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Tick size and spreads: The case of Nasdaq's decimalization ☆

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Abstract

Recent studies by the Securities and Exchange Commission (SEC) and by academics have provided empirical evidence that Nasdaq trade execution costs are still higher than, for instance, on the NYSE. Introducing decimal pricing is one concrete plan to enhance Nasdaq's competitiveness. However, effects of tick size changes are difficult to predict. For that reason models have been required for quite a while. Capital market synergetics is appropriate to investigate the effects of market microstructure changes.

In this paper, we examine the impact of a variation in Nasdaq's minimum price increment on quoted spreads. First, our findings confirm the numerical value of the decline in the average quoted spread in 1997 as an immediate effect of reducing the tick size from \$1/8 to \$1/16. This strongly affirms the reliability of our calculation results. Second, by applying the same research design again, we investigate the impact of a further reduction in the tick size to \$1/100. No such study is available at the moment. The expected changes in the average quoted spreads due to the reduction in the tick size from \$1/16 to \$1/100, the change through decimalization, range from an increase of 2.82% to a decrease of 15.51%. Derived by applying the same method again, the figures embody a reliable forecast of the real effects and are therefore of eminent importance to academics and to practitioners as well.

Keywords: Finance; Capital market synergetics; Nasdaq; Tick size; Spreads

1. Introduction

It has been a rocky road to decimal pricing. However, on April 9 2001 Nasdaq concluded the gradual conversion from fractions to dollars and cents trading, thereby meeting the deadline ordered by the Securities and Exchange Commission (SEC). The switch from the centuries-old system of fractions to decimals brings US stock markets in line with other exchanges around the world.

The effects of the reduction in Nasdaq's tick size from \$0.125 to \$0.0625 and the implementation of the SEC's Order Handling Rules in 1997 have been

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a subject of intense debate. [1] While existing studies have shown that the tick size change led to a significant decline in quoted spreads, there is still considerable debate among practitioners and academics about the extent of the spread-reducing effect. [2]

In this paper, we examine the impact of a variation in Nasdaq's minimum price increment on quoted spreads. Our investigation covers the whole range from lower to higher priced stocks as well as various market climates. Using capital market synergetics is appropriate and is implemented into the computer program KapSyn. Our results are consistent with previous studies of the effect of reducing the tick size from \$1/8 to \$1/16 and extend literature by forecasting the numerical value of the decline in the average quoted spread due to the implementation of decimal pricing.

The remaining part of the paper is organised as follows. Section 2 describes the analytical modelling of the stock exchange. Section 3 presents the market microstructure of Nasdaq as captured by KapSyn. In Section 4 the three capital market scenarios investigated are structured. Section 5 reports the simulation results of the effect of tick size changes on Nasdaq spreads, particularly the impact of the Nasdaq reforms in 1997 and 2001. It also offers a comparison of our results of the change in 1997 to other empirical studies based on actual observed data. Section 6 concludes the paper.

2. The analytical modelling of the stock exchange

Our approach to modelling capital markets is appropriate to follow Buchanan's (2001) call for catallactic modelling of economics. [3] The term catallactics was suggested by Archbishop Whately in 1838 and adopted particularly by the Austrian School economist Hayek denoting the *science of exchange*. [4] We explicitly consider the causal interdependencies given in reality: human activities cause changes in the state of the market within the framework of the stock exchange's rules and regulations. Transition rates are determined by the utility of each activity. The individual steps are explained below. Thereby we implement a "formal mathematics of human interaction" [5] demanded by Buchanan (2001).

Our realistic model of the market's microstructure, the so-called *KapSyn model*, pictures the forces governing the market participants' decisions to place buy or sell orders and provides a means to describe the market performance. [6] A basic introduction is provided by Loistl et al. (2001).

Capital market synergetics, or KapSyn for short, is a high-dimensional Markov model capable of structuring and implementing the rules and regulations of any market place. The name *synergetics* is borrowed from Herman Haken's famous book *Synergetics*. [7] Applying his ideas of connecting macro- and micro-level to capital markets his concept appropriate to explain phenomena in science had to be adapted in order to cope with the catallactic view. This will be realised by the combination of the following five different steps:

(i) The determination of a market place by means of a Markov process with a high-dimensional state space describing this market place.
(ii) The determination of each feasible activity according to the rules and regulations of the

[1] For studies of the impact of the Nasdaq reform on market quality see Barclay et al. (1999), Weston (2000), and Schultz (2000). Bessembinder (1999) examines post-reform trading costs on Nasdaq and the NYSE.

[2] Smith (1998) examines the different extent of the reform's impact across stocks. An investigation of the changes in spread components due to the market reform was conducted by McInish et al. (2000). Bessembinder (2000) provides an empirical analysis of tick size changes for Nasdaq securities near ten dollars. A study by Chung and Van Ness (2001) analyses the effect on intraday variation in spreads.

[3] See Buchanan (2001).

[4] See Devine (1999) and the literature cited therein. For an excellent outline of the Austrian approach to understanding market processes see Kirzner (1997).

[5] Buchanan (2001, p. 31).

[6] The *KapSyn model* was developed by Landes and Loistl and first published in 1989 with a condensed version published in 1992. See Loistl and Landes (1989) and Landes and Loistl (1992).

[7] See Haken (1983).

market place under consideration and the change of the state space it generates.

(iii) The determination of each activity's utility it provides to the actor.

(iv) The connection of each feasible activity with the change of a specific dimension of the state space; for example, an offer to buy the stock j changes the offer dimension of that stock j in the market place.

(v) The determination of the transition rate changing this specific state dimension under consideration by the utility the action generating the state space change provides to the actor.

The utility of any action may be computed according to the actor's specific utility function derived from the class of stochastic utility functions. The different details of the individual utility functions cause different reactions in the same market scenario, even if derived from the same class. [8]

The transition rate of a change in any dimension of the state space describing the specifics of any market place in detail is determined by the utility attached to any action feasible for any market participant according to the rules and regulations of the market place under consideration.

So the basic rule governing the actors' activities and the entire market performance works as follows:

> The higher the return of an action, for instance, an offer to buy or an acceptance of an offer to sell, the higher the action's utility and the higher the transition rate of the change in the specific dimension of the market the action under consideration generates.

Again, the key role of our approach lies in the modelling of the transition rates. They are not held constant but influenced by the behaviour of the individuals and the market state. This is achieved by observing that all changes of the state of the market are caused by activities of the market participants. These activities are governed by the preferences and information processing abilities of the individuals. This concept allows us to study the performance of the stock market as a self-organising social phenomenon at the micro-level regarding all the peculiar rules and regulations of the stock market under consideration.

The formal description of the basic structure of the concept is given in Appendix A, illustrating how the individual processing of information disclosed by the stock market performance is connected with the activities of individuals causing a change of the state of the market. A comprehensive description of the model is provided by Loistl and Vetter (2000), the basic structure is explained by Landes and Loistl (1992). The research report edited by Loistl and Landes (1989) points up the reliable imitation of capital market realities by the model applied.

KapSyn describes the behavioural attitudes of any market participant taking the specifics of the exchange under consideration, either electronic or floor trading, into account. The implementation of the actor's behavioural specifics due to the specifics of any exchange's rules and regulations has to be formulated individually. KapSyn provides the basic model and ideas, the implementation of a specific exchange module has to take the specific rules and regulations and, if necessary, the specific behavioural attitudes of the exchange's actors into consideration. For the moment being the rules and regulations of Frankfurt's Xetra and Nasdaq are implemented, the implementation of Tradepoint and NYSE is in progress. Loistl et al. (2001) document the benefits of modelling Xetra and Nasdaq by means of capital market synergetics to compare the organisational efficiency.

