



ELSEVIER

European Journal of Operational Research 135 (2001) 270–295

EUROPEAN  
JOURNAL  
OF OPERATIONAL  
RESEARCH

www.elsevier.com/locate/dsw

# Xetra efficiency evaluation and NASDAQ modelling by KapSyn

Otto Loistl<sup>a,\*</sup>, Bernd Schossmann<sup>b</sup>, Olaf Vetter<sup>c</sup>

<sup>a</sup> *Wirtschaftsuniversität Wien, Althanstrasse 39–45, 1090 Wien, Austria*

<sup>b</sup> *KPMG, Chicago, IL, USA*

<sup>c</sup> *DVFA, German Society of Investment Analysts and Asset Managers, Dreieich, Germany*

Received 20 November 1999; accepted 27 December 2000

## Abstract

In this paper we give an introduction to capital market synergetics, a model enabling us to compute transaction costs and investigate the operating efficiency of a stock market's microstructure. Frankfurt's trading system Xetra and NASDAQ are currently implemented into the computer program KapSyn. By using KapSyn we examine Xetra's behaviour in different market scenarios, particularly the designated sponsor's performance. The empirical evidence of the KapSyn parameter setting chosen is validated: connecting the parameter setting to economic data by using neural networks and analytical reasoning underlie the definition of market scenarios. The designated sponsor's eminent importance in non-liquid markets has been demonstrated very impressively. Finally, NASDAQ's transaction cost statistics are investigated in the above scenarios. Compared to Xetra, NASDAQ exhibits minimal transaction costs for mid-size trades, while transaction costs for small and block trades are nearly 100%/50% higher. © 2001 Elsevier Science B.V. All rights reserved.

**Keywords:** Finance; Capital market synergetics; Transaction costs; Stock market's microstructure

## 1. The KapSyn-model's basic structure<sup>1</sup>

### 1.1. Agents' decisions to buy and sell

#### 1.1.1. Rate of return and execution feasibility as the factors determining bids and asks

The distribution of individual assumptions of intrinsic value for a share  $j$  can be deduced from

the distribution of individual price expectations for it. An agent  $i$ 's assumption as to a share  $j$ 's intrinsic value arises from discounting his/her price expectation at the time at which the estimate is made. If the market participants behave in a risk-neutral manner, if interest rates are linear and if the observation is for a single period, then the following simple relationship applies exactly:

$$p_{ij}^{\text{cut}} = \frac{\hat{p}_{ij}}{1 + r},$$

$r$  is the risk-free rate. We use the formula as a proxy. The expression  $p_{ij}^{\text{cut}}$  represents the present value of agent  $i$ 's stock price estimate  $\hat{p}_{ij}$ .

\* Corresponding author.

E-mail addresses: otto.loistl@wu-wien.ac.at (O. Loistl), bernd.schossmann@florsheim.com (B. Schossmann), ov@dvfa.de (O. Vetter).

<sup>1</sup> Adapted from the KapSyn user handbook (Loistl and Vetter, 2000).

A distribution of the market's assumptions as to a share  $j$ 's intrinsic value can be gained by aggregating all individual NPV estimates in the same way as for the distribution of the market's price expectation for this share:

At the start of trading the following distribution applies:

$$\left(p_{ij}^{\text{cut}}\right)_{i=1\dots N} = \left(p_{1j}^{\text{cut}}, \dots, p_{ij}^{\text{cut}}, \dots, p_{Nj}^{\text{cut}}\right),$$

$$p_{ij}^{\text{cut}} = \frac{\hat{p}_{ij}^{\text{ext}}}{1+r}.$$

During trading the following distribution applies:

$$\left(p_{ij}^{\text{cut}}\right)_{i=1\dots N} = \left(p_{1j}^{\text{cut}}, \dots, p_{ij}^{\text{cut}}, \dots, p_{Nj}^{\text{cut}}\right),$$

$$p_{ij}^{\text{cut}} = \frac{\hat{p}_{ij}}{1+r} = \frac{\hat{p}_{ij}^{\text{ext}} + \Delta\hat{p}_{ij}}{1+r}.$$

The distribution of subjective intrinsic value assumptions at the start of trading reflects the NPV estimates of all market participants for share  $j$  at the start of trading. This distribution can also be interpreted as the market's fundamental intrinsic value assumption with regard to the shares.

The distribution of subjective intrinsic value assumptions at a particular point in time during trading provides the basis to compute the market participant  $i$ 's current NPV  $p_{ij}^{\text{cut}}$  for a share  $j$ . The current NPV  $p_{ij}^{\text{cut}}$  of an agent  $i$  for a share  $j$  at a particular time forms the basis of his/her decision to buy or sell that share. The planning process involved here will be described in the following section.

Every market participant will develop a strategy in line with the courses of trading which s/he considers possible, and can then use this strategy to react to certain changes in the state of affairs during trading. A large number of different factors can be seen as causing a change in the state of affairs on the stock exchange.

The process by which an agent  $i$ ,  $i \in I = \{1, \dots, N\}$ , makes an individual decision to submit an order for a share  $j$ ,  $j \in J = \{1, \dots, M\}$ , at a time  $t$  during trading, involves the following steps:

1. Decision as to whether to buy or sell share  $j$ .
2. Establishment of permissible order prices.

3. Establishment of permissible order amounts.
4. Evaluation of defined buy or sell actions.
5. Selection of an action.

These decisions will be described in detail in the following sections.

The principal decision is based on considerations of rates of return and realization:

- An order is only considered if it promises a positive yield.
- An order is only considered if it has a chance of being realized.

The criterion for the yield of an order is the order's *expected* yield. The conditional equation for the expected yield of an order at price  $p$  by an agent  $i$  for a share  $j$  is as follows:<sup>2</sup>

$$p(1 + \hat{r}_{ij}) = \hat{p}_{ij} \iff \hat{r}_{ij} = \frac{\hat{p}_{ij}}{p} - 1.$$

It is assumed that every agent can invest in both shares and interest bearing cash holdings. With cash holdings, the market rate of return  $r$  is attained. Comparing  $\hat{r}_{ij}$  with  $r$  allows a decision to be made as to whether to buy or sell a share at price  $p$ . The following relationships result:

$$r < \hat{r}_{ij} \Rightarrow \text{bid}; \quad r \geq \hat{r}_{ij} \Rightarrow \text{ask}.$$

If the conditional equation above is employed by  $\hat{r}_{ij}$ , the following then follows:

$$r < \frac{\hat{p}_{ij}}{p} - 1 \Rightarrow \text{bid}; \quad r \geq \frac{\hat{p}_{ij}}{p} - 1 \Rightarrow \text{ask}.$$

The present value  $p_{ij}^{\text{cut}} = \hat{p}_{ij}/(1+r)$  is derived from the future expected price  $\hat{p}_{ij}$  discounted with the appropriate interest rate. The discount rate can be selected according to the specific market conditions.

This  $p_{ij}^{\text{cut}}$  provides the means to model the decision behavior of an agent  $I$  when defining bid/ask prices that are worth realizing for a share  $j$  as follows:

- Buy at all prices  $p$  that are smaller than the present value  $p_{ij}^{\text{cut}}$  of the expected share price  $j$ .
- Sell at all prices  $p$  that are greater than or equal to the present value  $p_{ij}^{\text{cut}}$  of the expected share price of share  $j$ .

<sup>2</sup> Cf. Landes and Loistl (1992, p. 217).

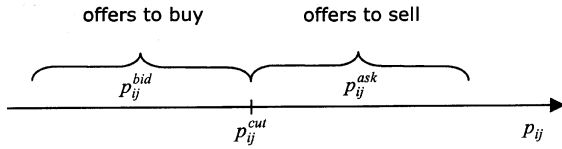


Fig. 1. Agent-specific valid range of bid/asks.

Feasible bid prices are defined by the interval  $(0, p_{ij}^{\text{cut}})$ , feasible ask prices are defined by the interval  $[p_{ij}^{\text{cut}}, \infty)$ . Fig. 1 illustrates this relationship.

An order's chance of realization acts as a criterion for its feasibility. The chance of realization can be assessed by comparing the order's price limit with the bids and asks already on the market. The following relations apply:

- A bid has a chance of realization if its price limit  $p$  is *higher* than the price limits of all bids already on the market.
- An ask has a chance of realization if its price limit  $p$  is *lower* than the price limits of all asks already on the market.

The chance of an offer for a share  $j$  being realized is examined by comparing the offer's price limit with the 'marginal market price'  $p_{ij}^{\text{cut}}$ . This marginal price separates all bids and asks already placed on the market for a share  $j$  according to their limits. It is calculated as follows:<sup>3</sup>

$$p_j^{\text{cut}} = \begin{cases} p_j^{\text{ask}} + 0,5 & \text{if } p_j \leq p_j^{\text{ask}} \\ p_j & \text{if } p_j^{\text{ask}} < p_j < p_j^{\text{bid}} \\ p_j^{\text{bid}} - 0,5 & \text{if } p_j \geq p_j^{\text{bid}} \end{cases},$$

where  $p_j^{\text{ask}}$  is the maximum bid price for a share  $j$  on the market;  $p_j^{\text{bid}}$  the minimum ask price for a share  $j$  on the market;  $p_j$  the current market price for a share  $j$  and  $p_j^{\text{cut}}$  is the marginal market price.

Fig. 2 illustrates the decisive significance of  $p_{ij}^{\text{cut}}$  as the watershed between supply and demand.

All limit orders on the market with limits below the marginal price  $p_j^{\text{cut}}$  represent bids, while all limit orders with limits above  $p_j^{\text{cut}}$  represent asks. The feasibility of order prices for a share  $j$  is then determined as follows:

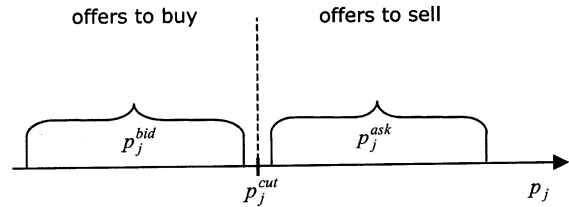


Fig. 2. Market-specific set of bid/asks.

- A bid for a share  $j$  has a chance of being realized if the order price  $p_j^{\text{bid}}$  is higher than the market's marginal price, in other words,  $p_j^{\text{bid}} > p_j^{\text{cut}}$ .
- An ask for a share  $j$  has a chance of being realized if the order price  $p_j^{\text{ask}}$  is lower than the market's marginal price, in other words,  $p_j^{\text{ask}} < p_j^{\text{cut}}$ .

Following the feasibility criterion, permissible bid prices are defined by the interval  $(p_j^{\text{cut}}, \infty)$ , while permissible ask prices are defined by the interval  $(0, p_j^{\text{cut}})$ .<sup>4</sup>

The simultaneous observation of offers which are worth realizing and offers which are feasible leads to agent  $i$  making a basic decision as to whether to buy or sell a share  $j$ .<sup>5</sup> There are two possible cases:

Situation	Market effect
I $p_{ij}^{\text{cut}} > p_j^{\text{cut}}$	Agent $i$ is a potential buyer of share $j$
II $p_{ij}^{\text{cut}} < p_j^{\text{cut}}$	Agent $i$ is a potential seller of share $j$

In situation I, the individual marginal price  $p_{ij}^{\text{cut}}$  is higher than the marginal market price. The agent  $i$  is a potential buyer of share  $j$ . Buy orders exist which are both feasible and worth realizing.

In situation II, the individual marginal price  $p_{ij}^{\text{cut}}$  is lower than the marginal market price. The agent  $i$  is a potential seller of share  $j$ . Sell orders exist which are both feasible and worth realizing.

Figs. 3 and 4 show the distribution of potential buyers and sellers of a share  $j$ , based on their

<sup>4</sup> Cf. Landes and Loistl (1992, p. 217).

<sup>5</sup> For the details of the integer condition of price offers, see Landes and Loistl (1992, p. 218).

<sup>3</sup> Cf. Landes and Loistl (1992, p. 217).

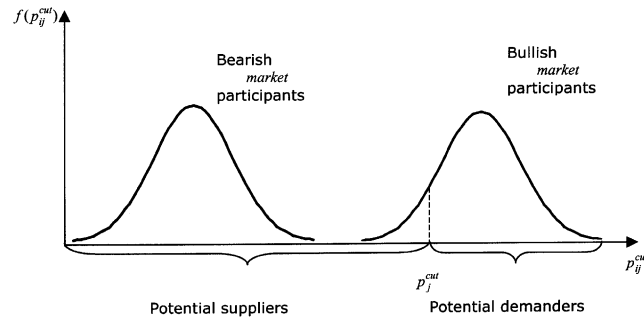


Fig. 3. Bearish market scenario.

