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COMPLEXITY MODELS IN FINANCIAL MARKETS

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SUMMARY

In this approach, the complexity of the self-organizing microstructure of the stock exchange is explicitly taken into consideration: the process of offers and trades as well as the adjustment of individual expectations are modelled with help of a (stochastic) jump process. Its abilities are illustrated by modelling the continuous quotations of asset prices at an auction type stock exchange. The functional form of the transition (hazard) rates is chosen to reflect the individual preferences and expectations as well as the economic environment. The model is described in detail and examples of Monte Carlo simulation results are presented.

KEY WORDS Market microstructure Markov processes Asset pricing in non-equilibrium Self-organization of open systems

1. INTRODUCTION

The number of articles dealing with the microstructure of stock exchanges increases continuously. The main reason might be that the organizational structure of most stock exchanges has changed and is still in progress. There is increasing discussion of main issues, such as, e.g., market maker auction markets, electronic screen based trading versus the floor trading, the abolition of dual trading and the installation of circuit breakers and several similar questions. Such a discussion becomes fruitful especially if it rests on the explicit rules governing the actions, on the organizational structure and the behavioural attitudes of the market participants.

The central problem is to determine the most suitable level of abstraction for the investigation of the functioning of the 'stock exchange' system (cf. De Bono, Reference 1, p. 30, slightly adjusted). The prevailing level hitherto has been the macrolevel. The increasing number of investigations at the microlevel strongly indicates that a macrolevel description is no longer the most appropriate one. Several microlevel approaches can be discerned.

Firstly, there is the econometric description of the stock quotations, in particular the behaviour of the share price relatives. Among the most advanced methods currently used are ARCH-models and their recent variants (cf. Bollerslev *et al.*[2]). However, there is no doubt that besides the eminence this econometric approach merits, its explaining power is confined to relations that are within the abilities of the underlying formal model. A different approach without these restrictions is that of artificial neural networks. This concept seems to be appropriate for handling questions of data-feeding raised by our approach.

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Secondly, the behavioural approach in finance has to be mentioned. It embodies an explicit (cf. Hunter and Coggin[3]) or implicit (cf. Bagwell[4]) criticism of the efficient capital market hypothesis (cf. LeRoy[5] for a general critical survey). Three basic aspects emerge: the market-clearing equilibrium, the no-arbitrage condition, and the rational-expectations approach, including learning and information processing.

Starting with the last, we can explain and also relate the other two concepts to our approach. A commonly cited implication of the rational expectations hypothesis is that an agent's expectation of a variable is an unbiased estimate of its future value (cf. Williams, Reference 6, and the references therein). The individual forecasts must be free of systematic and easily correctable biases (cf. Lucas[7] and Bullard[8]). These statements are derived from the basic maxim of Muth that the economy does not waste information (cf. Muth[9]).

The economy, or market, has to be comprehended as the ensemble of all its individual parts. Even if the market does not waste information, one cannot conclude that every individual is able to forecast the future quotations correctly. Everyday experience teaches the contrary. Muth's global statement implies that the market's efficient information processing yields a frequency distribution, not a point estimate, of the corporation's fundamental value that determines the actual quotation. This distribution can be understood as the universe of point estimates of all market participants. Even if every individual might have a different estimation, it is not necessary to estimate all individual valuations of the fundamentals; the shape of the distribution already suffices.

But trading is performed at the level of individuals. Therefore, the motives for trading have to be formulated at the individual level. They depend on the relation between the individual estimations and the actual quotations. We assume that every individual believes that his expected value will be equal to the later quoted price. We assume rational expectations at the subjective individual level, not at the whole market level. We are therefore able to handle the exploitation of arbitrage opportunities explicitly (cf. the illustrative description of arbitrage decision processes in a different context by Wilborg[10]) and are not forced to disregard their existence.

The basic rule for trading is derived from the oldest maxim of investment banking: buy low, sell dear. The market participant can buy now (or go short if that is possible according to the market rules) and sell at the end of the planning period (whatever that might be, one week, or month, etc.) at a price that he assumes to equal his expectation. This rule is justified by the following aspects.

1. An assumption about the individual forecasts has to be made to derive motivations for trading, viz. buying or selling.
2. The model must have regard to the adjustment of individual expectations according to the processing of information the market performance entails.
3. The adjustment of expectations and the formation of price quotations must be modelled so as to be independent.

It is not adequate in our opinion to disqualify market participants with estimates different from an equilibrium price as noise traders (cf. De Long *et al.*[11]).

The basic problem in modelling the capital market is the relation between subjective estimates and objective quotations emerging as a compromise between the buyer's and seller's subjective valuations. The value estimates have to be determined by aspects of their own, not with regard to price quotations. This paradigm is penetrating the capital market discussion. Duménil and Lévy[12] investigate the possibilities of micro-adjustments towards a long term equilibrium, even if they deal with markets of real goods, not confined to financial markets. As a further step in this direction, Morck *et al.*[13] have discussed the distribution of wealth between smart investors and noise traders. They start with the treatise of the importance of the market organization as such. The explicit addressing of the influence of institutional aspects requires explicit modelling of the market microstructure.

However, most of the models applied are only abstract descriptions of the real stock exchange. Greenwald and Stein[14] feature two types of trader and continuous trading, but do not cover efficient information processing. Pliska and Shalen[15] investigate the effects of regulations on trading activity, construct individual demand and supply functions, but determine the price by a global market clearing condition that is not encountered in real continuous quotations markets. Friedman[16] allows for learning and constructs a sort of acceptable buy-and-sell order determined with regard to an estimated mean of the distribution of value. He has to attack his complex model by means of simulations.

Another approach to investigating stock market performance relies on Markov models. Turner *et al.*[17] study the possibilities of learning. McQueen and Thorley[18] apply Markov chains to test the predictability of stock returns. Most of the analytical models employed in social sciences came originally from physics or other engineering sciences: there is a progression from hard to soft sciences (cf. Engelen[19]). With the help of those models one can explain self-organization (cf. Arthur[20]). Arthur is one of the prominent authors discussing the problem of self-organization. Another prominent scholar is Haken[21] who develops the concept of synergetics to describe the phenomena of self-organization systematically. Both scholars use the master equation as one of the central tools. Weidlich and Haag[22] introduce this concept in the modelling of social problems. Our approach leads also to a master equation, namely we are applying a Markov process in continuous time with discrete state space to describe the dynamics of the capital market.

The key role of our approach is played by the modelling of the transition rates. They are not held constant but influenced by the behaviour of the individuals and the market state. This is achieved by observing that all changes of the state of the market are caused by activities of the market participants. These activities are based on the preferences and information processing abilities of the individuals. This concept allows us to study the performance of the stock market as a self-organizing social phenomenon at the microlevel regarding all the peculiar rules and regulations of stock markets. It is also possible to rely on market participants behaving as real human beings and not on the artificial concept of theoretical agents (cf. Arthur[23]). It might also provide a solid basis for developing a description of phenomena at the macrolevel in terms of several equilibria or dynamics induced by self-organization (cf. Arthur *et al.*,[24] Durlauf[25]). Such phenomena may occur as underpricing or overpricing in issuing new equity rights.

In the following we describe the basic structure of the concept, illustrating how the individual processing of information disclosed by stock market performance is connected with the activities of individuals causing a change of the state of the market. The link is provided by the corresponding transition rates. The progress of the probability function of the market state is determined by the master equation, a linear differential equation. But, due to its vastly high dimension and the explicitly incorporated rules governing the activities at the stock market, it becomes so complex that it can be solved neither analytically nor numerically by discretization. Therefore, we pursue Monte Carlo simulations to gain an impression of the model's implications.

The organization of the paper is as follows. In Section 2, the structure of the model is briefly described, including a description of the state variables with their meaning, range and initial values (Section 2.1), the transitions and admissible actions together with their interrelations (Section 2.2), and the general structure of the transition rates (Section 2.3). In Section 3.1, the economic meaning of the state variables and the role they play are discussed. The determination of the admissible alternatives corresponding to market activities (offers, trades or cancels of valid offers) of the market participants is presented in Section 3.2. In Section 3.3, the computation of the quantity of shares to be traded or offered is described. The concrete modelling of the transition rates is defined in Section 3.4. In Section 4, the simulation technique and some typical results are demonstrated. The functioning of the process is analysed in Section 4.1. The simulation technique in accordance with the functioning of the process is described in Section 4.2. Some simulation results are prescribed in Section 4.3.

2. THE STRUCTURE OF THE MODEL

In this section, we briefly describe the mathematical structure of the model without the small detail. The detail and economic reasoning is the subject of Section 3.

We consider a stock market of continuous quotations of M single assets, labelled j ($j \in J = \{1, \dots, M\}$), constituted by N individuals, the agents, labelled i ($i \in I = \{1, \dots, N\}$). Formally, the market is assumed to be a *time homogeneous conservative Markov process* $(Z_t)_{t \geqslant 0}$ in *continuous time* with values in the discrete countable set Z, the *state space*. The process is completely determined by the set of possible transitions together with the corresponding transition rates when the initial distribution is given (for the relevant stochastic theory we refer to Feller[26]).