Using capital market synergetics is therefore appropriate for evaluating the operating efficiency of a stock market's microstructure and is implemented at the computer program KapSyn. To examine the efficiency of Nasdaq's tick size rules, capital market scenarios have to be structured. The market microstructure of Nasdaq as captured by KapSyn is given in the next section, the capital market's microstructure is defined by structuring a set of behavioural parameters reflecting the actor's

[8] For details see KapSyn user handbook provided by Loistl and Vetter (2000).

behaviour, e.g. different processing attitudes regarding fundamental or market-related information, and is described in Section 4.

3. The KapSyn Nasdaq module

The *KapSyn Nasdaq* module imitates the trading network of the Nasdaq stock market. [9] Apart from Nasdaq and Xetra, the KapSyn Modules Parameter Box can be used to activate the Market Maker, the FSE, imitating Frankfurt Stock Exchange's trading floor, and the NYSE module. See Fig. 1, where ATS stands for alternative trading system and MM for market maker.

Since the whole Nasdaq market represents a system of considerable complexity certain areas have been simplified and some parts have been left out of the model altogether. Table 1 shows the characteristics of the *Nasdaq* and the *KapSyn Nasdaq* module.

Basically the *Nasdaq* stock market is constructed as a dealer market where investors sell stocks at the buy prices offered by the market makers and buy stocks at the ask prices offered by the market makers. Market orders are executed at the best quote (the inside quote), that is the highest bid price and the lowest ask price of all market makers for the given stock. Limit orders are not exposed to the whole market, the market maker the order has been placed with has to display the limit order price in his quotes, though.

While negotiating over the phone or electronically with the investors the *KapSyn Nasdaq* market maker determines a first estimate of his ask and bid prices for the stocks he is making the market for. The *KapSyn* investment community is split up into two groups, the pessimists and the optimists. The size of both groups can be parametrically set before starting the simulation and allows to simulate different market scenarios. For details see KapSyn user handbook. [10] The initial value of the bid–ask spread is the maximal deviation of the initial stock price p_j from the price expectations of optimists and pessimists:

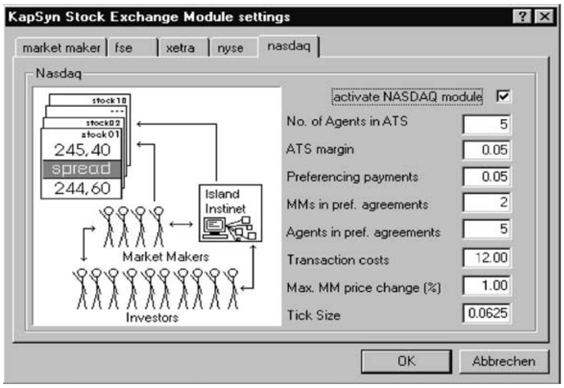

Fig. 1. Nasdaq's basic features.

$$\text{spread}_{\text{MM},j} = \max\left(|p_j - \bar{\hat{p}}_{ij}|_{i\in\text{opt}}, |p_j - \bar{\hat{p}}_{ij}|_{i\in\text{pess}}\right)$$

with the price expectations of optimists and pessimists

$$\bar{\hat{p}}_j^{\text{opt,pess}} = \frac{\sum_{i\in\text{opt,pess}}^{n/2} \hat{p}_{ij}}{n/2}.$$

The *KapSyn Nasdaq* market maker will set his quotes according to the quotes of the other market makers and adjust his quotes by the profit he has generated through his last trades. If he is not able to attract enough orders—the number of orders placed with him is below the average number of orders placed with all market makers—he will decrease his spread by adjusting his ask prices downward and/or his bid prices upward. By decreasing his spread he offers more competitive prices and is thus able to attract more orders. If, on the other hand, the profit he has generated with his trades in the given stock is below the average profit of all market makers for the stock he will try to increase his spread and thus his profit per trade. Since the market maker is not able to execute an arbitrary number of orders in a given time interval he has to balance his quote setting between profit per trade and number of trades. The market maker adjusts his ask and bid prices according to

$$\begin{pmatrix} p_{\text{MM},j}^{\text{buy}} \\ p_{\text{MM},j}^{\text{sell}} \end{pmatrix} = p_j + \delta \begin{pmatrix} -\text{range}_{\text{MM},j} \\ \text{range}_{\text{MM},j} \end{pmatrix}$$

[9] Adapted from Schossmann (2000).

[10] See Loistl and Vetter (2000).

Table 1
Comparison of the Nasdaq market and the corresponding KapSyn Nasdaq model

Nasdaq	KapSyn Nasdaq
Numerous different investment firms with different types of market makers (retail, wholesale, ...)	One type of market maker for regular market making and one market maker representing the alternative trading system, no difference between wholesale and retail
More than 10 different alternative trading systems	One alternative trading system
Three different access levels (Level 1: trading and all quotes, Level 2: all quotes, Level 3: only the best quotes)	Only two levels (Level 1: trading and all quotes, Level 2: all quotes). Some Internet trading firms also provide level 2 quotes for free, the separation into three different access level is historic
On 9 April 2001 the Nasdaq stock market switched to a decimal increment, i.e. reduced tick size to one cent	Tick size for all stocks can be fixed at the range between \$1 and \$0.001, this is a parameter which can be changed prior to starting the simulation
Over 6000 market maker and more than 6000 different stocks, potentially millions of investors	Both the number of market makers and the number of stocks and investors is limited. Currently not more than 50 market maker, 50 different stocks and a maximum of 500 investors. These limits are imposed by the capacity of the computers the simulation will run on and reasonably short simulation run times[a]
Different trading systems such as SOES, ACES and SelectNet, negotiations between investors and market makers over the phone	The information exchange resembles SelectNet, orders below 1000 shares are immediately executed (emulating SOES), no SOES or time lag resulting from phone negotiations

[a] The key aspect is that KapSyn investors may have different behavioural attitudes, the number of investors within a group with the same attitudes is not subject to any reasonable limitation.

by evaluating the profit from his trades against the average profit of all market makers. According to his success he will adjust his prices via

$$\text{range}_{\text{MM},j} = \begin{cases} \text{rev}_{\text{MM},j} > \frac{\sum_{\text{MM}} \text{rev}_{\text{MM},j}}{\text{MM}} : \text{range}_{\text{MM},j} + 3, \\ \text{rev}_{\text{MM},j} > 0 \wedge \text{trades}_{\text{MM},j} \\ \quad > \frac{\sum_{\text{MM}} \text{trades}_{\text{MM},j}}{\text{MM}} : \text{range}_{\text{MM},j} + 1, \\ \text{rev}_{\text{MM},j} \leqslant 0 \vee \text{trades}_{\text{MM},j} \\ \quad \leqslant \frac{\sum_{\text{MM}} trades_{\text{MM},j}}{\text{MM}} : \text{range}_{\text{MM},j} - 1, \end{cases}$$

where the profit $\text{rev}_{\text{MM},j}$ of the market maker MM for a given stock j is determined from all the trades of the market maker MM with stock j, $\text{trades}_{\text{MM},j}$ signifies the number of trades for market maker MM in stock j, and MM is the total number of market makers in stock j. The profit is then normalised by dividing by the current price of the stock to make all profits comparable. The initial value of *range* is determined from the price expectations of the investors:

$$\text{range}_{\text{MM},j} = \frac{|p_j - \bar{\hat{p}}_{ij}|_{i \in \text{opt,pess}}}{\delta_j}.$$

