



KapSyn

Computer-Modelled Stock Exchanges

User Manual

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1 Introduction

Over the past decades, trading has increasingly been regarded as an area for cutting costs. Several alternatives have evolved for placing orders, and transaction costs have become increasingly important as a means of evaluating them. Models for computing transaction costs have been needed for quite a while.

This Manual presents an approach to calculating transaction costs that is based on the capital market's microstructure. This model – called KapSyn for short – can imitate the microstructure of any stock exchange, especially with respect to its price-finding procedures. We have used the KapSyn model to calculate transaction costs for orders placed on four different stock exchanges: Frankfurt's XETRA, a computerized trading system (see Chapter 2.7.2); NASDAQ (Chapter 2.7.3); the NewYork Stock Exchange¹; and a market maker trading system².

The Manual is divided into 4 sections.

- Chapter 1 provides an introduction and quick overview of the KapSyn program package.
- Chapter 2 provides an in-depth description of the KapSyn model.
- Chapter 3 provides a detailed explanation of the KapSyn computer program, including the generation of transaction cost statistics.
- An extensive index helps the reader to find topics throughout the Manual.

¹ Not currently implemented; available in 2000

² Currently in the pre-beta phase

1.1 The KapSyn Computer Program

The KapSyn computer program is tailored to the Windows 95/98/NT operating systems. The latest release of the program and this Manual can be found on the Internet.

Furthermore, the program code and program documentation is available to programmers who wish to extend the program to include their own stock exchange modules.

Up-to-date information about these topics can be found on the Internet at the following address: <http://ifm.wu-wien.ac.at>

1.2 Quick Tour of the KapSyn Program

After installation, the program is started by clicking the  icon in the task bar.

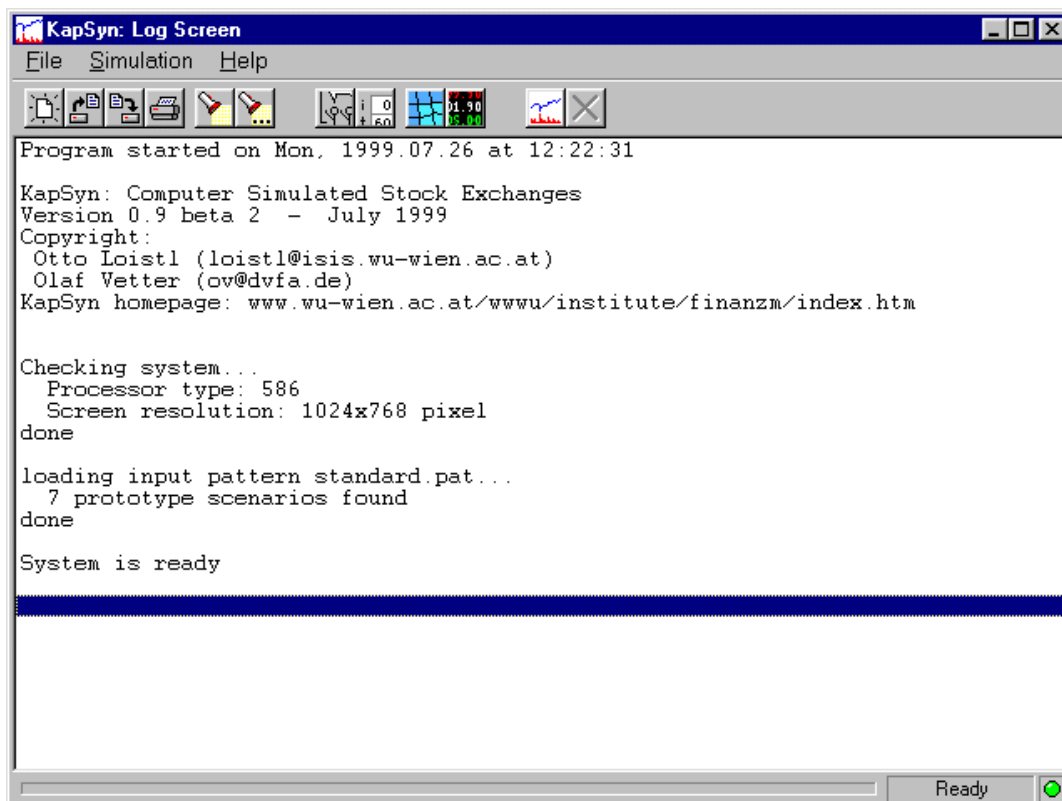


Fig 1 The main screen of the KapSyn application

The main screen of Fig 1 (the *KapSyn Log Screen*) is the "information center" for everything taking place, and is used to log all program messages and warnings.

The most common tasks are performed by clicking the appropriate icon on the task bar:

Log screen functions



Clears the current log screen



Opens a previously saved log screen or a parameter pattern file



Saves the current log screen to a text file



Prints the current log screen



Searches the log screen for a text fragment



Searches for the next occurrence of a text fragment

Parameter dialogs



Activation of and specific settings for the various stock exchange modules



View, change, save and reload all KapSyn parameter settings

Additional screens



Real-time view of the evolution of simulations and view of previous simulations



Ticker window

Program run



Starts a simulation with the current parameter settings



Stops a running simulation

The full set of program functions is discussed in Chapter 3.

1.2.1 Choosing a Stock Exchange Module

The modules dialog box is used to activate the stock exchange modules and set their behavior.

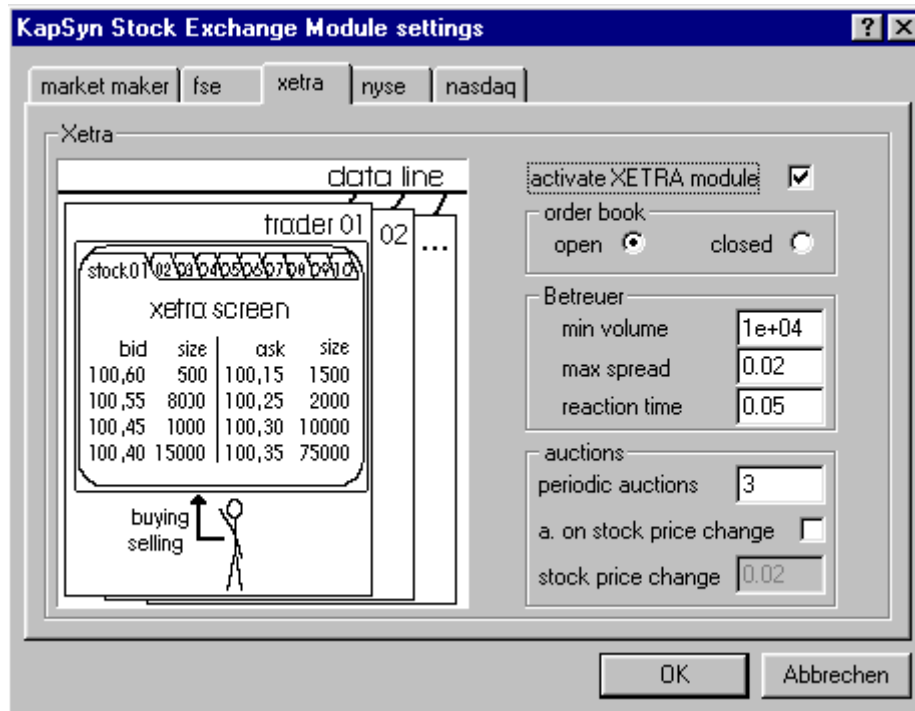


Fig 2 The stock exchange modules dialog box

An explanation of the stock exchange settings can be found at the beginning of the chapters describing the stock exchanges that are currently implemented.

1.2.2 Setting Parameters in KapSyn

Parameters can be set in KapSyn using the parameter dialog box, as shown in Fig 3.

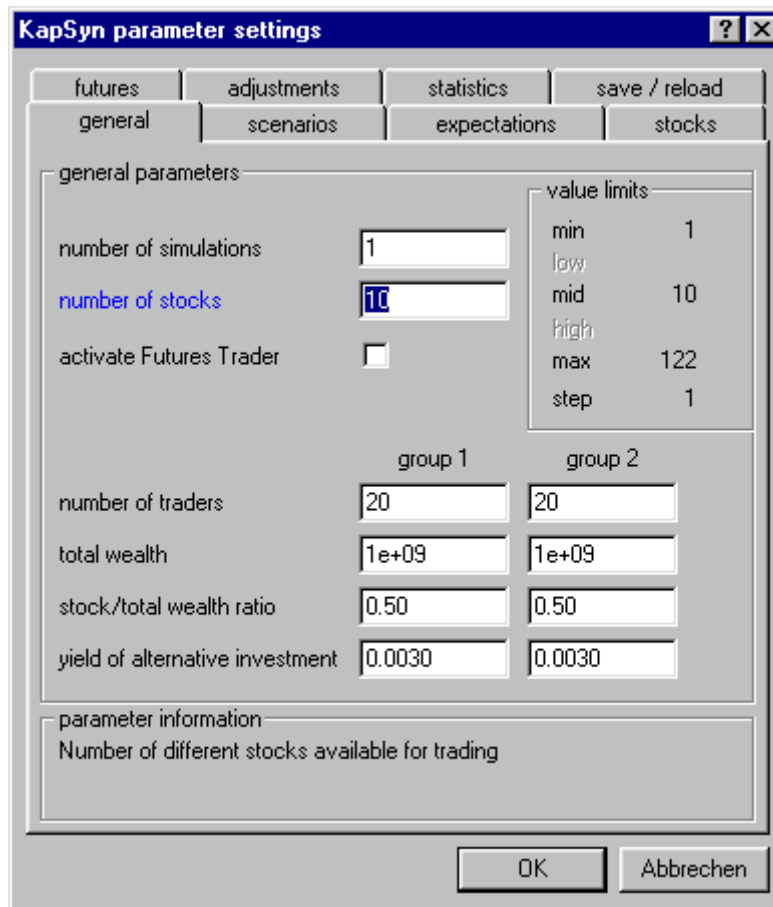


Fig 3 The KapSyn parameter settings dialog box

A rollover help system provides brief parameter information and suggests parameter values. See Chapter 3.4 for a further description of this system.

1.2.3 The Graphics Window

A graphics screen can be created at any time, even when a simulation is running, in order to view the evolution of stock prices and price expectations. An additional price/volume view shows the exact stock price path, the order book situation and a volume histogram. In addition, a detailed variable analysis view is provided for thorough scenario testing.

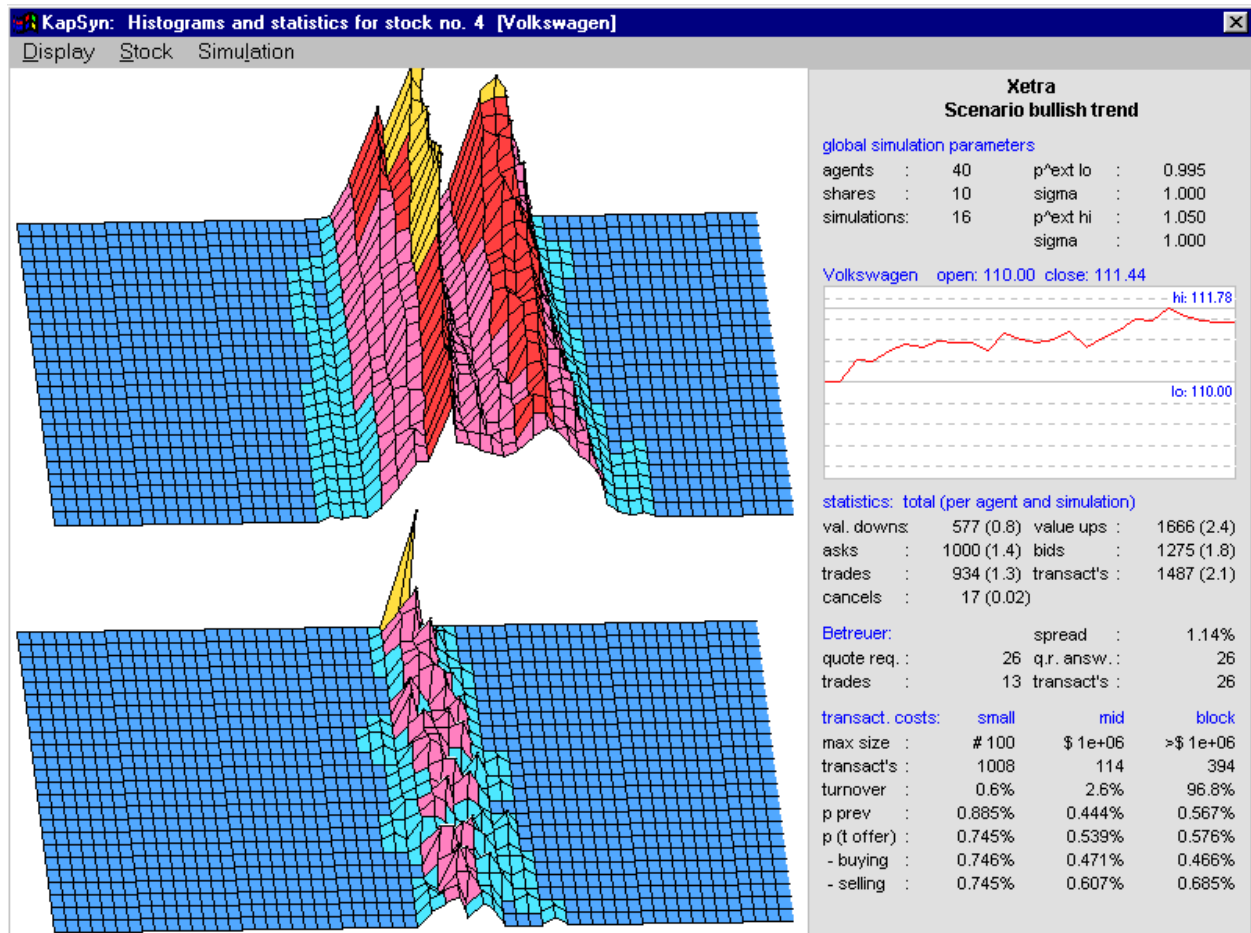


Fig 4 The graphics window

To the right of the graphics window a summary of simulation statistics, including transaction cost statistics, is provided.

An explanation of the graphics window is given in Chapter 3.5.

2 The KapSyn Model of the Market's Microstructure

2.1 Introduction

In an investment era in which margins are narrowing and performance pressures are growing, implementation issues are becoming increasingly important. In particular transaction costs are often understated, and institutions may be underestimating the cost of trading.

Securities trading is a relatively new topic for scientists and practitioners. In the 1960s, researchers believed in the importance of fundamental research. Trading was unimportant and inexpensive compared with the benefits of research. The impact of an individual trader's order on the market was low, and essentially the only way to execute an order was on the organized exchanges. The situation began to change in the 1970s and the evolution has continued in the 1990s.

During the last decades, trading has been increasingly regarded as area where costs could be saved: Several alternatives evolved for placing orders and transaction costs have become increasingly important to evaluate these alternatives. Models to compute transaction costs have been needed for a while.

Several alternatives such as NASDAQ and proprietary trading systems or ECNs and the national European Stock Exchanges or EASDAQ or the national New Markets evolved for placing orders. Electronic trading systems offering remote membership like XETRA provide even further opportunities.³

Calculating and measuring transaction costs are matters of intensive discussion and no small controversy. (Loistl (1996), pp. 84, 88.) Stoll (1993) provides a synopsis of different approaches.

The theoretical issue is even more complex. Chan/Lakonishok (1997, p. 715) assert it is an extremely difficult task to evaluate the operating efficiency of any market. There is no reason to believe that it is any easier to assess the quality of a stock market. There is no consensus on how to measure trading costs or how to control for other influences on trading activities and price movements.

³ A comprehensive description provide Domowitz/Steil (1999)

The differences emerge from several reasons. The most important seems to be that there does not exist a generally accepted analytical approach to structure market processes. The model has to capture those peculiarities addressed by Hausman/Lo/McKinlay (1992, p. 320):

Prices are quoted increments,
the timing is irregular and random,
price changes are conditioned on quantities such as volume, time between trades and the sequence of past prices.

The second aspect refers to the availability of data. For a long time, the data available were the quoted prices and the bid-ask spread. The now available data bases enable to compare trades with quotes (cf Blume/Goldstein (1997), p. 230, Bessembinder/Kaufman (1997), p. 291).

These data make the importance of the first reason even more obvious: Until now, there does not exist a framework to measure the efficiency of markets. The intensive empirical research deserves a theoretical guideline to avoid the pitfalls of data mining.

The pure commissions are not the transaction cost's major part. The problem is how to measure the costs of selling/buying liquidity or value or time. Measuring transaction costs is a problem that will not be settled as long as it is not possible to rely on approaches that model the traders' decision scenario in detail, that model the stock exchanges' microstructure frame for tradings' dynamic interactive process.⁴ Such a model has to provide endogeneously generated data by imitating the exchange's performance enabling to measure transaction costs covering the above mentioned tasks.

Such a model provides also a reliable comparison of different stock exchanges' transaction costs. We propose the KapSyn model to solve this problem. It is able to imitate the performance of any stock exchange. The model applies a markov process with continuous time and discrete state space to model the capital market's dynamics. It uses a stochastic logit utility approach to evaluate the traders' feasible activities' utility. The state transition probabilities are determined by these activities' utility figures. Like real stock exchanges, traders are able to place orders or revise their orders according to the processing of the information the market performance and the companies' fundamentals entails. In the version presented, the market orders are disclosed

to the market, but not the revision of beliefs. The model's basic structure is presented in Landes/Loistl (1992). A detailed description of the version implemented is given in this handbook.

This model – named KapSyn for short – is able to imitate the microstructure of any stock exchange, esp. regarding the price finding procedures.