2.1. The state variables

The state vector $z \in Z$, called the market state, is a multidimensional vector of several variables capturing the information attached to the current market status including also variables bearing the relevant information of the past:

$$z = (p, p^{\text{of}}, q^{\text{of}}, m, p^{\text{tr}}, d^{\text{tr}}, x, y, \hat{p}, \hat{p}^{\text{ext}}) \in Z$$

$$Z = \mathbb{N}^J \times \mathbb{N}^{I \times J} \times \mathbb{Z}^{I \times J} \times \mathbb{Z}^J \times \mathbb{Z}^J \times \mathbb{Z}^J \times \mathbb{Z}_+^{I \times J} \times \mathbb{Z}_+^I \times \mathbb{N}^{I \times J} \times \mathbb{N}^{I \times J}$$

The set $\mathbb{Z}$ denotes the set of integers, $\mathbb{N}$ the set of positive integers and $\mathbb{Z}_+$ the set of non-negative integers.

The constituent variables are defined as follows:

$p \quad = (p_j)_{j \in J} \in \mathbb{N}^J$
$p_j \quad$ current price of asset j
$p^{\text{of}} \quad = (p_{ij}^{\text{of}})_{i \in I, j \in J} \in \mathbb{N}^{I \times J}$
$p_{ij}^{\text{of}} \quad$ price of a valid asset-j-offer of agent i
$q^{\text{of}} \quad = (q_{ij}^{\text{of}})_{i \in I, j \in J} \in \mathbb{Z}^{I \times J}$
$q_{ij}^{\text{of}} \quad$ quantity of a valid asset-j-offer of agent i
$m \quad = (m_j)_{j \in J} \in \mathbb{Z}^J$
$m_j \quad$ market power of asset j
$p^{\text{tr}} \quad = (p_j^{\text{tr}})_{j \in} \in \mathbb{Z}^J$
$p_j^{\text{tr}} \quad$ price trend of asset j
$d^{\text{tr}} \quad = (d_j^{\text{tr}})_{j \in J} \in \mathbb{Z}^J$
$d_j^{\text{tr}} \quad$ counter-trend of asset j
$x \quad = (x_{ij})_{i \in I, j \in J} \in \mathbb{Z}_+^{I \times J}$
$x_{ij} \quad$ number of shares of asset j held by agent i
$y \quad = (y_i)_{i \in I} \in \mathbb{Z}_+^I$
$y_i \quad$ amount of cash (invested in bonds) of agent i
$\hat{p} \quad = (\hat{p}_{ij})_{i \in I, j \in J} \in \mathbb{N}^{I \times J}$

$\hat{p}_{ij}$ price expectation of agent i for asset j
$\hat{p}^{\text{ext}}$ $= (\hat{p}_{ij}^{\text{ext}})_{i\in I, j\in J} \in \mathbb{N}^{I\times J}$
$\hat{p}_{ij}^{\text{ext}}$ fundamental value estimation of agent i for asset j

The prices, amounts of cash, price expectations and fundamental value estimates are measured as integer multiples of a given unit. The same holds for the number of shares held and the quantities offered. The values of all variables except the fundamental value estimates are changed by transitions during the *trading session* (the interval $0 \leqslant t \leqslant t_{\max}$).

A maximum of one offer can be simultaneously valid for each agent and each asset. The valid offers are stored in the fields $\boldsymbol{p}^{\text{of}}$ (prices) and $\boldsymbol{q}^{\text{of}}$ (quantities) with the sign convention that positive quantities correspond to asks (offers to buy) and negative to bids (offers to sell).

At the beginning of the process ($t = 0$), the state variables have the following initial values:

$p_j = p_j^0$:	predetermined quotation, e.g. the closing quotation of the previous stock exchange day
$q_{ij}^{\text{of}} = 0$:	no offers valid (the value of p_{ij}^{of} is arbitrary)
$m_j = 0$:	no demand-supply pressure
$p_j^{\text{tr}} = d_j^{\text{tr}} = 0$:	no trend
x_{ij} and y_i:	predetermined individual start portfolio, e.g. sampled from a predetermined distribution
$\hat{p}_{ij} = \hat{p}_{ij}^{\text{ext}}$:	initial price expectations and fundamental value estimations coincide
$\hat{p}_{ij}^{\text{ext}}$:	sampled from a predetermined distribution and held fixed during the trading session

2.2. State transitions and activities

Every transition of the market state is brought about by agent activities. That is, there is a one-to-one correspondence between possible transitions $z \mapsto z'$ and admissible actions of the agents.

At each instant t, agent i has the choice between the following activities.

1. *Value adjustment* $V = (i, j, \delta)$:
   He may adjust his price expectation $\hat{p}_{ij}$ of asset j by $\delta = \pm 1$ units. If $\hat{p}_{ij} = 1$, the down-adjustment $(i, j, -1)$ is not admissible. This guarantees the positivity of price expectations.
2. *Offer* $O = (i, j, p, q)$, where O is an *ask* A if $q > 0$ or a *bid* B *if* $q < 0$:
   He may offer to buy or sell, respectively, the quantity $|q| \in \mathbb{N}$ of shares of asset j at the price $p \in \mathbb{N}$. We use the sign convention that positive quantities correspond to an ask (or purchase), negative to a bid (or sale). The ask $A = (i, j, p, q)$ is only admissible if $y_i \geqslant pq$ and no bid of the agent for that asset is valid. Similarly, the bid $B = (i, j, p, q)$ is only admissible if $x_{ij} \geqslant |q|$ and no ask of the agent for the asset is valid.
3. *Trade* $T = (i, j, O, q)$:
   If an ask or a bid of agent i meets (i.e. has the same price as) the valid offer $O = (i', j, p, q')$—ask or bid—of an other agent i', then $|q| \leqslant |q'|$ shares of asset j are traded: either i buys q shares of asset j from i' when $q > 0 > q'$, or i sells $-q$ shares to i' when $q < 0 < q'$. Trades underlie the same admissibility conditions as asks or bids, i.e. $y_i \geqslant pq$ and $x_{ij} \geqslant -q$.
4. *Cancel* $C = (i, j)$:
   He may cancel the valid offer $O = (i, j, p, q')$ of himself. This activity is admissible when an offer would be admissible (an ask if O is a bid—a bid if O is an ask) if O were not valid.

Activities are subdivided into two groups, the unobservable value adjustments and the *market events*, namely the publically observable offers, trades and cancels. The set of simultaneously admissible market events is further restricted. At the same time for the same asset, an agent cannot be both a demander and a supplier, i.e. he may either offer an ask, agree to a bid (buy) or cancel a bid or he may offer a bid, agree to an ask (sell) or cancel an ask. This fundamental ask–bid decision depends on his current portfolio, his price expectation, the current price and the currently valid offers. There are also restrictions concerning the price p of offers and the acceptability of valid offers, and there is a rule concerning the computation of the traded or offered quantity. The details can be found in Section 3. The state transitions corresponding to the activities are given in Table I.

The transitions of p_j^{tr} and d_j^{tr} in the case of a trade are as follows. We distinguish two cases, namely a trend continuation when the price change $\Delta p = p - p_j$ and p_j^{tr} have the same sign, and a counter-movement when they have opposite signs. First, Δp is added to both p_j^{tr} and d_j^{tr} in any case. If, in the case of a continuation, a previous counter-trend is overcompensated, i.e. d_j^{tr} now has the same sign as the trend, then d_j^{tr} is reset to 0. If, in the case of counter-movement, $|d_j^{\mathrm{tr}}|$ exceeds the critical value $d_{\max} |p_j^{\mathrm{tr}}|$, then a trend reversal is indicated and p_j^{tr} is set to d_j^{tr} and d_j^{tr} is reset to 0.

Table I

Activity	Transition
$V = (i, j, \delta)$	$\hat{p}_{ij} \mapsto \hat{p}_{ij} + \delta$
$O = (i, j, p, q)$	$p_{ij}^{\mathrm{of}} \mapsto p, q_{ij}^{\mathrm{of}} \mapsto q, m_j \mapsto m_j + \mathrm{sign}(q)$
$T = (i, j, (i', j, p, q'), q)$	$p_j \mapsto p, q_{i'j}^{\mathrm{of}} \mapsto q' + q, m_j \mapsto m_j + \mathrm{sign}(q)$
	$x_{ij} \mapsto x_{ij} + q, y_i \mapsto y_i - pq, x_{i'j} \mapsto x_{i'j} - q, y_{i'} \mapsto y_{i'} + pq$
	p_j^{tr} and d_j^{tr}: see below
$C = (i, j)$	$q_{ij}^{\mathrm{of}} \mapsto 0$

2.3. Transition rates

The theory of stochastic processes tells us that the transition rate of a transition is proportional to the probability that this transition is the next to occur. As transitions correspond to chosen activities, transition rates are proportional to the choice probabilities. So, we can and do use the logit model which, according to the economic theory of choice, is consistent with the random utility maximization hypothesis (cf. McFadden[27] and Börsch-Supan[28]). That is, the transition rate $\lambda(z', z)$ of the transition $z \mapsto z'$ called forth by the activity $\mathscr{A}$ of agent i in state z is written in the form

$$\lambda(z', z) = W \cdot \exp\{\Phi(\mathscr{A}, z)\} \tag{2.1}$$

where the constant W depends only on the type of activity, value adjustment or market event, i.e.