See initial value of the bid–ask spread given above. After the determination of the range value all ranges are divided by two since positive and negative adjustments are taken in relation to the current stock price:

$$\text{range}_{\text{MM},j} = \frac{\text{range}_{\text{MM},j}}{2}.$$

See symmetric quotation around the current stock price defined above. Since a maximum allowed price change for the market makers can be set parametrically the total adjustment of the new market maker quotes is checked against the price change parameter and, if necessary, adjusted to the maximum change allowed. In a final step the market maker has to adjust his spread to account with the limit orders placed with him. The SEC order handling rules state that a market maker is not allowed to quote a worse price than the prices of the open limit orders in his order book:

$$\begin{pmatrix} p_{\text{MM},j}^{\text{buy}} \\ p_{\text{MM},j}^{\text{sell}} \end{pmatrix}_{\text{quote}} = \begin{pmatrix} \min p_{\text{MM},j}^{\text{buy,orderbook}} \\ \max p_{\text{MM},j}^{\text{sell,orderbook}} \end{pmatrix}.$$

Transactions up to a size of 1000 stocks are automatically routed through the *small order execution system* (SOES) of the *Nasdaq*, which generally executes the order immediately at the current best price. Preferenced orders are also possible, i.e. SOES orders can be directed to a dealer who is not quoting the best price. For a discussion of preferencing, see Section 3.2. The lower limit for *Nasdaq* orders are 100 stocks.

The *KapSyn Nasdaq* module executes all orders up to 1000 stocks immediately, the stocks are transferred between the trading parties and the payment is immediately cleared, thus simulating the SOES. All other orders are treated as limit orders and are entered in the order book of the market maker the order is being placed with. After all quotes have been determined and the limit orders in the order book have been accounted for by adjusting the spreads the open transactions are executed and the order is deleted from the order book.

Currently the smallest change in value on the *Nasdaq* is one cent. In the *KapSyn model of the Nasdaq* the smallest price increment can be adjusted prior to simulation start and can be as low as \$0.001. The behaviour of the *KapSyn Nasdaq* traders and the underlying assumptions are elaborated in the KapSyn user handbook. [11]

3.1. The KapSyn Nasdaq ATS trades

Trades through an alternative trading system (ATS) such as Instinet and Island allow the investor to trade on the *Nasdaq* without directly contacting a market maker. The ATS acts as an electronic market maker who just passes on quotes and permits direct interaction between investors. Direct competition between the public and Nasdaq dealers led to a reduction in spreads without adversely affecting market quality. [12] The current implementation of the *KapSyn Nasdaq* module includes one ATS.

ATSs are also used by *Nasdaq* market makers and allow to extend trading beyond the regular trading times of the Nasdaq. Only a fraction of the market makers and investors take part in ATS trading, for the *KapSyn simulation of the market* the number of investors and market makers participating in ATS trades can be set prior to starting the simulation. The spread for ATS trades is calculated from the price expectations of the participating investors and from the ask and bid quotes of the contributing market makers. A minimal spread can be set prior to simulation start to account for trade commissions and other costs associated with ATS trading.

In *KapSyn* ATS trades can only be executed inside the current Nasdaq spread and only if both parties partake in ATS trading. Restricting trades over the ATS to those which are executed inside the Nasdaq spread is indeed a very strong condition. In reality investors and market makers might choose to trade over an ATS even if the quotes there are not better than the Nasdaq quotes. In this case, the ATS trade is just another electronically executed trade, much like a SOES, SelectNet or ACES trade. The simulation of the Nasdaq market through the KapSyn Nasdaq model might therefore slightly underestimate the number of ATS trades, the dynamic of the market simulation is nevertheless not affected by this simplification.

3.2. The KapSyn Nasdaq preferencing agreements

Since the profit of a market maker is directly related to the number of orders placed with him in a certain timeframe he will always try to attract more orders by offering competitive ask and bid prices. The market maker might not only use quotes to attract order flow but enter so-called *preferencing agreements* with certain investors. Under those agreements the market maker pays the investor a certain amount per share for orders placed with him. These payments might amount to between \$0.01 and \$0.02 per share. Through preferencing the investor has an additional monetary incentive to place orders with a particular market maker. Preferencing agreements are not formally regulated by the NASD or the SEC and are only subject to the individually negotiated terms. Since preferencing agreements are common but not directly part of the Nasdaq, market information

[11] See Loistl and Vetter (2000).

[12] See Barclay et al. (1999).

about the volume traded under preferencing and the exact terms is not widely available.

Preferencing agreements are intensely discussed in market microstructure research. On the one hand they are highlighted, for instance, by Bessembinder (1999) as a likely reason that trade execution costs are larger on Nasdaq than on NYSE, as there is little incentive left to improve quotes. But on the other hand, for instance, Ackert and Church (1999) report narrower spreads when market participants are given the opportunity to compete using payment for order flow. Battalio and Holden (2001) show that the opportunity for a sorting of orders based on objective characteristics of the traders or orders is sufficient to support the existence of payment for order flow.

The *KapSyn Nasdaq* module assumes that preferencing agreements exist between a fixed number of investors and a fixed number of market makers. The amount paid per share in an order under the agreement is also parametrically determined prior to simulation start. Since the negotiations necessary to establish preferencing agreements take a long time the preferencing agreements in the KapSyn Nasdaq simulation can be assumed to be constant over the simulation timeframe. The preferencing agreements directly influence the different offers among which an agent can choose. Whenever deciding among different offers the agent modifies the price by the preferencing agreement amount and chooses his partner market maker based on the modified price.

The peculiarities of Nasdaq's market microstructure are shown in Fig. 1. In our investigations five agents will take part in ATS trading, the margin for trading over the ATS is set to 5 cents. The order flow of five agents is preferenced to two market makers. Under such a preferencing agreement agents get 5 cents per share for placing orders with their market maker. There are no explicit transaction costs and market makers' price changes are limited to 5%. The minimum price increment is to be varied.

4. Structuring the capital market scenarios

To examine the impact of a variation in Nasdaq's tick size on quoted spreads, we structure three capital market scenarios. The most important aspects of the settings the investigations are based on are briefly described below. For an in-depth description of all parameters see KapSyn user handbook. [13]

To reduce stochastic influences fifteen subsequent simulations will be run for four stocks each: Palm, Cisco, Juniper, and Genzyme. The Nasdaq stocks chosen cover the whole range from lower to higher priced stocks as the price might influence the effects of tick size changes. [14] Each run represents a stock trading session. Start prices are set to the actual quotation in USD. The futures trader is activated, exploiting with his trades arbitrage opportunities between cash and futures markets.

In all market scenarios forty traders will deal in stocks. They have available funds of $1,000,000,000 each. At the trading session's beginning the fortune is invested 50% in stocks and 50% in interest bearing cash equivalents in order to simulate liquid markets. [15] The interest rate based on the traders' intended investment horizon of one month is 0.3%, i.e. an annual yield of 4%. Currently, two distinct compositions of the traders' start portfolios might be identified. However, the implementation of more groups of traders is possible.