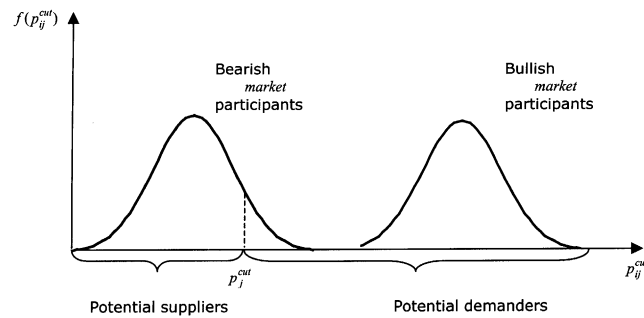


Fig. 4. Bullish market scenario.

subjective value estimations  $(p_{ij}^{\text{cut}})_{i=1\dots N}$ , according to the marginal market price  $p_j^{\text{cut}}$  of this share  $j$ .

All agents whose estimations' present value  $p_{ij}^{\text{cut}}$  lies to the left of  $p_j^{\text{cut}}$  are potential sellers of share  $j$ . All agents whose estimations' present value  $p_{ij}^{\text{cut}}$  lies to the right of  $p_j^{\text{cut}}$  are potential buyers of share  $j$ . In Fig. 3, the ratio of potential buyers to potential sellers is in favor of the sellers, and in Fig. 4 in favor of the buyers.

As mentioned before, generally market participants are represented by a (bimodal) distribution, in line with the parameters in the model.

The characterization of a bullish or bearish market for a share  $j$  using the distribution of the subjective value estimates  $(p_{ij}^{\text{cut}})_{i=1\dots N}$  and the state of the marginal market price  $p_j^{\text{cut}}$  also includes the level of interest rates on the interest bearing cash market. The higher the interest rate  $r$  – it is assumed to function as a calculatory interest rate for determining the present value of the future price estimate – the lower the expectations' present value of the market participants. Consequently, ris-

ing or falling interest rates will lead to a shift to the left or right in the distribution of the value estimates  $(p_{ij}^{\text{cut}})_{i=1\dots N}$  for a share  $j$ . The ratio of potential buyers to potential sellers also shifts accordingly.<sup>6</sup> This confirms the typical experience that rises in interest rates lead to drops in share prices.

#### 1.1.2. Establishing bid and ask prices

Following the decision in principle of an agent  $i$  to make a bid/ask for a share  $j$ , it is necessary to define the set of bid and ask prices. The following basic relationships apply:

1. An agent  $i$  is a potential buyer of share  $j$ , i.e.,  $p_{ij}^{\text{cut}} > p_j^{\text{cut}}$ . The sets of acceptable bid prices (expressed as integers) lie in the interval  $(p_j^{\text{cut}}, p_{ij}^{\text{cut}})$ .<sup>7</sup>

<sup>6</sup> See Casey (1998, p. 210).

<sup>7</sup> If the agent has already placed an ask on the market, he/she will only be permitted to cancel this order.

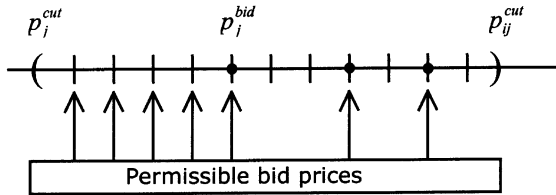


Fig. 5. Permissible bid and ask prices for a potential buyer.

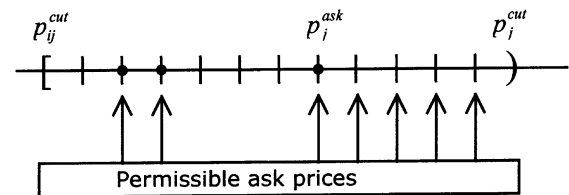


Fig. 6. Acceptable bid and ask prices for a potential seller.

2. An agent  $i$  is a potential seller of share  $j$ , i.e.,  $p_{ij}^{cut} < p_j^{cut}$ . The sets of acceptable ask prices (expressed as integers) lie in the interval  $[p_{ij}^{cut}, p_j^{cut}]$ .<sup>8</sup>

The bid and ask prices defined under 1 and 2 for buyers and sellers are both worth being realized and capable of being realized.<sup>9</sup> If bids and asks for a share  $j$  have already been placed on the market by other market participants, there will be a restriction on the acceptable bid/ask prices of an agent  $i$  for  $j$ .

1. An agent  $i$  is a potential buyer of share  $j$  and asks for this share  $j$  have already been submitted by other market participants on the market.

The prices for the asks on the market are represented by arrows in Fig. 5.<sup>10</sup>

In this case, the number of permissible bid and ask prices of agent  $i$  for share  $j$  is derived from

- the set of ask prices on the market, and
- the set of bid and ask prices located between the marginal market price  $p_j^{cut}$  and the lowest ask price on the market  $p_j^{bid}$ .

The maximum bid amount for those prices that lead to transactions is limited to the amount of the relevant ask. As regards the amount of bid and ask prices  $p \in (p_j^{cut}, p_j^{bid})$ , the only criterion is that  $q > 0$ .

2. An agent  $i$  is a potential seller of a share  $j$ , and other market participants have made bids for  $j$ .

The prices for the bids on the market are represented by arrows in Fig. 6.<sup>11</sup>

The set of acceptable asks for agent  $i$  for share  $j$  is now determined by

- the set of bid prices on the market, and
- the set of prices which lie between the highest bid price on the market  $p_j^{ask}$  and the marginal market price  $p_j^{cut}$ .

Just as in the case of the potential buyer, the maximum amount of order prices leading to transactions is restricted to the set of the relevant bids.<sup>12</sup> For the sets of the order prices  $p \in (p_j^{ask}, p_j^{cut})$ , all that is required is  $q < 0$ .

The fixing of the amounts the agents want to trade follows the decision to buy or sell. The following section outlines the basic calculation procedure for determining the amount which the agent will want to trade given a particular acceptable order price.

In concrete cases, this must be adjusted to take account of stock exchange regulations.

## 1.2. Calculating the transition rates

The transition rate of a permissible action  $a$  is generally calculated as follows:<sup>13</sup>

$$\lambda_a = W \cdot e^{\Phi_i(a)},$$

$W$  = speed of reaction parameter.

The preference function  $\Phi_i(a)$  and the speed of reaction parameter  $W$  are defined as follows, depending on the size of the action  $a$ :

<sup>8</sup> If the agent has already placed a bid on the market, he/she will only be permitted to cancel this order. Cf. Landes and Loistl (1992, p. 218).

<sup>9</sup> For  $p_{ij}^{cut} = p_j^{cut}$  agent  $i$  is indifferent as regards the issuing of a bid or ask for share  $j$ .

<sup>10</sup>  $p_j^{bid}$  indicates the minimum ask price for a share  $j$ .

<sup>11</sup>  $p_j^{ask}$  indicates the maximum bid price for a share  $j$ .

<sup>12</sup> The individual budget limits must be observed.

<sup>13</sup> Cf. Landes and Loistl (1992, p. 214).

- Action  $a = \text{Buy or sell action}$

$$W = W_E, \quad W_E > 0,$$

$$\Phi_i^E(a) = \xi_i^{ip} \phi_{ij}^{ip} + \xi_i^{\text{real}} \phi_{ij}^{\text{real}} + \xi_i^{\text{trd}} \phi_{ij}^{\text{trd}},$$

$$\phi_{ij}^{ip}, \phi_{ij}^{\text{real}}, \phi_{ij}^{\text{trd}} = \text{Single-value functions},$$

$$\xi_i^{ip}, \xi_i^{\text{real}}, \xi_i^{\text{trd}} = \text{Weighting factors},$$

$$\xi_i^{ip}, \xi_i^{\text{real}}, \xi_i^{\text{trd}} > 0.$$

- Action  $a = \text{Change in expectations}$

$$W = W_V, \quad W_V > 0,$$

$$\Phi_i^V(a) = \delta \left[ \eta_i^{\text{ext}} \phi_{ij}^{\text{ext}} + \eta_i^{\text{inf}} \phi_{ij}^{\text{inf}} + \eta_i^{\text{pot}} \phi_j^{\text{pot}} + \eta_i^{\text{tr}} \phi_j^{\text{tr}} \right],$$

$$\delta = \begin{cases} +1 & \text{if } a = \hat{p}_{ij} + 1 \\ -1 & \text{if } a = \hat{p}_{ij} - 1 \end{cases},$$

$$\phi_{ij}^{\text{ext}}, \phi_{ij}^{\text{inf}}, \phi_j^{\text{pot}}, \phi_j^{\text{tr}} = \text{Single-value functions},$$

$$\eta_i^{\text{ext}}, \eta_i^{\text{inf}}, \eta_i^{\text{pot}}, \eta_i^{\text{tr}} = \text{Weighting factors},$$

$$\eta_i^{\text{ext}}, \eta_i^{\text{inf}}, \eta_i^{\text{pot}}, \eta_i^{\text{tr}} > 0.$$

The ratio of the number of buy and sell actions to the number of changes to expectations can be governed by the speed of reaction parameters  $W_E$  and  $W_V$ .

When the utility of all of an agent's actions and the relevant transition rates have been calculated, the agent's stochastic selection behavior can be described. The following section examines this behavior in the context of market dynamics.

### 1.3. Market dynamics

#### 1.3.1. Determining the action to be implemented

**1.3.1.1. Systematic overview.** The following discussion of the process will focus on the market participants' decisions that result in a change of state. In order to describe the various relationships, a given point in time,  $t$ , will again be used. The following decision situations are of particular interest.

1. Decision behavior of an agent  $i$  with regard to a single activity  $a_{ijk}$ .

2. Decision behavior of an agent  $i$  with regard to the set of alternatives  $A_{ij}$ .
3. Decision behavior of an agent  $i$  with regard to the set of alternatives  $A_i$ .
4. Decision behavior of the market with regard to amount  $A$ .

The stochastic selection behavior with regard to this set of alternatives is discussed below. Table 1 helps to illustrate the various relationships involved.<sup>14</sup>

In the first row, the columns are numbered from left to right. Columns 4–7 deal with the agents' individual activities: every agent  $i$  has a specific number of action alternatives to choose from for a share  $j$ . These alternative actions are, on the one hand, buy or sell actions and, on the other hand, adjustments to expectations. The individual actions are entered in column 4 with the notation  $a_{ijk}$ . Column 5 gives the preference values  $\phi(a_{ijk})$  of the shares  $a_{ijk}$ , while column 6 gives the relevant transition rates  $\lambda_{a_{ijk}} = W e^{\phi(a_{ijk})}$ . The appropriate reaction time  $t_{a_{ijk}}$  is obtained on the basis of the transition rate (reaction rate). These times are listed in column 7. The reaction time  $t_{a_{ijk}}$  is the realization of an exponentially distributed random variable with the parameter  $\lambda_{a_{ijk}}$ .

Columns 8, 9 and 10 describe the selection problem regarding the set of actions in columns 1, 2 and 3.

The sets of actions  $A_{ij}$ ,  $i = 1 \dots N$ ,  $j = 1 \dots M$ , are given in column 3. The set of actions  $A_{ij}$  comprises the individual actions of an agent  $i$  for a share  $j$ . The selection of an action from  $A_{ij}$  is subject to the shortest reaction time of all actions from  $A_{ij}$ . The minimum reaction times  $t_{A_{ij}}^{\min}$ ,  $i = 1 \dots N$ ,  $j = 1 \dots M$ , are determined in column 8.

The sets of actions,  $A_i$ ,  $i = 1 \dots N$ , are given in column 2. The set of actions  $A_i$  comprises all actions of an agent  $i$  for all shares  $j$ ,  $j = 1 \dots M$ . This is given by the union  $A_i = A_{i1} \cup A_{i2} \cup \dots \cup A_{iN}$ . The choice of an action from  $A_i$  is again subject to the shortest reaction time of all actions from  $A_i$ . The relevant minimum reaction times  $t_{A_i}^{\min}$ ,  $i = 1 \dots N$ , are given in column 9.

<sup>14</sup> Adapted from Casey (1998, p. 230).