$$W = \begin{cases} W_V & \mathscr{A} \text{ is a value adjustment} \\ W_E & \mathscr{A} \text{ is a market event} \end{cases} \tag{2.2}$$

The term $\Phi(\mathscr{A}, z)$ corresponds to the deterministic part of the relative utility of the alternative chosen (Börsch-Supan[28]). We call it the *utility potential*. The utility potentials for different activities depend on different sets of state variables. These dependencies on the one hand and the effects of the transitions produced by the activities on the state variables on the other hand

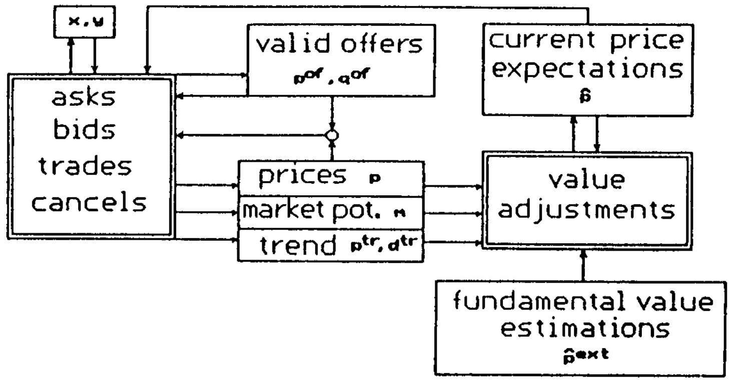

Figure 1. The interdependence of state and activity

are depicted in Figure 1. Activities appear in double framed boxes, state variables in single frames.

In our model, there are only finitely many admissible transitions in each state so that the process fulfils the condition of being conservative:

$$\sum_{z' \neq z} \lambda(z', z) = -\lambda(z, z) < \infty \tag{2.3}$$

The evolution of the market state z is then represented by the time evolution of the probability function of z, $P_t(z)$, obeying the so called *master equation* (cf. Haken,[21] Weidlich and Haag,[22] Landes and Loistl[29]):

$$\frac{\mathrm{d}}{\mathrm{d}t} P_t(z) = \sum_{z' \neq z} \lambda(z, z') P_t(z') - \sum_{z' \neq z} \lambda(z', z) P_t(z) \tag{2.4}$$

3. THE DETAILS

3.1. Detailed description of the state variables

At the beginning of the trading session, each agent i ($i \in I$) possesses a subjective estimation $\hat{p}_{ij}^{\text{ext}}$ of the fundamental value of each asset j ($j \in J$), which is based on the intrinsic value and the external information concerning the stock.

The fundamental value has to be understood as the present value of future annual earnings described as random variables. The factors influencing the fundamental situation of a corporation fluctuate much less frequently than actual market prices. We assume, therefore, that there are no external effects causing a change in the fundamental situation in the short run. Consequently, the estimates $\hat{p}_{ij}^{\text{ext}}$ are assumed to be predetermined by external factors and fixed during the trading session.

For each share, these subjective estimates may differ and, thus, constitute a distribution, which is to be interpreted as the market estimation of the share's intrinsic value. That is, we do not assume that the market is able to compress the intrinsic value into a unique scalar figure. Accordingly, we do not believe that there are rational expectations at the market level in the sense that all agents have the same expectations of the intrinsic value which are also equal to the realized equilibrium market price.

We do assume rational expectations at the individual level in the sense that every agent believes that *his* expectation will be realized at the end of the planning period. This assumption is the basis of the agent's trading decision rules: if the rate of return of investment in the asset based on his expectation is above the reference rate (bond rate) r, he wants to buy, if it is below, he wants to sell. This basic rule has to be modified for an operational implementation as explained below (cf. Section 3.2).

The individual expectations just mentioned are the current *price expectations* $\hat{p}_{ij}$ of the agents. Originating in $\hat{p}_{ij}^{\text{ext}}$ at $t=0$, they undergo a permanent revision in the course of time due to the observed market scenario. This accentuates the signal effect of stock market performance. The individual adjustment to market information structures, the internal value correction, is expressed as the difference $\Delta\hat{p}_{ij}^{\text{int}}$ between the current price expectation and the fundamental value estimation:

$$\hat{p}_{ij} = \hat{p}_{ij}^{\text{ext}} + \Delta\hat{p}_{ij}^{\text{int}}$$

The internal value correction is the result of learning from the information gained from the observable process of market activities. In the valuation of assets, the agents account both for the market climate independent externalities and the actual stock exchange performance.

To allow for strategic portfolio decisions, we explicitly include the current portfolio of each agent in the model: the field $\boldsymbol{y}$ of capital and $\boldsymbol{x}$ of share holdings. The initial portfolios for each agent are sampled from a predetermined distribution: the total wealth w_i, sampled from a Pareto distribution, is subdivided randomly into cash y_i and a predetermined number k_{opt} of investments x_{ij} in different shares:

$$w_i = y_i + \sum_{\nu=1}^{k_{\text{opt}}} x_{ij_\nu} p_{j_\nu}^0$$

The market events (offers, trades and cancels) are observed by all agents, they constitute the *explicitly observable* market process. They influence the internal factors, which account for the market performance. These are represented as follows.

1. The vector $\boldsymbol{p}$ of current stock prices consists of the quoted prices of the last trades of each asset except when there is no such trade, in which case p_j equals its predetermined initial value p_j^0.
2. The vector $\boldsymbol{m}$ of market powers of the assets measures the prevailing demand–supply pressure with regard to each asset. Purchase orders have a rising effect while sales orders induce a decrease of the value of m_j. In our present implementation, m_j is the difference between the numbers of demand and supply type actions that have occurred up to the present instant. It is initialized by 0.
3. The vector $\boldsymbol{p}^{\text{tr}}$ of price trends captures the current price tendencies. The current value of p_j^{tr} is the difference between the current price and the last trend reversal price of asset j. Initially, it is set to 0.
4. The vector $\boldsymbol{d}^{\text{tr}}$ of counter-trends (initially 0) is the vector of counter-movements of the price against the trend. It serves to determine a trend reversal of the price of asset j: if the counter-trend exceeds the predetermined critical part $d_{\max}$ of the magnitude $|p_j^{\text{tr}}|$ of the trend, then a trend reversal is indicated. In this case, the last trend reversal price becomes $p_j - d_j^{\text{tr}}$ and, hence, the trend becomes the counter-trend. The updating of $\boldsymbol{p}^{\text{tr}}$ and $\boldsymbol{d}^{\text{tr}}$ is described in Section 2.2.
5. The fields $\boldsymbol{q}^{\text{of}}$ and $\boldsymbol{p}^{\text{of}}$ contain the information concerning the set of valid offers:

$$q_{ij}^{\text{of}} \begin{cases} <0: & \text{the bid } B=(i,j,p_{ij}^{\text{of}},q_{ij}^{\text{of}}) \text{ is valid} \\ =0: & \text{there is no valid offer of agent } i \text{ for asset } j \\ >0: & \text{the ask } A=(i,j,p_{ij}^{\text{of}},q_{ij}^{\text{of}}) \text{ is valid} \end{cases} \tag{3.1}$$

Initially there are no offers valid, i.e. $q_{ij}^{\text{of}}=0$ at $t=0$. We define the sets $P_j^{\text{bid}} = \{p_{ij}^{\text{of}} \mid q_{ij}^{\text{of}} < 0\}$ and $P_j^{\text{ask}} = \{p_{ij}^{\text{of}} \mid q_{ij}^{\text{of}} > 0\}$ of valid market bid and ask prices, respectively. We then define the *maximal ask price* and *minimal bid price*

$$p_j^{\text{ask}} = \max P_j^{\text{ask}}, \quad p_j^{\text{bid}} = \min P_j^{\text{bid}} \tag{3.2}$$

if the corresponding set is not empty. Formally, we put $p_j^{\text{ask}} = 0$ if there is no valid ask and $p_j^{\text{bid}} = \infty$ (or a vastly high price assumed never to be reached) if there is no valid bid. The vector $p^{\text{cut}} = (p_j^{\text{cut}})_{j \in J}$ of current *market cut prices* is derived from p and the sets of valid offer prices P_j^{bid} and P_j^{ask}:

$$p_j^{\text{cut}} = \max\{p_j^{\text{ask}} + 0{\cdot}5, \min(p_j, p_j^{\text{bid}} - 0{\cdot}5)\} = \begin{cases} p_j^{\text{ask}} + 0{\cdot}5 & \text{if } p_j \leqslant p_j^{\text{ask}} \\ p_j^{\text{bid}} - 0{\cdot}5 & \text{if } p_j \geqslant p_j^{\text{bid}} \\ p_j & \text{otherwise} \end{cases} \tag{3.3}$$

The cut price cuts the range of prices for asset j into the *market demand interval* $(0, p_j^{\text{cut}}]$ of prices at which the market accepts a bid and the *market supply interval* $[p_j^{\text{cut}}, \infty)$ of prices at which the market accepts an ask. The cut price is to be understood as the reference price for the fundamental ask–bid decision.