The modelling of intra-day market activities is intended to simulate trading processes in a way that reflects the prevailing institutional environment. With respect to this institutional environment, our current work examines ongoing trading, which is organized as a public auction market. At any point during the trading session the market participants – henceforth referred to as 'agents' – can submit bids and asks. Whenever two orders match in volume and price, the quote is fixed. Under certain conditions, an auction may take place. The XETRA pricing process is a typical example of this hybrid system. However, we shall start by outlining the basic formal structure of the KapSyn model. This structure is not affected by the system being adapted to specific pricing organizations.

Budget constraints aside, any agent can, in principle, submit bids and asks for any price or volume for all stocks being traded during the trading session, or accept existing bids and asks on the market. Nevertheless, limits are provided in the parameter dialog box in order to ensure that the system functions sensibly. Such bid patterns must be reproduced using a reliable model structure. To do this, the following questions need to be clarified:⁵

- What are the motives driving the placement of bids and asks, and how can the individual agent's decision-making process in this regard be adequately modelled while at the same time taking the prevailing institutional environment into account?
- What repercussions, direct or indirect, do market prices have on the market participant's assumptions about intrinsic value?

During the trading session, every agent considers certain activities relating to execution which they feel are advantageous, depending on the number of shares or amount of cash assets at their disposal, their risk attitude, the

⁴ A detailed description of the real trading process provide Handa/Schwartz/Tiwari (1998) and Handa/Schwartz/Tiwari (1999)

⁵ cf. Battistini/Ferrari/Peccati (1994) p.140 ff., Ferrari/Luciano/Peccati (1993) p.118 ff., Landes/Loistl (1992) p.210 ff. and the literature cited in these publications.

information at their disposal, their information processing capacity, and the particular market conditions.

A distinction can be drawn between two types of information – market information and fundamental information.⁶ Fundamental signals are obtained by collecting information regarding a company's fundamental situation, or that of a sector, country, etc.

Market signals are obtained from the observation of market activities. They are manifested in the form of price trends, bid-ask spreads, transaction volumes, and indeed any information which can be gleaned from market activities.

The identification and subsequent processing of market and fundamental signals can lead to modifications being made to expectations, for example with regard to expected future share prices and/or dividends. These changes in expectations lead in turn to a revision of existing portfolios. The portfolio adjustment process can be structured as follows:

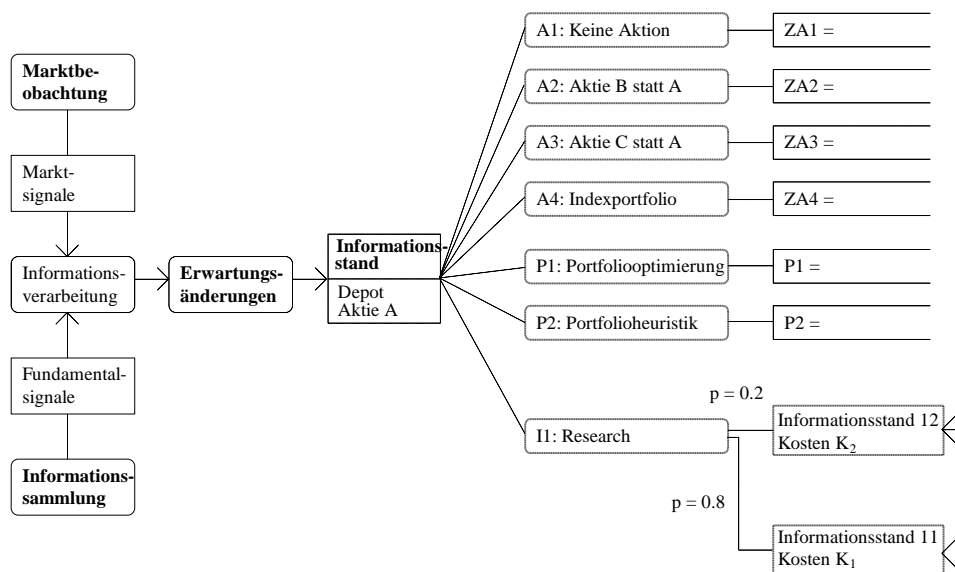


Fig 5 The information component of the portfolio decision-making process

The concept of a 'change in expectations' needs explaining in a little more depth. Market participants do not just have expectations

- with regard to prices/dividends which the investors believe they will be able to obtain in the future, but also
- with regard to the prices at which the investors believe they can place bids and asks on the market at the present moment.

⁶ cf. Landes/Loistl (1992) p.215 f. The frequently-used concept of 'noise traders' is not considered here.

Both categories of expectations have a major influence on expected earnings from particular investment activities. Thus, fundamental and/or market signals can cause investors

- on the one hand, to revise their future price and/or dividend expectations;
- on the other hand, to revise their expectations regarding the probability that their current offers and bids will be realized, and to change their buy / sale limits accordingly.

In both cases an examination of the investor portfolio is required. As the figure above makes clear, many different considerations can come into play here. Investors can decide to optimize their portfolios, they can use portfolio heuristics, but they can also gather further information and only then make an investment decision. KapSyn imitates the investment planning process.

A signal-dependent planning process is appropriate for modelling daily market activities. A re-appraisal of investor portfolios takes place if new information on the market climate or a company's fundamental situation is made available. An examination of the composition does not necessarily have to lead to a revision. As the planning system is signal-dependent, the planning process will always be initiated when a signal occurs. The times at which signals occur thus also determine the lengths of the planning periods, which will not now directly correspond to each other. By taking a short-term view of market activities we can concentrate solely on market signals. In this case, the fundamental expectations of the market participants can be taken to be constant in the short term. The implementation of a sudden shift in the fundamental assessment e.g. following appraisal of the latest business news does not pose a major problem within the KapSyn framework. Furthermore, when a planned action is actually executed depends on the point in time at which other market participants act. If another market participant performs an action (i.e. a signal) before the agent who is planning to do the same, then the latter's planning process will be triggered all over again. Simultaneous implementation of two different activities is not possible.

The economic success of a financial investment is usually measured as the return on capital employed (ROCE) over the planning period. If the planning period assumed does not have a fixed time limit, then an assumption as to the duration of the planning period is needed to measure the success of an investment. Determining investment performance for this assumed planning period requires knowledge of future returns, in other words of the future stream of earnings. Future income is determined by future dividends and/or

the prices that the investor believes he or she can obtain in the future. The length of this planning period is set by the relevant forecast horizon (e.g. 1 day, 1 week, 1 month, 1 year, etc.). Any dividend payments are taken into account in a simplified manner, by including them in an adjusted price.

In the case of signal-dependent planning, the time structure of the forecast period will regularly deviate from that of the planning period. As a result, when a signal occurs, the investment decision will be made on the basis of the forecast horizon as shortened by the time elapsed since the last signal. This situation can be simplified by the assumption of a 'rolling forecast' – i.e. a dynamic planning process with a rolling forecast. The following figure illustrates the interaction of the various factors in a single-period forecast. Of note is the asynchronous timing of the signal occurrences, and the constant planning periods.⁷

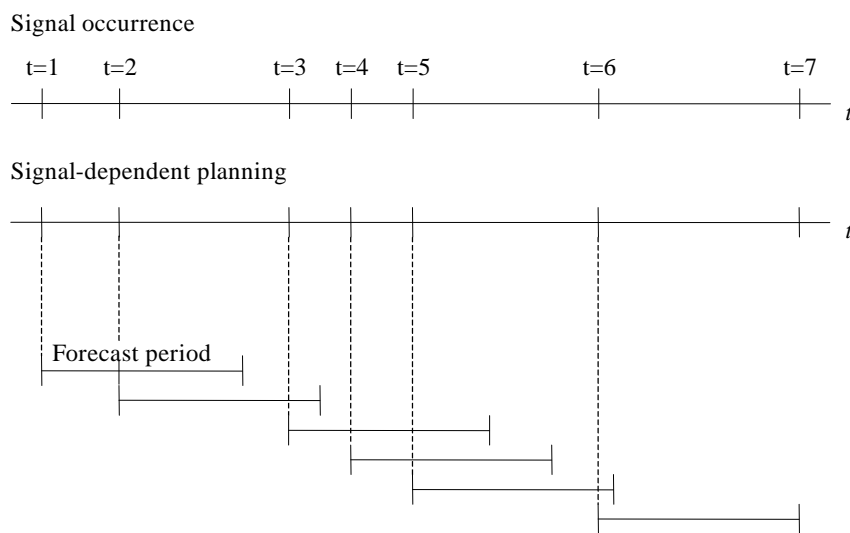


Fig 6 Signal-dependent planning with rolling forecast

Following this short description of the basic ideas behind the system, we shall now describe the approach in formally correct terms. Nonetheless, the emphasis is on providing an easily understandable explanation.

⁷ See Loistl/Landes (1989) p.115

2.2 The KapSyn Model

2.2.1 Overview of the Model

2.2.1.1 Market Participants

The explicit simulation of market activity, or more specifically of a *single* trading session, is performed using a Markov process which is homogeneous as regards time $\{\tilde{Z}_t, t \geq 0\}$. The random variable \tilde{Z}_t describes the state of the market at time t . The process has a continuous time and discrete state space.

As a result, prices can be quoted in increments and the timing of actions can be nonsynchronous. Skillful modelling of state variables enables price changes to be made conditional on quantities such as volume, the time between trades, the sequence of past price changes, etc.

1. Take a stock exchange with $j = 1, \dots, J$ securities and $i = 1, \dots, I$ traders. KapSyn version 3.0 provides a range of max 122 securities and of max 122 traders and one arbitrageur between the spot market and the futures market described by the "SYNTAX" Future futures contract. The range of acceptable figures is given in the dialog box. The example given below refers to 10 stocks and 40 traders. The SYNTAX ("synthetic *Aktienindex*") is calculated according to the DAX formula. The arbitrageur influences the prices on the spot market by his/her spot buy and sell orders. The arbitrageur has been added to the version described in Landes/Loistl (1992) in view of the increasing importance of futures/spot arbitrage to price finding on the spot market. The arbitrageur trades the difference between the "SYNTAX" and the "SYNTAX Future". The SYNTAX Future reverts to the mean around the SYNTAX spot figures. The arbitrageur accordingly acts as the buyer/seller of the individual stocks on the spot market.
2. The detailed description of the stock exchange's status z entails that each trader is endowed with each of the securities and with cash. The following tables give examples of this. These figures have to be adjusted to the real capital market investigated. In addition, the figures have to be adjusted to reflect the actual situation: a market characterized by a huge amount of cash reacts differently to a highly invested market.

Share Trader	1	2	3	4	5	6	7	8	9	10
1	1100	1700	1100	1000	3000	1700	2200	1400	2500	3700
2	2000	3000	2000	2000	6000	3000	4000	2000	5000	...
3	7000	1000	6000	6000	1900	1000	1300	8000
4	8000	1200	7000	7000	2100	1200	1500
5	3000	6000	1200	2000	1400	3000
6	2000	3000	2000	1000	6000
7	8000	1300	8000	7000
8	3000	4000	2000
9	8000	1200
10	9000
...
35	7000
36	6000	9000
37	1100	2000	3000
38	2200	1400	2500	3800
39	1300	1600	1000	1900	2800
40	1500	8000	1000	6000	1200	1800

Tab 1 Traders' share endowments at the beginning of the session

Trader	Cash	Trader	Cash
1	2878	21	941
2	8599	22	5443
3	5230	23	8244
4	4484	24	1560
5	10159	25	5226
6	8753	26	9759
7	4603	27	2186
8	8723	28	6360
9	4986	29	6388
10	4396	30	5675
11	4170	31	7371
12	3959	32	7330
13	8925	33	5433
14	5155	34	3633
15	1010	35	10145
16	680	36	9365
17	6866	37	4349
18	443	38	3865
19	7546	39	4526
20	4240	40	6685

Tab 2 Traders' cash endowments at the beginning of the session (random allocations)

- The detailed description entails the trader's estimates of the intrinsic value of each of the stocks. The set of all intrinsic value estimates for each stock is condensed into a frequency distribution of the individual estimates.

A bimodal distribution is used for the \hat{p}^{ext} estimates. At least for the moment, this consists of two truncated normal distributions, although any other shape might be possible. The bimodal shape suggests that there are optimistic and pessimistic traders. If all the traders had the same expectations, the distribution would shrink to a single point.

The optimistic and the pessimistic parts might have different shapes, depending on the m and s of this specific truncated normal distribution. In the current version, the m and s of both the bimodal truncated parts of \hat{p}^{ext} distribution are determined according to each capital market scenario defined. For a detailed description, see Chapter 2.4.1.8, 2.4.1.9 and 2.6.3.

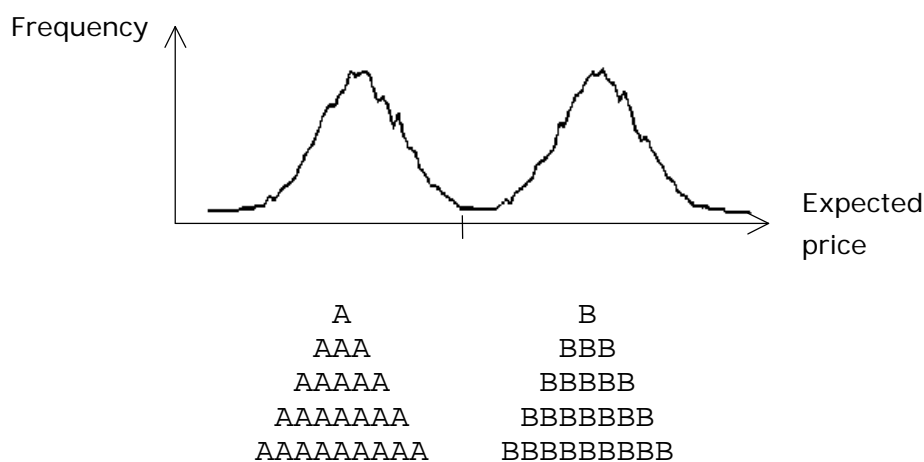


Fig 7 Trader groups A and B

Describing the set of all intrinsic value estimates using a frequency distribution allows us to describe the market estimates by a range of values. No unique estimation figure is required. Furthermore, only the shape and the characteristic parameters of the frequency distribution need be estimated.

Although this procedure also entails an estimation bias, the latter is less important than that of the capital market equilibrium approach, as market participants revise their estimates during the trading session according to the information they are processing on the shares' market performance. The way in which this information is processed will be different for market technicians and for fundamentalists: For example technicians believe in the market and assume that prices reflect the available information correctly. They will therefore adjust their estimate to the market. Fundamentalists, however, will stick to their estimates. In addition, the prevailing overall capital market scenario in each case - bullish, bearish, etc. - influences market performance.

For example, an insider situation is characterized by the fact that one person (the insider) is immune to market opinion and will stick to his or her value estimate.

The relation of the opening quotation to \hat{p}^{ext} frequency distribution will generate different stockmarket performances, especially dynamic behavior like a bifurcation. In addition, this depends on the climate on the day concerned, which is determined by the set of parameter values.

Sometimes, even fundamentalists adjust quickly, and even technicians do not trust the market. This depends on the specifics of the behavioral parameter set, which are specified for the moment by 7 scenarios providing a general guideline. Users can specify the parameters in line with their preferences.

4. Every trader believes that, at the end of the planning period (one day, one week or one year), the price of a security will equal his or her individual estimate. We apply rational expectations at the level of the individuals, not at the level of the market as a whole. This assumption leads to the basic rule of trading: **traders aim to buy securities whose share price is above their individual intrinsic value estimates, and they aim to sell those securities whose share price is below their individual estimates and invest the proceeds elsewhere.** Of course, transaction costs and the opportunity costs of alternative investments also have to be taken into account.

2.2.1.2 Evaluating the Utility of Actions

1. Each trader can make offers to buy or to sell securities at specified prices and quantities fixed by him/her; alternatively, he/she can accept such offers. Placing or accepting offers is generally known as “**event**”. Each trader can also adjust his/her intrinsic value estimate; such changes are known as “**value adjustments**”. In putting these actions into practice, the trading rules of the marketplace concerned must be observed.⁸ The disclosure of events depends on the rules governing the marketplace; for example, there may be an open or closed order book, or any other rule. Different pricing systems may also apply, e.g. a hybrid system such as XETRA or a quote-driven system with a complex routing structure such as

⁸ Value adjustments are not disclosed to the market.

NASDAQ. The following outline provides a general description only. The possibilities implemented for a specific organization are described in the relevant chapters.

In formal terms, the set of traders' actions k is K . The action A_{ijk} of trader i on security j specifying the alternative k will be valued according to the stochastic utility function⁹ concept. In other words, the utility m_{ijk} ascribed to action A_{ijk} given the market status z will be determined by the formula

$$m_{ijk} = F_{ij}(A_{ijk}, z) + e_{jk}$$

A stochastic component is added to the deterministic part. The stochastic component e_{jk} is assumed to be independent and identically distributed.

2. Developing a specific model for valuing and selecting the alternatives requires that the function F and the distribution function of the stochastic component e_{jk} be specified. The function F will not be described in detail here.¹⁰

Assuming a normal distribution for the stochastic term leads to the *probit model*¹¹. The extreme value distribution leading to the *logit model* is, in principle, another potential choice.¹²

3. Under the assumption that the stochastic component e_{jk} follows an extreme value distribution and neglecting the suffix for trader i and security j , the probability for selecting action A_k is given by

$$(1) \quad R(k|K) = \frac{\exp(F(A_k, z))}{\sum \exp(F(A_l, z))}$$

The probability of selecting action A_k is proportional to the deterministic part $F(A_k)$ of the utility.¹³

⁹ cf. A. Börsch-Supan (1987) p.13

¹⁰ Interested readers should refer to Landes/Loistl (1992).

¹¹ See D. McFadden (1984) p.1418

¹² See D. McFadden (1984) p.1418

¹³ cf. Landes/Loistl (1992)

4. The probability correspondence can be written as follows:

$$(2) \quad P(k|K) := W * \exp(F(A_k, z))$$

This formula represents the fundamental axiom at the heart of KapSyn:

**The greater the utility of an action,
the higher the probability that it will be realized.**

The innovative aspect of KapSyn is that it links an action's utility with the market transition probability.