Table 1  
Reaction rates and times

1	2	3	4	5	6	7	8	9	10
$A$	$A_i$	$A_{ij}$	$a_{ijk}$	$\phi(a_{ijk})$	$\lambda_{a_{ijk}} = W e^{\phi(a_{ijk})}$	$t_{a_{ijk}}$	$t_{A_{ij}}^{\min}$	$t_{A_i}^{\min}$	$t_A^{\min}$
$A$	$A_1$	$A_{11}$	$a_{111}$	$\phi(a_{111})$	$\lambda_{a_{111}}$	$t_{a_{111}}$	$t_{A_{11}}^{\min}$		
			$a_{112}$	$\phi(a_{112})$	$\lambda_{a_{112}}$	$t_{a_{112}}$			
			$\vdots$	$\vdots$	$\vdots$	$\vdots$			
		$A_{12}$	$a_{121}$	$\phi(a_{121})$	$\lambda_{a_{121}}$	$t_{a_{121}}$	$t_{A_{12}}^{\min}$		
			$a_{122}$	$\phi(a_{122})$	$\lambda_{a_{122}}$	$t_{a_{122}}$			
			$\vdots$	$\vdots$	$\vdots$	$\vdots$			
		$A_{1M}$	$a_{1M1}$	$\phi(a_{1M1})$	$\lambda_{a_{1M1}}$	$t_{a_{1M1}}$	$t_{A_{1M}}^{\min}$	$t_{A_1}^{\min}$	
			$a_{1M2}$	$\phi(a_{1M2})$	$\lambda_{a_{1M2}}$	$t_{a_{1M2}}$			
			$\vdots$	$\vdots$	$\vdots$	$\vdots$			
		$A_{N1}$	$a_{N11}$	$\phi(a_{N11})$	$\lambda_{a_{N11}}$	$t_{a_{N11}}$	$t_{A_{N1}}^{\min}$		
			$a_{N12}$	$\phi(a_{N12})$	$\lambda_{a_{N12}}$	$t_{a_{N12}}$			
			$\vdots$	$\vdots$	$\vdots$	$\vdots$			
	$A_N$	$A_{N2}$	$a_{N21}$	$\phi(a_{N21})$	$\lambda_{a_{N21}}$	$t_{a_{N21}}$	$t_{A_{N2}}^{\min}$		
			$a_{N22}$	$\phi(a_{N22})$	$\lambda_{a_{N22}}$	$t_{a_{N22}}$			
			$\vdots$	$\vdots$	$\vdots$	$\vdots$			
		$A_{NM}$	$a_{NM1}$	$\phi(a_{NM1})$	$\lambda_{a_{NM1}}$	$t_{a_{NM1}}$	$t_{A_{NM}}^{\min}$	$t_{A_N}^{\min}$	$t_A^{\min}$
			$a_{NM2}$	$\phi(a_{NM2})$	$\lambda_{a_{NM2}}$	$t_{a_{NM2}}$			
			$\vdots$	$\vdots$	$\vdots$	$\vdots$			

The sets of actions  $A_i$  of all agents  $i$ ,  $i \dots N$  are summarized in column 1 by combining them to form a total alternative amount  $A = A_1 \cup A_2 \cup \dots \cup A_N$ . Column 10 gives the minimum reaction time  $t_A^{\min}$  for all actions from  $A$ . The action with which the minimum is reached will be performed next. In this way, only one action can be performed at a time.

*1.3.1.2. The basis in probability theory of selection behavior.* The section below examines the agents' selection behavior, which is based on probability theory as mentioned above.

*1.3.1.2.1. Decision behavior of an agent  $i$ .* Let us take any alternative action  $k$  (Table 1, column 4) of an agent  $i$  for a share  $j$ :  $a_{ijk}$ . Let the random variable  $\tilde{T}_{a_{ijk}}$  describe the waiting time of agent  $i$  until  $a_{ijk}$  is implemented. This corresponds to the time which agent  $i$  will have to wait before implementation, if s/he does not observe any other action on the

market in the meantime. This time is described as *agent  $i$ 's reaction time for the action  $a_{ijk}$* .

The distribution function  $F_{a_{ijk}}(\tau) = P(\tilde{T}_{a_{ijk}} \leq \tau)$  gives the probability that agent  $i$ 's waiting or reaction time before the implementation of  $a_{ijk}$  will not exceed  $\tau$  time units. In the capital market model discussed here, which has time-homogeneous transition rates, the waiting or reaction time of an agent before implementation of action  $a_{ijk}$  is exponentially distributed with the parameter  $\lambda_{a_{ijk}}$ .<sup>15</sup>

$$F_{a_{ijk}}(\tau) = P(\tilde{T}_{a_{ijk}} \leq \tau) = 1 - e^{-\lambda_{a_{ijk}} \tau}.$$

In this context, the transition rate  $\lambda_{a_{ijk}} = W \cdot e^{\phi(a_{ijk})}$  (Table 1, column 6) is also described as agent  $i$ 's reaction rate for action  $a_{ijk}$ .<sup>16</sup> It is a function of the

<sup>15</sup> See in general Breiman (1969, p. 37, p. 211) and Feller (1968). The concrete presentation follows Casey (1998, p. 232 f).

<sup>16</sup> See Montgomery and Runger (1999, p. 179).

utility  $\phi(a_{ijk})$  (column 5) of  $a_{ijk}$ . The complementary distribution function

$$R_{a_{ijk}}(\tau) = 1 - F_{a_{ijk}}(\tau),$$

describes the probability that agent  $i$ 's waiting or reaction time before implementation of  $a_{ijk}$  exceeds  $\tau$  time units. It is described as a reaction function. Fig. 7 gives an impression of the reaction function curves with varying reaction rates  $\lambda_{a_{ijk}}$ .

The reaction function curves clearly illustrate the fact that the agent's individual waiting time until the implementation of a particular action decreases as the utility  $\phi(a_{ijk})$ , i.e., the reaction rate  $\lambda_{a_{ijk}} = We^{\phi(a_{ijk})}$ , increases. This relationship is also expressed in the length of the expected reaction time. The mean reaction time for the action  $a_{ijk}$  is defined by

$$E(\tilde{T}_{a_{ijk}}) = 1/\lambda_{a_{ijk}},$$

and is inversely proportional to the reaction rate: the higher the reaction rate, the lower the expected reaction time until the implementation of  $a_{ijk}$ . The reaction time realized by an agent  $i$  for an action  $a_{ijk}$  is described by  $t_{a_{ijk}}$ . The following is then true:

$$t_{a_{ijk}} = \text{Realization of the random variables } \tilde{T}_{a_{ijk}}.$$

The reaction times implemented by all agents for all actions are given in column 7 in Table 1.

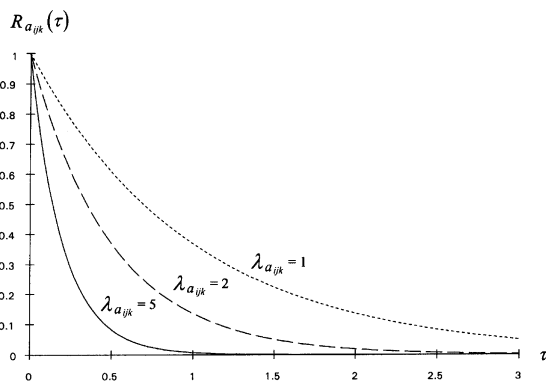


Fig. 7. Reaction time's probability distribution.

### 1.3.1.2.2. Selection of the action to be implemented. Probability theory relationships

Column 3 gives the set  $A_{ij}$  of possible actions. It is now necessary to provide a more detailed description of the process starting with the selection of the action from  $A_{ij}$  which the agent  $i$  wants to carry out next. The primary criterion for the selection is the utility of the action  $a_{ijk} \in A_{ij}$ . The higher the utility  $\phi(a_{ijk})$  of an action  $a_{ijk}$ , the more urgently its execution will be evaluated. The following notation stating full particulars is used to describe the process:

Random variable  $\tilde{Y}_{A_{ij}}$  describes the alternative action that the agent selects from a given set of alternatives,  $A_{ij}$ .

Let random variable  $\tilde{T}_{A_{ij}}$  describe the agent's planned waiting or reaction time until the implementation of the selected alternative. The following relation applies for the probability distributions  $\tilde{Y}_{A_{ij}}$  and  $\tilde{T}_{A_{ij}}$ :

$$P(\tilde{Y}_{A_{ij}} = a_{ijk} / A_{ij}) = \frac{\lambda_{a_{ijk}}}{\sum_{a_{ijk} \in A_{ij}} \lambda_{a_{ijk}}}.$$

The probability that an agent  $i$  will choose the alternative action  $a_{ijk} \in A_{ij}$  from a given set of alternatives  $A_{ij}$  is directly proportional to the sum of the reaction rates of all actions in  $A_{ij}$ :

$$P(\tilde{T}_{A_{ij}} \leq \tau) = 1 - e^{-\sum_{a_{ijk} \in A_{ij}} \lambda_{a_{ijk}} \tau}.$$

The agent's waiting or reaction time until the implementation of the selected alternative is exponentially distributed with the parameter  $\sum_{a_{ijk} \in A_{ij}} \lambda_{a_{ijk}}$ . It can be shown that the selection probability  $P(\tilde{Y}_{A_{ij}} = a_{ijk} / A_{ij})$  is equal to the probability that the observed action  $a_{ijk}$ , will exhibit the shortest reaction time of all the alternative actions from  $A_{ij}$ .<sup>17</sup> As shown in column 8, the determination of the alternative actions with the minimum reaction time  $t_{A_{ij}}^{\min}$  is obtained from a comparison of the realized reaction times of all alternative actions from  $A_{ij}$ :

$$t_{A_{ij}}^{\min} = \min \{t_{a_{ijk}} \mid a_{ijk} \in A_{ij}\}.$$

<sup>17</sup> See Loistl and Landes (1989, p. 72 f, p. 94 ff).



The minimum reaction time  $t_{A_{ij}}^{\min}$  is the realization of the random variables  $\tilde{T}_{A_{ij}}^{\min} = \min\{\tilde{T}_{a_{ijk}} \mid a_{ijk} \in A_{ij}\}$ . This random variable is exponentially disturbed with the parameter  $\sum_{a_{ijk} \in A_{ij}} \lambda_{a_{ijk}}$ . It is given in column 8, line 1.<sup>18</sup> The expression  $\lambda_{ij} = \sum_{a_{ijk} \in A_{ij}} \lambda_{a_{ijk}}$  is described as agent  $i$ 's reaction rate for share  $j$ .

The following relation applies:

The greater the benefit gained from the action of an agent  $i$  for a share  $j$ , the higher the reaction rate  $\lambda_{ij}$  of this agent  $i$  for this share  $j$  will be, and the shorter the expected reaction time of this agent  $i$  before implementing an action from  $A_{ij}$  will be.<sup>19</sup>

Following the above explanations, the alternative action  $k'$  selected by an agent  $i$  for a share  $j$  and the waiting time until its implementation can be determined either as a realization of the distributions  $P(\tilde{Y}_{A_{ij}} = a_{ijk'} / A_{ij})$  and  $P(\tilde{T}_{A_{ij}} \leq \tau)$ , or as a realization of the random variables  $\tilde{T}_{A_{ij}}^{\min} = \min\{\tilde{T}_{a_{ijk}} \mid a_{ijk} \in A_{ij}\}$ . Both random experiments deliver the same result. In KapSyn, the latter variant is used for selecting the action: the waiting time until the implementation of an action from  $A_{ij}$ , estimated by agent  $i$ , is determined by  $t_{A_{ij}}^{\min} = \min\{t_{a_{ijk}} \mid a_{ijk} \in A_{ij}\}$ . The selected action is that with which the minimum  $t_{A_{ij}}^{\min}$  is achieved.

#### Implementation in practice taking a random component into account

In addition to the determination of the benefit from the action i.e., its utility, a random experiment is involved in determining the minimum reaction time. In concrete terms, the action with the greatest utility will probably, but not definitely, be assigned the shortest reaction time by the agent. The following procedure applies:

A random figure between 0 and 1 is chosen for every reaction function and, in this way, the reaction time for the relevant reaction function is determined.

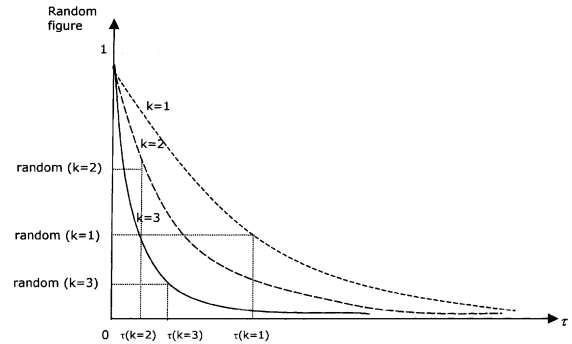


Fig. 8. Random element in determining shortest ratio time.

From Fig. 8, it emerges that the random figure does not in fact allocate the shortest reaction time to action  $k = 3$  – which is in fact rated more highly – but rather to action  $k = 2$ , which is not rated so highly.

In individual cases, the random experiment can lead to a change in the order of waiting time derived from the action's utility. It is probable that the actions with higher utility values will lead to shorter reaction rates.

#### Selection of the action to be implemented from all possible actions $A_i$ of agent $i$

In line with the principle that an agent can only carry out a single action, the share in which the agent wants to become active now needs to be established. All the shares in which the agent is able to trade must be observed. Every agent  $i$  then has to evaluate  $M$  sets of actions  $A_{ij}$ ,  $j = 1 \dots M$ . The behavior of an agent  $i$  with regard to the selection of an action from  $A_i = A_{i1} \cup A_{i2} \cup \dots \cup A_{iM}$  will be described below.

The aim is to find which of the alternative actions  $a_{ik} \in A_i$  the agent  $i$  plans to implement next, as well as the waiting time until implementation.<sup>20</sup> In line with the notations used thus far, the following variable declarations will be employed.

The random variable  $\tilde{Y}_{A_i}$  describes the alternative actions that the agent selects for a given alternative number  $A_i$ .

<sup>18</sup> See Landes and Loistl (1992, p. 223 f).

<sup>19</sup> An agent  $i$ 's expected reaction time for the implementation of an action from  $A_{ij}$  is  $1/\lambda_{ij}$ .