These variables reflect the information on the current capital market performance explicitly perceptible for each agent. The vectors p, m and p^{tr} in conjunction with the individual expectations of the fundamental values establish the driving forces of the unobservable process of individual value adjustments by which the evolution of the field $\hat{p}$ of current price expectations is driven. The price expectations in turn govern in conjunction with p^{cut}, p^{of} and q^{of} the explicit process of market events (cf. Figure 1).

3.2. The fundamental ask–bid decision and the set of admissible prices

For the discussion in this section we fix an agent i and an asset j. The decision of whether the agent appears as a demander or a supplier of the asset rests on the question whether there are prices which are acceptable for both the agent and the market. These prices then form the set of admissible prices. Each such price determines an admissible market event which, except for a cancel, is accompanied with a definite quantity q, the positive or negative volume of the (intended) transaction, which depends deterministically on that price as well as on the price expectation $\hat{p}_{ij}$ and the portfolio of the agent. The determination of these quantities is described in Section 3.3.

The agent decides to invest or disinvest at a price p on the basis of the expected one-period rate of return $\hat{r}_{ij}$ corresponding to his price expectation $\hat{p}_{ij}$, which is just the end-of-period price he expects:

$$p(1 + \hat{r}_{ij}) = \hat{p}_{ij} \tag{3.4}$$

We assume that the agent has the opportunity to invest idle capital in bonds with a market rate of return of r. He therefore wants to sell at all prices for which his expected rate of return is below r and to buy at all prices for which $\hat{r}_{ij}$ is above r:

$$\hat{r}_{ij} < r \Rightarrow \text{bid}; \quad \hat{r}_{ij} > r \Rightarrow \text{ask} \tag{3.5}$$

Consequently, the whole range of prices is cut by the *individual cut price*

$$p_{ij}^{\text{cut}} = \frac{\hat{p}_{ij}}{1 + r} \tag{3.6}$$

into the set $(0, p_{ij}^{\text{cut}})$ of individually (for i) acceptable ask prices and the set $(p_{ij}^{\text{cut}}, \infty)$ of individually acceptable bid prices:

$$p > p_{ij}^{\text{cut}} \Rightarrow \text{bid}; \quad p < p_{ij}^{\text{cut}} \Rightarrow \text{ask} \tag{3.7}$$

On the other hand, as described in Section 3.1, the price range is also cut at the market side into the set $(0, p_j^{\text{cut}}]$ of prices at which the market accepts a bid and the set $[p_j^{\text{cut}}, \infty)$ of prices at which the market accepts an ask.

Admissible prices must be acceptable for both the agent and the market. As prices are positive integers, admissible ask prices belong to the set $[p_j^{\text{cut}}, p_{ij}^{\text{cut}}) \cap \mathbb{N}$ and admissible bid prices to the set $(p_{ij}^{\text{cut}}, p_j^{\text{cut}}] \cap \mathbb{N}$. It is obvious that these sets cannot both be simultaneously non-empty. There are admissible ask prices if the *maximal individual ask price*

$$p_{ij}^{\text{ask}} = \max\{p \in \mathbb{N} \mid p < p_{ij}^{\text{cut}}\} \tag{3.8}$$

is not smaller than p_j^{cut} while there are admissible bid prices if the *minimal individual bid price*

$$p_{ij}^{\text{bid}} = \min\{p \in \mathbb{N} \mid p > p_{ij}^{\text{cut}}\} \tag{3.9}$$

is not greater than p_j^{cut}. This determines the *fundamental ask–bid decision*:

$$p_{ij}^{\text{ask}} \geqslant p_j^{\text{cut}} \Rightarrow \text{ask}; \quad p_{ij}^{\text{bid}} \leqslant p_j^{\text{cut}} \Rightarrow \text{bid} \tag{3.10}$$

In the former case, we want to say that the decision is to ask, in the latter to bid. It may happen that the decision is neither to ask nor to bid. In that case, no market event actions (i.e. only value adjustments) are admissible for the agent with regard to the asset. If, otherwise, the decision is to ask (bid, respectively) and there is a valid bid (ask, respectively) of i for j, i.e. if $q_{ij}^{\text{of}} < 0$ (>0, respectively), then the only admissible market event action of the agent for the asset is the cancelling $C = (i, j)$ of that bid (ask, respectively). Let us now assume that this situation is not true.

The central reasoning is that, firstly, each trade of a valid offer whose price is in the range of corresponding admissible prices is admissible and that, secondly, offers at prices in the range of admissible prices are admissible if and only if there is no admissible trade which is more favourable for the agent with respect to the expected rate of return. This leads to the following rules.

We consider first the case that the decision is to ask, i.e. that

$$P_{ij}^{\text{ask}} = [p_j^{\text{cut}}, p_{ij}^{\text{ask}}] \cap \mathbb{N} \neq \emptyset$$

For each price $p_{i'j}^{\text{of}}$ of a valid bid in $P_j^{\text{bid}} \cap P_{ij}^{\text{ask}}$, the purchase $T = (i, j, (i', j, p_{i'j}^{\text{of}}, q_{i'j}^{\text{of}}), q)$ of the definite quantity (as mentioned above) of $q, 0 < q \leqslant |q_{ij}^{\text{of}}|$, shares of the asset to agent i' at that price is admissible. For all prices p in P_{ij}^{ask} which are not greater than or equal to the price of any valid bid, i.e.

$$p_j^{\text{cut}} \leqslant p \leqslant \min\{p_{ij}^{\text{ask}}, p_j^{\text{bid}} - 1\}$$

the corresponding admissible action is an ask $A = (i, j, p, q)$, $q > 0$. If $p_j^{\text{bid}} - 1 < p_j^{\text{cut}}$, no ask is admissible.

It remains to discuss the case that the decision is to bid, i.e. that

$$P_{ij}^{\text{bid}} = [p_{ij}^{\text{bid}}, p_j^{\text{cut}}] \cap \mathbb{N} \neq \emptyset$$

The situation is completely analogous to that in the ask case. The sale of $|q|$, $0 < -q \leqslant q_{i'j}^{\text{of}}$, shares of the asset at the price $p_{i'j}^{\text{of}}$ to agent i', i.e. $T = (i, j, (i', j, p_{i'j}^{\text{of}}, q_{i'j}^{\text{of}}), q), q < 0$, is admissible if $p_{i'j}^{\text{of}} \in P_{ij}^{\text{bid}} \cap P_j^{\text{ask}}$ while the bid $B = (i, j, p, q), q < 0$, is admissible for each price $p \in P_j^{\text{bid}}$ which is not smaller than or equal to the price of a valid ask, i.e.

$$p_j^{\text{cut}} \geqslant p \geqslant \max\{p_{ij}^{\text{bid}}, p_j^{\text{ask}} + 1\}$$

No bid is admissible if $p_j^{\text{ask}} + 1 > p_j^{\text{cut}}$.

3.3. The determination of the quantity

The quantity q associated with an admissible offer or trade of asset j of agent i is that signed (positive for ask/purchase, negative for bid/sale) volume of the (intended) transaction which is optimal with respect to a certain individual objective function under certain restrictions. There are the budget restrictions of agent i not to invest for credit, i.e. $y_i \geqslant pq$, and not to make future sales, i.e. $x_{ij} \geqslant -q$. In the case of a trade, the volume is also not allowed to exceed the volume $|q_{i'j}^{\text{of}}|$ of the corresponding valid offer. In case of a sale, the no-credit restriction of the trading partner, agent i', has also to be kept.

The individual objective function is the *risk-adjusted portfolio return*, i.e. the expected rate of return of the agent's 'intended' portfolio, which results from performing the intended transaction, adjusted by a risk measure. In our model we do not use the traditional risk adjustment proportional to the covariance or correlation between the share and a market portfolio. We rather assume that risk is measured as the deviation of the (intended) portfolio from a certain predetermined target portfolio, which is given by the aspired distribution $(s_{ij}^{\text{ext}})_{j \in J \cup \{0\}}$ of the portions of the portfolio positions on the total wealth w_i of the portfolio. The index $j = 0$ corresponds to capital (invested in bonds).