2.2.1.3 State Transition Rates and the Utility of Actions

1. Every market participant is able to realize one of a set of several actions in a specific market state. Each action generates a specific change of that state. Of course, he or she can take one action at a time only. The new state z' depends on the action A chosen. The transition time T' depends only on the state z , according to the Markov rule. The connection of transition time and the action's utility is provided by means of the parameter of the exponential distribution of the transition time. The transition time of trader i in state z is an exponential distributed random variable that takes the traders' utility as a parameter (cf. Loistl/Landes (1989), p. 112).
2. The exponential distribution of the transition time has the parameter (cf. Loistl/Landes (1989), p. 94):

$$I(z) = \sum_{z' \neq z} I(z, z')$$

The attached transition time τ is determined by the exponential distribution

$$(3) \quad R(T \leq t) = 1 - \exp(-I(z, z') * t)$$

The transition time t is equal to time \mathbf{t} until the realization of action A , assuming that no other observable market action takes place in the meantime. If there is no action with a shorter transition time, this action will be realized and the related transition from z to z' will be generated.

3. The state transitions are governed by the principle of the shortest reaction time, i.e. the highest utility (motivation potential). This principle can be realized by selecting the transition probabilities appropriately $I(z, z')$.

Starting from the state z , the probability $p(z, z')$ that the next state will be z' is proportional to the transition intensity:

$$(4) \quad p(z, z') := \frac{I(z, z')}{I_z}.$$

4. The following relation provides the link between the logit approach and the stochastic process (cf. the detailed reasoning in Landes/Loistl (1992)):

$$(5) \quad I(z, z'(A_{ijk})) = W * \exp(F_{ij}(A_{ijk}, z)).$$

This fundamental connection is possible since both approaches have the same structure:

The probability of selecting action A_{ijk} is (assuming the logit model) proportional to $\exp(F_{ij}(A_{ijk}, z))$. The transition probability $p(z, z')$ is proportional to the transition intensity $I(z, z')$. This relation provides transition times for each action. The expected transition time of an action will be:

$$I(z, z'(A_{ijk}))^{-1}.$$

The basic principle of KapSyn can be specified more precisely: **The greater the utility (motivation potential), the shorter the reaction time and the higher the probability that an action will be realized.**

2.3 Transition Rates as Control Elements in the Market Process

2.3.1 Formal Definition of Transition Rates

Market transitions are brought about by agents' actions. Every agent can carry out the following set of activities:

Buying and selling

- Submission of the agent's own bid or ask
- Acceptance of another bid or ask
- Cancellation of the agent's own bid or ask

Changes in price expectations

- Positive change in expectations
- Negative change in expectations

The acceptance of an existing bid or ask on the market results in a trade with a quoted price. This action is observed by all market participants. The same is true when an agent submits an own bid or ask, or when an agent cancels his/her bids or asks placed at an earlier point. In addition to executing bids and asks, the model allows all agents to think over their opinions as to their future price expectations, and revise these if necessary. Changes in expectations also lead to a transformation of the current market conditions. However, in contrast to buying and selling, such actions are not divulged to other market participants.

The basic model conforms to disclosure rules, and its rules should be adapted to the relevant rules applicable in concrete stock exchanges' cases. The individual decision-making processes by market participants as to whether or not to execute an action are governed by the market condition z applicable at a particular moment, in accordance with the Markov property. When an action is executed, the Markov process goes from the current state z to the state z' generated by the action. The following diagram makes this clear:

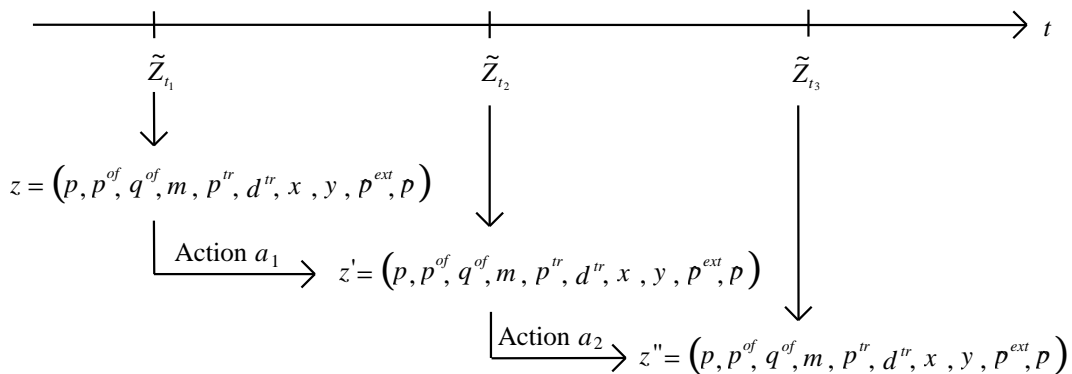


Fig 8 Transformation of the market condition vector by actions

The following section gives an overview of how the model operates in formal terms.

As has already been mentioned, market dynamics are described by a Markov process $\{\tilde{Z}_t, t \geq 0\}$ which is time-homogeneous. The process has continuous time and discrete state space. By providing the initial distribution of \tilde{Z}_0 and all transition probabilities $P(\tilde{Z}_{t+\Delta t} = z' / \tilde{Z}_t = z) = P_{z,z'}(t, t + \Delta t)$ the process can be described in full; in other words, all n-dimensional distribution functions can be derived.¹⁴

The specification of the transition probabilities of a Markov process with constant time and discrete state space is generally achieved by means of a system of differential equations. This system is represented in the present capital market model by the 'Master equation'.¹⁵

To draw up the Master equation, all transition rates in the process must be given:¹⁶ The transition rate from z to z' at time t can be obtained by assuming that the transition function $P_{z,z'}(t, t + \Delta t)$ at the point $\Delta t = 0$ is differentiable after Δt :

$$\left. \frac{dP_{z,z'}(t, t + \Delta t)}{d \Delta t} \right|_{\Delta t=0} = I_{z,z'}(t)$$

The probability theory interpretation of the transition rates can be clearly represented using the definition of the derivation as a limit of the difference quotient.¹⁷ The following relations are produced for $z \neq z'$ and $z = z'$:

$$z \neq z': \quad \lim_{\Delta t \rightarrow 0} \frac{P_{z,z'}(t, t + \Delta t) - P_{z,z'}(t, t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{P_{z,z'}(t, t + \Delta t)}{\Delta t} = I_{z,z'}(t) \geq 0$$

¹⁴ Cf. Breiman (1968) p.129, p.319 f. For the requirements under which the transition functions are valid as transition probabilities (Chapman-Kolmogorov equations, etc.) cf. Breiman (1969) p.198 f., Loistl/Landes (1989) p.96

¹⁵ See Haken (1983) for a discussion of the master equation as a version of Kolmogorov's forward equation. The master equation represents one of the displacement equations which are usually employed for describing dynamic systems. The following types of displacement equation exist, depending on the size of the state space and the number of parameters:

- The *Fokker-Planck equation* for continuous state space and continuous parameter space;
- The *Chapman-Kolmogorov equation* for discrete state space and discrete parameter space;
- The *master equation* for discrete state space and continuous parameter space.

See Loistl/Landes (1989) p.10; see Haken (1983) for further details.

¹⁶ e.g. See Haken (1983) p.82.

¹⁷ See Haken (1983) p.82f

The probability $P_{z,z'}(t, t + \Delta t)$ of state z being transformed into state z' in the time interval $[t, t + \Delta t]$ is approximately given by $I_{z,z'}(t)\Delta t$ where Δt is sufficiently small:

$$z = z' : \quad \lim_{\Delta t \rightarrow 0} \frac{P_{z,z}(t, t + \Delta t) - P_{z,z}(t, t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{P_{z,z}(t, t + \Delta t) - 1}{\Delta t} = I_{z,z}(t) \leq 0$$

The KapSyn model assumes time-homogeneous transition probabilities: ¹⁸

$$P_{z,z'}(t, t + \Delta t) = P_{z,z'}(\Delta t)$$

The transition probabilities do not depend on the exact position of the process on the time axis, but rather merely on the period Δt . In this case, the transition rates $I_{z,z'}(t)$ and $I_{z,z}(t)$ can be simplified to $I_{z,z'}$ and $I_{z,z}$.

If all transition rates are given, then the transition probabilities $P_{z,z}(t)$ - and thus also the one-dimensional boundary probabilities $P_z(t) = P(\tilde{Z}_t = z)$ of the process being modelled here - can be specified by solving the Master equation. The complexity of the Master equation, however, makes an explicit analytical solution very difficult. Nonetheless, an approximate solution can be found by simulating the process. This is the basis of the KapSyn Simulation: It imitates the stock exchange's performance by solving numerically via simulation the Master equation modelling the stock exchange performance.

To do this, let us assume that the process is viewed at a particular moment, time t . The market conditions at time t are represented by z . The length of time spent in state z needs to be found, as does the new state once state z has been left. The following notations will be used:

- Let the random variable \tilde{T} describe the length of time spent in state z . The distribution function $F(t) = P(\tilde{T} \leq t)$ determines the probability that the length of time is less than or equal to t time units. It represents the probability that the market will not leave its current state within the time interval $[t, t + t]$.

¹⁸ Time-homogeneous transitions imply independence of a particular location on the time axis, but not, however, from the current stochastic realization.

- Let the random variable \tilde{Y} describe the new state after leaving state z . The distribution $f(z') = P(\tilde{Y} = z')$ determines the probability that the market process will move towards state z' after leaving state z .

From Markov process theory we know that the distributions of \tilde{T} and \tilde{Y} are characterized as follows:¹⁹

- Distribution of the time spent in state z :

$$P(\tilde{T} \leq t) = 1 - e^{-\sum_{z'} I_{z,z'} t}$$

The time spent in state z is exponentially distributed with the parameter $\sum_{z'} I_{z,z'}$. The following equation is used to determine the mean period spent in state z :

$$(6) \quad E(\tilde{T}) = 1 / \sum_{z'} I_{z,z'}$$

- Probability of the new state after leaving state z :

$$(7) \quad P(\tilde{Y} = z') = \frac{I_{z,z'}}{\sum_{z'} I_{z,z'}}$$

The probability of moving from state z to state z' is determined by the relationship between the transition rate $I_{z,z'}$ of the change of state $z \rightarrow z'$ to the sum $\sum_{z'} I_{z,z'}$ of all transition rates defined in state z .

The random variables of the time spent \tilde{T} and of the next state \tilde{Y} are independent of each other.²⁰ The following equation thus applies to the joint probability distribution of the random variables \tilde{T} and \tilde{Y} :

$$(8) \quad P(\tilde{Y} = z', \tilde{T} \leq t) = \frac{I_{z,z'}}{\sum_{z'} I_{z,z'}} \left(1 - e^{-\sum_{z'} I_{z,z'} t} \right)$$

¹⁹ cf. Breiman (1969) p.211f

²⁰ See Loistl/Landes (1989) p.95f; see Breiman (1969) p.211 for a general overview.

The process dynamics can be simulated in exactly the same way to the rules described here: in accordance with an exponentially distributed random variable with the parameter $\sum_{z'} I_{z,z'}$, the market is in state z , and then moves to state z' with a probability $P(\tilde{Y} = z')$. The probabilities of state z'' being the state that z' will next move toward and the distribution of the time spent in state z' are now determined by the set of transition rates defined in state z' .

This general description of the way in which the process operates illustrates the fact that the transition rates exercise the greatest influence over the market status. The following section explains how the transition rates can be determined from an evaluation of alternative actions.

2.3.2 Linking Actions and Transition Rates

Each transition from state z to state z' is determined by the action $a_{k'}$ that causes state z' . The distributions $f(z')$ and $F(t)$ therefore describe the probability that action $a_{k'}$, which causes the state change $z \rightarrow z'$, will be performed, and the decision-making time until the realization of $a_{k'}$ of the agent concerned, measured by the elapsed time. The elapsed time of an action $a_{k'}$ is the time that passes until $a_{k'}$ is executed. However, if another activity is performed on the market in the meantime, this requires a review of the chosen action in the context of the information resulting from the new market conditions.²¹

The transition rate $I_{z,z'}$ therefore represents the probability of execution and execution time of the variable principally guiding action $a_{k'}$. The probability of execution of an action $a_{k'}$ can be formulated as follows in accordance with (7) with $I_{z,z'} = I_{a_{k'}}$:

$$(9) \quad P(a_{k'} \text{ is the next to be performed}) = \frac{I_{a_{k'}}}{\sum_{a_k \in A} I_{a_k}}$$

The probability that the next action to be carried out is $a_{k'}$ (for instance a bid by agent 3 for security 7), is given by the relationship between the transition rate of $a_{k'}$ to the sum of the transition rates of all activities A defined in the

²¹ See Loistl/Landes (1989) p.68.

current market state. The formulation of market state transition rates is thus linked to the formulation of transition rates via individual actions. The following diagram illustrates the way in which actions and transition rates are linked.

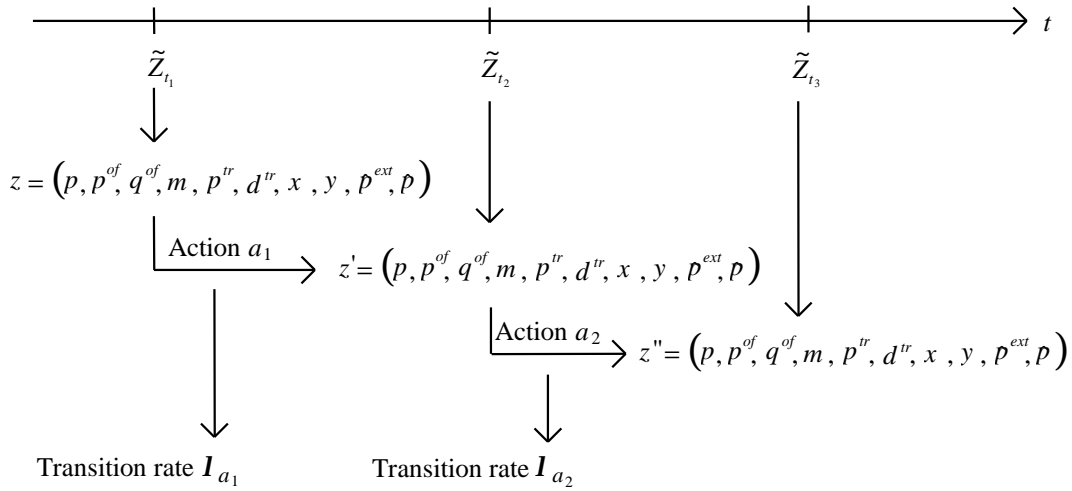


Fig 9 Transformation of the market state vector via actions²²

The market state z at time t_1 is transformed into state z' by action a_1 . The transition rate for the state shift from z to z' is I_{a_1} . Each possible activity a_k has a change of market state, a transition rate I_{a_k} , and an elapsed time, which is the period of time between t_1 and t_2 . Such an activity could be, for example, the placing of a bid or ask for a security by a market participant. The precise determinants also depend on the particular stock exchange organization.

The proportionality characteristic of the probability of execution described in (9) continues to apply even the probabilities are not formulated in terms of the set A of the activities of all agents on all stocks, but rather in terms of a subset of A .²³ Such a subset is represented, for example, by the set A_i of permissible activities by an agent i in the current market state. The contingent probability of execution of an activity $a_{ik} \in A_i$ is defined as follows:

$$(10) \quad P(a_{ik} \text{ is the next to be performed} \mid a_{ik} \in A_i) = \frac{I_{a_{ik}}}{\sum_{a_{ik} \in A_i} I_{a_{ik}}}$$

The probability that activity a_{ik} will be the next to be executed from the set of possible actions A_i is given by the relationship of the transition rate of a_{ik} to

²² Graphic according to Casey (1998) p.189. For the symbols used, see section 2.4.1.1

²³ See Loistl/Landes (1989) p.67 ff.

the sum of the transition rates of all activities A_i . This probability can also be interpreted as the probability that agent i will decide in favor of the alternative action $a_{ik} \in A_i$ from among a given set of actions A_i .

The problem of how to specify the *execution* probabilities of activities can thus be interpreted as a problem of how to specify the *selection* probabilities of activities. To determine the selection probabilities of an agent i with regard to a set of actions A_i , every action in A_i must be assigned a transition rate. Discussing the related problems in general terms, Haken (1983a) writes:

„The crux to derive a Master equation is not so much writing down the expressions [...], which are rather obvious, but to determine the transition rates I explicitly. This can be done in two ways. Either we can write down the I 's by means of plausibility arguments. [...] Another way, however, is to derive the I 's from first principles where mainly quantum statistical methods have to be used.“²⁴

In accordance with this last principle, the transition rates are specified in the Landes/Loistl model (1992) on the basis of aspects of decision theory. One area in which decision theory aspects relating to the formulation of transition rates are expressed is in the exponential formulation of the transition rates:²⁵

$$(11) \quad I_{a_{ik}} = e^{\Phi_i(a_{ik})}$$

Φ_i = Preference function of agent i with regard to action a_{ijk}

This formulation provides a foundation in decision theory for the individual execution probabilities in (9). They result in

$$(12) \quad P(\text{Agent } i \text{ selects action } a_{ik} \text{ for a given alternative amount } A_i) = \frac{e^{\Phi_i(a_{ik})}}{\sum_{a_{ik} \in A_i} e^{\Phi_i(a_{ik})}}$$

and are known as logit probabilities.²⁶ Decision theory principles are also expressed in the formulation of the preference function Φ_i in the exponent of

²⁴ Haken (1983) p.88. The variable w_i used by Haken, was replaced by I .

²⁵ See Landes/Loistl (1992) p.214

²⁶ cf. the logit model in section 2.2.1.2

(11). Despite the concrete formulation of Φ_i , the basic control principle is as follows:²⁷

- The greater the utility $\Phi_i(a_{ik'})$ of an action $a_{ik'}$ for an agent i , the higher its transition rate $I_{a_{ik'}} = e^{\Phi_i(a_{ik'})}$, the greater the probability of its execution and the shorter the mean wait until execution.
- The smaller the utility $\Phi_i(a_{ik'})$ of an action $a_{ik'}$ for an agent i , the lower its transition rate $I_{a_{ik'}} = e^{\Phi_i(a_{ik'})}$, the smaller the probability of its execution and the longer the mean wait until execution.