<sup>20</sup> The index  $j$ , used earlier to allot an action  $k$  by an agent  $i$  to a share  $j$ , will no longer be referred to here, for reasons of simplicity.

The random variable  $\tilde{T}_{A_i}$  describes the agent's planned waiting or reaction time until the implementation of the selected alternative. The distributions  $\tilde{Y}_{A_i}$  and  $\tilde{T}_{A_i}$  are characterized as follows:

$$P(\tilde{Y}_{A_i} = a_{ik'}/A_i) = \frac{\lambda_{a_{ik'}}}{\sum_{a_{ik} \in A_i} \lambda_{a_{ik}}}.$$

The probability that an agent  $i$  will select the action  $a_{ik'} \in A_i$  from a given number of possible actions  $A_i$  is proportional to the sum of the reaction rates of all actions from  $A_i$ .

$$P(\tilde{T}_{A_i} \leq \tau) = 1 - e^{-\sum_{a_{ik} \in A_i} \lambda_{a_{ik}} \tau}.$$

The agent's waiting or reaction time until the implementation of the selected alternative is exponentially distributed with the parameter  $\sum_{a_{ik} \in A_i} \lambda_{a_{ik}}$ .

The following relation applies here, too, as before: the probability,  $P(\tilde{Y}_{A_i} = a_{ik'}/A_i)$ , is equal to the probability that the action  $a_{ik'}$  exhibits the shortest reaction time of all the alternative actions from  $A_i$ . This minimum reaction time can also be taken as the minimum  $t_{A_i}^{\min}$  (given in column 9) of all *share*-specific reaction times  $t_{A_{ij}}^{\min}$ ,  $j = 1 \dots M$  (given in column 8) of agent  $i$ :

$$t_{A_i}^{\min} = \min \{t_{A_{ij}}^{\min} \mid j = 1 \dots M\}.$$

The reaction time  $t_{A_i}^{\min}$ ,  $i \in I = \{1 \dots N\}$  is the realization of the random variables  $\tilde{T}_{A_i}^{\min} = \min\{\tilde{T}_{A_{ij}}^{\min} \mid j = 1 \dots M\}$ . This random variable is also exponentially distributed. Its distribution parameter is given by  $\lambda_i = \sum_{a_{ik} \in A_i} \lambda_{a_{ik}}$  (see column 9, line 1). It corresponds to the reaction rate of agent  $i$ .<sup>21</sup> Once more, the following applies: the greater the benefit (utility) gained from an agent  $i$ 's actions for all shares  $j$ ,  $j = 1 \dots M$ , the faster his/her reaction rate  $\lambda_i$ , and the shorter his/her expected reaction time for the implementation of an action will be.<sup>22</sup>

#### Market decision behavior for an amount $A$

Finally, it is necessary to determine the agent who will trade next from the set of all agents  $i$ ,  $i = 1 \dots N$ . At this level, too, trading cannot take place simultaneously. This selection is once again made on the basis of the greatest achievable benefit (utility).

Let  $A = A_1 \cup A_2 \cup \dots \cup A_N$  be the number of alternative actions by all agents for all shares. The action sought,  $a_k \in A$ , is the one which will be implemented next. The selection also determines the time of its realization.<sup>23</sup> The usual notations apply:

The random variable  $\tilde{Y}_A$  describes the action that will be realized next.

The random variable  $\tilde{T}_A$  describes the waiting time until the implementation of the chosen action. The distributions  $\tilde{Y}_A$  and  $\tilde{T}_A$  are linked to each other as follows:

$$P(\tilde{Y}_A = a_{k'}/A) = \frac{\lambda_{a_{k'}}}{\sum_{a_k \in A} \lambda_{a_k}}.$$

The probability that, for a given set of actions  $A$ , the action  $a_{k'}$ , will be the next to be implemented, is proportional to the sum of the reaction rates of all actions from  $A$ .

$$P(\tilde{T}_A \leq \tau) = 1 - e^{-\sum_{a_k \in A} \lambda_{a_k} \tau}.$$

The waiting time until the implementation of the action to be carried out is exponentially distributed with the parameter  $\sum_{a_k \in A} \lambda_{a_k}$ .

It is equal to the probability that the action  $a_{k'}$  will exhibit the minimum reaction time out of all the alternatives from  $A$ . This reaction time can be given as the minimum  $t_A^{\min}$  given in column 10 of all *agent*-specific reaction times  $t_{A_i}^{\min}$ ,  $i = 1 \dots N$ , given in column 9:

$$t_A^{\min} = \min \{t_{A_i}^{\min} \mid i = 1 \dots N\}.$$

The reaction time  $t_A^{\min}$  is the realization of the random variables  $\tilde{T}_A^{\min} = \min\{\tilde{T}_{A_i}^{\min} \mid i = 1 \dots N\}$ . It is exponentially distributed with the parameter

<sup>21</sup> See Landes and Loistl (1992, p. 223 f).

<sup>22</sup> Agent  $i$ 's expected reaction time is  $1/\lambda_i$ .

<sup>23</sup> The index  $i$ , used earlier to allot an action  $k$  by an agent  $i$  is no longer required with this question and is therefore, not given.

$\lambda = \sum_{a_k \in A} \lambda_{a_k}$ , given in column 10, line 1. This parameter corresponds to the reaction rate for the entire market. The higher the market reaction rate  $\lambda$ , the shorter the average waiting time until the implementation of the next action overall.<sup>24</sup>

The stochastic selection behavior of the market participants, and hence the market dynamics, can now be described as follows: the number of alternative actions by an agent  $i$ ,  $i \in I = \{1 \dots N\}$ , for a share  $j$ ,  $j \in J = \{1 \dots M\}$ , is combined in amount  $A_{ij}$ . For share 1 the amount  $A_{i1}$  applies, for share 2 the amount  $A_{i2}, \dots$ , and, finally, for share  $M$  the amount  $A_{iM}$  applies.

In his/her first step, agent  $i$  selects an action  $a_{ijk} \in A_{ij}$  for every share  $j$  that s/he would implement if observing share  $j$  in isolation. This action is described as a potential action of agent  $i$  with regard to share  $j$  and is determined subject to the shortest reaction time  $t_{A_{ij}}^{\min} = \min\{t_{a_{ijk}} | a_{ijk} \in A_{ij}\}$ . It is probable that agent  $i$  will choose the action from  $A_{ij}$  that offers him the greatest gain.

After this selection step, agent  $i$  must select from the total  $M$  alternative actions the one which should be realized in the end, from his or her point of view. This action is once again determined by the criterion of the shortest reaction time:  $t_{A_i}^{\min} = \min\{t_{A_{ij}}^{\min} | j = 1 \dots M\}$ . Again, it is probable that the alternative selected will be that which most stimulates the agent, in other words, which promises the greatest gain.

A total of  $M$  agents are active on the stock exchange. The agents' decision-making processes are independent of each other; i.e., they take place in parallel. As a result, every agent will determine the action most advantageous for him, and that s/he will want to realize. The reaction times

$t_{A_i}^{\min}$ ,  $i = 1 \dots N$ , determine the agent whose turn it will actually now be to act, whose action will be implemented and who will change the observed market conditions. Of all the agents, it is the one with the shortest reaction time who will be the next to act:  $t_A^{\min} = \min\{t_{A_i}^{\min} | i = 1 \dots N\}$ .

#### 1.4. Price trends

$$\mathbf{p}^{\text{tr}} = (p_j^{\text{tr}})_{j=1 \dots M} = (p_1^{\text{tr}}, \dots, p_j^{\text{tr}}, \dots, p_M^{\text{tr}}),$$

$$\mathbf{d}^{\text{tr}} = (d_j^{\text{tr}})_{j=1 \dots M} = (d_1^{\text{tr}}, \dots, d_j^{\text{tr}}, \dots, d_M^{\text{tr}}).$$

The vector  $\mathbf{p}^{\text{tr}}$  comprises any price trends  $p_j^{\text{tr}}$ ,  $p_j^{\text{tr}} \in \{\dots, -1, 0, +1, \dots\}$ , of all shares  $j$ ,  $j \in J = \{1, \dots, M\}$ , at time  $t$ . The vector  $\mathbf{d}^{\text{tr}}$ ,  $d_j^{\text{tr}} \in \{\dots, -1, 0, +1, \dots\}$ , comprises their trend reversal indicators. These indicators focus on technical chart developments.

A rise in price of a share  $j$  results in a positive price trend, a fall in price in a negative price trend  $p_j^{\text{tr}}$ .<sup>25</sup> A recalculation of the price trend  $p_j^{\text{tr}}$  of share  $j$  always takes place if a trade in share  $j$  occurs. A change in direction of the current quoted price can either confirm or interrupt the previous trend. If the trend in direction of the current market price is in line with the trend in direction of the previous price trend  $p_j^{\text{tr}}$  (i.e., if the price difference between the quoted price at time  $t$  and the previous market price of share  $j$  has the same sign as the previous price trend  $p_j^{\text{tr}}$ ), the current price trend  $p_j^{\text{tr}}$  is confirmed. In this case, the previous price trend  $p_j^{\text{tr}}$  is updated in line with the strength of the change in price.

However, if the price change and price trend are prefixed with different signs, this may signal a trend reversal. The trend reversal indicator  $d_j^{\text{tr}}$  of share  $j$  is used to check whether this involves a 'real' trend reversal, or merely a short-term consolidation of the stock price. Share  $j$ 's trend reversal indicator  $d_j^{\text{tr}}$  is a technical chart parameter that indicates when an existing trend is continued or interrupted. This figure is calculated in parallel

<sup>24</sup> The mean market reaction time  $E(\tilde{T}_A^{\min})$  is equal to  $1 / \sum_{a_k \in A} \lambda_{a_k}$ . Using the figure, the reaction rates of all actions from A can also be described as follows:

$$\lambda_{a_{k'}} = \frac{\lambda_{a_{k'}}}{\sum_{a_k \in A} \lambda_{a_k}} \sum_{a_k \in A} \lambda_{a_k} = P(\tilde{Y}_A = a_{k'} / A) \frac{1}{E(\tilde{T}_A^{\min})}.$$

This formula can be useful if the reaction rates cannot be measured directly. If the selection probabilities and the mean reaction times are available, then the reaction rates can be calculated from the latter (see Karlin and Taylor (1975, p. 150 ff) and the literature cited).

<sup>25</sup> The inclusion of backward-looking state variables lessens the "amnesia" of the Markov process.

to the price trend. A trend reversal is always assumed for a directional price movement if the stock price breaks out of a pre-defined band in the opposite direction to the previous price movement. The band can be set to any width. If the predefined band is crossed in the relevant direction, the price trend and trend reversal indicator are adapted to the price's new direction. In the case of short-term consolidation, on the other hand, the previous price trend  $p_j^t$  is confirmed and merely corrected up or down, depending on the size of the current price change.

### 1.5. Program output and simulation results

#### 1.5.1. The graphics window

During a simulation run, a lot of information can be obtained about what happens. First, general information, warnings and errors are logged to the KapSyn log screen. The status bar shows the progress for the current simulation and the simulation series. The graphics window presents a real-time view of the simulation evolution, and the ticker window shows every ask, bid or trade that is made while the simulation runs.

Previous simulation results can be reviewed with the graphics window, and all simulation statistics are appended to a text file.

The graphics window is opened and closed with the *Graphics* command found in the *Simulation* menu, or with a shortcut on the symbol bar. There are three views available, the general, price/volume and parameters view (found in the *Display* menu of the graphics screen).

All data shown in the general view is permanently stored, so this view can be restored when necessary. All other views are temporary: they are created only during a simulation run and are not permanently stored due to the large amount of data necessary. As soon as the window view is changed, these graphics disappear and their historic parts cannot be recreated.

The graphics window can be opened or closed at any time, even during a simulation run. It needs

at least a screen resolution of  $800 \times 600$  pixels ( $1024 \times 768$  is recommended).

*1.5.1.1. The general view.* The general view graphics window is divided into different sections. Results can be viewed during or after a simulation. The simulation data needed to create the general view is available until a new simulation is started. If not available, the last simulation results can be reloaded with *Simulations/Load results* (File type set to *Last Simulation Results*). Data from automated simulation runs can be loaded in the same way.

Whenever a result file is loaded, all current parameter settings are overwritten by the settings that were used during the loaded simulation.

The graphics and statistics are displayed for a single stock, which can be chosen with the stock menu items.

The upper left 3D-graphic illustrates the variation of the traders price expectations in time. The horizontal lines represent the traders subjective price expectations, going from low (left) to high (right) values. Time leads from the back to the front of the graphic: the backmost horizontal layer of the mountain structure shows the  $\hat{p}$ -distribution at the beginning, the frontmost layer illustrates the  $\hat{p}$ -distribution at the end of the trading session. The altitude indicates the number of traders (frequency) that had this price expectation at that point in time: the higher the peak of the “mountain”, the more traders shared the respective price expectation. Values from a simulation series are cumulated.