Our risk adjustment term corresponding to a portfolio $y_i, (x_{ij})_{j \in J}$ at prices $(p_j)_{j \in J}$ is proportional to the sum S_i of the squared differences of the portfolio portions

$$s_{i0} = \frac{y_i}{w_i}, \quad s_{ij} = \frac{x_{ij} p_j}{w_i}, \quad j \in J \tag{3.11}$$

on the portfolio wealth

$$w_i = y_i + \sum_{j \in J} x_{ij} p_j \tag{3.12}$$

and the target portfolio portions s_{ij}^{ext}:

$$S_i = \sum_{j=0}^{M} (s_{ij} - s_{ij}^{\text{ext}})^2 \tag{3.13}$$

The risk-adjusted rate of return of the portfolio is then the term

$$R_i = \frac{\hat{w}_i}{w_i} - 1 - \tfrac{1}{2} \xi_i^{\text{risk}} S_i; \quad \hat{w}_i = y_i(1 + r) + \sum_{j \in J} x_{ij} \hat{p}_{ij} \tag{3.14}$$

The parameter ξ_i^{risk} measures the risk aversion of agent i. The objective function is then the term R_i evaluated at the intended portfolio

$$y_i - pq; \quad x_{ij} + q; \quad (x_{ij'})_{j' \neq j}$$

with current prices for the assets $j' \neq j$ and the intended price p for the asset j under consideration. It becomes a function of q depending on the fixed price p. Only the terms $\hat{w}_i$, s_{i0} and s_{ij} depend on q but not w_i. Denoting the relative wealth of the intended investment by $s = pq/w_i$, we obtain

$$\hat{w}_i = y_i(1 + r) + \sum_{j \in J} x_{ij} \hat{p}_{ij} + q(\hat{p}_{ij} - p(1 + r)) = \hat{w}_i^0 + w_i(\hat{r}_{ij} - r)s$$

$$s_{i0} = (y_i - pq)/w_i = s_{i0}^0 - s \quad \text{and} \quad s_{ij} = (x_{ij} + q)p/w_i = s_{ij}^0 + s$$

as functions of s (the superscript 0 corresponds to $q = 0$). The objective is then to maximize

$$R_i(s) = \frac{\hat{w}_i^0}{w_i} - 1 + (\hat{r}_{ij} - r)s - \tfrac{1}{2} \xi_i^{\text{risk}} \left((s_{i0}^0 - s_{i0}^{\text{ext}} - s)^2 + (s_{ij}^0 - s_{ij}^{\text{ext}} + s)^2 + \sum_{j' \in J \setminus \{j\}} (s_{ij'} - s_{ij'}^{\text{ext}})^2 \right)$$

After a little straightforward algebra, we obtain

$$R_i(s) = R_i^* - \xi_i^{\text{risk}} (s - s^*)^2 \tag{3.15}$$

where $R_i^* = R_i(s^*)$ is the global maximum of R_i (whose value is not needed further) attained at

$$s^* = \frac{1}{2}\left[(s_{i0}^0 - s_{i0}^{\text{ext}}) - (s_{ij}^0 - s_{ij}^{\text{ext}}) + \frac{\hat{r}_{ij} - r}{\xi_i^{\text{risk}}}\right] \tag{3.16}$$

The corresponding quantity is then given by

$$q^* = \frac{w_i}{2p}\left[(s_{i0}^0 - s_{i0}^{\text{ext}}) - (s_{ij}^0 - s_{ij}^{\text{ext}}) + \frac{\hat{r}_{ij} - r}{\xi_i^{\text{risk}}}\right] \tag{3.17}$$

What is really needed is not the globally optimal quantity but the integer valued quantity which maximizes R_i in a given interval $[q_1, q_2], q_1, q_2 \in \mathbb{N}$. As R_i is a quadratic function the solution is to choose that integer $q \in [q_1, q_2]$ which is nearest to q^*:

$$q = \begin{cases} q_1 & \text{if } q^* \leqslant q_1 \\ \text{round}(q^*) & \text{if } q_1 < q^* < q_2 \\ q_2 & \text{if } q^* \geqslant q_2 \end{cases} \tag{3.18}$$

The just mentioned interval of admissible quantities is to choose so that the admissibility conditions for the corresponding activity are met. These are, in particular:

ask: positive quantity and no credit allowed:

$$q_1 = 1, \quad q_2 = \max\{n \in \mathbb{N} \mid n \leqslant y_i/p\}$$

bid: negative quantity and no future sales allowed:

$$q_1 = -x_{ij}, \quad q_2 = -1$$

purchase: positive quantity, no credit allowed and trading volume bounded by the supplied quantity of the corresponding valid bid of agent i':

$$q_1 = 1, \quad q_2 = \min(|q_{i'j}^{\text{of}}|, \max\{n \in \mathbb{N} \mid n \leqslant y_i/p\})$$

sale: negative quantity, no future sales, no credit allowed (for the trading partner i') and trading volume bounded by the asked quantity of the corresponding valid ask of agent i':

$$q_1 = -\min(x_{ij}, q_{i'j}^{\text{of}}, \max\{n \in \mathbb{N} \mid n \leqslant y_{i'}/p\}), \quad q_2 = -1$$

In our modelling, we have assumed that the agents aspire to a target portfolio of $k_{\text{opt}} + 1$ equal portions. That is, for each agent i there is a subset J_i of k_{opt} elements of J such that

$$s_{i0}^{\text{ext}} = s_{ij}^{\text{ext}} = \frac{1}{k_{\text{opt}} + 1}, \; j \in J_i; \quad s_{ij}^{\text{ext}} = 0, \; j \in J \setminus J_i$$

The set J_i is not determined explicitly but assumed to be variable. The asset under consideration is assumed to belong to J_i if and only if the activity is an ask or purchase or the number k_i of different assets held by the agent (the size of the set of all $j \in J$ with positive x_{ij}) is not greater than k_{opt}. This yields:

$$q^* = \frac{1}{2}\left[\frac{y_i}{p} - x_{ij} + \frac{w_i}{p}\left(\frac{\hat{r}_{ij} - r}{\xi_i^{\text{risk}}} - \Delta\right)\right], \quad \Delta = \begin{cases} 0 & \text{ask/purchase or } k_i \leqslant k_{\text{opt}} \\ \dfrac{1}{k_{\text{opt}} + 1} & \text{bid/sale and } k_i > k_{\text{opt}} \end{cases} \tag{3.19}$$

3.4. The transition rates in detail

In this section we want to present our modelling of the transition rates. As mentioned in Section 2.3, we use the logit model to determine the structure of the transition rate $\lambda(z', z)$

of the transition $z \mapsto z'$ (cf. (2.1)): transition rates are proportional to an exponential function whose exponent is the utility potential of the activity causing the transition.

By adequate formulation of the transition rates, some activities may be favoured while others are suppressed according to the market and the individual situation expressed by the value of z. Our effort lies in an attempt to capture all relevant facts concerning the capital market—the institutional regulations as well as the pattern of individual behaviour—and incorporate them appropriately into the model. This is achieved by setting up restrictions which manifest themselves in the admissibility of activities (cf. Sections 2.2 and 3.2) and in defining suitable utility potentials, which take account of preferences of individual behaviour in a formal manner.

Our approach is based decidedly on the differentiation between value adjustments which are not known to the public and market events which are openly observable. The dynamics of these different types of activity are different, which led us to choose different proportionality constants W in formula (2.1), the *reagibility parameters* W_V and W_E (cf. (2.2)) describing the speed of reacting, for the two types. Also the functional form of the utility potentials and the variables on which they depend differ. According to formula (2.1) it suffices to describe the utility potentials.

The utility potentials for each activity are the result of the superposition of a number of relevant factors. This is achieved by the representation of the utility potentials as the weighted sum of partial utility potentials, as we will see below. We call the weights the *coupling parameters*. We assume that the set I of agents is partitioned into a small number g of groups $I_1, \ldots, I_g$ which are homogeneous with respect to the coupling parameters, i.e. each coupling parameter may have g different values, one for each group.

We start our description with the utility potential $\Phi_V(i, j, \delta, z)$ for a **value adjustment** $\mathbf{V} = (i, j, \delta)$. It is decomposed into four parts and signed with the sign of δ because influences which favour upwards adjustments disfavour at the same time downwards adjustments:

$$\Phi_V(i, j, \delta, z) = \delta[\eta_l^{\text{ext}} \Phi_{ij}^{\text{ext}}(z) + \eta_l^{\text{inf}} \Phi_{ij}^{\text{inf}}(z) + \eta_l^{\text{pot}} \Phi_j^{\text{pot}}(z) + \eta_l^{\text{tr}} \Phi_j^{\text{tr}}(z)], \quad i \in I_l \tag{3.20}$$

The four partial potentials are:

1. The external potential

$$\Phi_{ij}^{\text{ext}}(z) = \frac{|\hat{p}_{ij}^{\text{ext}} - \hat{p}_{ij}|^{\varkappa} \operatorname{sign}(\hat{p}_{ij}^{\text{ext}} - \hat{p}_{ij})}{\hat{p}_{ij}^{\text{ext}}} \tag{3.21}$$

We assume that the intrinsic value estimation of the agent remains always the basis for the corresponding price expectation. This means that there is a stimulus to reattract $\hat{p}_{ij}$ to $\hat{p}_{ij}^{\text{ext}}$, which is the stronger the more these two terms diverge from each other. The parameter $\varkappa > 0$ is the degree of the external force. It is near one (usually $\geqslant 1$) where $\varkappa = 1$ corresponds to the linear case.

2. The market information potential

$$\Phi_{ij}^{\text{inf}}(z) = \frac{p_j - \hat{p}_{ij}}{p_j} \tag{3.22}$$

High prices increase the readiness for an upwards adjustment, low prices for a downwards adjustment. The market information potential favours adjustments which decrease the distance between p_j and $\hat{p}_{ij}$.