Our investigations are based on three capital market scenarios named bullish, volatile, and bearish. The pessimistic group's mean of the stock's intrinsic value distribution is set to 99.5% of the opening quotation in all scenarios, while the optimistic group's mean of the stock intrinsic value distribution is gradually reduced from 105% of the opening quotation in the scenario bullish to 101% in the scenario bearish. [16] For both groups the

[13] Both the KapSyn user handbook, Loistl and Vetter (2000), and the KapSyn computer program, Loistl and Vetter (1999), are available at the Internet http://ifm.wu-wien.ac.at.

[14] See, for example, Smith (1998, p. 5).

[15] An examination of different portfolio compositions, particularly in order to simulate the impact of tick size changes on trading characteristics of thinly traded stocks, is left for future research.

[16] The parameter values chosen for the individual scenarios reflect the behaviour and information-processing of the market participants as described in the literature about behavioural finance. See, for example, Shiller (2000). The detailed reasoning of the parameter selection is given in Loistl and Vetter (2000).

fundamental expectations' standard deviation is 1% in the scenario bullish, 0.8% in the scenario volatile, and 0.6% in the scenario bearish. The traders' behaviour is controlled by relevant setting of the specific parameters. In the scenario bullish, for instance, investors are strongly orientated towards a price trend. In the scenario bearish, by contrast, investors stick to their fundamental value estimate. A comprehensive presentation of the behavioural parameters is given in the KapSyn user handbook. [17]

The following investigation of the effects of tick size changes on quoted spreads is based on the above defined capital market scenarios.

5. Tick size changes and quoted spreads

To investigate the change in quoted spreads with a variation of the tick size 15 simulations were run for each of the four stocks investigated (Palm, Cisco, Juniper, and Genzyme) for three different market scenarios (bullish, volatile, and bearish) and for each tick size. The tick size was varied over \$0.5 (\$1/2), \$0.25 (\$1/4), \$0.125 (\$1/8), \$0.0625 (\$1/16), \$0.03125 (\$1/32), \$0.015625 (\$1/64) to \$0.01. To understand the dependency of the alterations in the quoted spreads under variation of the tick size the stocks for our investigation were selected from different price bands. The initial price for Palm was set to \$7, for Cisco to \$18, for Juniper to \$44 and for Genzyme to \$96. These prices roughly correspond to the quoted prices on the Nasdaq at the beginning of April 2001.

The quoted spreads resulting from the simulations for the different market scenarios, stocks and tick sizes are presented in Table 2. The quoted spreads shown in the table are the average spreads from the set of 15 simulations, where the absolute spread refers to the actual spread value in dollars and the relative spread represents the quoted spread divided by the initial stock price expressed as a percentage. The average absolute spread ranges from \$0.02981 for Juniper in the volatile market scenario with a tick size of \$1/100 to \$0.86237 for Genzyme in the bullish market scenario with a tick size of \$1/2. The average relative spread falls to a minimum of 0.056% for Cisco in the bearish market scenario with a tick size of \$1/100 and reaches a maximum of 2.778% again for Cisco in the bearish market scenario given a tick size of \$1/2. The relative spread allows a comparison of results across different stocks.

Additionally, the minimum and maximum of the absolute quoted spreads from the simulated data set in the bearish, the volatile, and the bullish market scenario are given in Table 3. Table 3 shows the stock in the leftmost column, then the tick size in the next column and the minimum as well as the maximum spread for all market scenarios in the following columns. The widest range (\$0.05–\$0.56154) can be observed for Juniper in the bearish market scenario with a tick size of \$1/2. The presentation of the min/max range points up the variability of the simulation results, which include a stochastic component. [18]

The average relative quoted spreads for all stocks and market scenarios over the tick size are displayed in Fig. 2. [19] Note that not only the tick size but also the different stocks influence the average quoted spreads. Generally, the range of the average quoted spread decreases with a lower tick size. The quoted spreads in the bullish scenario are, when compared to the other market scenarios, more clearly following a downward trend with the changed tick size. [20] The setting for the bullish scenario leads to a clear trend in the price evolution of the stock where the volatility is higher in the volatile and the bearish scenario. Bessembinder (2000) documents the different effects of tick size changes in volatile markets. [21]

The calculation results given in the figures and tables above clearly demonstrate that, on the av-

[17] See Loistl and Vetter (2000).

[18] The variability of the simulation results can be shown by the min/max range of the relative quoted spreads as well. For details of the stochastic component in the simulation results, see Section 2.

[19] Note the logarithmic scale of the tick size.

[20] The comparatively good values of the diagnostic statistics of the regressions for this scenario in Table 4 confirm this observation.

[21] See Bessembinder (2000, pp. 219ff).

Table 2
Average absolute and relative quoted spreads

Stock	Tick size ($)	Average absolute spread ($)			Average relative spread (%)		
		Scenario			Scenario		
		Bearish	Volatile	Bullish	Bearish	Volatile	Bullish
Palm	0.5	0.05000	0.05000	0.08282	0.714	0.714	1.183
Initial price:	0.25	0.06400	0.05200	0.09770	0.914	0.743	1.396
$7	0.125	0.09743	0.09197	0.11273	1.392	1.314	1.610
	0.0625	0.09177	0.09316	0.09703	1.311	1.331	1.386
	0.03125	0.05897	0.05881	0.05931	0.842	0.840	0.847
	0.015625	0.05194	0.05170	0.05218	0.742	0.739	0.745
	0.01	0.04374	0.04166	0.04485	0.625	0.595	0.641
Cisco	0.5	0.12272	0.07672	0.26602	2.778	0.682	0.426
Initial price:	0.25	0.15373	0.19241	0.30506	1.389	0.854	1.069
$18	0.125	0.20747	0.20356	0.22537	0.694	1.153	1.131
	0.0625	0.11197	0.11577	0.11735	0.347	0.622	0.643
	0.03125	0.13034	0.13089	0.13246	0.174	0.724	0.727
	0.015625	0.13954	0.13343	0.14547	0.087	0.775	0.741
	0.01	0.13658	0.14820	0.15935	0.056	0.759	0.823
Juniper	0.5	0.34109	0.32595	0.56317	0.775	0.741	1.280
Initial price:	0.25	0.41780	0.42318	0.44393	0.950	0.962	1.009
$44	0.125	0.37933	0.36907	0.40024	0.862	0.839	0.910
	0.0625	0.32645	0.31876	0.34760	0.742	0.724	0.790
	0.03125	0.31188	0.30751	0.31714	0.709	0.699	0.721
	0.015625	0.21615	0.21360	0.22849	0.491	0.485	0.519
	0.01	0.03005	0.02981	0.03107	0.068	0.068	0.071
Genzyme	0.5	0.82351	0.82725	0.86237	0.858	0.862	0.898
Initial price:	0.25	0.72602	0.74916	0.78638	0.756	0.780	0.819
$96	0.125	0.68122	0.66959	0.71854	0.710	0.697	0.748
	0.0625	0.72955	0.73451	0.74592	0.760	0.765	0.777
	0.03125	0.50661	0.49870	0.53709	0.528	0.520	0.559
	0.015625	0.06142	0.06147	0.06146	0.064	0.064	0.064
	0.01	0.05928	0.05933	0.05925	0.062	0.062	0.062

erage, the quoted spread drops with a reduction of the tick size. In order to make the structure of the results, which include a stochastic component, [22] evident and thereby quantifying the reduction in the quoted spreads with a decreased tick size we have applied a simple linear regression model to the calculation results for each stock i, $i \in I = \{1,2,3,4\}$, and market scenario j, $j \in J = \{1,2,3\}$. [23] Eq. (1) shows the linear model applied.

$$\text{spread}_{ij\tau} = k_{ij} \times \text{tick}_{ij\tau} + d_{ij} \quad \text{with} \quad \tau \in T = \{1,\ldots,7\} \quad \text{and} \quad \text{tick}_{ij\tau} = \text{tick}_\tau \; \forall i,j \tag{1}$$

where *spread* is the average absolute quoted spread, k the slope, *tick* the tick size, and d the intercept of the regression. Table 4 shows the results of the regression analysis. Note that the spread for tick size $1/2 for both the bearish and the volatile scenario was excepted from the regression for Palm since no trade occurred in these simulations. The spread displayed results from ATS-trades only.