The evaluation of the action utility is generally focused on extractable information (current market prices, price trends, excess supply or demand, etc.) as well as on agents' price expectations at the start of trading.

The following section provides an overview of the interdependence of evaluation and action on the basis of the rules described in this section.

2.3.3 Interdependence of Evaluation and Action

The information/action cycle can be clearly illustrated using the following diagram.

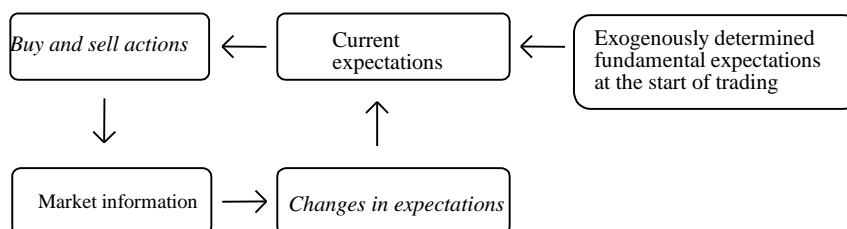


Fig 10 Interdependence of evaluation and action²⁸

When trading starts, every agent i , $i \in I = \{1..N\}$, has a fundamental, exogenously determined expectation of the future prices of each share j , $j \in J = \{1..M\}$. This price expectation is shaped by fundamental data such as, for example, the economic prospects of the company j or the industry in which j is active, the quality of j 's management, j 's expected sales growth, etc. Each agent diversifies his/her portfolio exogenously at the start of trading on the basis of his/her fundamental price expectations. The result of this

²⁷ See Loistl/Landes (1989) p.70f

diversification is manifested in the number of shares of each share type and in the amount of capital invested in bonds.

During trading, agents constantly observe what happens on the market. If necessary, an agent will adapt his/her portfolio to current market developments. In turn, bids and asks placed on the market are signals which are manifested in a number of ways, such as in current market prices, price trends, and excess supply or demand. These visible market signals contain objective information about market conditions. They influence an agent's expectations, particularly with regard to the prices that investors believe they will be able to realize on the market in the future. In turn, changes in agents' expectations influence the submission of current buy or sell orders.

2.4 The Model in Detail

This section presents the KapSyn model in detail. The first subsection describes the components of the market condition vector. The next subsection looks at the market state components \hat{p}^{ext} and \hat{p} in more detail. The third subsection describes the agent's planning process for executing actions, while the fourth and final subsection explores the process dynamics from a behavioral theory point of view.

2.4.1 The Market Condition Vector

2.4.1.1 Definition and Abbreviations

The state variable vector z , which comprises the multi-stock, multi-trader scenario discussed here, is characterized by:

$$z=(p, p^{of}, q^{of}, m, p^T, \Delta^T, x, y, \hat{p}, \hat{p}^{ext}, f_{S_{max}}) \in \mathbf{R}$$

\mathbf{Z} denotes the set of integers, \mathbf{N} the set of positive integers, \mathbf{Z}_+ the set of non-negative integers, \mathbf{R} the set of real numbers and \mathbf{R}_+ the set of non-negative real numbers.

The constituent variables are defined as follows:

$$p = (p_j)_{j \in J} \in \mathbf{R}_+^J$$

²⁸ cf. Landes/Loistl (1992) p.215., Casey (1998) p.192.

p_j current price of share j

$$p^{of} = (p_{ij}^{of})_{i \in I, j \in J} \in \mathbb{R}_+^{I \times J}$$

p_{ij}^{of} price of a valid share j offer of trader i

$$q^{of} = (q_{ij}^{of})_{i \in I, j \in J} \in \mathbb{Z}^{I \times J}$$

q_{ij}^{of} quantity of a valid share j offer of trader i

$$m = (m_j)_{j \in J} \in \mathbb{Z}^J$$

m_j market power of share j

$$p^T = (p_j^T)_{j \in J} \in \mathbb{Z}^J$$

p_j^T price trend of share j

$$D^T = (D_j^T)_{j \in J} \in \mathbb{Z}^J$$

D_j^T counter-trend of share j

$$x = (x_{ij})_{i \in I, j \in J} \in \mathbb{Z}_+^{I \times J}$$

x_{ij} number of shares j held by trader i

$$y = (y_i)_{i \in I} \in \mathbb{R}_+^I$$

y_i amount of cash (invested in bonds) held by trader i

$$\hat{p} = (\hat{p}_{ij})_{i \in I, j \in J} \in \mathbb{R}_+^{I \times J}$$

\hat{p}_{ij} price expectation of trader i for share j

$$\hat{p}^{ext} = (\hat{p}_{ij}^{ext})_{i \in I, j \in J} \in \mathbb{R}_+^{I \times J}$$

\hat{p}_{ij}^{ext} = intrinsic value estimate of trader i for share j

at the start of a trading session

$$f_{synax} = (f_{synax}) \in \mathbb{R}_+$$

f_{synax} futures quotation; future based on given stock index. The "SYNAX" futures

quotation is calculated by $p_{synax} = \sum_j \frac{1}{\text{number of stocks}} p_j \cdot$

The prices, amounts of cash, price expectations and intrinsic value estimates are measured as integer multiples of a given unit. The same is true of the number of shares held and the quantities offered. As is the case for p , p^o , y , \hat{p} , \hat{p}^{ext} and f_{synax} , the given unit can be a fraction of 1. As a result, these variables belong to \mathbf{R}_+ . The values of all variables, except the intrinsic value estimates, are changed by traders' actions during the trading session (the interval $0 \leq t \leq t_{max}$).

Only one offer can be simultaneously valid for each trader/share combination. The valid offers are stored in the fields p^o (prices) and q^o (quantities), with positive quantities corresponding to asks (offers to buy), and negative quantities to bids (offers to sell).

At the beginning of the process ($t=0$), the state variables have the following initial values:

$p_j = p_j^0$: initial quotation, e.g.
the closing quotations of the previous day of trading

$q_{ij}^{of} = 0$: no offers valid

$m_j = 0$: no demand-supply pressure

$p_j^T = \Delta_j^T = 0$: no trend

x_{ij} and y_i : initial individual start portfolio, (sample from an initial distribution, for example)

$\hat{p}_{ij} = \hat{p}_{ij}^{ext}$: initial price expectations coincide with intrinsic value estimations

\hat{p}_{ij}^{ext} : sample taken from an initial distribution and kept fixed during the trading session

$f_{synax} = p_{synax}^0$: with the initial SYNAX quotation $p_{synax}^0 = \sum_j const_j p_j^0$

- i indicates the number of the corresponding trader, including special traders, i.e.
 - the futures trader

- the market makers (depending on the stock exchange module)

all of whom have their own cash portfolio and stock portfolio.

- All η , ρ and ξ variables are weighting factors and can be viewed and changed in the parameters dialog box.

2.4.1.2 Market Prices

$$\mathbf{p} = (p_j)_{j=1..M} = (p_1, \dots, p_j, \dots, p_M)$$

The vector \mathbf{p} comprises the current market prices²⁹ of shares j , $j \in J = \{1, \dots, M\}$, at time t . Quotations are subject to the integer condition $p_j \in \{1, 2, \dots\}$.

2.4.1.3 Order Book

Price limit

$$\mathbf{p}^{of} = (p_{ij}^{of})_{i=1..N; j=1..M} = \begin{pmatrix} p_{11}^{of} & \cdots & p_{1M}^{of} \\ \vdots & p_{ij}^{of} & \vdots \\ p_{N1}^{of} & \cdots & p_{NM}^{of} \end{pmatrix}$$

and volume

$$\mathbf{q}^{of} = (q_{ij}^{of})_{i=1..N; j=1..M} = \begin{pmatrix} q_{11}^{of} & \cdots & q_{1M}^{of} \\ \vdots & q_{ij}^{of} & \vdots \\ q_{N1}^{of} & \cdots & q_{NM}^{of} \end{pmatrix}$$

The matrices \mathbf{p}^{of} and \mathbf{q}^{of} contain all order prices and their amounts on the market at time t . These matrices represent the market's order book at time t , which can be inspected by every market participant. The suffix *of* stands for 'offer'.

It is assumed that an agent i can only be active on the market with one valid order for a particular share j at any one time t during trading. The element matrix, p_{ij}^{of} , $p_{ij}^{of} \in \{1, 2, \dots\}$, comprises a price offer made by an agent i , $i \in I = \{1..N\}$, for a share j , $j \in J = \{1, \dots, M\}$, and the element $q_{ij}^{of} \in \{\dots, -1, 0, +1, \dots\}$ the appropriate order amount. Bid amounts are prefixed with a plus sign, ask

²⁹ For details of the calculation cf. Landes/Loistl (1992) p.212 f. and chapter 2.4.3.1. of this handbook.

amounts with a minus sign. If an agent i has a valid bid on the market for a share j at a time t , then $q_{ij}^{of} > 0$. Similarly, in the case of a valid ask, $q_{ij}^{of} < 0$. If an agent i has neither a valid bid nor a valid ask on the market for a share j at a time t , then $q_{ij}^{of} = 0$.

2.4.1.4 Excess Supply and Demand as Market Potential

$$\mathbf{m} = (m_j)_{j=1..M} = (m_1, \dots, m_j, \dots, m_M)$$

The vector \mathbf{m} comprises excess supply or demand for a share j that may exist at time t . $m_j \in \{\dots, -1, 0, +1, \dots\}$ applies. A positive market potential m_j for a share j signals excess buy orders, while a negative market potential signals excess sell orders. The excess supply/demand is measured by the difference between the number of bids on the market and the number of asks on the market. Thus, bids placed on the market raise the market potential m_j of a share j by one unit, while asks placed on the market lower the potential by one unit. Bids and asks which lead to transactions reduce or raise the market potential (by one unit) in keeping with the buy or sell order then removed from the market.

2.4.1.5 Price Trends

$$\mathbf{p}^{tr} = (p_j^{tr})_{j=1..M} = (p_1^{tr}, \dots, p_j^{tr}, \dots, p_M^{tr})$$

$$\mathbf{d}^{tr} = (d_j^{tr})_{j=1..M} = (d_1^{tr}, \dots, d_j^{tr}, \dots, d_M^{tr})$$

The vector \mathbf{p}^{tr} comprises any price trends $p_j^{tr}, p_j^{tr} \in \{\dots, -1, 0, +1, \dots\}$, of all shares j , $j \in J = \{1, \dots, M\}$, at time t . The vector \mathbf{d}^{tr} , $d_j^{tr} \in \{\dots, -1, 0, +1, \dots\}$, comprises their trend reversal indicators. These indicators focus on technical chart developments.

A rise in price of a share j results in a positive price trend, a fall in price in a negative price trend p_j^{tr} .³⁰ A recalculation of the price trend p_j^{tr} of share j always takes place if a trade in share j occurs. A change in direction of the current quoted price can either confirm or interrupt the previous trend. If the trend in direction of the current market price is in line with the trend in

³⁰ The inclusion of backward-looking state variables lessens the "amnesia" of the Markov process.

direction of the previous price trend p_j^{tr} (i.e. if the price difference between the quoted price at time t and the previous market price of share j has the same sign as the previous price trend p_j^{tr}), the current price trend p_j^{tr} is confirmed. In this case the previous price trend p_j^{tr} is updated in line with the strength of the change in price.

However, if the price change and price trend are prefixed with different signs, this may signal a trend reversal. The trend reversal indicator d_j^{tr} of share j is used to check whether this involves a 'real' trend reversal, or merely a short-term consolidation of the stock price.

Share j 's trend reversal indicator d_j^{tr} is a technical chart parameter that indicates when an existing trend is continued or interrupted. This figure is calculated in parallel to the price trend. A trend reversal is always assumed for a directional price movement if the stock price breaks out of a pre-defined band in the opposite direction to the previous price movement. The band can be set to any width. If the predefined band is crossed in the relevant direction, the price trend and trend reversal indicator are adapted to the price's new direction. In the case of short-term consolidation, on the other hand, the previous price trend p_j^{tr} is confirmed and merely corrected up or down, depending on the size of the current price change.

2.4.1.6 Securities Portfolios

$$\mathbf{x} = (x_{ij})_{i=1..N; j=1..M} = \begin{pmatrix} x_{11} & \cdots & x_{1M} \\ \vdots & x_{ij} & \vdots \\ x_{N1} & \cdots & x_{NM} \end{pmatrix}$$

Matrix \mathbf{x} comprises the agent's current stocks at time t . More precisely, the element x_{ij} , $x_{ij} \in \{1,2,\dots\}$ comprises the number of shares held by agent i , $i \in I = \{1..N\}$, in the company j , $j \in J = \{1,\dots,M\}$, at time t . Every share purchase leads to an increase in the holdings of the agents involved in the trade, while every share sale leads to a reduction in these holdings.³¹

³¹ Although short sales have been excluded in this version, the implementation of such a feature is possible.

2.4.1.7 Cash (Interest paying) holdings

$$\mathbf{y} = (y_i)_{i=1..N} = (y_1, \dots, y_j, \dots, y_N)$$

Vector \mathbf{y} comprises the cash held by the agent at time t . It is assumed that freely available cash can be invested in a fixed-interest investment. Each stock purchase leads to a reduction of agent i 's amount of cash, every stock sale leads to an increase in agent i 's amount of cash.³²

2.4.1.8 Fundamental Price Expectations

$$\hat{\mathbf{p}}^{ext} = (\hat{p}_{ij}^{ext})_{i=1..N; j=1..M} = \begin{pmatrix} \hat{p}_{11}^{ext} & \cdots & \hat{p}_{1M}^{ext} \\ \vdots & \hat{p}_{ij}^{ext} & \vdots \\ \hat{p}_{N1}^{ext} & \cdots & \hat{p}_{NM}^{ext} \end{pmatrix}$$

The matrix $\hat{\mathbf{p}}^{ext}$ comprises what are known as the agent's fundamental price expectations with regard to the shares traded on the market. It is assumed that every agent i , $i \in I = \{1..N\}$, can aggregate his/her expectations of the future performance of a share j , $j \in J = \{1, \dots, M\}$ to a single estimate, \hat{p}_{ij}^{ext} , at the start of trading. This single estimate represents the price of the share j expected by agent i at the end of the period, i.e. the future price which the agent attaches to the security. The agents' fundamental price expectations are exogenously determined at the start of trading and are assumed to be constant during trading.³³ For this reason, they are also described as the agent's external price expectations. However, they can change during trading as a result of reports on changes in the corporation's fundamental situation.

The KapSyn model assumes a multifactor model for evaluating a share's intrinsic value. The following influencing factors are included: changes in market prices, price trends, and expectations of future prices and order prices. Of course, additional factors or other factors cited in the literature can also be

³² Although long sales have been excluded in this version, the implementation of such a feature is possible.

³³ Landes/Loistl (1992) p.215: "The factors influencing the fundamental situation of a corporation fluctuate much less frequently than market prices. We assume, therefore, that there are not external effects causing a change in the fundamental situation in the short run. Consequently, the estimates \hat{p}_{ij}^{ext} are assumed to be predetermined by external factors and fixed during the trading session."

included. For instance, the factors named by Fama/French (1992) are compatible with the factors used here.³⁴

The agents' fundamental price expectations are built on internal company data (such as the quality of the management) and external data (such as general economic developments). The price expectation's determination by fundamental data does not rule out the existence of other market participants who base their price expectations using other methods, such as technical analysis, for example. Technical analysts, like fundamentalists, will have an expectation of the general price development of a share that is formed on the basis of external data, but they will take current market activities into account when forming their expectations to a greater extent than other groups of investors. The inclusion of a group of investors guided by technical market analysis is expressed in the fact that this group of agents is prepared to adapt their price expectations to current market activities during trading. The concrete behaviour is controlled by relevant setting of the specific KapSyn parameters. Detail see chapter 3.4.

The current price expectation \hat{p}_{ij} of an agent i with regard to a share j at a time t results from the fundamental price expectation \hat{p}_{ij}^{ext} plus the adjustments made to expectation as a result of market developments up to time t .

$$\begin{array}{ccc}
 \text{Fundamental} & & \text{Technical} \\
 \text{indicators} & & \text{indicators} \\
 \text{influence} & & \text{influence} \\
 \downarrow & & \downarrow \\
 \hat{p}_{ij} & = & \hat{p}_{ij}^{ext} + \Delta \hat{p}_{ij}
 \end{array}$$

$\Delta \hat{p}_{ij}$ = Amount of all expectation adjustments up to time t

The current price expectations of all agents i , $i \in I = \{1..N\}$, with regard to all shares j , $j \in J = \{1, \dots, M\}$, at a time t during trading are contained in the matrix $\hat{\mathbf{p}}$:

$$\hat{\mathbf{p}} = (\hat{p}_{ij})_{i=1..N; j=1..M} = \begin{pmatrix} \hat{p}_{11} & \cdots & \hat{p}_{1M} \\ \vdots & \hat{p}_{ij} & \vdots \\ \hat{p}_{N1} & \cdots & \hat{p}_{NM} \end{pmatrix}$$

³⁴ cf. Fama/French (1992)

2.4.1.9 Determining Distribution comprising Subjective Price Expectations

At the beginning of a trading session every trader has a price expectation regarding each of the available shares. The question whether his or her discounted price expectation is higher or lower than the current share price has an impact on the trader's decision to buy or sell. The set of individual traders' price expectations is described by means of a frequency distribution. Currently, the frequency distribution consists of two truncated normal distributions, each described by mean and standard deviation. Each of the means is expressed as percentage deviation from the opening quotation.

Traders can revise their subjective expectations. They do this when the transition time of their expectation adjustment is shorter than the minimum transition times of market events. The adjustment of price expectations plays an important role in the development of share prices. This is used when drawing up prototypical KapSyn parameter patterns.