3. The market power potential

$$\Phi_j^{\text{pot}}(z) = \rho \frac{m_j}{p_j} + (1 - \rho) \frac{1}{M} \sum_{j' \in J} \frac{m_{j'}}{p_{j'}} \tag{3.23}$$

accounts for the influence of the prevailing demand–supply pressure, expressed as the vector $\boldsymbol{m}$, on the adjustment of price expectations. A positive climate at the stock exchange, represented by a demand pressure (positive market power), favours upwards adjustments, a negative climate (supply pressure) downwards adjustments. The total potential is decomposed into a part which represents the relative market power of the asset under consideration and a part which measures the average relative power of the whole market. The parameter ρ weights the influence of the single asset against that of the whole market.

4. The price trend potential

$$\Phi_j^{\mathrm{tr}}(z) = \rho \frac{p_j^{\mathrm{tr}}}{p_j - p_j^{\mathrm{tr}}} + (1-\rho)\frac{1}{M}\sum_{j' \in J} \frac{p_{j'}^{\mathrm{tr}}}{p_{j'} - p_{j'}^{\mathrm{tr}}} \tag{3.24}$$

The same argumentation as for the market power potential is valid also for the price trends. The trends come in relative to the last trend reversal price $p_j - p_j^{\mathrm{tr}}$. The same weighting parameter ρ is used.

The coupling parameters η_l^{ext}, η_l^{inf}, η_l^{pot} and η_l^{tr} measure the relative influence of the corresponding potential for an agent in the respective group I_l.

We now describe the utility potentials for **market events**. In our modelling, the quantity q is not part of the basic decision of which activity at which price is to be chosen, so that the utility potential does not depend on q. For the purpose of defining the utility potential, it suffices to characterize a market event by the quadrupel $E = (i, j, p, a)$, where i is the agent, j the asset and p the price which, in case of a cancel, is defined by p_j^{cut}, and $a \in \{-3, -2, -1, 1, 2, 3\}$ characterizes the activity:

$$|a| = \begin{cases} 1 & \text{offer} \\ 2 & \text{trade} \\ 3 & \text{cancel} \end{cases} \qquad \mathrm{sign}(a) = \begin{cases} +1 & \text{ask, purchase, cancel of bid} \\ -1 & \text{bid, sale, cancel of ask} \end{cases}$$

For each market event $E = (i, j, p, a)$, we distinguish three partial potentials:

$$\Phi(E, z) = \xi_l^{\mathrm{trade}} \Phi^{\mathrm{trade}}(a) + \xi_l^{\mathrm{ip}} \Phi_{ij}^{\mathrm{ip}}(a, p, z) + \xi_l^{\mathrm{real}} \Phi_{ij}^{\mathrm{real}}(a, p, z), \quad i \in I_l \tag{3.25}$$

The coupling parameters of agents of group I_l for the partial potentials are ξ_l^{real}, ξ_l^{ip} and ξ_l^{trade}.

The trade potential

$$\Phi^{\mathrm{trade}}(a) = \begin{cases} 0 & \text{if } |a| = 1 \\ 1 & \text{if } |a| > 1 \end{cases} \tag{3.26}$$

increases the probability of a trade or cancel as compared with an offer because, as we assume, the agents prefer a sure trade now to an uncertain trade in the future. A cancel is treated here like a trade with himself.

The other two potentials must be analysed together. They are designed to treat the ask-type ($a > 0$) and bid-type ($a < 0$) activities symmetrically. The range of admissible prices is an interval $[p', p'']$ whose extreme points p' and p'' are the prices p_j^{cut} and p_{ij}^{cut} (cf. Section 3.2):

$$[p', p''] = [p_j^{\mathrm{cut}}, p_{ij}^{\mathrm{cut}}] \text{ if } a > 0; \quad [p', p''] = [p_{ij}^{\mathrm{cut}}, p_j^{\mathrm{cut}}] \text{ if } a < 0$$

Considered as a function of p, the *realization potential* $\Phi_{ij}^{\mathrm{real}}$ measures how likely it is that the market accepts the price p, while the *individual preference potential* Φ_{ij}^{ip} expresses the individual preference of the agent to realize high returns. Thus, the realization potential is increasing (decreasing) and the individual preference potential is decreasing (increasing) in p if $a > 0$ ($a < 0$). The strength of both is the relative positive difference of the price p and the corresponding extreme price, p_j^{cut} for the realization potential and p_{ij}^{cut} for the individual preference potential. The difference is to be taken relative to the greater of the two prices which are subtracted. We obtain:

$$\Phi_{ij}^{\mathrm{real}}(a, p, z) = \begin{cases} (p - p_j^{\mathrm{cut}})/p & a > 0 \\ (p_j^{\mathrm{cut}} - p)/p_j^{\mathrm{cut}} & a < 0 \end{cases} \tag{3.27}$$

$$\Phi_{ij}^{\mathrm{ip}}(a, p, z) = \begin{cases} (p_{ij}^{\mathrm{cut}} - p)/p_{ij}^{\mathrm{cut}} & a > 0 \\ (p - p_{ij}^{\mathrm{cut}})/p & a < 0 \end{cases} \tag{3.28}$$

By this modelling, both potentials vary from 0 to $(p'' - p')/p''$, the extrema being attained at the extreme points of the interval. The increasing potential is strictly concave and the decreasing is linear in p. The additive weighted (by the coupling parameters) superposition of both is a strictly concave function of price p, which has a maximum in the interval (p', p'') at $\sqrt{\gamma p' p''}$ if and only if the quotient γ of the coupling parameter of the increasing and that of the decreasing potential is in the interval $(p'/p'', p''/p'')$.

4. SIMULATION

The time evolution of the market state can be described by its single time probability distributions which obey the master equation. Although this is a linear equation, it is practically impossible to find an explicit theoretical solution as well as a direct numerical solution because of the vastly high dimension of the master equation.

A way out of this dilemma is a solution by Monte Carlo simulation. This also helps us to understand the dependence of the structure of the solutions on the set of parameter values.

The simulations are designed in accordance with the functioning of the process, which will be described first.

4.1. The functioning of the process

As is known from the theory of jump processes, the waiting time between two transitions, i.e. the time the process stays in state z until it jumps, is exponentially distributed with parameter $\lambda(z)$, the sum of the transition rates of all possible transitions. The probability that the state then jumps to z' is just $\lambda(z', z)/\lambda(z)$.

This structure can already be used to simulate the process. But this would not reflect the underlying economic structure. The above mathematical structure is exactly the same when the process development is described as follows.

Let t_0 be the start time, i.e. the time at which a market event has occurred, and let z_0 be the state at t_0. Let $T(i, j, \mathscr{A})$ be independent exponentially distributed random reaction times with parameter $\lambda(z(i, j, \mathscr{A}), z_0)$, the transition rate of the transition from z_0 to $z(i, j, \mathscr{A})$ forced by the admissible action $\mathscr{A}$. For each i and j, let $T(i, j)$ be the minimum over all $T(i, j, \mathscr{A})$ when $\mathscr{A}$ runs through the set of all admissible actions for agent i and asset j in state z_0, and let $\mathscr{A}_{ij}$ be the action at which the minimum is attained. This can be interpreted so that agent i decides to carry out action $\mathscr{A}_{ij}$ at time $t_0 + T(i, j)$. Further, let T^* be the minimum of all $T(i, j)$ attained at (i^*, j^*). Then the next transition is produced by the action $\mathscr{A}_{i^*j^*}$ of agent i^* for asset j^*. We must distinguish two cases.

The first case is that this action is a market event. Then it is observed by all agents, and their decisions based on the information given by z_0 lose their validity. The process starts anew in the same situation as at t_0 but at the new state $z(i^*, j^*, \mathscr{A}_{i^*, j^*})$.

The second case is that this action is a value adjustment. Then it is not observed by the other agents. The set of admissible actions and their transition rates of all pairs (i, j) except (i^*, j^*)

do not change. The remaining reaction times $T(i, j) - T^*$ for all these pairs and a new independent reaction time for (i^*, j^*) obtained by the same method as at t_0 now compete for the shortest reaction time to elect the action which is carried out next and produces the next transition.

Again, this action could be a market event (Case 1 applies) or a value adjustment (Case 2 applies), and so on. One can show that such a progress exactly defines the same distribution of waiting times and next-jump probabilities as those theoretically demanded (cf. Landes *et al.*, Reference 30, p. 100). But this description is much more well-adapted to the real economic situation.

4.2. The simulation technique

As already mentioned, the simulation technique follows the running scheme described above. At the beginning, the initial values are set as described in Section 3.1. In particular, the internal value estimates are sampled from a given distribution which, in the present implementation, is a mixture of truncated normal distributions.

The following algorithm starting at Step 1 with $t = 0$ describes the simulation.

Step 1. For all pairs (i, j), all admissible actions and the corresponding transition rates are computed. For all these actions, uniformly distributed pseudo random numbers on $(0, 1]$ are generated. Their negative logarithms divided by the corresponding transition rate then yield simulated realizations of the corresponding reaction times. Separately for each pair (i, j), the minima t_{ij} of these reaction times are computed. Let $\mathscr{A}_{ij}$ be the actions at which these minima are attained. Proceed with Step 2.