For all stocks and market scenarios investigated the intercept of the regression d_{ij} is statistically

[22] See Footnote 18.

[23] A question for future research is whether a non-linear regression has more explanatory power.

Table 3
Min/max range of absolute quoted spreads from the simulated data set

Stock	Tick size ($)	Absolute spread ($)					
		Scenario					
		Bearish		Volatile		Bullish	
		Min	Max	Min	Max	Min	Max
Palm	0.5	0.05000	0.05000	0.05000	0.05000	0.05000	0.20833
Initial price:	0.25	0.05000	0.11000	0.05000	0.08000	0.05000	0.18500
$7	0.125	0.05000	0.13571	0.05000	0.11000	0.05000	0.16818
	0.0625	0.07500	0.10000	0.08261	0.10294	0.08879	0.10515
	0.03125	0.05723	0.05983	0.05338	0.06063	0.05775	0.06063
	0.015625	0.04970	0.05471	0.05075	0.05355	0.04817	0.05442
	0.01	0.03732	0.05061	0.03395	0.04815	0.03706	0.05318
Cisco	0.5	0.05000	0.32143	0.05000	0.15000	0.05000	0.43000
Initial price:	0.25	0.05000	0.23947	0.14474	0.25000	0.05000	0.42241
$18	0.125	0.18846	0.22879	0.17759	0.23000	0.21389	0.23313
	0.0625	0.10455	0.11853	0.11189	0.12132	0.11204	0.12151
	0.03125	0.12019	0.13640	0.12240	0.13931	0.12319	0.14106
	0.015625	0.12340	0.20066	0.12007	0.16842	0.11842	0.17950
	0.01	0.12603	0.16962	0.12105	0.21707	0.13089	0.18298
Juniper	0.5	0.05000	0.56154	0.05000	0.48542	0.40625	0.70000
Initial price:	0.25	0.37368	0.44176	0.38113	0.46429	0.42929	0.45556
$44	0.125	0.34276	0.42687	0.34861	0.38924	0.37256	0.43763
	0.0625	0.27017	0.35739	0.27247	0.34797	0.30786	0.37708
	0.03125	0.27676	0.34404	0.28543	0.33305	0.29340	0.34620
	0.015625	0.18076	0.24878	0.19327	0.24636	0.18894	0.28123
	0.01	0.02551	0.03281	0.02302	0.03481	0.02434	0.03551
Genzyme	0.5	0.75847	0.93910	0.68333	0.91364	0.79882	0.93736
Initial price:	0.25	0.65465	0.79333	0.68000	0.80597	0.70500	0.88056
$96	0.125	0.62500	0.74719	0.60082	0.73025	0.68178	0.76947
	0.0625	0.64804	0.78638	0.68741	0.78698	0.67964	0.80114
	0.03125	0.41867	0.60886	0.44529	0.58810	0.44375	0.78294
	0.015625	0.06090	0.06182	0.06102	0.06207	0.06083	0.06187
	0.01	0.05885	0.05965	0.05908	0.05959	0.05896	0.05951

significant at a 5% level, indicating the existence of a minimum bid–ask spread even with an infinitesimal price-quoting grid. Market maker's costs of trading (i.e., fixed costs, inventory, and adverse selection) constitute an economic limit. Increasing the tick size exhibits a different picture for lower and higher priced stocks. For the three higher-priced stocks in the bullish scenario and Genzyme in all scenarios investigated the slope of the regression k_{ij} is at least marginally significant (t-statistics ranging from 3.059 to 2.007 with p-values from 0.028 to 0.101), while for the three lower-priced stocks in the bearish and volatile scenario and Palm in the bullish scenario the hypothesis that the slope k_{ij} is equal to zero cannot be rejected at conventional levels. From the economic point of view, the increase in spreads made possible by regulators through higher ticks is a potential additional burden on market participants. Naturally, there is an upper limit to how much transaction costs market participants are prepared to accept. As market participants certainly regard transaction costs in relation to the stock price the economic limit to transaction costs in dollar terms is higher (lower) for higher (lower) priced stocks. Therefore the regulatory possibility of an increase

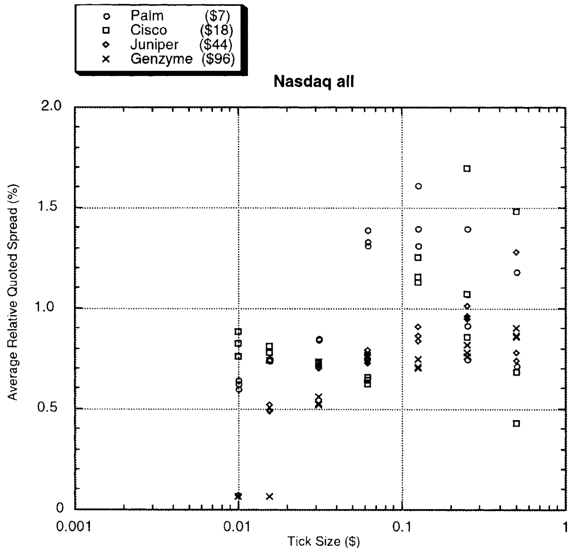

Fig. 2. Average relative quoted spreads shown over the tick size.

in spreads through higher ticks could still be exploited by market makers for higher priced stocks, while for lower priced stocks the economic limit accepted by market participants has already been reached. To allow for a discussion of the spread behaviour due to tick size changes across stocks priced differently and various scenarios we include all regression models estimated in the calculation below.

The value of the coefficient of determination R^2 confirms the observation described above. For the three higher-priced stocks in the bullish scenario and Genzyme in all scenarios the R^2 ranges from 0.652 to 0.446, while for the other market scenarios and stocks the R^2 reaches a maximum of 0.243. Additionally, the F-statistics are given in Table 4. [24]

To test for autocorrelation, heteroskedasticity, and normality of residuals the Durbin–Watson test, White test, and Jarque–Bera test were performed, respectively. In all market scenarios investigated for Palm, Juniper, and Genzyme the Durbin–Watson test cannot reject the null hypothesis of no autocorrelation (significance level of 5%), for Cisco the absence of autocorrelation is indicated (at 5% level; Durbin–Watson statistic of 2.264, 1.764, and 1.975 for the bearish, volatile, and bullish scenario, respectively). The White test cannot reject the null hypothesis of no heteroskedasticity at conventional levels for all market scenarios and stocks investigated with the exception of Cisco in the volatile and bullish scenario (statistically significant at the 5% level and the 10% level, respectively). For all scenarios and stocks investigated the Jarque–Bera statistic is insignificant at conventional levels (maximum value of 1.577 with minimum p-value of 0.455), the null hypothesis of normality of residuals cannot be rejected.

We have then used the regression model to determine the decrease in the spread with a given change of the tick size. Table 5 shows the change in spread, both absolute ($) and relative (%) to the original spread at the initial tick. The spread changes due to the Nasdaq market reforms in 1997 and 2001 shown result from a reduction of the tick size from $1/8 to $1/16 and a change in the tick size from $1/16 to $1/100, respectively. The regression results are given for each stock and market scenario to allow for an analysis of the dependency of the changes in the quoted spread on the general market conditions and the price of the stock.