The \hat{p}^{ext} distribution (i.e. the distribution of stock price expectations at the start of a simulation) is calculated as follows (omitting the specification for stock j):

$$\begin{aligned}
 &\text{mean of lower segment of } \hat{p}^{ext} \text{ distribution,} &&= p_0 \cdot \hat{p}_{factor1}^{ext} \\
 &\text{(representing the pessimists, for example)} \\
 &\text{mean of upper segment of } \hat{p}^{ext} \text{ distribution,} &&= p_0 \cdot \hat{p}_{factor2}^{ext} \\
 &\text{(representing the optimists, for example)} \\
 &\text{standard deviation of} && \mathbf{s}_{rel1} = \frac{p_0 \cdot \mathbf{s}_{fak1}}{100}, \text{ with } p_0 \text{ as initial stock price} \\
 &\text{lower segment} \\
 &\text{standard deviation of} && \mathbf{s}_{rel2} = \frac{p_0 \cdot \mathbf{s}_{fak2}}{100}, \text{ with } p_0 \text{ as initial stock price} \\
 &\text{upper segment}
 \end{aligned}$$

The calculation of any individual trader's estimate is based on these figures and derived from the formula below, which approximates the exact formula for truncated normal distribution figures.

$$\hat{p}_{trader\ i(i=1..20),stock\ j}^{ext} = \left(\frac{p_0 \cdot \hat{p}_{factor1}^{ext} + p_0}{2} + \sqrt{\frac{-2 \log(i/20)}{1,5}} - 1 \right) \mathbf{s}_{rel1} - random(0..1) \frac{\mathbf{s}_{rel1}}{5} + \frac{\mathbf{s}_{rel1}}{10}$$

$$\hat{p}_{\text{trader } i(i=21..40), \text{stock } j}^{\text{ext}} = \left(\frac{p_0 \cdot \hat{p}_{\text{factor } 2}^{\text{ext}} + p_0}{2} + \sqrt{\frac{-2 \log\left(\frac{i-20}{20}\right)}{1,5}} - 1 \right) \mathbf{s}_{\text{rel}2} - \text{random}(0..1) \frac{\mathbf{s}_{\text{rel}2}}{5} + \frac{\mathbf{s}_{\text{rel}2}}{10}$$

$\hat{p}_{\text{trader } i(i=1..20), \text{stock } j}^{\text{ext}}$ Individual trader i 's own estimation of the intrinsic value, belonging to the lower segment of the entire p^{ext} distribution (comprising the estimates of traders 1 to 20)

$\hat{p}_{\text{trader } i(i=21..40), \text{stock } j}^{\text{ext}}$ Individual trader i 's own estimation of the intrinsic value, belonging to the upper segment of the entire p^{ext} distribution (comprising the estimates of traders 21 to 40)

The square root sections of the formula describe truncated normal distributions. The range of these truncated normal distributions varies due to factors $\mathbf{s}_{\text{factor}1}$ and $\mathbf{s}_{\text{factor}2}$ respectively. Additionally, a random term

$$- \text{random}(0..1) \frac{\mathbf{s}_{\text{rel}k}}{5} + \frac{\mathbf{s}_{\text{rel}k}}{10}, k \in \{1,2\}$$

is added. This is responsible for variations in the truncated normal distributions.

Fig 11 gives two examples of \hat{p}^{ext} distributions. The left-hand side has quite a large sigma, while the right-hand side has a small sigma. The first line gives the unique opening quotation $p_0 = 200.00$. The second line characterizes the two groups of traders in the \hat{p}^{ext} distribution. The names given refer to the distribution segment's mean values, which are given in the next line. They are the same for both examples. The lower segment has mean of 0.97 while the upper segment has a mean of 1.03.

The explicit shape of the frequency distributions comprising the individual trader's explicit fundamental value estimates is given in Table 4 below. The exact numbers are computed according to the formula given to calculate \hat{p}_i^{ext} . It should be stressed once again that the individual estimates are computed by the frequency distribution's parameters and not by individual estimates.

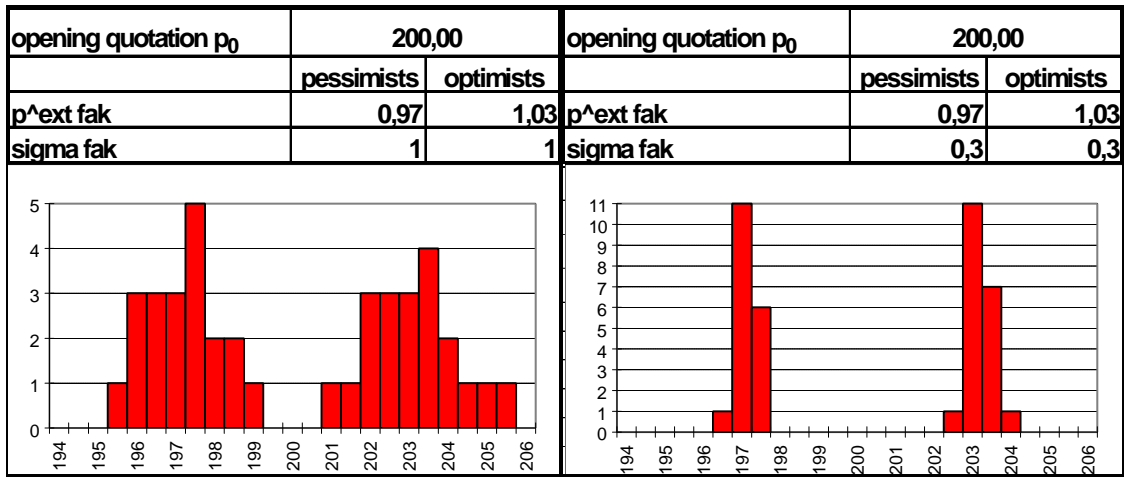


Fig 11 Two \hat{p}^{ext} distributions

Trader No.		Individual Price Expectation p^{ext}			
		Example 1		Example 2	
Pess.	Opt.	Pessimists	Optimists	Pessimists	Optimists
1	21	198.90	204.88	197.64	203.56
2	22	198.45	204.73	197.48	203.42
3	23	198.05	204.08	197.37	203.31
4	24	197.95	203.77	197.27	203.24
5	25	197.65	203.81	197.26	203.22
6	26	197.42	203.36	197.19	203.19
7	27	197.42	203.21	197.06	203.10
8	28	197.09	203.07	197.05	203.03
9	29	196.90	202.92	196.99	203.00
10	30	196.78	202.73	196.95	202.92
11	31	196.98	202.87	196.93	202.94
12	32	196.60	202.80	196.94	202.93
13	33	196.54	202.51	196.84	202.86
14	34	196.21	202.26	196.78	202.79
15	35	196.45	202.33	196.77	202.78
16	36	196.23	202.08	196.72	202.70
17	37	195.82	201.87	196.64	202.66
18	38	195.79	201.83	196.62	202.59
19	39	195.46	201.34	196.50	202.53
20	40	195.17	200.78	196.35	202.45

Tab 3 \hat{p}_i^{ext} individual value estimates for the examples above

2.4.1.10 The Futures Trader

In addition to the traders, the KapSyn model also comprises a *futures trader*, who performs arbitrage transactions between the spot and the derivatives market. Professionals requested that futures traders be implemented as they are an important supply/demand factor.

Above and beyond this, the formation of a "SYNTAX" price composed of KapSyn share prices depicts an analogue to the DAX. Furthermore, a SYNTAX futures price is also calculated (see below). If the SYNTAX future quotes above or below the current SYNTAX listing, it offers the futures trader an incentive to either buy or sell SYNTAX contracts. Like the SYNTAX price, these contracts consist of KapSyn shares. In the basic version, the SYNTAX is calculated according to the Laspeyre index formula, like the DAX.

Fluctuations in the SYNTAX price can be described by a jump process that reverts to the mean. This process embodies a market trend reversal directed towards the current SYNTAX price. The SYNTAX transition rate W_{syntax} is determined by

$$w_{syntax}^{\pm} = W_{syntax} e^{\pm(d_f + ((p_{syntax} - f_{syntax})))}$$

where d_f constitutes the trend continuation component, p_{syntax} the current SYNTAX price and f_{syntax} the current SYNTAX futures price.

Trade in blocks of SYNTAX shares is influenced by the cost of the carry rate c_{syntax} by:

$$\begin{aligned} f_{syntax} &\geq (1 + c_{syntax}) p_{syntax} \text{ (bid)} \\ f_{syntax} &\leq (1 - c_{syntax}) p_{syntax} \text{ (ask)} \end{aligned}$$

The quantity of the SYNTAX share blocks q is determined by:

$$q = \mathbf{x}_F (r_{syntax} - c_{syntax})$$

The transition rate w_F of futures traders' arbitrage activity is as follows:

$$w_F = W_F e^{0.5x_F (r_{synax} - c_{synax})^2}$$

where r_{synax} is the yield of an alternative investment.

A transition period is calculated with the help of w_F , just as is the case with spot traders' transition rates. This transition period determines the point in time at which the futures trader carries out transactions on the market.

2.4.2 Adapting Expectations

In contrast to the usual rational expectation hypothesis, it is assumed here that market participants can have divergent expectations of future developments, and hence individual price expectations may also differ. We do assume rational expectations at the individual level in the sense that every agent believes that *his/her* expectation will be realized at the end of the planning period. However, market participants are assumed to have subjective rational expectations inasmuch as it is assumed that they are convinced that their expectation of a share's price at the end of the planning horizon will correspond to the actual quotation at the end of the planning horizon. The existence of varying expectations means that market participants with high price expectations will buy from participants with low expectations, in the hope of making a market profit. The following frequency distribution results from the aggregation of all individual price expectations for a share j :

At the start of trading:

$$\left(\hat{p}_{ij}\right)_{i=1..N} = \left(\hat{p}_{ij}^{ext}\right)_{i=1..N} = \left(\hat{p}_{1j}^{ext}, \dots, \hat{p}_{ij}^{ext}, \dots, \hat{p}_{Nj}^{ext}\right)$$

During trading:

$$\left(\hat{p}_{ij}\right)_{i=1..N} = \left(\hat{p}_{1j}, \dots, \hat{p}_{ij}, \dots, \hat{p}_{Nj}\right)$$

$$\hat{p}_{ij} = \hat{p}_{ij}^{ext} + \Delta\hat{p}_{ij}$$

The market participants constantly monitor their subjective price expectations $\left(\hat{p}_{ij}\right)_{i=1..N}$ for a share j during trading and revise if necessary. Positive factors influencing market sentiment lead to positive adjustments of expectations by the agents, while events having a negative impact on market climate encourage negative adjustments of expectations. Depending on its formation,

the distribution $(\hat{p}_{ij})_{i=1..N}$ can be taken as the market's current price expectation for a share j .

2.4.3 Agents' Decisions to Buy and Sell

2.4.3.1 Rate of Return and Execution's Feasibilities as the Factors Determining Bids and Asks

The distribution of individual assumptions of intrinsic value for a share j can be deduced from the distribution of individual price expectations for it. An agent i 's assumption as to a share j 's intrinsic value arises from discounting his/her price expectation at the time at which the estimate is made. If the market participants behave in a risk-neutral manner, if interest rates are linear and if the observation is for a single period, then the following simple relationship applies exactly:

$$p_{ij}^{cut} = \frac{\hat{p}_{ij}}{1+r}$$

r = risk-free rate

We use the formula as a proxy. The expression p_{ij}^{cut} represents the present value of agent i 's stock price estimate \hat{p}_{ij} .

A distribution of the market's assumptions as to a share j 's intrinsic value can be gained by aggregating all individual NPV estimates in the same way as for the distribution of the market's price expectation for this share:

At the start of trading the following distribution applies:

$$(p_{ij}^{cut})_{i=1..N} = (p_{1j}^{cut}, \dots, p_{ij}^{cut}, \dots, p_{Nj}^{cut}), \quad p_{ij}^{cut} = \frac{\hat{p}_{ij}^{ext}}{1+r}$$

During trading the following distribution applies:

$$(p_{ij}^{cut})_{i=1..N} = (p_{1j}^{cut}, \dots, p_{ij}^{cut}, \dots, p_{Nj}^{cut}), \quad p_{ij}^{cut} = \frac{\hat{p}_{ij}}{1+r} = \frac{\hat{p}_{ij}^{ext} + \Delta\hat{p}_{ij}}{1+r}$$

The distribution of subjective intrinsic value assumptions at the start of trading reflects the NPV estimates of all market participants for share j at the start of trading. This distribution can also be interpreted as the market's fundamental intrinsic value assumption with regard to the shares.

The distribution of subjective intrinsic value assumptions at a particular point in time during trading provides the basis to compute the market participant i 's current NPV p_{ij}^{cut} for a share j . The current NPV p_{ij}^{cut} of an agent i for a share j at a particular time forms the basis of his/her decision to buy or sell that share. The planning process involved here will be described in the following section.

Every market participant will develop a strategy in line with the courses of trading which s/he considers possible, and can then use this strategy to react to certain changes in the state of affairs during trading. A large number of different factors can be seen as causing a change in the state of affairs on the stock exchange.

The process by which an agent i , $i \in I = \{1..N\}$, makes an individual decision to submit an order for a share j , $j \in J = \{1, \dots, M\}$, at a time t during trading, involves the following steps:

1. Decision as to whether to buy or sell share j
2. Establishment of permissible order prices
3. Establishment of permissible order amounts
4. Evaluation of defined buy or sell actions
5. Selection of an action

These decisions will be described in detail in the following sections.

The principal decision is based on considerations of rates of return and realization:

- An order is only considered if it promises a positive yield.
- An order is only considered if it has a chance of being realized.

The criterion for the yield of an order is the order's *expected* yield. The conditional equation for the expected yield of an order at price p by an agent i for a share j is as follows:³⁵

³⁵ cf. Landes/Loistl (1992) p.217.

$$(13) \quad p(1 + \hat{r}_{ij}) = \hat{p}_{ij} \quad \Leftrightarrow \quad \hat{r}_{ij} = \frac{\hat{p}_{ij}}{p} - 1$$

It is assumed that every agent can invest in both shares and interest bearing cash holdings. With cash holdings, the market rate of return r is attained. Comparing \hat{r}_{ij} with r allows a decision to be made as to whether to buy or sell a share at price p . The following relationships result:

$$r < \hat{r}_{ij} \Rightarrow \text{bid}; \quad r \geq \hat{r}_{ij} \Rightarrow \text{ask}$$

If the conditional equation (13) is employed by \hat{r}_{ij} , the following then follows:

$$(14) \quad r < \frac{\hat{p}_{ij}}{p} - 1 \Rightarrow \text{bid}; \quad r \geq \frac{\hat{p}_{ij}}{p} - 1 \Rightarrow \text{ask}$$

The present value $p_{ij}^{cut} = \hat{p}_{ij} / (1+r)$ is derived from the future expected price \hat{p}_{ij} discounted with the appropriate interest rate. The discount rate can be selected according to the specific market conditions.

This p_{ij}^{cut} provides the means to model the decision behavior of an agent I when defining bid/ask prices that are worth realizing for a share j as follows:

- Buy at all prices p that are smaller than the present value p_{ij}^{cut} of the expected share price j .
- Sell at all prices p that are greater than or equal to the present value p_{ij}^{cut} of the expected share price of share j .

Feasible bid prices are defined by the interval $(0, p_{ij}^{cut})$, feasible ask prices are defined by the interval $[p_{ij}^{cut}, \infty)$. Fig 12 illustrates this relationship.

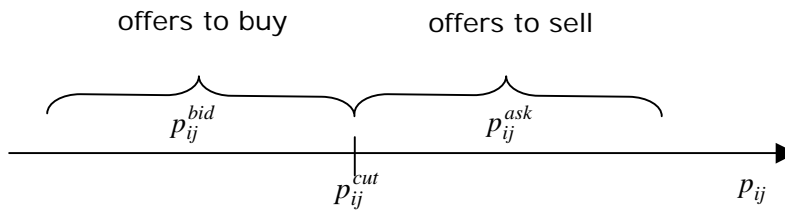


Fig 12 Agent-specific valid range of bid/asks

An order's chance of realization acts as a criterion for its feasibility. The chance of realization can be assessed by comparing the order's price limit with the bids and asks already on the market. The following relations apply:

- A bid has a chance of realization if its price limit p is *higher* than the price limits of all bids already on the market.
- An ask has a chance of realization if its price limit p is *lower* than the price limits of all asks already on the market.

The chance of an offer for a share j being realized is examined by comparing the offer's price limit with the 'marginal market price' p_j^{cut} . This marginal price separates all bids and asks already placed on the market for a share j according to their limits. It is calculated as follows:³⁶

$$(15) \quad p_j^{cut} = \begin{cases} p_j^{ask} + 0,5, & \text{if } p_j \leq p_j^{ask} \\ p_j, & \text{if } p_j^{ask} < p_j < p_j^{bid} \\ p_j^{bid} - 0,5, & \text{if } p_j \geq p_j^{bid} \end{cases}$$

p_j^{ask} = maximum bid price for a share j on the market

p_j^{bid} = minimum ask price for a share j on the market

p_j = current market price for a share j

p_j^{cut} = marginal market price

The following diagram illustrates the decisive significance of p_j^{cut} as the watershed between supply and demand.

³⁶ cf. Landes/Loistl (1992) p.217.