Step 2. The minimum t^* over all t_{ij} is computed. Let (i^*, j^*) be the corresponding pair. The reaction time t^* is added to t. If the resulting t is greater than or equal to the maximal time t_{max}, then stop. Otherwise, if $\mathscr{A}^* = \mathscr{A}_{i^*j^*}$ is a market event then proceed with Step 4 else with Step 3.

Step 3. The transition corresponding to the value adjustment $\mathscr{A}^*$ is performed. For all pairs $(i, j) \neq (i^*, j^*)$ the reaction times t_{ij} are replaced by $t_{ij} - t^*$. For the pair (i^*, j^*) new reaction times for the corresponding admissible actions, their minimum t_{i^*,j^*} and the corresponding action $\mathscr{A}_{i^*j^*}$ are computed as in Step 1. Proceed with Step 2.

Step 4. The transition corresponding to the market event $\mathscr{A}^*$ is performed. Proceed with Step 1.

At equally spaced times, the current states of the simulated paths are stored. For each parameter set, a number of paths are generated. The parameters are as follows.

1. The coupling parameters $\eta_l^{ext}, \eta_l^{inf}, \eta_l^{pot}, \eta_l^{tr}$ of agents of group I_l, $l = 1, \ldots, g$, of the partial potentials of the utility potential for value adjustments.
2. The coupling parameters ξ_l^{real}, ξ_l^{ip}, ξ_l^{trade} of agents of group I_l, $l = 1, \ldots, g$, of the partial potentials of the utility potential for market events.
3. The risk aversion parameters ξ_i^{risk} which are also group specific, i.e. equal for members i of the same group I_l.
4. The reagibility parameters W_V and W_E of the adjustment and market event process.
5. The specific impact ρ of asset j on the potentials Φ_j^{pot} and Φ_j^{tr}.
6. The degree $\varkappa$ of the external potential.
7. The market rate of return r.
8. The critical value d_{max} of the counter-trend.

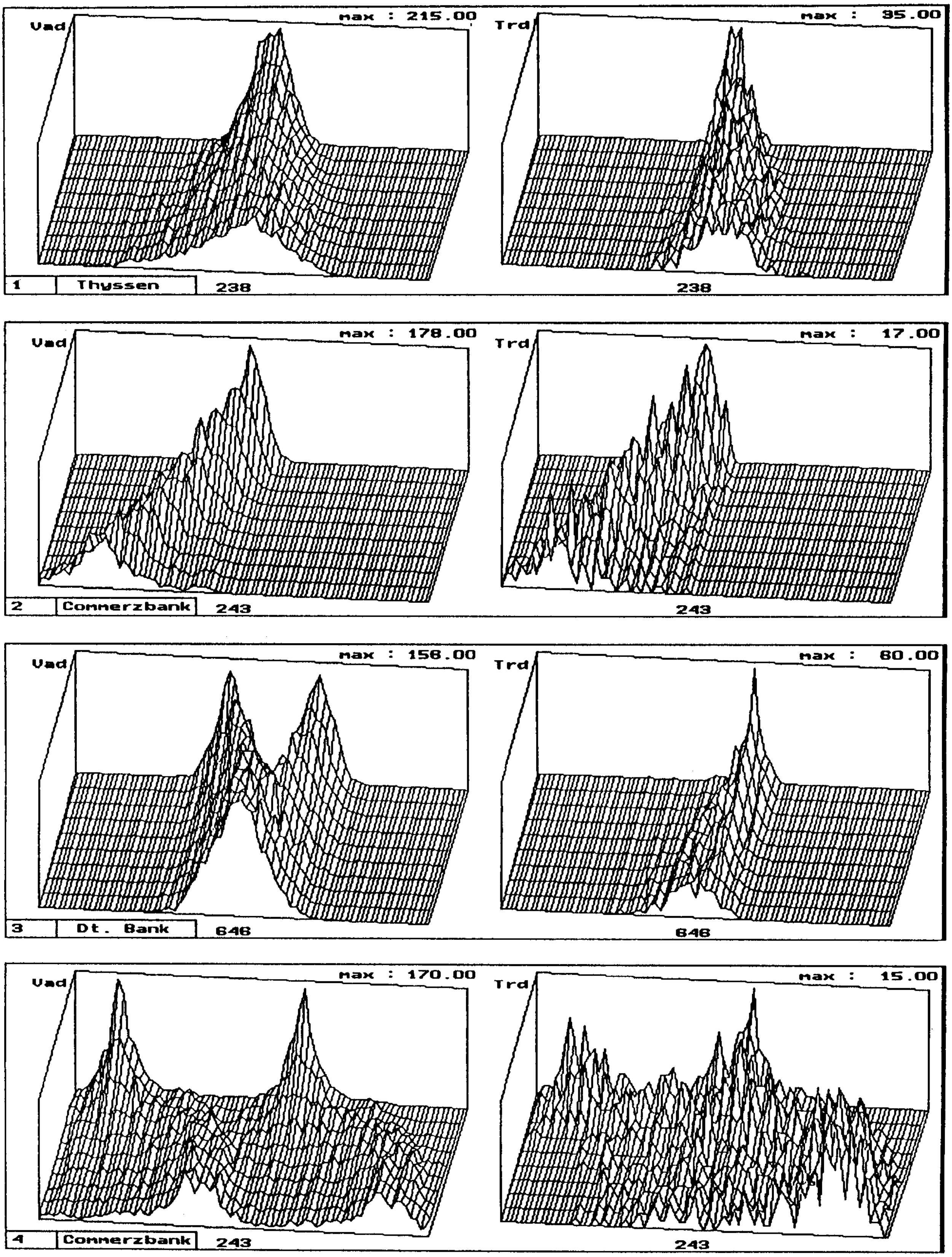

Figure 2. Four simulations of distributions of price expectations (left) and prices (right)

9. The number k_{opt} of different assets in the target portfolio.
10. The parameters w_0 and ζ_w of the Pareto distribution of the wealth of the agents at $t = 0$.
11. The maximal time span t_{max}.

The computations are carried out on an eight-transputer network. Nevertheless, the computation times are still so high that only a limited number of simulation results are available as yet.

4.3. Some simulation results

We now present a few simulation results in graphical form. We worked with 10 shares and 40 agents subdivided into five groups, labelled A to E. Each group represents a certain kind of stock market speculator characterized by a set of group specific parameter values.

For each parameter constellation, 200 paths with $t_{max} = 10$ are simulated. We thus obtain at each time $t \leqslant 10$ for each asset j a distribution of 200 quoted prices and a distribution of 8000 price estimates. In the graphics depicted in Figure 2, these simulated distributions are shown at 10 time points (the left for the price expectations, the right for the quoted prices).

We show here examples of four different types of scenario. In the first, we observe a slight increase of both mean prices and expectations, the distributions remain unimodal but become more and more diffuse. In the second, there is a strong decline of both mean prices and expectations, the distributions are unimodal again. In the third, the prices decrease and remain unimodal while the initially bimodal expectations distribution becomes unimodal. In the fourth, we see a strong increase of mean prices and expectations, the expectations remain unimodal while the prices become diffuse.

We hope to detect classes of parameter constellations leading to typical performance scenarios. As different constellations correspond to different behavioural attitudes of the agents, this leads to a methodology for characterizing these attitudes and relating them to typical phenomena at the macrolevel. This goes beyond the scope of this paper.

Table II

No.	Par.	Val	Group	η^{ext}	η^{inf}	η^{pot}	η^{tr}	ξ^{ip}	ξ^{trade}	ξ^{real}	ξ^{risk}
1	$\ln(W_E)$	−3·8	A	0·1	0·2	0·0	0·3	0·8	0·0	0·1	1·0
	$\ln(W_V)$	−4·4	B	0·2	0·2	0·0	0·3	0·8	0·0	0·1	1·0
	ρ	0·5	C	0·3	0·2	0·0	0·3	0·8	0·0	0·1	1·0
	$\varkappa$	1·0	D	0·4	0·2	0·0	0·3	0·8	0·0	0·1	1·0
	d_{max}	0·05	E	0·5	0·2	0·0	0·3	0·8	0·0	0·1	1·0
2	$\ln(W_E)$	−3·8	A	0·1	54	0·0	1·0	0·8	0·0	0·1	1·0
	$\ln(W_V)$	−4·4	B	0·1	54	0·0	1·0	0·8	0·0	0·1	1·0
	ρ	0·5	C	0·1	54	0·0	1·0	0·8	0·0	0·1	1·0
	$\varkappa$	1·0	D	0·1	54	0·0	1·0	0·8	0·0	0·1	1·0
	d_{max}	0·05	E	0·1	54	0·0	1·0	0·8	0·0	0·1	1·0
3	$\ln(W_E)$	−3·8	A	0·0	20	0·0	0·0	0·8	0·0	0·1	1·0
	$\ln(W_V)$	−4·4	B	0·0	20	0·0	0·0	0·8	0·0	0·1	1·0
	ρ	0·5	C	0·0	20	0·0	0·0	0·8	0·0	0·1	1·0
	$\varkappa$	1·0	D	0·0	20	0·0	0·0	0·8	0·0	0·1	1·0
	d_{max}	0·02	E	0·0	20	0·0	0·0	0·8	0·0	0·1	1·0
4	$\ln(W_E)$	−6·0	A	3·1	1·0	0·3	2·0	0·0	0·0	0·0	1·0
	$\ln(W_V)$	−6·5	B	3·1	1·0	0·3	2·0	0·0	0·0	0·0	1·0
	ρ	0·8	C	3·1	1·0	0·3	2·0	0·0	0·0	0·0	1·0
	$\varkappa$	2·0	D	3·1	1·0	0·3	2·0	0·0	0·0	0·0	1·0
	d_{max}	0·10	E	3·1	1·0	0·3	2·0	0·0	0·0	0·0	1·0

The parameter values are as follows. Common to all four simulations are $r = 0{\cdot}01$, $k_{opt} = 5$, $w_0 = 30\,000$, and $\zeta_w = 1$. The others are collected in Table II.