The change in the quoted spread due to the Nasdaq reform in 1997 ranges from 3.47%, i.e., an increase, for Cisco in the volatile market scenario to −15.58%, i.e., a decrease, for Genzyme in the volatile market scenario.

The results obtained from our stock market simulations with the KapSyn Nasdaq program fall into the same range of results from investigations of the real Nasdaq stock market. Van Ness et al. (1999) and Chung and Van Ness (2001) examine the reduction in tick size on the Nasdaq in 1997 for stocks priced at more than $10. [25] They observe

[24] As we have a simple linear regression, of course, the global test on the regression equation corresponds to the individual test on the slope k_{ij}. The comments on the significance of the slope k_{ij} already made above therefore hold good for the results of the global test.

[25] See Van Ness et al. (1999, p. 9), and Chung and Van Ness (2001, p. 147f).

Table 4
Regression analysis of the reduction in quoted spreads due to tick size changes

Stock i	Regression (average absolute quoted spread)			
		Scenario j		
		Bearish ($j = 1$)	Volatile ($j = 2$)	Bullish ($j = 3$)
Palm ($i = 1$)	d_{ij}	0.06139967	0.06236029	0.06961548
Initial price: $7	k_{ij}	0.07980311	0.03061525	0.05965264
	Observations	6	6	7
	t-Statistic d_{ij}	4.689*** (0.009)	4.450** (0.011)	5.353*** (0.003)
	t-Statistic k_{ij}	0.718 (NS)	0.258 (NS)	1.001 (NS)
	R^2	0.114	0.016	0.167
	F-Statistic	0.516 (NS)	0.066 (NS)	1.002 (NS)
	Durbin–Watson	1.377	1.322	0.853
	White	3.546 (0.170)	2.598 (NS)	0.791 (NS)
	Jarque–Bera	0.575 (NS)	0.547 (NS)	0.585 (NS)
Cisco ($i = 2$)	d_{ij}	0.14470792	0.15437874	0.14800660
Initial price: $18	k_{ij}	−0.01064960	−0.08012580	0.31682513
	Observations	7	7	7
	t-Statistic d_{ij}	8.506*** (0.000)	6.840*** (0.001)	5.961*** (0.002)
	t-Statistic k_{ij}	−0.137 (NS)	−0.775 (NS)	2.785** (0.039)
	R^2	0.004	0.107	0.608
	F-Statistic	0.019 (NS)	0.600 (NS)	7.754** (0.039)
	Durbin–Watson	2.264**	1.764**	1.975**
	White	1.498 (NS)	6.349** (0.042)	5.516* (0.063)
	Jarque–Bera	1.577 (NS)	0.764 (NS)	0.438 (NS)
Juniper ($i = 3$)	d_{ij}	0.23801308	0.23540947	0.22446403
Initial price: $44	k_{ij}	0.35867160	0.34194663	0.76470088
	Observations	7	7	7
	t-Statistic d_{ij}	3.851** (0.012)	3.804** (0.013)	4.115*** (0.009)
	t-Statistic k_{ij}	1.266 (NS)	1.206 (NS)	3.059** (0.028)
	R^2	0.243	0.225	0.652
	F-Statistic	1.604 (NS)	1.454 (NS)	9.358** (0.028)
	Durbin–Watson	0.917	0.975	0.852
	White	0.728 (NS)	0.532 (NS)	1.610 (NS)
	Jarque–Bera	0.799 (NS)	0.735 (NS)	1.319 (NS)
Genzyme ($i = 4$)	d_{ij}	0.34113217	0.33957401	0.35616068
Initial price: $96	k_{ij}	1.20647522	1.22991841	1.28512227
	Observations	7	7	7
	t-Statistic d_{ij}	2.600** (0.048)	2.587** (0.049)	2.589** (0.049)
	t-Statistic k_{ij}	2.007 (0.101)	2.045* (0.096)	2.038* (0.097)
	R^2	0.446	0.455	0.454
	F-Statistic	4.027 (0.101)	4.182* (0.096)	4.155* (0.097)
	Durbin–Watson	0.819	0.840	0.829
	White	3.563 (0.168)	3.269 (0.195)	3.759 (0.153)
	Jarque-Bera	0.460 (NS)	0.454 (NS)	0.533 (NS)

For each stock and market scenario the means of the seven (six) sets of 15 simulation runs for each tick size are used for regression analysis. Significance on a 10% (*), 5% (**), and 1% (***) level is indicated. Two tail p-values are reported in parentheses. 'NS' indicates p-values greater than 0.20. The critical values (5% significance level) for the Durbin–Watson test when $n = 6/k = 1$ and $n = 7/k = 1$ are $d^*_L = 0.610/d^*_U = 1.400$ and $d^*_L = 0.700/d^*_U = 1.356$, respectively.

Table 5
Spread change due to Nasdaq reforms in 1997 and 2001

Stock		Nasdaq market reform					
		1997			2001		
		Scenario			Scenario		
		Bearish	Volatile	Bullish	Bearish	Volatile	Bullish
Palm	Spread change ($)	−0.0049877	−0.0019134	−0.0037283	−0.0041897	−0.0016073	−0.0031318
Initial price $7	Spread change (%)	−6.99	−2.89	−4.83	−6.31	−2.50	−4.27
Cisco	Spread change ($)	0.0006656	0.0050079	−0.0198016	0.00055911	0.00420661	−0.0166333
Initial price $18	Spread change (%)	0.46	3.47	−10.55	0.39	2.82	−9.91
Juniper	Spread change ($)	−0.0224170	−0.0213717	−0.0477938	−0.0188303	−0.0179522	−0.0401468
Initial price $44	Spread change (%)	−7.92	−7.68	−14.93	−7.23	−6.99	−14.74
Genzyme	Spread change ($)	−0.0754047	−0.0768699	−0.0803201	−0.0633399	−0.0645707	−0.0674689
Initial price $96	Spread change (%)	−15.33	−15.58	−15.54	−15.21	−15.51	−15.46

spread reductions following the change to 1/16 by 10.94% and 14.72%, respectively. [26] For the corresponding scenario we find a decrease of 13.67% in the quoted spreads. [27] The decrease of 18.67% in the average quoted spread as an effect of a change in the tick size from $1/8 to $1/16 documented by McInish et al. (2000) rests on a sample of 95 stocks priced at $36 on average. [28] For the price range and market scenario investigated by McInish et al. (2000) we show a decline by 14.93%. [29] Bessembinder (2000) finds an average decrease in the spread size of 8.14% for stocks priced at about $10 and a reduction in the tick size from $1/8 to $1/32 due to the stock price falling below the $10 mark. [30] In accordance with Bessembinder (2000), we report a spread reduction by 7.36% for stocks priced at about $10 following a tick size change from $1/8 to $1/32. [31] The results are summarised in Table 6.

The lower reduction in the quoted spread through a decreased tick size for lower priced stocks and a more pronounced reduction for higher priced stocks reported by Van Ness et al. (1999) can also be seen in our results as presented in Table 5. [32]

According to some authors there are other effects which explain the different effect of the tick

[26] See Van Ness et al. (1999, p. 22), and Chung and Van Ness (2001, p. 150).

[27] The spread reductions in the corresponding scenario bullish for all stocks priced at over $10 average 13.67%. See Table 5. For the adequacy of the bullish market scenario see Section 4 and the literature cited therein.

