interest Rate Swap Yields", Journal of Finance, 52, 1297-1321

Duffie, D., and K. Singleton, 1999, "Modeling the Term Structure of Defaultable Bonds", Review of Financial Studies, 12, 1999

Duffie, D., L. Pedersen, and K. Singleton, 2000, Modeling Sovereign Yield Spreads: A case studiy of Russian Debt, working paper

Düllmann, K., and M. Windfuhr, 2000, "Credit Spreads Between German and Italian Sovereign Bonds - Do Affine Models Work?", working paper, University of Mannheim

Geyer, A., and S. Pichler, 1999, "A state-space approach to estimate and test multi-factor Cox-Ingersoll-Ross models of the term structure", The Journal of Financial Research, 22, 107-130

Grinblatt, M., 1994, An analytic solution for interest rate swap spreads, Centre for Economic Policy Research European Science Foundation Working Paper: 53

Harvey, A. C., Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press, 1989

Kempf, A., and M. Uhrig-Homburg, 2000, Liquidity and its impact on bond prices, Schmalenbach Business Review, 52, 26-44

Lando, D., 1998, "On Cox Processes and Credit Risky Securities", Review of Derivatives Research 2: 99-120

Lund, J., 1999, A model for studying the effects of EMU on European yield curves, European Finance Review, 2, 321-363

Pearson, N., and T. Sun, 1994, "Exploiting the Conditional Density in Estimating the Term Structure: An Application to the Cox, Ingersoll, and Ross Model", Journal of Finance, 49, 1279-1304

Vasicek, O., 1977, An equilibrium characterization of the term structure, Journal of Financial Economics, 5, 177-188