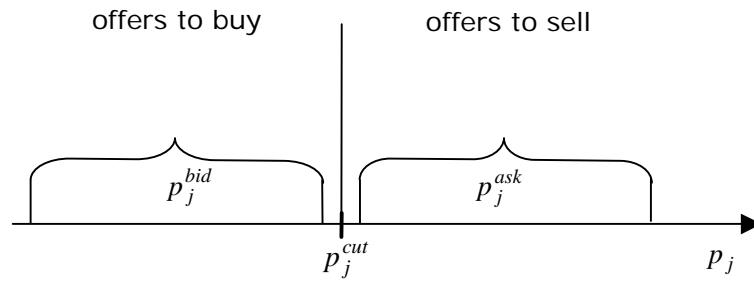


Fig 13 Market-specific set of bid/asks

All limit orders on the market with limits below the marginal price p_j^{cut} represent bids, while all limit orders with limits above p_j^{cut} represent asks. The feasibility of order prices for a share j is then determined as follows:

- A bid for a share j has a chance of being realized if the order price p_j^{bid} is higher than the market's marginal price, in other words, $p_j^{bid} > p_j^{cut}$.
- An ask for a share j has a chance of being realized if the order price p_j^{ask} is lower than the market's marginal price, in other words, $p_j^{ask} < p_j^{cut}$.

Following the feasibility criterion, permissible bid prices are defined by the interval (p_j^{cut}, ∞) , while permissible ask prices are defined by the interval $(0, p_j^{cut})$.³⁷

The simultaneous observation of offers which are worth realizing and offers which are feasible leads to agent i making a basic decision as to whether to buy or sell a share j :³⁸ There are two possible cases:

Situation		Market effect
I	$p_{ij}^{cut} > p_j^{cut}$	Agent i is a potential buyer of share j .
II	$p_{ij}^{cut} < p_j^{cut}$	Agent i is a potential seller of share j .

In situation I, the individual marginal price p_{ij}^{cut} is higher than the marginal market price. The agent i is a potential buyer of share j . Buy orders exist which are both feasible and worth realizing.

³⁷ cf. Landes/Loistl (1992) p.217.

In situation II, the individual marginal price p_{ij}^{cut} is lower than the marginal market price. The agent i is a potential seller of share j . Sell orders exist which are both feasible and worth realizing.

The following diagrams show the distribution of potential buyers and sellers of a share j , based on their subjective value estimations $(p_{ij}^{cut})_{i=1..N}$, according to the marginal market price p_j^{cut} of this share j .

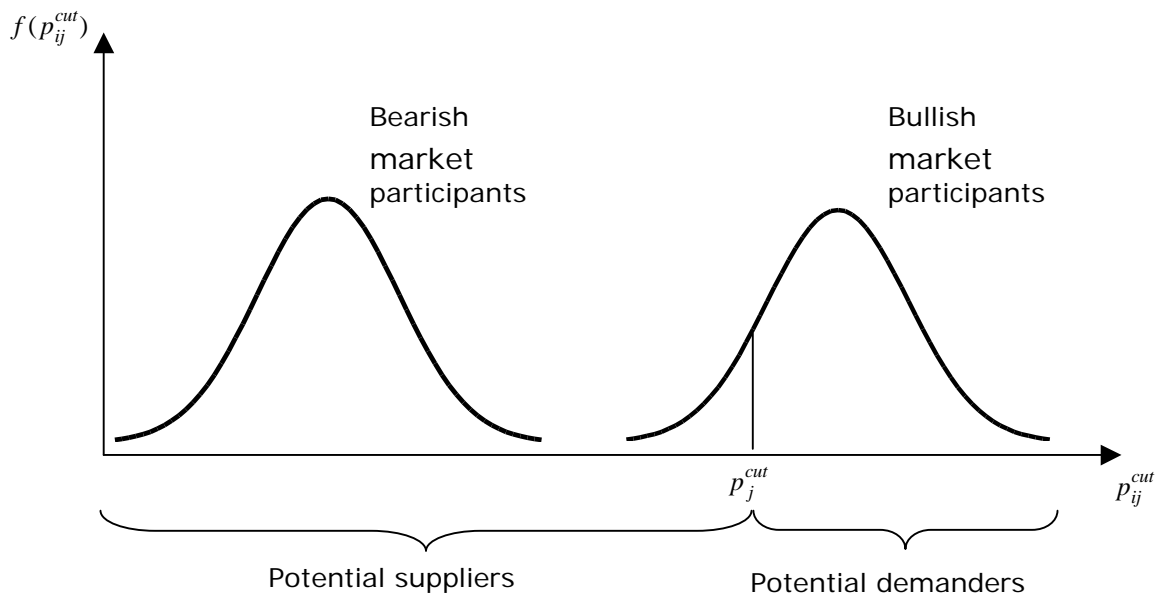


Fig 14 Bearish market scenario

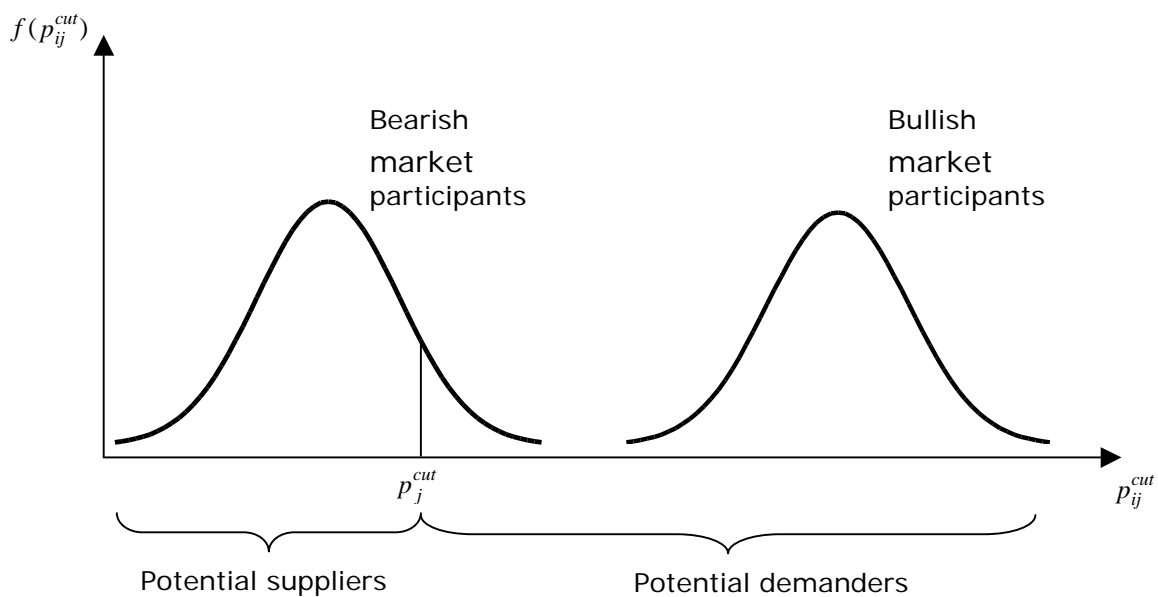


Fig 15 Bullish market scenario

³⁸ For the details of the integer condition of price offers see Landes/Loistl (1992) p.218.

All agents whose estimations' present value p_{ij}^{cut} lies to the left of p_j^{cut} are potential sellers of share j . All agents whose estimations' present value p_{ij}^{cut} lies to the right of p_j^{cut} are potential buyers of share j . In fig 15, the ratio of potential buyers to potential sellers is in favor of the sellers, and in fig 16 in favor of the buyers.

As mentioned before, generally market participants are represented by a (bimodal) distribution, in line with the parameters in the model. The fixing of the parameters was described in detail in section 2.4.1.9.

The characterization of a bullish or bearish market for a share j using the distribution of the subjective value estimates $(p_{ij}^{cut})_{i=1..N}$ and the state of the marginal market price p_j^{cut} also includes the level of interest rates on the interest bearing cash market. The higher the interest rate r – it is assumed to function as a calculatory interest rate for determining the present value of the future price estimate – the lower the expectations' present value of the market participants. Consequently, rising or falling interest rates will lead to a shift to the left or right in the distribution of the value estimates $(p_{ij}^{cut})_{i=1..N}$ for a share j . The ratio of potential buyers to potential sellers also shifts accordingly.³⁹ This confirms the typical experience that rises in interest rates lead to drops in share prices.

2.4.3.2 Establishing Bid and Ask Prices

Following the decision in principle of an agent i to make a bid/ask for a share j , it is necessary to define the set of bid and ask prices. The following basic relationships apply:

1. An agent i is a potential buyer of share j , i.e., $p_{ij}^{cut} > p_j^{cut}$. The sets of acceptable bid prices (expressed as integers) lie in the interval $(p_j^{cut}, p_{ij}^{cut})$.⁴⁰
2. An agent i is a potential seller of share j , i.e., $p_{ij}^{cut} < p_j^{cut}$. The sets of acceptable ask prices (expressed as integers) lie in the interval $[p_{ij}^{cut}, p_j^{cut})$.⁴¹

³⁹ See Casey (1998) p.210.

⁴⁰ If the agent has already placed an ask on the market, he/she will only be permitted to cancel this order.

The bid and ask prices defined under 1 and 2 for buyers and sellers are both worth being realized and capable of being realized.⁴² If bids and asks for a share j have already been placed on the market by other market participants, there will be a restriction on the acceptable bid/ask prices of an agent i for j .

1. An agent i is a potential buyer of share j and asks for this share j have already been submitted by other market participants on the market.

The prices for the asks on the market are represented by arrows in Fig 17.⁴³

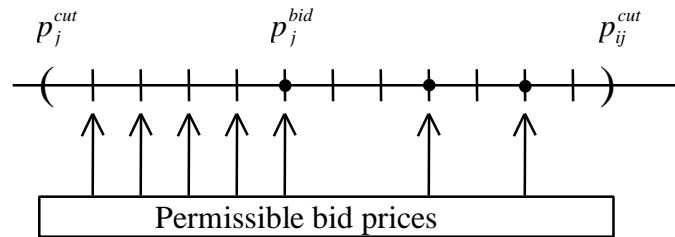


Fig 16 Permissible bid and ask prices for a potential buyer

In this case, the number of permissible bid and ask prices of agent i for share j is derived from

- the set of ask prices on the market, and
- the set of bid and ask prices located between the marginal market price p_j^{cut} and the lowest ask price on the market p_j^{bid} .

The maximum bid amount for those prices which lead to transactions is limited to the amount of the relevant ask. As regards the amount of bid and ask prices $p \in (p_j^{cut}, p_j^{bid})$, the only criterion is that $q > 0$.

2. An agent i is a potential seller of a share j , and other market participants have made bids for j .

The prices for the bids on the market are represented by arrows in the following diagram.⁴⁴

⁴¹ If the agent has already placed a bid on the market, he/she will only be permitted to cancel this order.. cf. hierzu Landes/Loistl (1992) p.218.

⁴² For $p_{ij}^{cut} = p_j^{cut}$ agent i is indifferent as regards the issuing of a bid or ask for share j .

⁴³ p_j^{bid} indicates the minimum ask price for a share j .

⁴⁴ p_j^{ask} indicates the maximum bid price for a share j .

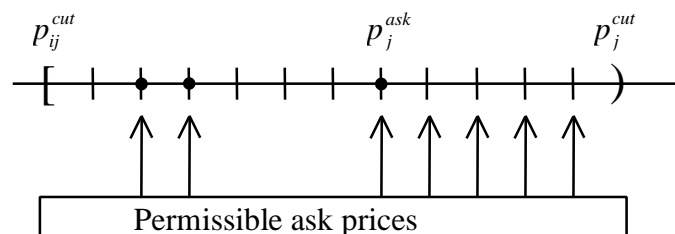


Fig 17 Acceptable bid and ask prices for a potential seller

The set of acceptable asks for agent i for share j is now determined by

- the set of bid prices on the market, plus:
- The set of prices which lie between the highest bid price on the market p_j^{ask} and the marginal market price p_j^{cut} .

Just as in the case of the potential buyer, the maximum amount of order prices leading to transactions is restricted to the set of the relevant bids.⁴⁵ For the sets of the order prices $p \in (p_j^{ask}, p_j^{cut})$, all that is required is $q < 0$.

The fixing of the amounts the agents want to trade follows the decision to buy or sell. The following section outlines the basic calculation procedure for determining the amount which the agent will want to trade given a particular acceptable order price.

In concrete cases this must be adjusted to take account of stock exchange regulations.

2.4.3.3 Determining Permissible Bid/Ask Amounts

Calculation of the amount belonging to a given order price p is performed in a simplified manner by means of an objective function based on the portfolio's return and risk. In the current version, transition rates are not employed in these calculations, due to limited processing capacity. Nevertheless, in the future the goal is to simultaneously assess prices and amounts when evaluating bids and asks. Operationalization of the portfolio return is performed on the basis of the subjective expected portfolio yield. Operationalization of the portfolio risk is performed using benchmark portfolios. At the beginning of the planning period, every market participant has a benchmark portfolio in mind, which they must try to work towards. Thus, the measure of risk relates to the deviation of the actual values from the targets. These deviations are laid down

⁴⁵ The individual budget limits must be observed.

as part of efficient portfolio management. Finally, portfolio return and risk are aggregated to a multi-attribute target function, and the two attributes are linked and added together. The formal relationships are presented below.⁴⁶

The portfolio return is calculated on the basis of current market values. The return results from the quotient of the subjectively expected final portfolio value \hat{V}_i and the current portfolio market value V_i :

$$(16) \quad \hat{R}_i = \frac{\hat{V}_i}{V_i} - 1 = \frac{y_i(1+r) + \sum_{j=1}^M x_{ij} \hat{p}_{ij}}{y_i + \sum_{j=1}^M x_{ij} P_j} - 1$$

This quantity describes the subjectively expected portfolio return if the agent were to form a portfolio in line with his/her actual shareholdings in the current market situation.

The portfolio risk is defined as the deviation of the actual portfolio structure from the desired portfolio structure. The portfolio structure is formulated on the basis of current market values, in a similar way to the way in which the portfolio return is calculated. The target market value holdings for all investments traded on the market are denoted by $s_{ij}^{ext}, j=0, \dots, M$. In this context, $0 \leq s_{ij}^{ext} \leq 1$ and $\sum_{j=0}^M s_{ij}^{ext} = 1$: The market value holdings that are actually

realized are calculated as follows:

$$s_{i0} = \frac{y_i}{V_i} = \text{Proportion of market value of capital invested in bonds}$$

$$s_{ij} = \frac{x_{ij} P_j}{V_i} = \text{Proportion of market value of capital invested in share } j$$

These holdings vary over time in line with the price fluctuations of the shares held in the portfolio. Over and above this, every transaction leads to a change in the share composition of the portfolio. The measure of risk used in KapSyn is as follows:

⁴⁶ The parallel views for active portfolio management are obvious. For a discussion of active portfolio management see Grinold/Kahn (1995).

$$(17) \quad S_i = \sum_{j=0}^M (s_{ij} - s_{ij}^{ext})^2$$

Total variance has not been taken into account here. This concept follows again active portfolio management, investigating the risk due to deviations from the benchmark.

The target function results from the linking and adding of portfolio return and portfolio risk in (16) and (17):

$$(18) \quad R_i = \hat{R}_i - \frac{1}{2} \mathbf{x}_i^{risk} S_i$$

$$\mathbf{x}_i^{risk} = \text{behavioural parameter, } \mathbf{x}_i^{risk} > 0$$

R_i can be described as the risk-adjusted yield potential. This potential varies during the process; the variation is initially independent of any transactions and entirely the result of price changes in the shares contained in the portfolio. Furthermore, every restructuring of the portfolio leads to a further change in the potential.

The following sections examines the price and amount orders of an agent i for a share j . This order is represented by (p, q) . The variable t is used as a summation index in the following summands. If the (p, q) orders placed by the agent i for the share j is accepted by the market, this will lead to a change in agent i 's cash in hand by pq monetary units and to a change in the shareholdings of share j by q units.

Agent i 's potential price limits and amount for share j are evaluated with regard to the change in the risk-adjusted return potential which they cause when the order is accepted by the market. The yield potential \hat{R}_i changes due to a (p, q) transaction of agent i for share j as follows:

$$\hat{R}_i = \frac{\hat{V}_i}{V_i} - 1 \quad \Rightarrow \quad \hat{R}_i(p, q) = \frac{\hat{V}_i(p, q)}{V_i(p)} - 1$$

The function $\hat{R}_i(p, q)$ results as follows:

$$(19) \quad \hat{R}_i(p, q) = \frac{y_i(1+r) + \sum_t x_{ij} \hat{p}_{ij} + q(\hat{p}_{ij} - p(1+r))}{y_i + \sum_{t \neq j} x_{it} p_t + x_{ij} p} - 1$$

It is clear that the portfolio price induced by a (p, q) order – contained in the denominator of (19) – depends solely on the size of the order price p , and not on the order's amount q . On the other hand, the subjectively expected future value of the portfolio – in the numerator of (19) – is determined both by the size of the order's price p and by the size of the order's amount q .

The degree of portfolio risk induced by the (p, q) order for the share j is:

$$(20) \quad S_i(p, q) = (s_{i0}(p, q) - s_{i0}^{ext})^2 + (s_{ij}(p, q) - s_{ij}^{ext})^2 + \sum_{t \neq j} (s_{it}(p) - s_{it}^{ext})^2 =$$

$$= \left(\frac{y_i - pq}{V_i(p)} - s_{i0}^{ext} \right)^2 + \left(\frac{(x_{ij} + q)p}{V_i(p)} - s_{ij}^{ext} \right)^2 + \sum_{t \neq j} \left(\frac{x_{ij} p_j}{V_i(p)} - s_{it}^{ext} \right)^2$$

The risk-adjusted yield potential, which is dependent on the order, is then determined as follows:

$$(21) \quad R_i(p, q) = \hat{R}_i(p, q) - \frac{1}{2} \mathbf{x}_i^{risk} S_i(p, q)$$

For a given order price p for a share j agent i will now calculate whatever order amount q offers the greatest goal achievement. Therefore the partial derivation of the function $R_i(p, q)$ must be solved for q and equated to zero:

$$\frac{\mathcal{R}_i(p, q)}{\mathcal{Q}} = \frac{1}{V_i(p)} \frac{\mathcal{N}_i(p, q)}{\mathcal{Q}} - \frac{1}{2} \mathbf{x}_i^{risk} \frac{\mathcal{S}_i(p, q)}{\mathcal{Q}} = 0$$

If the resulting equation is solved for q , the result is an order amount function for agent i for share j independent of p :

$$(22) \quad q_{ij}^*(p) = \frac{1}{2p} \left[\left(\frac{\hat{p}_{ij} - p(1+r)}{\mathbf{x}_i^{risk} p} + (s_{ij}^{ext} - s_{i0}^{ext}) \right) V_i(p) + y_i - px_{ij} \right]$$

The function $q_{ij}^*(p)$, for a given order price p for share j , produces the value for q that maximizes the target function value $R_i(p, q)$ in conjunction with p .⁴⁷

⁴⁷ The function $-0,5 \mathbf{x}_i^{risk} S_i(p, q)$ is concave, while the function $\hat{R}_i(p, q)$ is linear in q . As a result, the linking and adding of the two functions has a maximum.