[28] See McInish et al. (2000, p. 68).

[29] See Juniper in the bullish market scenario, Table 5. The reasoning on the adequacy of the market scenario chosen is given in Section 4 and the literature cited therein.

[30] See Bessembinder (2000, pp. 215 and 220). The reduction of the average quoted spread from 47.9 to 44.0 cents equals a 8.14% decrease.

[31] We use the regression models for Palm shown in Table 4 to determine the decrease in the quoted spread with a given tick size change from $1/8 to $1/32 for stocks priced at around $10. The spread decrease in the three market scenarios investigated averages 7.36%.

[32] See Van Ness et al. (1999, p. 22).

Table 6
Comparison of simulation results to empirical studies

Nasdaq stock sample characteristics	Average change in quoted spread (%) due to tick size reduction				
	Van Ness et al. (1999)	Chung and Van Ness (2001)	McInish et al. (2000)	Bessembinder (2000)	Simulation results
804 stocks; price at least \$10; tick from \$1/8 to \$1/16	−10.94				−13.67
134 stocks; price at least \$10; tick from \$1/8 to \$1/16		−14.72			−13.67
95 stocks; average price of \$36; tick from \$1/8 to \$1/16			−18.67		−14.93
765 stocks; price around \$10; tick from \$1/8 to \$1/32				−8.14	−7.36

size reduction on the quoted spreads with the variation of the price of the underlying stock. The Nasdaq order handling rules (OHR) and trades over the ATS have a significant influence on the spread size. [33] The relatively little use of odd \$1/16 or \$1/32 quotes observed by Bessembinder (2000) might, especially with stocks priced at \$10 or below, inhibit spread reductions made possible through smaller ticks. [34]

Our results confirm that the average reduction in quoted spreads is depressed with lower priced stocks and more pronounced with higher-priced stocks.

Furthermore, Table 5 shows the results of the regression analysis for a reduction in the tick size from \$1/16 to \$1/100 to allow for conclusions on the effect of the decimalization at the Nasdaq stock market in 2001. The expected changes in the average quoted spreads due to this reduction in the tick size range from an increase of 2.82% for Cisco in the volatile market scenario to a decrease of 15.51% for Genzyme in the volatile market scenario. Both the stock price and the market climate affect the extent of the change in quoted spreads. Although a slight increase in a few quoted spreads is possible, on average Nasdaq's decimalization will lead to a reduction in Nasdaq's quoted spreads of 7.91%. This may indicate that the initial minimum price increment prevents liquidity suppliers from quoting competitive spreads. [35] Therefore the switch to decimal pricing presents an increased savings potential to investors, thereby enhancing the competitiveness of the Nasdaq stock market.

There are currently no results available on the effect of the decimalization of the Nasdaq stock market on the size of the quoted spread. The NASD expects a decrease in the quoted spreads, [36] but definite figures are not yet available. Without any doubt a closer examination of changes in the spread due to decimalization will be the focus of future research.

6. Conclusions

Our emulation of the Nasdaq stock market using the KapSyn Nasdaq program allowed us to investigate the effect of decimal price increments on the size of the quoted spreads.

According to our results the expected changes in the average quoted spreads due to the reduction in the tick size from \$1/16 to \$1/100, the change through decimalization, range from an increase of

[33] See Smith (1998) with reference to the order handling rules affecting the quoted spreads.

[34] See Bessembinder (2000, p. 222).

[35] For a discussion see Bacidore (2001).

[36] See NASD (2001).

2.82% to a decrease of 15.51%. Both the stock price and the market climate affect the extent of the change in quoted spreads. Although a slight increase in a few quoted spreads is possible, on average Nasdaq's decimalization will lead to a reduction in Nasdaq's quoted spreads of 7.91%. This may indicate that the initial minimum price increment prevents liquidity suppliers from quoting competitive spreads. Therefore the switch to decimal pricing presents an increased savings potential to investors, thereby enhancing the competitiveness of the Nasdaq stock market.

The reliability of our calculation results is strongly affirmed by the fact that, by applying the same research design, we are able to confirm the numerical value of the decline in the average quoted spread in 1997 as an immediate effect of reducing the tick size from \$1/8 to \$1/16.

Our calculation results extend literature by forecasting the numerical value of the change in the average quoted spread due to the implementation of decimal pricing. Currently there are no definite results available. The findings strongly indicate that various dimensions of market quality have to be considered when analysing the relation between quoted spreads and tick size rules. Without any doubt a closer examination of changes in the spread due to decimalization will be the focus of future research.

Appendix A. Catallactic modelling of capital markets by KapSyn

We consider a stock market of $\dot{M}$ single assets, labelled j ($j \in J = \{1, \dots, \dot{M}\}$), constituted by $\dot{N}$ individuals, the agents, labelled i ($i \in I = \{1, \dots, \dot{N}\}$). [37] Formally, the stock market is assumed to be a time homogenous conservative Markov process $(\dot{Z}_t)_{t \geqslant 0}$ in continuous time with values in the discrete countable set $\dot{Z}$, the state space. The process is completely determined by the set of possible transitions together with the corresponding transition rates when the initial distribution is given.

The state vector $z \in \dot{Z}$, called the market state, is a multidimensional vector of several variables capturing the information attached to the current market status including also variables bearing relevant information of the past:

$$z = (p, p^{\text{of}}, q^{\text{of}}, m, p^{\text{tr}}, d^{\text{tr}}, x, y, \widehat{p}, \widehat{p}^{\text{ext}}) \in \dot{Z},$$

$$\dot{Z} = N^J \times N^{I\times J} \times Z^{I\times J} \times Z^J \times Z^J \times Z^J \times Z_+^{I\times J} \times Z_+^I \times N^{I\times J} \times N^{I\times J}.$$

The set Z denotes the set of integers, N the set of positive integers and Z^+ the set of non-negative integers. The constitutes variables are defined as follows:

$p = (p_j)_{j\in J} \in N^J$,

p_j current price of asset j,

$p^{\text{of}} = (p_{ij}^{\text{of}})_{i\in I, j\in J} \in N^{I\times J}$,

p_{ij}^{of} price of a valid asset-j-offer of agent i,

$q^{\text{of}} = (q_{ij}^{\text{of}})_{i\in I, j\in J} \in Z^{I\times J}$,

q_{ij}^{of} quantity of a valid asset-j-offer of agent i,

$m = (m_j)_{j\in J} \in Z^J$,

m_j market power of asset j,

$p^{\text{tr}} = (p_j^{\text{tr}})_{j\in J} \in Z^J$,

p_j^{tr} price trend of asset j,

$d^{\text{tr}} = (d_j^{\text{tr}})_{j\in J} \in Z^J$,

d_j^{tr} counter-trend of asset j,

$x = (x_{ij})_{i\in I, j\in J} \in Z_+^{I\times J}$,

x_{ij} number of shares of asset j held by agent i,

$y = (y_i)_{i\in I} \in Z_+^I$,

y_i amount of cash (invested in bonds) of agent i,

$\widehat{p} = (\widehat{p}_{ij})_{i\in I, j\in J} \in N^{I\times J}$,

$\widehat{p}_{ij}$ price expectation of agent i for asset j,

$\widehat{p}^{\text{ext}} = (\widehat{p}_{ij}^{\text{ext}})_{i\in I, j\in J} \in N^{I\times J}$,

$\widehat{p}_{ij}^{\text{ext}}$ fundamental value estimation of agent i for asset j.