The lower left 3D-graphic has the same structure as the above, but illustrates the variation of stock prices in time. Since every new simulation of a series creates a new stock price path, the frequency of all the paths can be shown. The higher the “mountain”, the more simulations had the same stock price at that point in time.

For calculation purposes, the frequencies are measured by a equidistant grid. However, the individual path quotations during simulation are asynchronous, not equidistant. In the general view

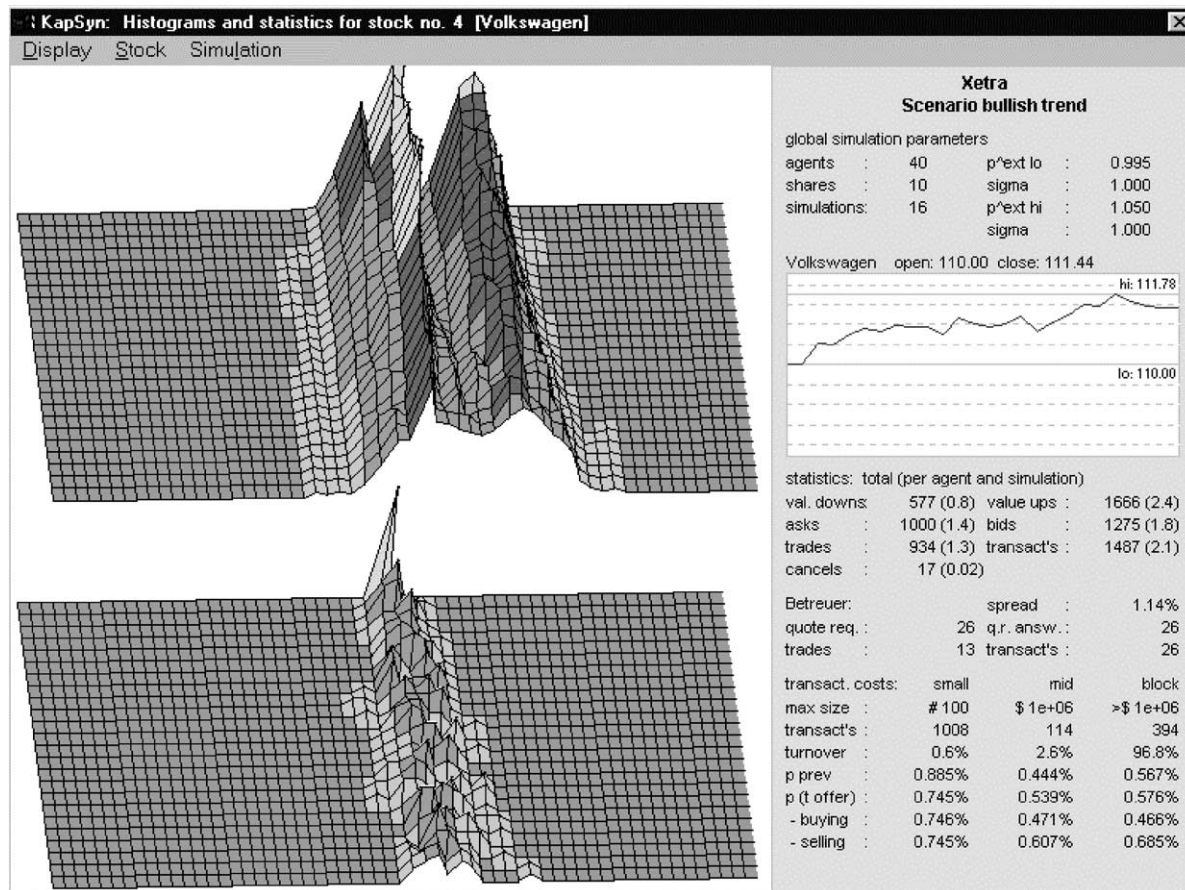


Fig. 9. Graphics window: general view.

graphic, the quotation at a grid point is the last real quotation preceding the grid's point in time.

For perspective-related reasons the 3D-view is slightly turned to the right. This is why the movement in time to the left, i.e., towards lower price expectations, is over-estimated at first sight. The lines drawn from the back to the front can be helpful when trying to estimate whether the distribution moves towards higher or lower prices.

The global simulation parameters on the right of Fig. 9 present the basic parameters of the simulation. The  $\hat{p}_{ext}$  distribution is characterized by the lower mean of  $\hat{p}_{ext} = 0.995$  and the upper mean of  $\hat{p}_{ext} = 1.05$ , with each standard deviation set to 1.0. The chart below shows the average of the 16 simulated price paths.

The lower right corner displays market statistics: the traders adjusted their  $\hat{p}$ -estimations 577 times downwards, 1666 times upwards, placed 1000 offers to sell and 1275 offers to buy, resulting in 934 trades with 1487 transactions.<sup>26</sup> Seventeen times an offer (ask or bid) was cancelled. The values in brackets show the statistics per agent and simulation.

Depending on the simulated stock exchange, some specific statistics are given in the next

<sup>26</sup> The number of transactions may be higher than the number of asks, bids or trades. This can happen, if a large volume ask is met by numerous smaller bids, and vice versa. Only one trade, but each individual transaction is counted.

section. In Fig. 9 the statistics display the Kap-Syn Xetra designated sponsor. There were 26 quote requests and the same amount of quote request answers with an average quote spread of 1.14%, resulting in 13 trades with 26 transactions. For NASDAQ, this section exhibits ATS statistics.

The calculation of transaction costs is split into three categories according to *small*, *mid* and *block* trades. The maximum size is given for each category. In Fig. 9, *small* means all trades up to 100 shares (#), *block* all trades with more than 1 billion currency units, and *mid* anything in between. The next lines display transaction statistics for each category.

First, the number of transactions and their relative turnover is calculated. Here, for instance, 1008 small transactions account for 0.6% of total turnover.

Second, the transaction costs are given as *previous price* and *price at offer*. All transaction cost statistics are based on the complete execution of offers. In most cases, transaction costs calculated on prices at *offer* (i.e., prices valid at the moment the order is placed on the market) are higher than those calculated on *previous price*. This is the case because it takes some time to settle an entire order, so the *price at offer* can differ – sometimes considerably – from the actual price. In a market maker module, there is no time

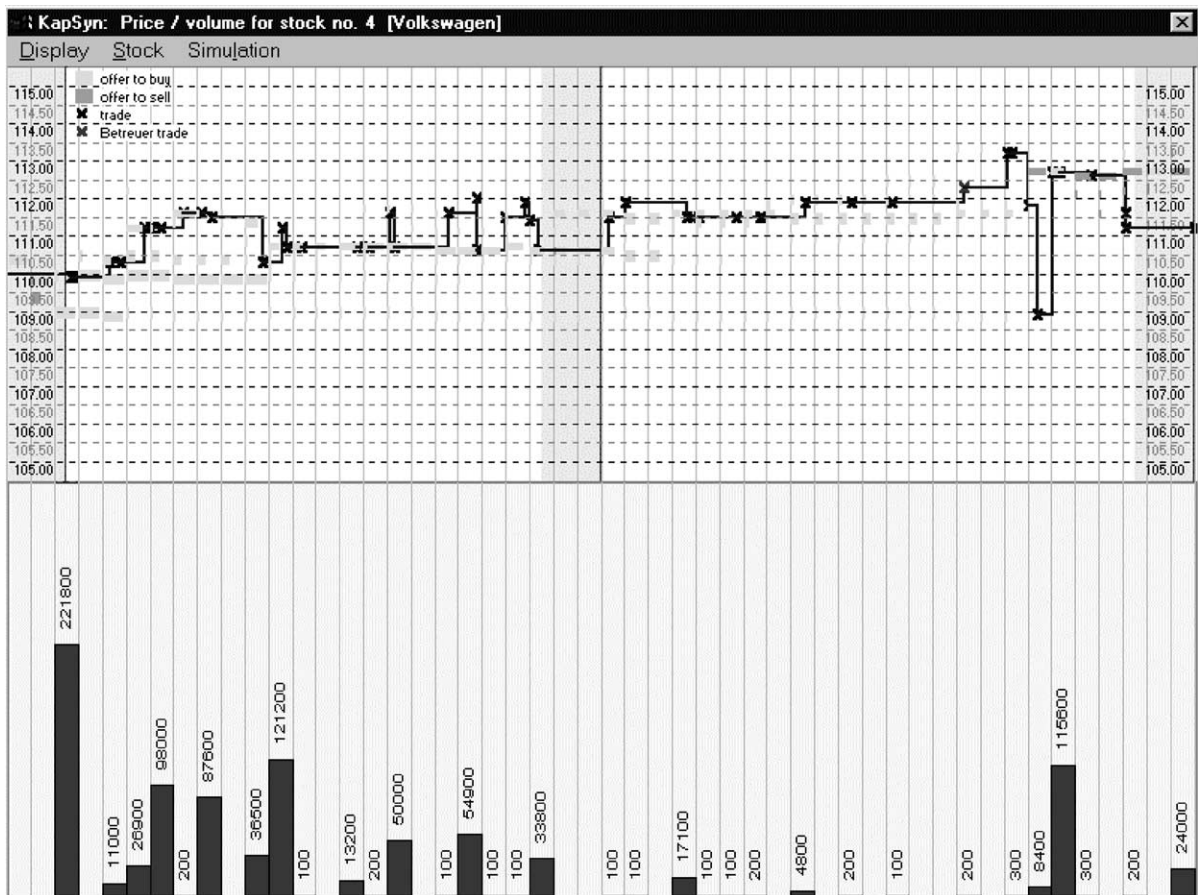


Fig. 10. Graphics window: price/volume view.

delay between order and settlement, therefore transaction costs calculated on *prices at offer* are zero.

**1.5.1.2. The price/volume view.** The *price/volume* view (Fig. 10) is drawn only during a simulation run. The upper half shows the exact stock price evolution (blue line), and every trade is marked by a black *x*, whereas Xetra designated sponsor trades are marked with a red *x*. The light blue areas represent Xetra call for auction periods, while the red lines indicate the auction itself. The green and red rectangles are order book histograms, drawn in 48 equidistant time steps on the price (*y*-) axis. The higher (in *x*-axis direction) the

rectangle, the more supply (red) or demand (green) for the given price is in the order book.

The lower half of the window produces a trade volume histogram in 48 equidistant time intervals.

**1.5.1.3. The variables view.** The *variables* view is a tool that graphically represents all KapSyn parameter values for all stocks at 48 intervals. This view is useful to study the effects of KapSyn parameter variations on the stock exchange performance.

Four small windows are provided for each stock, the first showing the traders expectations distribution in blue. A deeper color symbolizes more traders with the same stock price expectation

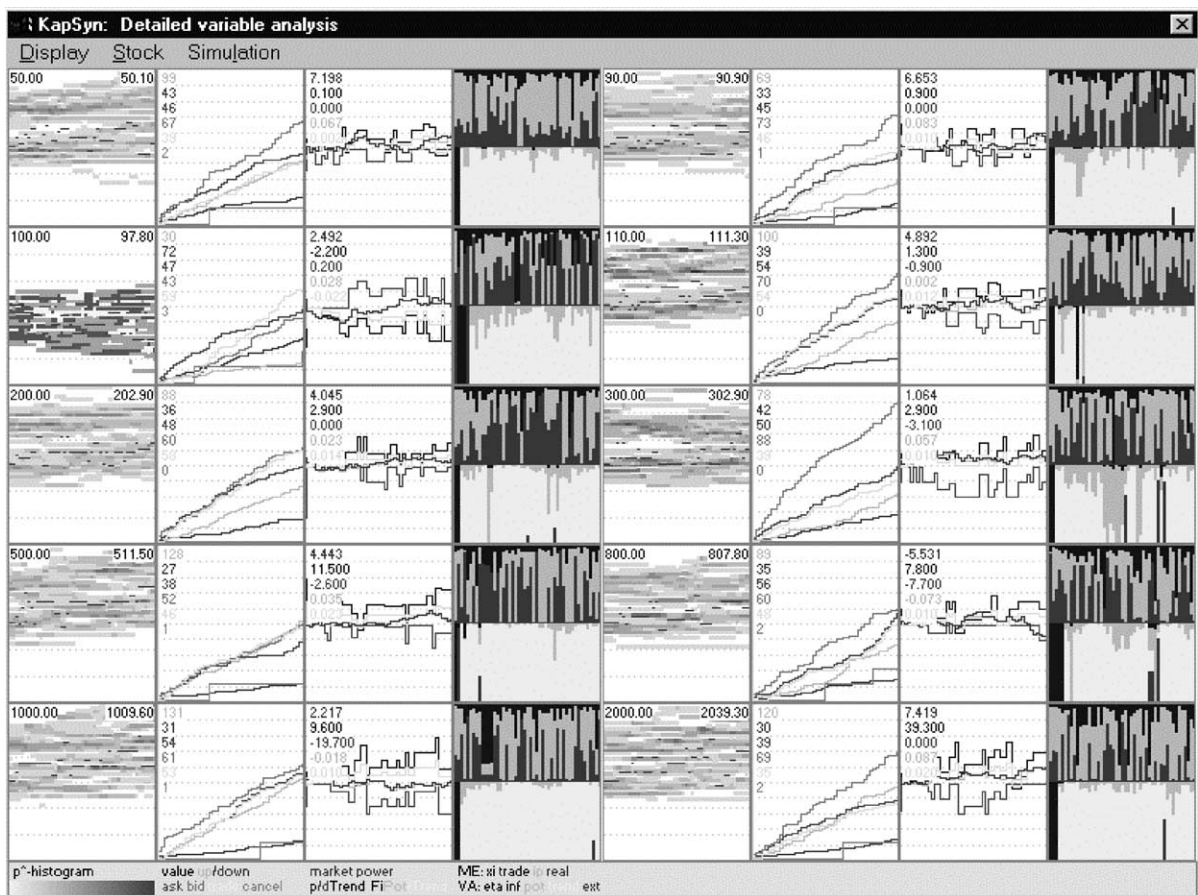


Fig. 11. Graphics window: variables view.