Regardless of the number of different shares held in the portfolio, an agent will generally want to hold a share j at a given order price p (i.e. $s_{ij}^{ext} > 0$) if the potential order price promises a (subjectively expected) yield which is higher than the alternate yield (cases a and b).

On the other hand, if the yield of the order price p is lower than the alternative interest rate, then the portfolio should be examined with regard to the number of desired and actually held shares.

The function of the optimum order amount $q_{ij}^*(p)$ can assume both positive and negative values, *regardless of whether the underlying order price p for a share j represents a buy price or a sell price*. For a potential buy price $p < p_{ij}^{cut}$ and $q_{ij}^*(p) < 0$ the relevant bid amount of share j is increased by $+1$. The maximum bid amount will be limited by the agent's available cash, assuming that he or she does not have a debt facility.⁴⁸ Similarly, for a potential ask price $p \geq p_{ij}^{cut}$ and $q_{ij}^*(p) > 0$ the relevant ask amount is decreased by -1 . If short sales are ruled out, then the maximum amount that can be offered is restricted to the number of shares concerned in the portfolio. The agent's supply and demand function, which is dependent on the order, is thus:⁴⁹

$$(23) \quad p < p_{ij}^{cut} : \quad q_{ij}^N(p) = \left\{ \begin{array}{ll} 1, & \text{if } q^*(p) \leq 1 \\ \text{round} q^*(p), & \text{if } 1 < q^*(p) < \text{trunc}(y_i / p) \\ \text{trunc}(y_i / p), & \text{if } q^*(p) \geq \text{trunc}(y_i / p) \end{array} \right\}$$

$$(24) \quad p > p_{ij}^{cut} : \quad q_{ij}^A(p) = \left\{ \begin{array}{ll} -x_{ij}, & \text{if } q^*(p) \leq -x_{ij} \\ \text{round} q^*(p), & \text{if } -x_{ij} < q^*(p) < -1 \\ -1, & \text{if } q^*(p) \geq -1 \end{array} \right\}$$

The following section examines the procedure for evaluating previously determined order's price and amount in more detail.

⁴⁸ Here, free funds are tied up in the bonds paying interest at interest rate r

⁴⁹ The "round" function rounds the real argument $q_{ij}^*(p)$ off to the nearest integer; the "trunc" function transforms $q_{ij}^*(p)$ into an integer by omitting the figures after the decimal indicator.

2.4.3.4 Evaluation of Buy and Sell Actions

Every permissible order price and amount $a = (p, q)$ submitted by an agent i for a share j is allotted a utility value with the following valuation function:

$$\Phi_i^E(a) = \mathbf{x}_i^{ip} \mathbf{f}_{ij}^{ip} + \mathbf{x}_i^{real} \mathbf{f}_{ij}^{real} + \mathbf{x}_i^{trd} \mathbf{f}_{ij}^{trd}$$

$$\mathbf{f}_{ij}^{ip}, \mathbf{f}_{ij}^{real}, \mathbf{f}_{ij}^{trd} = \text{Single-value functions}$$

$$\mathbf{x}_i^{ip}, \mathbf{x}_i^{real}, \mathbf{x}_i^{trd} = \text{Weighting factors, } \mathbf{x}_i^{ip}, \mathbf{x}_i^{real}, \mathbf{x}_i^{trd} > 0$$

The valuation function Φ_i^E comprises the "individual preference potential" \mathbf{f}_{ij}^{ip} , the "realization potential" \mathbf{f}_{ij}^{real} and the "trading potential" \mathbf{f}_{ij}^{trd} . The action's entire utility consists of several pieces, each called "potential". These functions for individual values are formulated in different ways, regardless of the size of the (buy or sell) action to be evaluated. Order amounts are not explicitly taken into account in this process.⁵⁰ The following relationships apply:⁵¹

1. Individual preference potential \mathbf{f}_{ij}^{ip} :

$$\mathbf{f}_{ij}^{ip}(a) = \begin{cases} \frac{p_j^{cut} - p}{p_{ij}^{cut}}, & a = \text{bid with } p \in (p_j^{cut}, p_{ij}^{cut}) \\ \frac{p - p_{ij}^{cut}}{p}, & a = \text{ask with } p \in [p_{ij}^{cut}, p_j^{cut}) \end{cases}$$

The individual preference potential \mathbf{f}_{ij}^{ip} , $0 < \mathbf{f}_{ij}^{ip} < 1$ comprises the yields which can be achieved with a single buy/ask. The basic behavioral rule is as follows: the higher the yield which can be achieved from a buy or ask, the higher will be the preference for this buy/ask. The potential $\mathbf{f}_{ij}^{ip}(a)$ defined by Landes/Loistl (1992) has to be seen according to buy and sell activity, assuming $p_{ij}^{cut} = 100$:

a) Buy action: $\mathbf{f}_{ij}^{ip} = \frac{p - 100}{p}$

b) Sell action: $\mathbf{f}_{ij}^{ip} = \frac{100 - p}{100}$

⁵⁰ Landes/Loistl (1992) p.222

The function curve for the individual preference potential is shown in the following diagram for an agent i with a present value expectation $p_{ij}^{cut} = 100$:

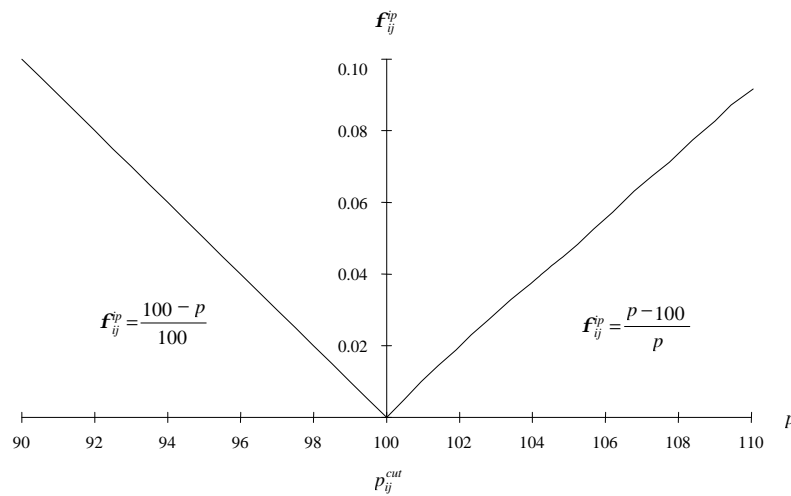


Fig 18 Individual preference potential f_{ij}^p depending on the order's price

The individual preference potential f_{ij}^p is a linear curve for bids $p \in (p_j^{cut}, p_{ij}^{cut})$ and a concave curve for asks $p \in [p_{ij}^{cut}, p_j^{cut})$. As can easily be seen from the graph, the preference potentials become increasingly asymmetrical the further they move from p_{ij}^{cut} . It is advisable to limit the possible bid-ask range. Within a range of $p_{ij}^{cut} \pm 5\%$, the difference is neglectable. As the model is dealing with continuous trading, the proposed range is clearly no restriction.

2. Realization potential f_{ij}^{real} :

$$f_{ij}^{real}(a) = \left\{ \begin{array}{l} \frac{p - p_j^{cut}}{p}, \quad a = \text{bid}, p \in (p_j^{cut}, p_{ij}^{cut}) \\ \frac{p_{ij}^{cut} - p}{p_{ij}^{cut}}, \quad a = \text{ask}, p \in [p_{ij}^{cut}, p_j^{cut}) \end{array} \right\}$$

The realization potential f_{ij}^{real} , $0 \leq f_{ij}^{real} < 1$ comprises the realization components of a bid/ask. This potential follows the maxim 'the greater the chance of realization of an order, the higher the preference for this order will be'. Continuing the example above, the following diagram shows the function curve

⁵¹ cf. Landes/Loistl (1992) p.222 f.

of the realization potential for the marginal market prices $p_j^{cut} = 90$ and $p_j^{cut} = 110$.

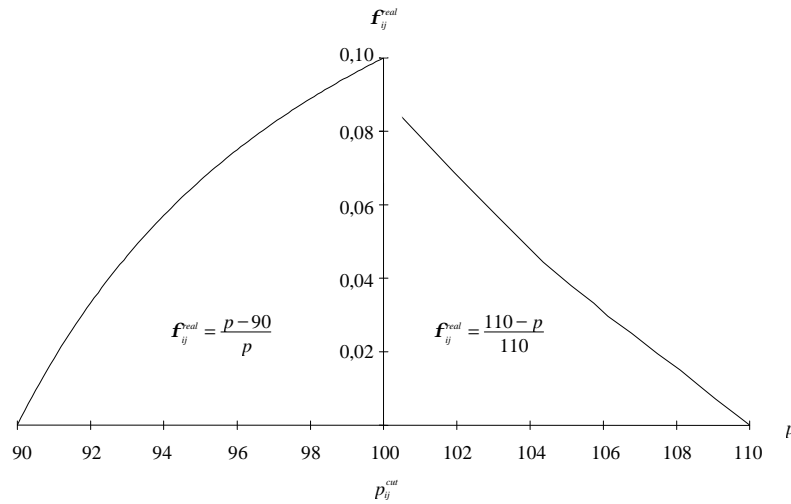


Fig 19 Realization potential f_{ij}^{real} depending on the order's price limit

The realization potential f_{ij}^{real} has a concave curve for bids $p \in (p_j^{cut}, p_{ij}^{cut})$ and a linear curve for asks $p \in [p_{ij}^{cut}, p_j^{cut})$.

To sum up, it can be reiterated that, for potential bids, the function f_{ij}^{p} is linear, while the function f_{ij}^{real} is concave in p . For potential asks, the function f_{ij}^{p} is concave, while the function f_{ij}^{real} is linear in p . At this stage it should be mentioned that the chosen function curves allow a large number of different orders of agents' preferences to be depicted. The possible bid/ask range should therefore be limited for the realization potential as well. Within a range of $p_{ij}^{cut} \pm 5\%$, the difference is neglectable. As the model is dealing with continuous trading, the proposed range is clearly no restriction.

3. Trade potential f_{ij}^{rd} :

The trade potential is modelled as an indicator function:

$$f_{ij}^{rd}(a) = \begin{cases} 1, & \text{Action } a \text{ leads to execution of a trade} \\ 0, & \text{Otherwise} \end{cases}$$

The trade function enables certain trades – transactions in the example above – to be preferred.

Given these notations, the evaluation functions for buy/sell actions is as follows:

Preference function for buy actions: Buy actions $a = (p, q)$, $p \in (p_j^{cut}, p_j^{cut})$, $q = q^N(p)$ are evaluated using the valuation function:

$$\Phi_i^E(a) = \mathbf{x}_i^{ip} \left(\frac{p_j^{cut} - p}{p_j^{cut}} \right) + \mathbf{x}_i^{real} \left(\frac{p - p_j^{cut}}{p} \right) + \mathbf{x}_i^{trd} \begin{cases} 1, \text{Execution of a trade} \\ 0, \text{Otherwise} \end{cases}$$

Preference function for sell actions: Sell actions $a = (p, q)$, $p \in [p_j^{cut}, p_j^{cut})$, $q = q^A(p)$ are evaluated using the valuation function:

$$\Phi_i^E(a) = \mathbf{x}_i^{ip} \left(\frac{p - p_j^{cut}}{p} \right) + \mathbf{x}_i^{real} \left(\frac{p_j^{cut} - p}{p_j^{cut}} \right) + \mathbf{x}_i^{trd} \begin{cases} 1, \text{Execution of a trade} \\ 0, \text{Otherwise} \end{cases}$$

The intensity of an agent i 's preference for individual valuation criteria can be determined using the weighting factors \mathbf{x}_i^{ip} , \mathbf{x}_i^{real} and \mathbf{x}_i^{trd} .⁵²

2.4.4 Evaluation of Value Adjustments

The individual planning process of an agent i , $i \in I = \{1..N\}$, regarding the adjustment of his/her expectations for a share j , $j \in J = \{1, \dots, M\}$, at a time t during trading, can be described as follows:⁵³

1. Establishment of permissible intrinsic value changes
2. Evaluation of defined expectation changes
3. Selection of an action

These points will be explored in the following sections.

As regards the establishment of permissible expectation changes, an agent i is permitted two trading alternatives for a share j :⁵⁴

1. Correction upwards of \hat{p}_{ij} by one unit: $a = \hat{p}_{ij} + \mathbf{d}$

⁵² The simulations by Landes/Loistl (1992) p.223 ff. distinguish between five groups of agents, which are assigned different weighting factors (i.e. behavioral patterns).

⁵³ See Landes/Loistl (1992) p.213, p.221 ff.

⁵⁴ A negative expectation correction is not permitted for $\hat{p}_{ij} = 1$.

2. Correction downwards of \hat{p}_{ij} by one unit: $a = \hat{p}_{ij} - \mathbf{d}$

The preference function used to evaluate these actions is:

$$\Phi_i^V(a) = \mathbf{d}[\mathbf{h}_i^{ext} \mathbf{f}_{ij}^{ext} + \mathbf{h}_i^{inf} \mathbf{f}_{ij}^{inf} + \mathbf{h}_i^{pot} \mathbf{f}_j^{pot} + \mathbf{h}_i^{tr} \mathbf{f}_j^{tr}]$$

$$\mathbf{d} = \begin{cases} +1, & \text{if } a = \hat{p}_{ij} + 1 \\ -1, & \text{if } a = \hat{p}_{ij} - 1 \end{cases}$$

$$\mathbf{f}_{ij}^{ext}, \mathbf{f}_{ij}^{inf}, \mathbf{f}_j^{pot}, \mathbf{f}_j^{tr} = \text{Single-value functions}$$

$$\mathbf{h}_i^{ext}, \mathbf{h}_i^{inf}, \mathbf{h}_i^{pot}, \mathbf{h}_i^{tr} = \text{Weighting factors, } \mathbf{h}_i^{ext}, \mathbf{h}_i^{inf}, \mathbf{h}_i^{pot}, \mathbf{h}_i^{tr} > 0$$

The valuation function Φ_i^V comprises what is called the 'external potential' \mathbf{f}_{ij}^{ext} , the market information potential \mathbf{f}_{ij}^{inf} , the market trend potential \mathbf{f}_j^{pot} and the price trend potential \mathbf{f}_j^{tr} . The following relationships apply:⁵⁵

1. External potential \mathbf{f}_{ij}^{ext} :

$$\mathbf{f}_{ij}^{ext} = \begin{cases} + \frac{|\hat{p}_{ij}^{ext} - \hat{p}_{ij}|^k}{\hat{p}_{ij}^{ext}}, & \hat{p}_{ij}^{ext} > \hat{p}_{ij} \\ - \frac{|\hat{p}_{ij}^{ext} - \hat{p}_{ij}|^k}{\hat{p}_{ij}^{ext}}, & \hat{p}_{ij}^{ext} < \hat{p}_{ij} \end{cases}$$

This potential favors changes to expectations that are moving in the direction of the *fundamental price expectation* \hat{p}_{ij}^{ext} . The greater the relative deviation of the current price expectation \hat{p}_{ij} of an agent i for a share j from the agent's original fundamental expectation, the greater will be the agent's readiness (preference) to change his/her expectation in the direction of \hat{p}_{ij}^{ext} . Similarly, changes to expectations that imply moving away from \hat{p}_{ij}^{ext} are suppressed by the potential \mathbf{f}_{ij}^{ext} . The growth behavior of the external potential can be managed by the k parameter.

⁵⁵ See Landes/Loistl (1992) p.221 f.

2. Market information potential $\mathbf{f}_{ij}^{\text{inf}}$:

$$\mathbf{f}_{ij}^{\text{inf}} = \left\{ \frac{p_j - \hat{p}_{ij}}{p_j} \right\}$$

This potential favours changes to expectations in the direction of the *current market price* p_j : the greater the relative deviation of the current price expectation \hat{p}_{ij} from the current market price p_j , the greater will be the agent's readiness (preference) to change his/her expectation in the direction of p_j . Similarly, changes to expectations that imply moving away from the current market price p_j are suppressed by the potential $\mathbf{f}_{ij}^{\text{inf}}$.

3. Market power potential $\mathbf{f}_j^{\text{pot}}$:

$$\mathbf{f}_j^{\text{pot}} = r \frac{m_j}{p_j} + (1-r) \frac{1}{M} \sum_{j' \in J} \frac{m_{j'}}{p_{j'}}$$

This potential favours positive changes to expectations given market demand pressure and negative changes to expectations given market supply pressure. The higher the general market demand pressure – expressed as the relationship between excess demand and the current market prices of the relevant securities – the more (or less) the agents will be inclined to adjust their expectations upwards (or downwards). Similarly, the greater the general market supply pressure – expressed as the relationship between excess supply and the current market prices of the relevant securities – the more (or less) the agents will be prepared to adjust their expectations upwards (or downwards). The parameter r can be used to manage the influence of market demand pressure or market supply pressure for a share j in relation to market demand or supply pressure for all shares.