[37] The following presentation is adapted from Landes and Loistl (1992).

The prices, amounts of cash, price expectations and fundamental value estimates are measured as integer multiples of a given unit. The same holds for the number of shares held and the quantities offered. The values of all variables except the fundamental value estimates are changed by transitions during the trading session (the interval $0 \leqslant t \leqslant t_{\max}$).

A maximum of one offer can be simultaneously valid for each agent and each asset. The valid offers are stored in the fields p^{of} (prices) and q^{of} (quantities) with the sign convention that positive quantities correspond to bids (offers to buy) and negative to asks (offers to sell). At the beginning of the process ($t = 0$), the state variables have the following initial values:

- $p_j = p_j^0$ predetermined quotation, e.g. the closing quotation of the previous stock exchange day,
- $q_{ij}^{\text{of}} = 0$ no offers valid (the value of p_{ij}^{of} is arbitrary),
- $m_j = 0$ no demand–supply pressure,
- $p_j^{\text{tr}} = d_j^{\text{tr}} = 0$ no trend,
- x_{ij} and y_i predetermined individual start portfolio, e.g. sampled from a predetermined distribution,
- $\widehat{p}_{ij} = \widehat{p}_{ij}^{\text{ext}}$ initial price expectations and fundamental value estimates coincide;derived from market observation, e.g. analysts' recommendations,
- $\widehat{p}_{ij}^{\text{ext}}$ sampled from a predetermined distribution and held fixed during the trading session.

Even if until now the modelling of the market's microstructure in such a realistic dimension is a rare exception in the literature, it is not the most innovative aspect. The real innovation in modelling the market's microstructure lies in the fact that any change of the state vector is related to activities changing this specific dimension. Every transition of the market state is brought about by agents' activities. That is, there is a one-to-one correspondence between possible transitions $z \mapsto z'$ and admissible actions of the agents.

At each instant t, agent i has the choice between the following activities:

(i) Value adjustment $V = (i, j, \delta)$:
He may adjust his price expectation $\widehat{p}_{ij}$ of asset j by $\delta = \pm 1$ units. If $\widehat{p}_{ij} = 1$, the down-adjustment $(i, j, -1)$ is not admissible. This guarantees the positivity of price expectations.

(ii) Offer $O = (i, j, p, q)$, where O is a bid B if $q > 0$ or an ask A if $q < 0$:
He may offer to buy or sell, respectively, the quantity $|q| \in N$ of shares of asset j at the price $p \in N$. We use the sign convention that positive quantities correspond to a bid (or purchase), negative to an ask (or sale). The bid $B = (i, j, p, q)$ is only admissible if $y_i \geqslant pq$ and no ask of the agent for that asset is valid. Similarly, the ask $A = (i, j, p, q)$ is only admissible if $x_{ij} \geqslant |q|$ and no bid of the agent for that asset is valid.

(iii) Trade $T = (i, j, O, q)$:
If a bid or an ask of agent i meets (i.e. has the same price as) the valid offer $O = (i', j, p, q')$ – bid or ask–of an other agent i', then $|q| \leqslant |q'|$ shares of asset j are traded: either i buys q shares of asset j from i' when $q > 0 > q'$, or i sells $-q$ shares to i' when $q < 0 < q'$. Trades underlie the same admissibility conditions as bids or asks, i.e. $y_i \geqslant pq$ and $x_{ij} \geqslant q$.

(iv) Cancel $C = (i, j)$:
He may cancel the valid offer $O = (i, j, p, q')$ of himself. This activity is admissible when an offer would be admissible (a bid if O is an ask–an ask if O is a bid) if O were not valid.

Activities are subdivided into two groups, the unobservable value adjustments and the market events, namely the publicly observable offers, trades and cancels. The set of simultaneously admissible market events is further restricted. At the same time for the same asset, an agent cannot be both a demander and a supplier, i.e., he may either offer an ask, agree to a bid or cancel a bid or he may offer a bid, agree to an ask or cancel an ask. This fundamental bid–ask decision depends on his current portfolio, his price expectation, the current price and the currently valid offers. [38] There are

[38] For a detailed explanation of the fundamental bid–ask decision see Loistl et al. (2001).

Table 7
Feasible activities and corresponding state transitions

Activity	Transition
$V = (i, j, \delta)$	$\widehat{p}_{ij} \mapsto \widehat{p}_{ij} + \delta$
$O = (i, j, p, q)$	$p_{ij}^{\text{of}} \mapsto p, q_{ij}^{\text{of}} \mapsto q, m_j \mapsto m_j + \text{sign}(q)$
	$p_j \mapsto p, q_{i'j}^{\text{of}} \mapsto q' + q, m_j \mapsto m_j + \text{sign}(q)$
$T = (i, j, (i', j, p, q'), q)$	$x_{ij} \mapsto x_{ij} + q, y_i \mapsto y_i - pq, x_{i'j} \mapsto x_{i'j} - q, y_{i'} \mapsto y_{i'} + pq$, p_j^{tr} and d_j^{tr} : see text
$C = (i, j)$	$q_{ij}^{\text{of}} \mapsto 0$

also restrictions concerning the price p of offers and the acceptability of valid offers, and there is a rule concerning the computation of the traded or offered quantity. The state transitions corresponding to the activities are given in Table 7.

The transitions of p_j^{tr} and d_j^{tr} in the case of a trade are as follows. We distinguish two cases, namely a trend continuation when the price change $\Delta p = p - p_j$ and p_j^{tr} have the same sign, and a counter-movement when they have opposite signs. First, Δp is added to both p_j^{tr} and d_j^{tr} in any case. If, in the case of a continuation, a previous counter-trend is overcompensated, i.e., d_j^{tr} now has the same sign as the trend, then d_j^{tr} is reset to 0. If, in the case of counter-movement, $|d_j^{\text{tr}}|$ exceeds the critical value $d_{\max}|p_j^{\text{tr}}|$, then a trend reversal is indicated and p_j^{tr} is set to d_j^{tr} and d_j^{tr} is reset to 0.

The state's transition rate is determined by the utility the specific activity generating this state change entails to the actor. The utility of any action is computed according to the actor's specific utility function derived from the class of stochastic utility functions. The different details of the individual utility functions cause different reactions in the same market scenario, even if derived from the same class. A detailed description is given in Loistl et al. (2001).

In our model, there are only finitely many admissible transitions in each state z so that the process fulfils the condition of being conservative [39]:

$$\sum_{z' \neq z} \lambda(z', z) = -\lambda(z, z) < \infty.$$

The evolution of the market state z is then represented by the time evolution of the probability function of z, $P_t(z)$, obeying the so-called *master equation*, a linear differential equation. Therefore the entire market performance is given by the *master equation* integrating the entire set of transition rates λ which is explicitly formulated in the research report edited by Loistl and Landes (1989):

$$\frac{\mathrm{d}}{\mathrm{d}t} P_t(z) = \sum_{z' \neq z} \lambda(z, z') P_t(z') - \sum_{z' \neq z} \lambda(z', z) P_t(z).$$

Even if this *master equation* basically exhibits a simple linear structure, it has not yet been solved analytically due to its high-dimensional structure and the many restrictions imposed according to the explicit rules and regulations of the stock exchange under consideration. We therefore perform Monte Carlo simulations guided by the master equation above. So the results derived by KapSyn, for instance, the impact of tick size changes on Nasdaq spreads in different market scenarios, are based on this master equation.

[39] See Landes and Loistl (1992, p. 215).

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