CONCLUSIONS

Even if we assume that we are breaking new ground, there are previous results that we are relying upon and to which our approach is related. The comparison to the literature helps to point out the peculiarities of our approach. A detailed elaboration of the relation to the main body of the literature is provided by Landes and Loistl,[29] and Loistl and Reiß.[34] A condensed version of the one stock model is presented in Landes and Loistl.[32] The computation of the fundamental value distribution on annual report figures is illustrated in Landes and Loistl.[33]

The synergetic view has been developed by Hermann Haken.[21] Weidlich and Haag[22] applied the concept in the modelling of social phenomena. Both publications are mentioned by Brian Arthur[23] (p. 23); he stresses the independence of the transitions: they can be made only one unit at a time. He underlines that models of this fixed-size, Markov-transition kind are standard in genetics, epidemiology and in areas of physics. This statement must not be misunderstood as though the application of such a model to problems in economics requires no major changes. The concept of synergetics fundamentally relies on the dominance of individuals' activities and their interactions at the microlevel.

Phenomena at the macrolevel are consequences of micro-events. A description of economic interdependencies at the macrolevel must rest on the activities at the microlevel. Establishing macrorelations without that basis might lead to economic theorems endanged by untrustworthiness. Both the rising scepticism about (monetary) macroeconomics and the rising investigations of the capital markets' microstructure underline the discomfort with the prevailing global concepts. We believe that, for example, a valid description of the market microstructure has to rest on the individuals' activities and consistent integration into institutional rules. The activities have to be determined by explicit behavioural attitudes and the continuous processing of information entailed in the market performance. The main difference between our approach and Brian Arthur's positioning of that sort of Markov models is the discussion of the transition rates. We break up the global given transition rates (a concept that is, of course, well known in science) into single components determined by behavioural attitudes. As the transition rates govern the realization of activities we are thus able to integrate the acting agents into the factors causing a change in the state of the capital market.

Thus, the detailed elaboration of factors determining the transition rates provides the possibility of explicitly modelling the variability and richness of (capital) market scenarios at an extent that goes far beyond the hitherto prevailing description by stochastic processes given by the results of probability theory. e.g. Duffie.[31] The integration of both approaches might be very promising.

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REFERENCES

1. E. De Bono, *The Mechanism of Mind*, reprint, Middlesex, 1981.
2. T. Bollerslev, R. T. Chow, N. Jayaraman and K. F. Kroner, 'ARCH modelling in finance: a selective review of theory and empirical evidence', *J. Econometrics*, to be published.

3. J. E. Hunter and D. T. Coggin, 'Analyst judgment: the efficient market hypothesis versus a psychological theory of human judgment', *Organiz. Behavior Human Decision Processes*, **42**, 284–302 (1988).
4. L. S. Bagwell, 'Shareholder heterogeneity: evidence and implications', *Amer. Econ. Rev.* **81**, pp. 218–221 (1991).
5. S. F. LeRoy, 'Efficient capital markets and martingales', *J. Econ. Literature*, **27**, 1583–1621 (1989).
6. A. W. Williams, 'The formation of price forecasts in experimental markets', *J. Money, Credit, Banking*, **19**, 1–18 (1987).
7. R. E. Lucas, Jr., 'Understanding business cycles', in K. Brunner and A. H. Meltzer (eds), *Stabilization of the Domestic and International Economy*, Carnegie-Rochester Conference Series on Public Policy, 1977, pp. 215–239.
8. J. B. Bullard, 'Learning, rational expectations and policy: a summary of recent research', *Federal Reserve Bank of St. Louis Review*, **73**, 50–60 (1991).
9. J. F. Muth, 'Rational expectations and the theory of price movements', *Econometrica*, **29**, 315–335 (1961).
10. C. Wilborg, 'Arbitrage, equilibrium and market efficiency, a parable', *J. Econ. Behavior Organiz.* **15**, 297–302 (1991).
11. B. J. De Long, A. Shleifer, L. H. Summers and R. J. Waldmann, 'Noise trader risk in financial markets', *J. Polit. Econ.*, **98**, 703–738 (1990).
12. G. Duménil and D. Lévy, 'Micro adjustment toward long-term equilibrium', *J. Economic Theory*, **53**, 369–395 (1991).
13. R. Morck, A. Shleifer and R. W. Vishny, 'The stock market and investment: is the market a sideshow?' *Brookings Papers on Economic Activity*, **2**, 157–215 (1990).
14. B. C. Greenwald and J. C. Stein, 'Transactional risk, market crashes, and the role of circuit breakers', *J. Business*, **64**, 443–462 (1991).
15. S. R. Pliska and C. T. Shalen, 'The effects of regulations on trading activity and return volatility', *J. Futures Mkts*, **11**, 135–151 (1991).
16. D. Friedman, 'A simple testable model of double auction markets', *J. Econ. Behavior Organiz.*, **15**, 47–70 (1991).
17. C. M. Turner, R. Startz and C. R. Nelson, 'A Markov model of heteroskedasticity, risk, and learning in the stock market', *J. Financial Econ.*, **25**, 3–22 (1989).
18. G. McQueen and S. Thorley, 'Are stock returns predictable? A test using Markov chains', *J. Finance*, **46**, 239–263 (1991).
19. G. Engelen, 'The theory of self-organization and modelling complex urban systems', *Europ. J. Operational Res.*, **37**, 42–57 (1988).
20. B. W. Arthur, 'Self-reinforcing mechanisms in economics', in P. W. Anderson, K. J. Arrow and D. Pines (eds), *The Economy as an Evolving Complex System*, Addison-Wesley, Redwood City, 1988, pp. 9–31.
21. H. Haken, *Synergetics*, Springer-Verlag, Berlin, 1978.
22. W. Weidlich and G. Haag, *Concepts and Models of a Quantitative Sociology*, Springer-Verlag, Berlin, 1983.
23. B. W. Arthur, 'Learning and adaptive economic behavior, designing economic agents that act like human agents: a behavioral approach to bounded rationality', *Amer. Econ. Rev.*, **81**(2), (1988).
24. B. W. Arthur, Y. M. Ermoliev and Y. M. Kaniovski, 'Path-dependent processes and the emergence of macro-structure', *Europ. J. Operational Res.*, **30**, pp. 294–303 (1987).
25. S. N. Durlauf, 'Multiple equilibria and persistence in aggregate fluctuations', *Amer. Econ. Rev.*, **81**, 70–74 (1991).
26. W. Feller, *An Introduction to Probability Theory*, Vol. II, Wiley, New York, 1966.
27. D. McFadden, 'Conditional logit analysis of qualitative choice behaviour', in P. Zarembka (ed.), *Frontiers in Econometrics*, Academic Press, New York, pp. 105–142, 1973.
28. A. Börsch-Supan, *Econometric Analysis of Discrete Choice*, Springer-Verlag, Berlin, 1987.
29. T. Landes and O. Loistl, 'The market performance at the microlevel', in O. Loistl and T. Landes (eds), *The Dynamic Pricing of Financial Assets*, McGraw-Hill, Hamburg, pp. 1–12 (1989).
30. T. Landes, O. Loistl and W. Reiß, 'The market microstructure and the dynamic pricing of financial assets', in O. Loistl and T. Landes (eds), *The Dynamic Pricing of Financial Assets*, McGraw-Hill, Hamburg, pp. 53–108, 1989.
31. D. Duffie, *Security markets: Stochastic Models*, Academic Press, Boston, 1988.
32. T. Landes and O. Loistl, 'The continuous quotations at an auction market', *Rivista di Matematica per le Scienze Economiche e Sociali*, **12**(1), 73–89 (1990).
33. T. Landes and O. Loistl, 'Capital structure, principal/agency-relation and the value of the corporation', *Omega, Int. J. Mgmt Sci.*, **19**, 291–303 (1991).
34. O. Loistl, and W. Reiß, 'Price building mechanisms and their modelling in the literature, in O. Loistl and T. Landes (eds), *The Dynamic Pricing of Financial Assets*, McGraw-Hill, Hamburg, pp. 13–51, 1989.