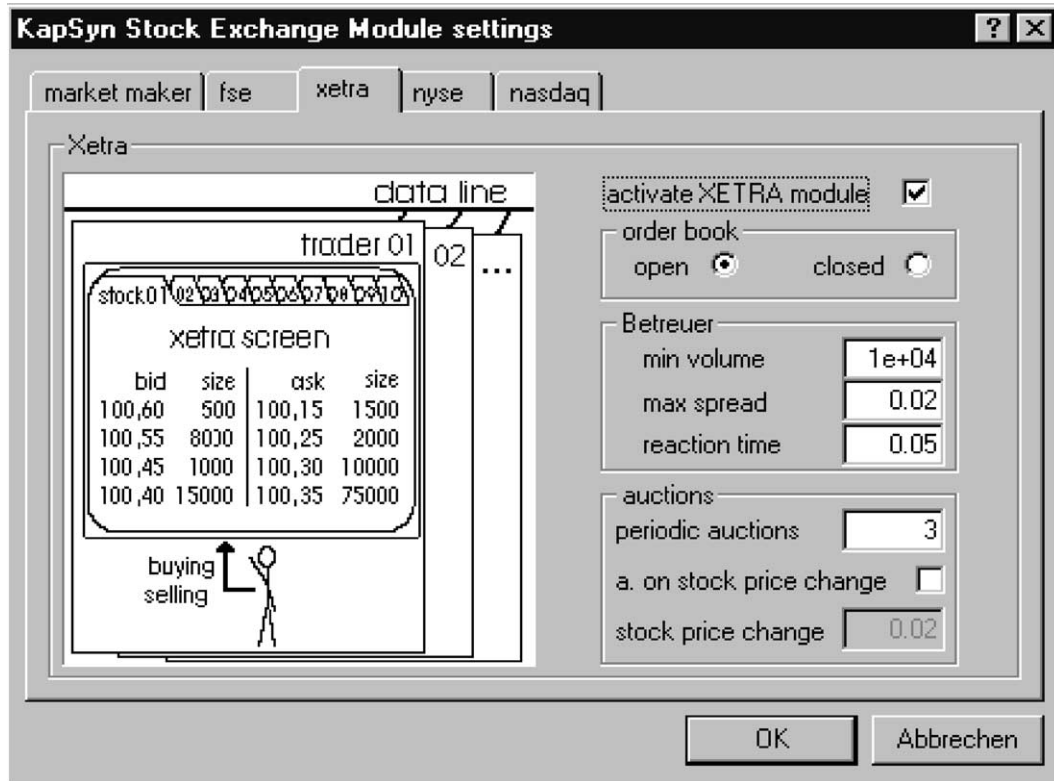


Fig. 12. Xetra's basic features.

in a certain price range ( $y$ -axis) at a point in time ( $x$ -axis). The stock price itself is given in red, open and close price is given on top.

The second window shows the evaluation of event counters for price expectation value ups and value downs, for asks, bids, trades and cancels. The final values are given in the line colour.

In the third window market, power with related  $pTrend$  and  $dTrend$  as well as  $fi$  pot and  $fi$  trend are drawn, and final values are given in the variable colors.

The fourth window is divided in two sections. The upper half shows the relative weight of all summands that make the market event transition rate. The lower half is used to show the relative weight of all value adjustment summands. In Fig. 11 for instance the lower half of the windows show that the value adjustment transition rates are dominated by *eta trend* most of the time (the yellow colour).

## 2. Simulation results

### 2.1. Designated sponsor actions in different scenarios

The following pages first provide information about the KapSyn<sup>27</sup> capital market scenario the investigations are based upon. Of course many others settings are possible according to the description in the user handbook. A comprehensive information about the efficiency of a set of trading and price settlement rules requires of course testing several scenarios. The following presentation's major goal is to inform about the KapSyn facilities besides providing remarkable insight into the

<sup>27</sup> Downloading of the computer program KapSyn (Loistl and Vetter, 1999) from website <http://ifm.wu-wien.ac.at>.



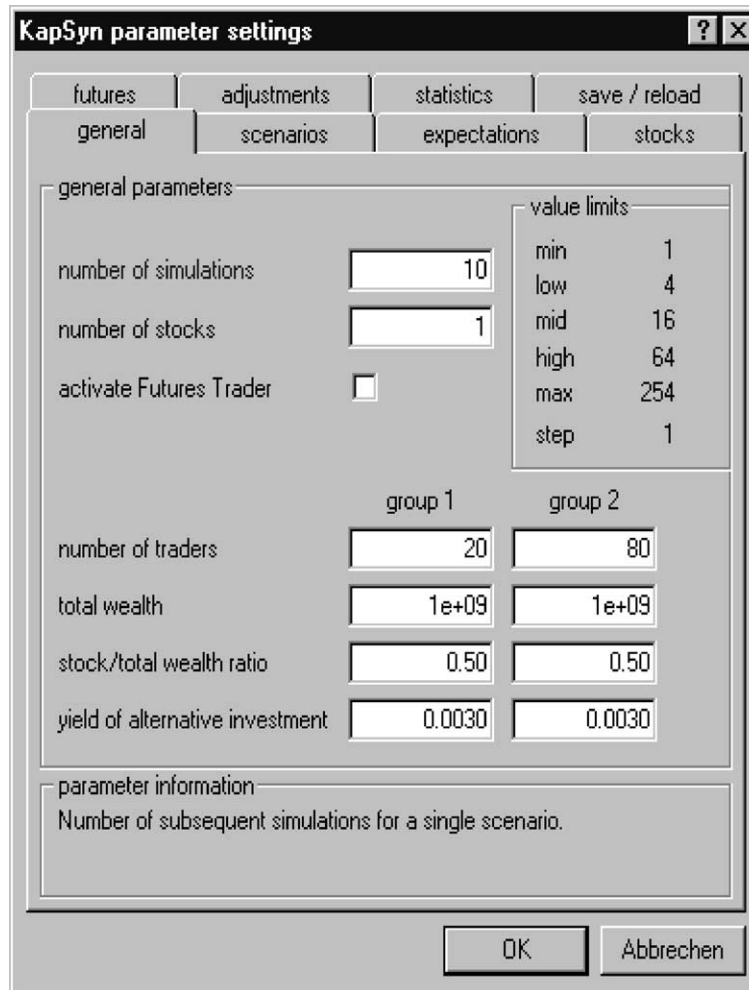


Fig. 13. Traders' designation.

Xetra system.<sup>28</sup> Second, the simulation results are discussed.

The scenario presented in Fig. 12 provides Xetra's basic features: The open order book facility is activated. The betreuers offers' minimum volume amounts to 1000, the maximum spread to 2% and the reaction time is scheduled an 0.05 time units, which may be adjusted towards the specific requirements. The periodic auction feature is ac-

tivated. The number of auctions increases from 3–9 auctions. The auction's initiating feature due to predefined price changes is not activated for this run.

Fig. 13 provides the basic organizational structure investigated: the investigation is based on 10 simulations for 1 stock. The simulations may treat a portfolio of several stocks too. The results for one and several stocks may be similar, even if the portfolio structure influences the individual stock's performance. The futures trader is not activated in the scenario investigated. The empirical evidence of the following KapSyn parameter set-

<sup>28</sup> For an introduction to the Xetra trading system see, e.g., Deutsche Börse AG (1999).

**KapSyn parameter settings** [?] [X]

futures adjustments statistics save / reload  
general scenarios expectations stocks

scenario dependant parameters (page 1 of 2): greeks

scenario:  1 of 7

	group 1	group 2		group 1	group 2
eta ext	80.0000	5.0000	xi ip	3.0000	3.0000
eta inf	0.1000	3.0000	xi trade	0.0030	0.0030
eta pot	0.5000	3.0000	xi real	2.0000	2.0000
eta trend	5.0000	80.0000	xi risk	3.0000	1.0000

rho:  lamda:   
kappa:  delta max:

value limits:  
min: 0  
low: 0.5  
mid: 1.0  
high: 3.0  
max: 10.0

parameter information:  
Linking of  $p^$  to  $ppot(1+r)$  ('group 2')

Fig. 14. Traders' behavioral parameters.

ting is validated: connecting the parameter setting to economic data by using neural networks and analytical reasoning underlie the definition of market scenarios.<sup>29</sup>

KapSyn separates two groups of traders. In this simulation, the bullish group comprises 20 traders, the bearish group 80 traders. Each trader disposes of a quantity of 9,000,000,000 units to be invested. In the first scenario investigated, the fortune is invested 50% in stocks and 50% in interest bearing cash investment. The interest rate is 0.3%. In the

second scenario, the market participants own cash only.

Fig. 14 provides the market participant's reaction parameter settings, again different of bullish and bearish traders. The scenario's overall character is named 'bullish trend'. Without going into full detail, let us mention some striking differences between the two groups of investors. The values of eta ext exhibit a major difference: 80,000 for group 1 and 5000 for group 2. Group 1 investors stick to their fundamental value estimate, group 2 investors do not. This is the typical situation of fundamental oriented vs. market oriented investors. The eta ext figures correspond to the figures for eta inf (0.1 vs. 3.0), eta pot (0.5 vs. 3.0) and eta trend

<sup>29</sup> For details see Loistl (1994, p. 422 ff).

(5.0 vs. 80): eta inf governs the orientation towards the market performance, eta pot governs the investors' weight put at the market potential and eta trend controls the investors' orientation towards a market trend.

This scenario can be seen as a typical bullish scenario. The figures governing the immediate behavior of buying and selling are not that much different. The auctions' dynamic results are gen-

erated by the fundamental values controlling revision (market potential).

The figures rho, kappa, lambda and delta max controlling price trends contribute to the value adjustment motivation potential. For details see user handbook.

The first simulation results presented in Fig. 15 exhibits Xetra behaviour for three auctions and the bearish group's expected fundamental value

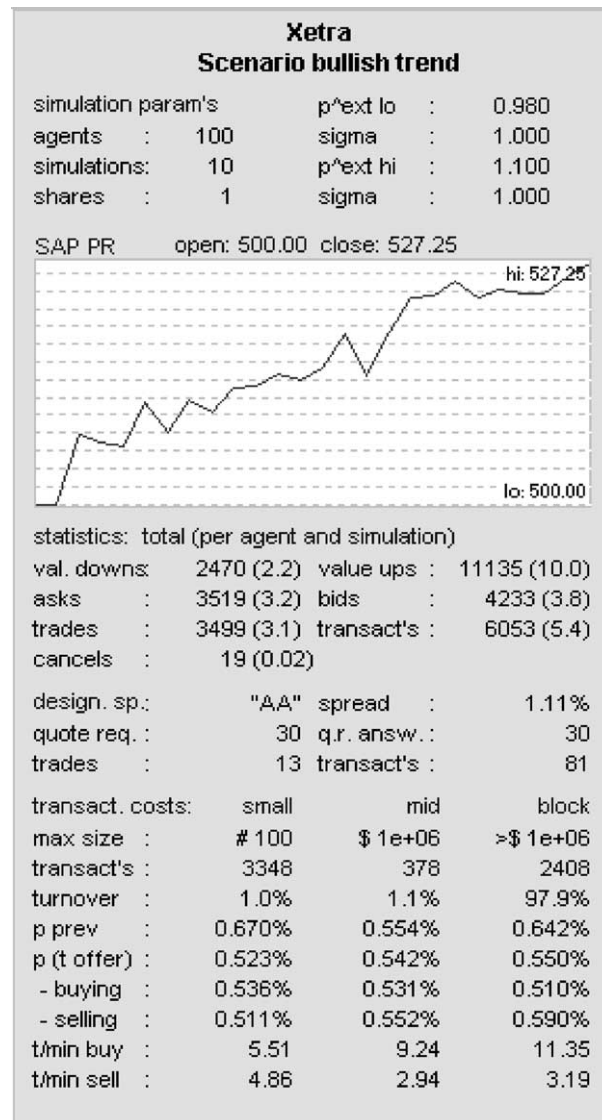


Fig. 15. Xetra transaction cost statistics and designated sponsor's evaluation.

98% of the opening quotation, the bullish group's estimation 1.10% of the opening quotation with each group having the same standard deviation of 100%. The opening quotation starts at 500 and ends at 527.25 confirmed by the number of value adjustments upwards (11,135) compared to value adjustments downwards.

Slightly more bids (4233) than asks (3519) generated 3499 trades split up in 6053 transactions. In this scenario, the market does not ask the designated sponsor and he behaves quite appropriately. According to Deutsche Börse criteria, he gets an AA rating: the spread is only 1.11% and he answers 100% of the quote requests (30). It's interesting to note that the

transaction costs measured by the change between the price prevailing at the offers' time and the execution price is lowest for buying block trades. However, the average time for executing a buy block trade (11.35) compared to the average time to sell a block trade (3.19) clearly indicates that block trade buyers sell time for value and block trade sellers act in the opposite direction.

Fig. 16 exhibits that the designated sponsor is not requested in a very active and liquid market. The scenario assumes 50% endowment in stocks and cash each. Three auctions take place.

Fig. 17 exhibits a similar result, despite the nine actions established, even if the designated sponsor

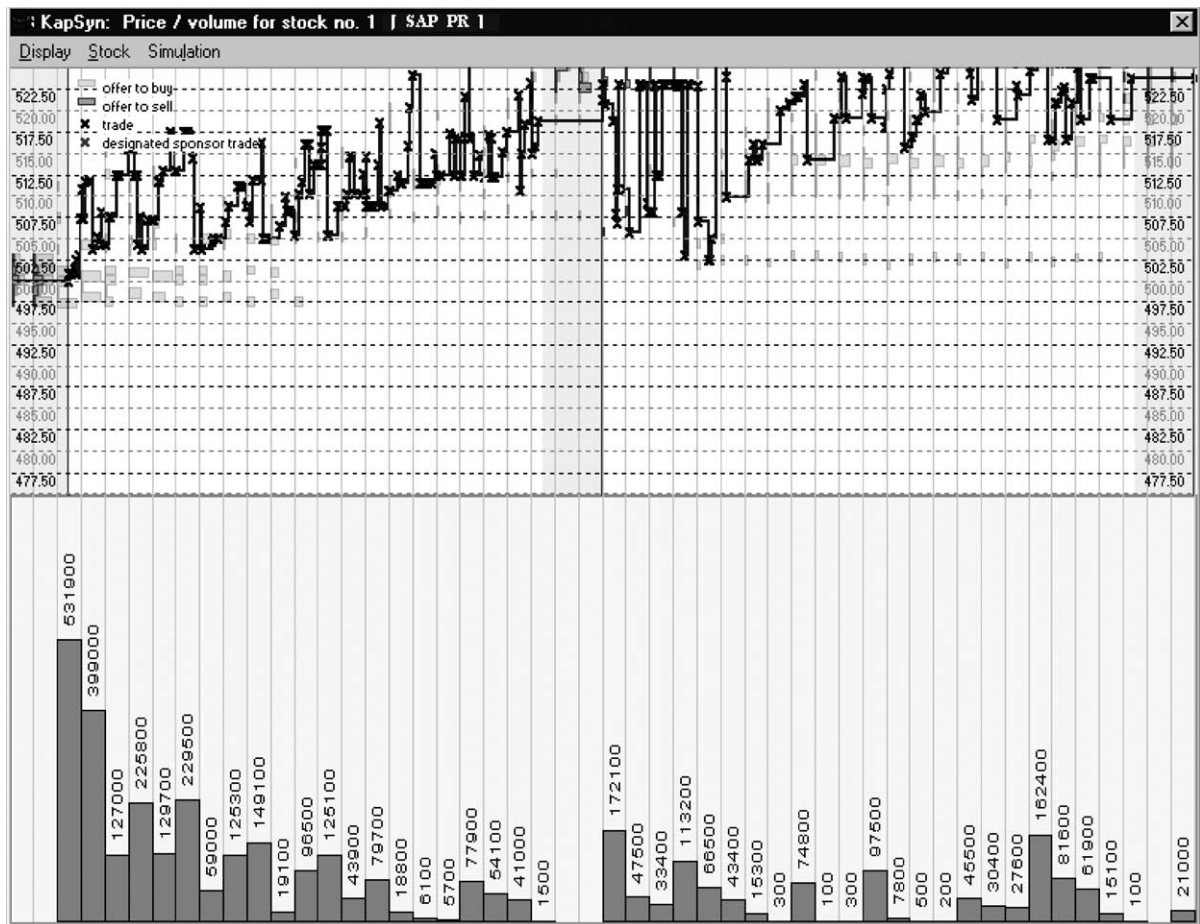


Fig. 16. Market performance during the session.

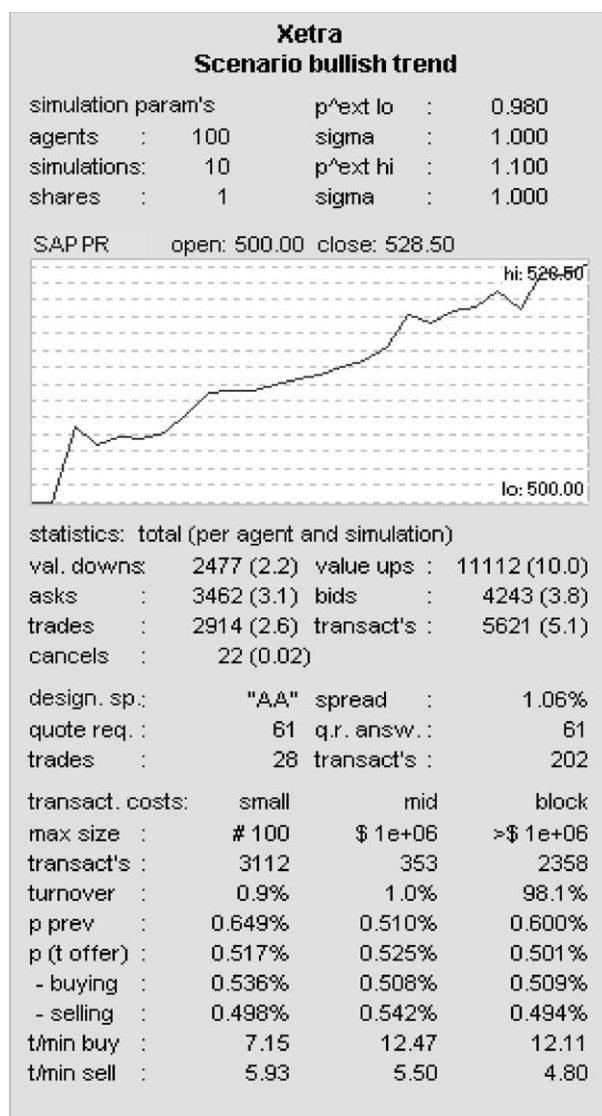


Fig. 17.

performs a greater number of actions. His quote is requested 93 times and he responds to all requests. The transaction costs increased as well as the execution time. Obviously the number of nine auctions is beyond the optimum.

Fig. 18 indicates designated sponsors' low level of activities, even if the market participants trade marking and the designated sponsors trade marking is difficult to identify.

Referring to auction number six it turned out that also intra-day auctions can be highly liquid. The lower half of the window produces a trade volume histogram in 48 equidistant time intervals, while the vertical lines in the upper half indicate the auction itself.

Fig. 19 provides the market performance based on a change of the market participants endowment only. They dispose of a great quan-

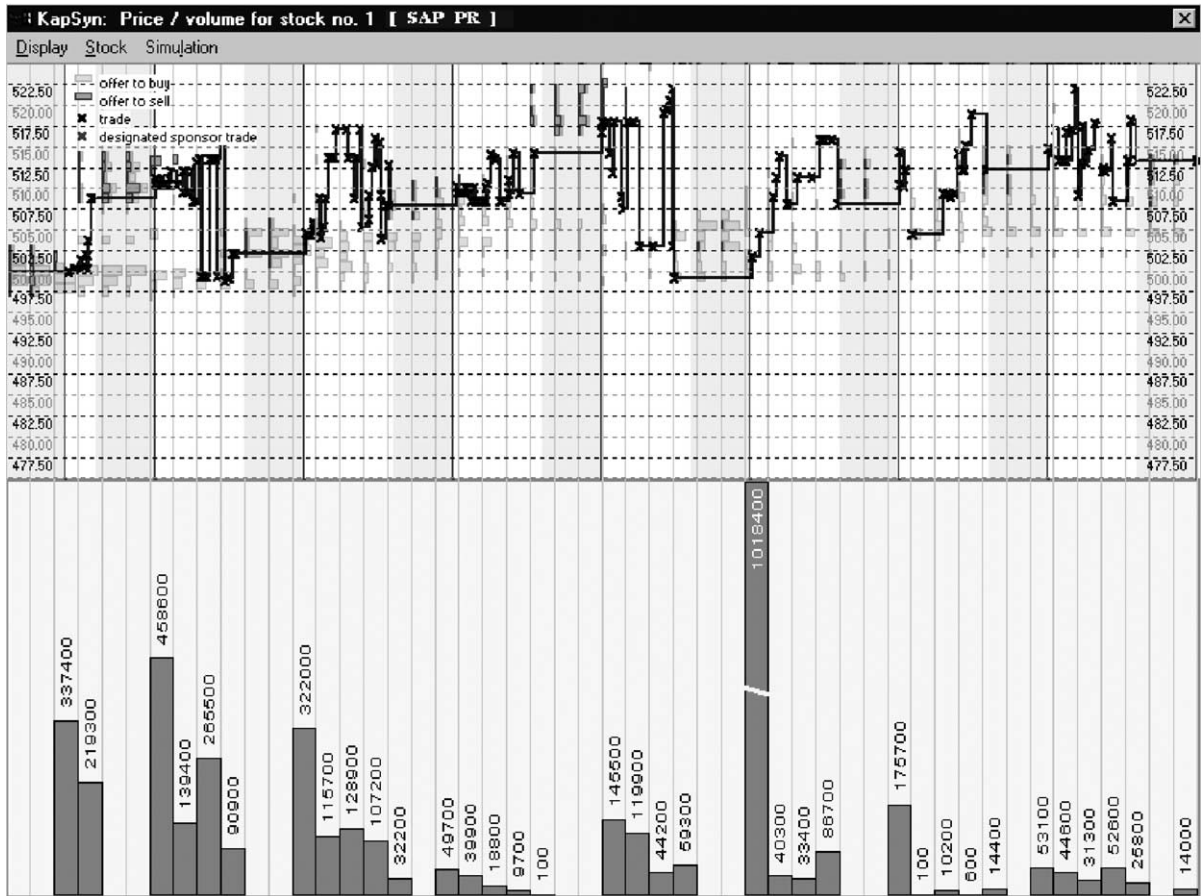


Fig. 18.

tity of cash but no stocks. Everything else remains unchanged. The number of value adjustments decreases as well as the number of trades. However, the price climbs even higher to 537.50. The designated sponsor's spread decreases to 1.03%. The transaction costs experience a dramatic change: they decrease for block selling activities to 0.034% while waiting time is reduced to zero. It decreases for block buying activities to 0.469% but the average waiting time increases to 19.59. There is the typical hausse scenario: it is a seller market.

Fig. 20 underlines the designated sponsor's main function in non-liquid markets: he provides the liquidity, the stocks requested for trading. In

liquid markets, his importance remains well below this level.

## 2.2. NASDAQ transaction costs

The KapSyn NASDAQ module simulates the electronic trading system of NASDAQ (Fig. 21).<sup>30</sup>

<sup>30</sup> For an introduction to the electronic trading system of the NASDAQ, see Kandel and Marx (1997) and literature cited. For further discussions about the implications of market microstructure, see Cohen et al. (1986), Coughenour and Shastri (1999), Glosten and Harris (1988), Huang and Stoll (1996), Jarnecic (1999), Neal (1992) and Stoll (1989).

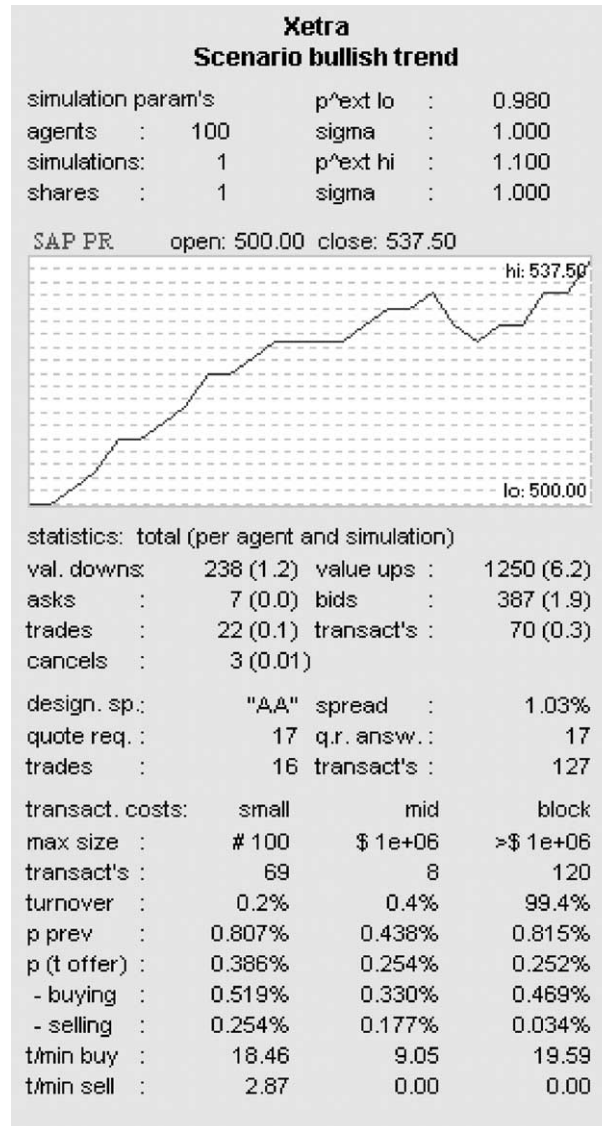


Fig. 19.

There are two specific features for this KapSyn module: ATS trades and preferencing agreements. The implementation of the module simplifies truly complex NASDAQ trading, but it does cover the main features of this stock market. Compared to Xetra, NASDAQ exhibits minimal transaction

costs for midsize trades. For small and block trades the transaction costs are almost 100% /50% higher than for Xetra. The Xetra number of small and block transactions is higher than NASDAQ's but NASDAQ exhibits a larger number of midsize trades.

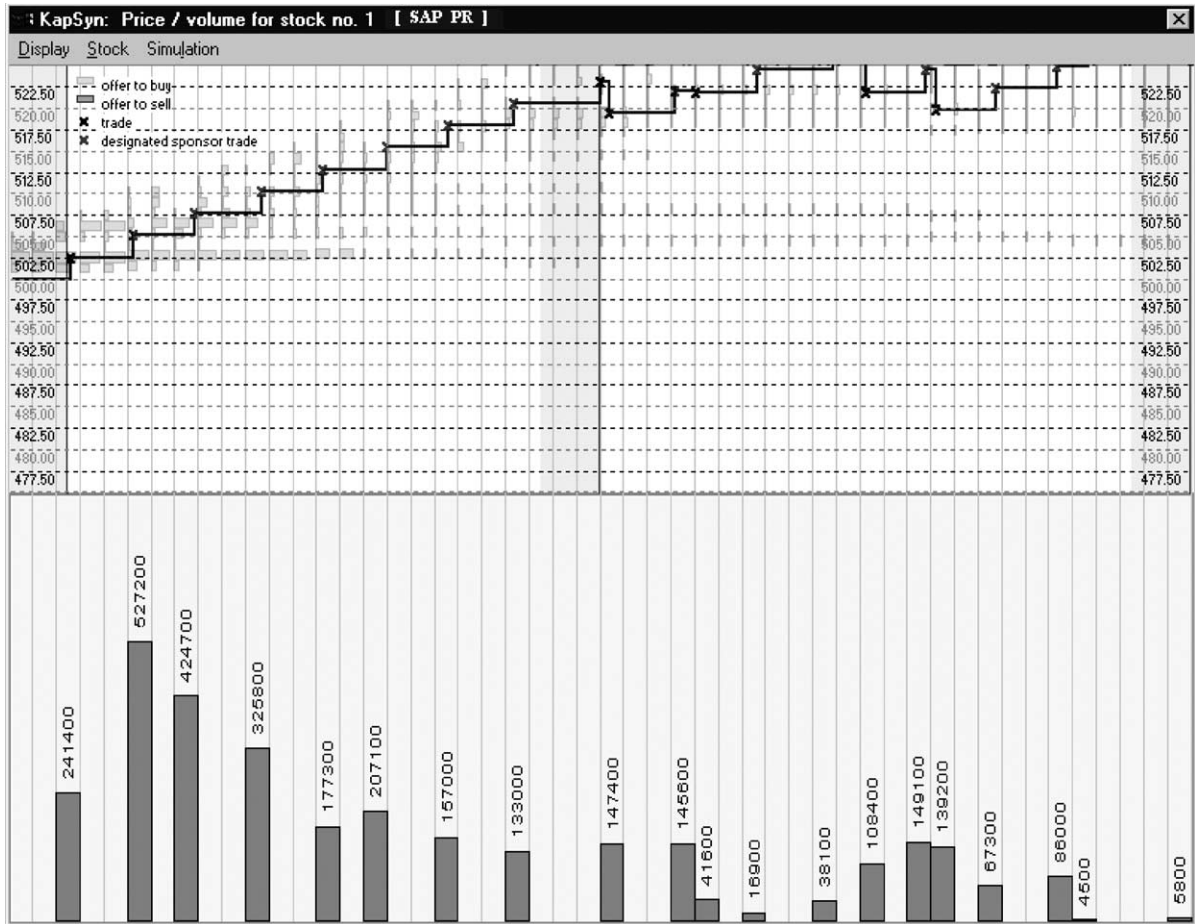


Fig. 20. Designated sponsor's domination in non-liquid markets.

### 3. Summary

Currently the computer program KapSyn provides the facilities to investigate the efficiency of Xetra's trading rules and of NASDAQ's electronic trading system. By using KapSyn we examine Xetra's behaviour and NASDAQ's transaction cost statistics in different market scenarios. The empirical evidence of the KapSyn parameter setting chosen is validated: connecting the parameter setting to economic data by using neural networks and analytical reasoning underlie the definition of

market scenarios. We demonstrate very impressively, the designated sponsor's eminent importance in non-liquid markets: he enables in such a scenario trading. By contrast, in highly liquid markets, trading takes place among the market participants nearly without contacting the designated sponsor. In addition, we investigate transaction costs on Xetra and on NASDAQ: compared to Xetra, NASDAQ exhibits minimal transaction costs for mid-size trades, while transaction costs for small and block trades are nearly 100%/50% higher.



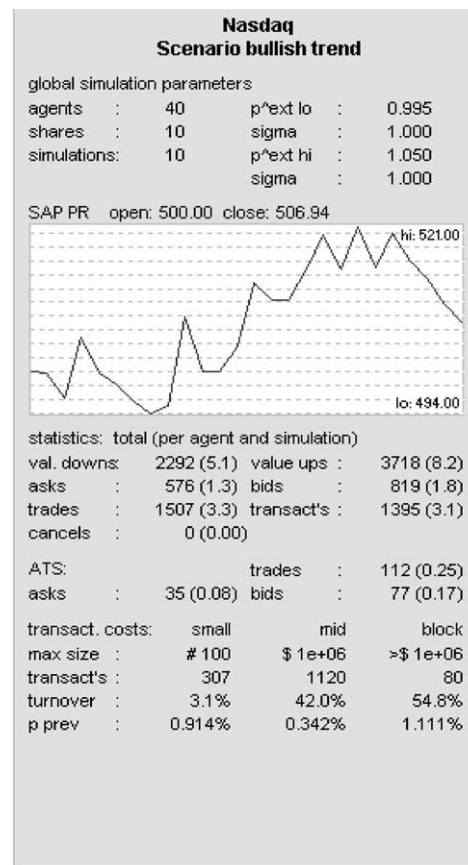
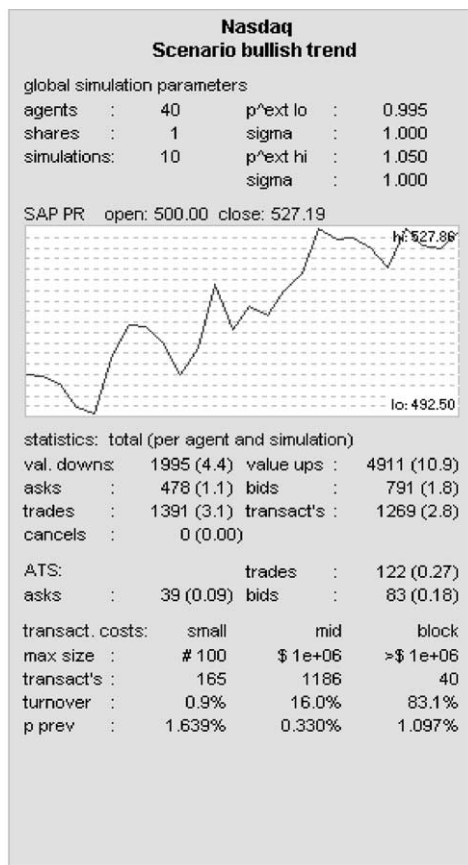
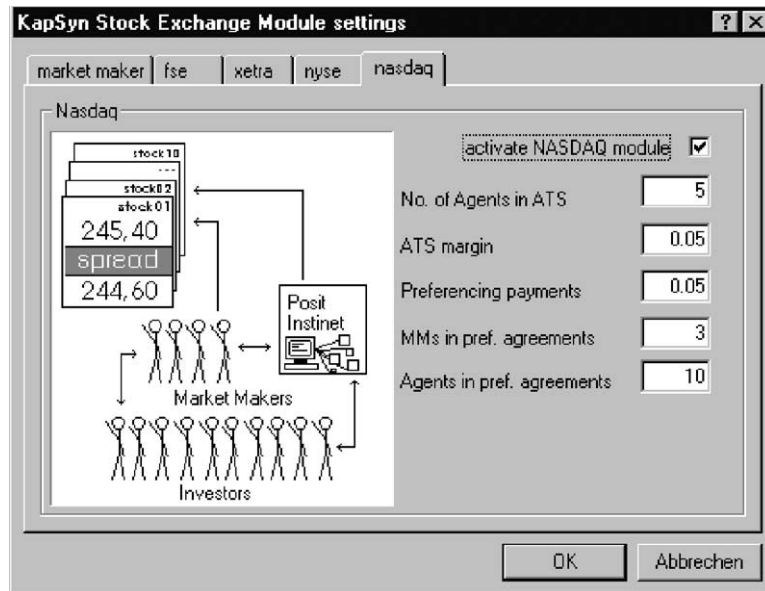


Fig. 21. NASDAQ-structure and transaction cost statistics.

## References

- Breiman, L., 1969. *Probability and Stochastic Processes*. Mifflin, Boston.
- Casey, C., 1998. Zur Modellierung von Unternehmenswerten und Aktienpreisen auf dem Kapitalmarkt: Die Mikrostruktur des Kapitalmarktes, Diss., Paderborn.
- Cohen, K.J., Maier, S.F., Schwartz, R.A., Whitcomb, D.K., 1986. *The Microstructure of Securities Markets*, Englewood Cliffs, NJ.
- Coughenour, J., Shastri, K., 1999. Symposium on market microstructure: A review of empirical research. *The Financial Review* 34, 1–28.
- Deutsche Börse AG, 1999. Xetra Release 3-Leading Edge, brochure, January, Frankfurt am Main, 33 pp.
- Feller, W., 1968. *An Introduction to Probability Theory and its Applications*, third ed., vol. 1. Wiley, New York.
- Glosten, L., Harris, L., 1988. Estimating the components of the bid–ask spread. *Journal of Financial Economics* 21, 123–142.
- Huang, R.D., Stoll, H.R., 1996. Dealer versus auction markets: A paired comparison of execution costs on NASDAQ and the NYSE. *Journal of Financial Economics* 41, 313–357.
- Jarnecic, E., 1999. Trading volume lead/lag relations between the ASX and ASX option market: Implications of market microstructure. *Australian Journal of Management* 24 (1), 77–94.
- Kandel, E., Marx, L.M., 1997. NASDAQ market structure and spread patterns. *Journal of Financial Economics* 45, 61–89.
- Karlin, S., Taylor, H.M., 1975. *A First Course in Stochastic Processes*, second ed. Academic Press, Boston.
- Landes, T., Loistl, O., 1992. Complexity models in financial markets. *Applied Stochastic Models and Data Analysis*, Special Issue in Finance 8, 209–228.
- Loistl, O., 1994. *Kapitalmarkttheorie*, third ed., München.
- Loistl, O., Landes, T., 1989. *The Dynamic Pricing of Financial Assets*. McGraw-Hill, Hamburg.
- Loistl, O., Vetter, O., 1999. KapSyn. Computerprogramm zur Effizienzmessung von Börsenorganisationen Version 3.0 (Downloadable from website <http://ifm.wu-wien.ac.at>).
- Loistl, O., Vetter, O., 2000. KapSyn. Computer-Modelled Stock Exchanges, User Handbook Version 3.01, 130 pages (Downloadable from website <http://ifm.wu-wien.ac.at>).
- Montgomery, D.C., Runger, G.C., 1999. *Applied Statistics and Probability for Engineers*, second ed. Wiley, New York.
- Neal, R., 1992. A comparison of transaction costs between competitive market maker and specialist market structures. *Journal of Business* 65, 317–334.
- Stoll, H.R., 1989. Inferring the components of the bid–ask spread: Theory and empirical tests. *Journal of Finance* 44, 115–134.