4. Price trend potential \mathbf{f}_j^{tr} :

$$\mathbf{f}_j^{\text{tr}} = r \frac{p_j^{\text{tr}}}{p_j - p_j^{\text{tr}}} + (1-r) \frac{1}{M} \sum_{j' \in J} \frac{p_{j'}^{\text{tr}}}{p_{j'} - p_{j'}^{\text{tr}}}$$

This potential favours positive changes to expectations in the case of rising price levels and negative changes to expectations in the case of falling price levels. The stronger the positive market price trend, the greater will be the

agents' readiness to raise their expectations. Similarly, the stronger the negative market price trend, the greater the agents' readiness to lower their expectations will be. The parameter r can be used to manage the influence of the price trend of a share j in relation to the price trend for all shares.

Just as with the valuation function for buy and sell actions, the degree of preference of an agent i for the individual valuation criteria can be managed using the weighting factors h_i^{ext} , h_i^{inf} , h_i^{pot} and h_i^{tr} .

In the final planning phase, the decision phase, every agent must select an alternative from the set of trading alternatives available to him, ranging from buy or sell actions to positive and negative expectation changes.

If deterministic behavioral structures are in place, an agent will give preference to whichever trading alternative offers the greatest preference value. In the case of stochastic modelling, as is used here, the individual selection behavior can only be forecast with probability and not with certainty. Here, the logit model functions as a stochastic selection model. For the logit selection probabilities to be calculated, the transition rates for the defined actions must also be calculated.

2.5 Calculating the Transition Rates

The transition rate of a permissible action a is generally calculated as follows:⁵⁶

$$I_a = W \cdot e^{\Phi_i(a)}, \quad W = \text{speed of reaction parameter}$$

The preference function $\Phi_i(a)$ and the speed of reaction parameter W are defined as follows, depending on the size of the action a :

- Action a = Buy or sell action

$$W = W_E, \quad W_E > 0$$

$$\Phi_i^E(a) = x_i^{ip} f_{ij}^{ip} + x_i^{real} f_{ij}^{real} + x_i^{trd} f_{ij}^{trd}$$

$$f_{ij}^{ip}, f_{ij}^{real}, f_{ij}^{trd} = \text{Single-value functions}$$

$$x_i^{ip}, x_i^{real}, x_i^{trd} = \text{Weighting factors, } x_i^{ip}, x_i^{real}, x_i^{trd} > 0$$

- Action a = Change in expectations

$$W = W_V, \quad W_V > 0$$

$$\Phi_i^V(a) = \mathbf{d} \left[\mathbf{h}_i^{ext} \mathbf{f}_{ij}^{ext} + \mathbf{h}_i^{inf} \mathbf{f}_{ij}^{inf} + \mathbf{h}_i^{pot} \mathbf{f}_j^{pot} + \mathbf{h}_i^{tr} \mathbf{f}_j^{tr} \right]$$

$$\mathbf{d} = \begin{cases} +1, & \text{falls } a = \hat{p}_{ij} + 1 \\ -1, & \text{falls } a = \hat{p}_{ij} - 1 \end{cases}$$

$$\mathbf{f}_{ij}^{ext}, \mathbf{f}_{ij}^{inf}, \mathbf{f}_j^{pot}, \mathbf{f}_j^{tr} = \text{Single-value functions}$$

$$\mathbf{h}_i^{ext}, \mathbf{h}_i^{inf}, \mathbf{h}_i^{pot}, \mathbf{h}_i^{tr} = \text{Weighting factors, } \mathbf{h}_i^{ext}, \mathbf{h}_i^{inf}, \mathbf{h}_i^{pot}, \mathbf{h}_i^{tr} > 0$$

The ratio of the number of buy and sell actions to the number of changes to expectations can be governed by the speed of reaction parameters W_E and W_V .

When the utility of all of an agent's actions and the relevant transition rates have been calculated, the agent's stochastic selection behavior can be described.⁵⁷ The following section examines this behavior in the context of market dynamics.

2.6 Market Dynamics

2.6.1 Determining the Action to be Implemented

2.6.1.1 Systematic Overview

The following discussion of the process will focus on the market participants' decisions that result in a change of state. In order to describe the various relationships, a given point in time, t , will again be used. The following decision situations are of particular interest:

1. Decision behavior of an agent i with regard to a single activity a_{ijk}
2. Decision behavior of an agent i with regard to the set of alternatives A_{ij}
3. Decision behavior of an agent i with regard to the set of alternatives A_i

⁵⁶ cf. Landes/Loistl (1992) p.214.

⁵⁷ See the introductory discussion in section 2.6.1.2.2.

4. Decision behavior of the market with regard to amount A

The stochastic selection behavior with regard to this set of alternatives is discussed below. The following table helps illustrate the various relationships involved.⁵⁸

⁵⁸ Adapted from Casey (1998) p.230.

1	2	3	4	5	6	7	8	9	10
A	A_i	A_{ij}	a_{ijk}	$f(a_{ijk})$	$\mathbf{l}_{a_{ijk}} = We^{f(a_{ijk})}$	$t_{a_{ijk}}$	$t_{A_{ij}}^{\min}$	$t_{A_i}^{\min}$	t_A^{\min}
A	A_1	A_{11}	a_{111}	$f(a_{111})$	$\mathbf{l}_{a_{111}}$	$t_{a_{111}}$	$t_{A_{11}}^{\min}$	$t_{A_1}^{\min}$	
			a_{112}	$f(a_{112})$	$\mathbf{l}_{a_{112}}$	$t_{a_{112}}$			
			:	:	:				
		A_{12}	a_{121}	$f(a_{121})$	$\mathbf{l}_{a_{121}}$	$t_{a_{121}}$	$t_{A_{12}}^{\min}$		
			a_{122}	$f(a_{122})$	$\mathbf{l}_{a_{122}}$	$t_{a_{122}}$			
			:	:	:	:			
		:	:	:	:	:	:		
		:	:	:	:	:	:		
		A_{1M}	a_{1M1}	$f(a_{1M1})$	$\mathbf{l}_{a_{1M1}}$	$t_{a_{1M1}}$	$t_{A_{1M}}^{\min}$		
			a_{1M2}	$f(a_{1M2})$	$\mathbf{l}_{a_{1M2}}$	$t_{a_{1M2}}$			
			:	:	:	:			
		:	:	:	:	:	:		
	:	:	:	:	:	:			
	A_N	A_{N1}	a_{N11}	$f(a_{N11})$	$\mathbf{l}_{a_{N11}}$	$t_{a_{N11}}$	$t_{A_{N1}}^{\min}$		
			a_{N12}	$f(a_{N12})$	$\mathbf{l}_{a_{N12}}$	$t_{a_{N12}}$			
			:	:	:				
		A_{N2}	a_{N21}	$f(a_{N21})$	$\mathbf{l}_{a_{N21}}$	$t_{a_{N21}}$	$t_{A_{N2}}^{\min}$		
			a_{N22}	$f(a_{N22})$	$\mathbf{l}_{a_{N22}}$	$t_{a_{N22}}$			
:			:	:					
:		:	:	:	:	:			
:		:	:	:	:	:			
A_{NM}		a_{NM1}	$f(a_{NM1})$	$\mathbf{l}_{a_{NM1}}$	$t_{a_{NM1}}$	$t_{A_{NM}}^{\min}$			
		a_{NM2}	$f(a_{NM2})$	$\mathbf{l}_{a_{NM2}}$	$t_{a_{NM2}}$				
		:	:	:	:				
							$t_{A_N}^{\min}$	t_A^{\min}	

Tab 4 Reaction rates and times

In the first row, the columns are numbered from left to right. Columns 4 to 7 deal with the agents' individual activities: every agent i has a specific number of actions alternatives to choose from for a share j . These alternative actions are, on the one hand, buy or sell actions and, on the other hand, adjustments to expectations. The individual actions are entered in column 4 with the notation a_{ijk} . Column 5 gives the preference values $f(a_{ijk})$ of the shares a_{ijk} (as calculated in sections 2.4.3.4 and 2.4.4), while column 6 gives the relevant transition rates $\mathbf{I}_{a_{ijk}} = We^{f(a_{ijk})}$. The appropriate reaction time $t_{a_{ijk}}$ is obtained on the basis of the transition rate (reaction rate). These times are listed in column 7. The reaction time $t_{a_{ijk}}$ is the realization of an exponentially distributed random variable with the parameter $\mathbf{I}_{a_{ijk}}$.

Columns 8, 9 and 10 describe the selection problem regarding the set of actions in columns 1, 2 and 3:

The sets of actions A_{ij} , $i = 1..N$, $j = 1..M$, are given in column 3. The set of actions A_{ij} comprises the individual actions of an agent i for a share j . The selection of an action from A_{ij} is subject to the shortest reaction time of all actions from A_{ij} . The minimum reaction times $t_{A_{ij}}^{\min}$, $i = 1..N$, $j = 1..M$ are determined in column 8.

The sets of actions A_i , $i = 1..N$ are given in column 2. The set of actions A_i comprises all actions of an agent i for all shares j , $j = 1..M$. This is given by the union $A_i = A_{i1} \cup A_{i2} \dots \cup A_{iN}$. The choice of an action from A_i is again subject to the shortest reaction time of all actions from A_i . The relevant minimum reaction times $t_{A_i}^{\min}$, $i = 1..N$ are given in column 9.

The sets of actions A_i of all agents i , $i = 1..N$ are summarized in column 1 by uniting them to form a total alternative amount $A = A_1 \cup A_2 \dots \cup A_N$. Column 10 gives the minimum reaction time t_A^{\min} for all actions from A . The action with which the minimum is reached will be performed next. In this way, only one action can take place at a time.

2.6.1.2 The Basis in Probability Theory of Selection Behavior

The section below examines the agents' selection behavior, which is based on probability theory as mentioned above.

2.6.1.2.1 Decision behavior of an agent i

Let us take any alternative action k (table 4, column 4) of an agent i for a share j : a_{ijk} . Let the random variable $\tilde{T}_{a_{ijk}}$ describe the waiting time of agent i until a_{ijk} is implemented. This corresponds to the time which agent i will have to wait before implementation, if s/he does not observe any other action on the market in the meantime. This time is described as *agent i 's reaction time for the action a_{ijk}* .

The distribution function $F_{a_{ijk}}(t) = P(\tilde{T}_{a_{ijk}} \leq t)$ gives the probability that agent i 's waiting or reaction time before the implementation of a_{ijk} will not exceed t time units. In the capital market model discussed here, which has time-homogeneous transition rates, the waiting or reaction time of an agent before implementation of action a_{ijk} is exponentially distributed with the parameter $\lambda_{a_{ijk}}$:⁵⁹

$$F_{a_{ijk}}(t) = P(\tilde{T}_{a_{ijk}} \leq t) = 1 - e^{-\lambda_{a_{ijk}} t}$$

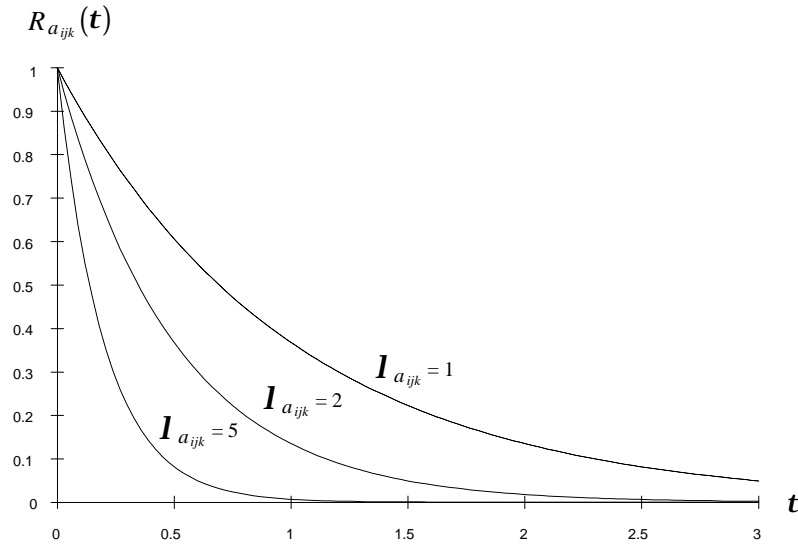
In this context, the transition rate $\lambda_{a_{ijk}} = W e^{f(a_{ijk})}$ (table 4, column 6) is also described as agent i 's reaction rate for action a_{ijk} .⁶⁰ It is a function of the utility $f(a_{ijk})$ (column 5) of a_{ijk} . The complementary distribution function

$$R_{a_{ijk}}(t) = 1 - F_{a_{ijk}}(t)$$

describes the probability that agent i 's waiting or reaction time before implementation of a_{ijk} exceeds t time units. It is described as a reaction function. The following diagram gives an impression of the reaction function curves with varying reaction rates $\lambda_{a_{ijk}}$:

⁵⁹ See in general Breiman (1969) p.37, p.211, Feller (1968). The concrete presentation on pp.76/77 follows Casey (1998) p.232 f.

⁶⁰ See Montgomery/Runger (1999) p.179.



The reaction function curves clearly illustrate the fact that the agent's individual waiting time until the implementation of a particular action decreases as the utility $f(a_{ijk})$, i.e. the reaction rate $I_{a_{ijk}} = We^{f(a_{ijk})}$, increases. This relationship is also expressed in the length of the expected reaction time. The mean reaction time for the action a_{ijk} is defined by

$$E(\tilde{T}_{a_{ijk}}) = 1 / I_{a_{ijk}}$$

and is inversely proportional to the reaction rate: the higher the reaction rate, the lower the expected reaction time until the implementation of a_{ijk} . The reaction time realized by an agent i for an action a_{ijk} is described by $t_{a_{ijk}}$. The following is then true:

$$t_{a_{ijk}} = \text{Realization of the random variables } \tilde{T}_{a_{ijk}}$$

The reaction times implemented by all agents for all actions are given in column 7 in Tab 4.

2.6.1.2.2 Selection of the Action to be Implemented

2.6.1.2.2.1 Probability Theory Relationships

Column 3 gives the set A_{ij} of possible actions. It is now necessary to provide a closer description of the process starting with the selection of the action from A_{ij} which the agent i wants to carry out next. The primary criterion for the selection is the utility of the action $a_{ijk} \in A_{ij}$. The higher the utility $f(a_{ijk})$ of an

action a_{ijk} , the more urgently its execution will be evaluated. The following notation is used to more accurately describe the process:

Random variable $\tilde{Y}_{A_{ij}}$ describes the alternative action that the agent selects from a given set of alternatives, A_{ij} .

Let random variable $\tilde{T}_{A_{ij}}$ describe the agent's planned waiting or reaction time until the implementation of the selected alternative. The following relation applies for the probability distributions $\tilde{Y}_{A_{ij}}$ and $\tilde{T}_{A_{ij}}$:

$$P(\tilde{Y}_{A_{ij}} = a_{ijk} / A_{ij}) = \frac{I_{a_{ijk}'}}{\sum_{a_{ijk} \in A_{ij}} I_{a_{ijk}}}$$

The probability that an agent i will choose the alternative action $a_{ijk} \in A_{ij}$ from a given set of alternatives A_{ij} is directly proportional to the sum of the reaction rates of all actions in A_{ij} :

$$P(\tilde{T}_{A_{ij}} \leq t) = 1 - e^{-\sum_{a_{ijk} \in A_{ij}} I_{a_{ijk}} t}$$

The agent's waiting or reaction time until the implementation of the selected alternative is exponentially distributed with the parameter $\sum_{a_{ijk} \in A_{ij}} I_{a_{ijk}}$. It can be

shown that the selection probability $P(\tilde{Y}_{A_{ij}} = a_{ijk} / A_{ij})$ is equal to the probability that the observed action a_{ijk} will exhibit the shortest reaction time of all the alternative actions from A_{ij} .⁶¹ As shown in column 8, the determination of the alternative actions with the minimum reaction time $t_{A_{ij}}^{\min}$ is obtained from a comparison of the realized reaction times of all alternative actions from A_{ij} :

$$t_{A_{ij}}^{\min} = \min\{t_{a_{ijk}} \mid a_{ijk} \in A_{ij}\}$$

⁶¹ See Landes/Loistl (1989) p.72 f., p.94 ff.

The minimum reaction time $t_{A_{ij}}^{\min}$ is the realization of the random variables $\tilde{T}_{A_{ij}}^{\min} = \min\{\tilde{T}_{a_{ijk}} \mid a_{ijk} \in A_{ij}\}$. This random variable is exponentially distributed with the parameter $\sum_{a_{ijk} \in A_{ij}} I_{a_{ijk}}$. It is given in column 8, line 1.⁶² The expression

$I_{ij} = \sum_{a_{ijk} \in A_{ij}} I_{a_{ijk}}$ is described as agent i 's reaction rate for share j .

The following relation applies:

The greater the benefit gained from the action of an agent i for a share j , the higher will be the reaction rate I_{ij} of this agent i for this share j , and the shorter the expected reaction time of this agent i before implementing an action from A_{ij} will be.⁶³

Following the above explanations, the alternative action k' selected by an agent i for a share j and the waiting time until its implementation can be determined either as a realization of the distributions $P(\tilde{Y}_{A_{ij}} = a_{ijk} / A_{ij})$ and

$P(\tilde{T}_{A_{ij}} \leq t)$, or as a realization of the random variables $\tilde{T}_{A_{ij}}^{\min} = \min\{\tilde{T}_{a_{ijk}} \mid a_{ijk} \in A_{ij}\}$.

Both random experiments deliver the same result. In KapSyn, the latter variant is used for selecting the action: the waiting time until the implementation of an action from A_{ij} , estimated by agent i , is determined by

$t_{A_{ij}}^{\min} = \min\{t_{a_{ijk}} \mid a_{ijk} \in A_{ij}\}$. The selected action is that with which the minimum $t_{A_{ij}}^{\min}$

is achieved.

⁶² See Landes/Loistl (1992) p.223 f.

⁶³ An agent i 's expected reaction time for the implementation of an action from A_{ij} is $1/I_{ij}$.

2.6.1.2.2.2 Implementation in Practice Taking a Random Component into Account

In addition to the determination of the benefit from the action i.e. its utility, a random experiment is involved in determining the minimum reaction time. In concrete terms, the action with the greatest utility will probably, but not definitely, be assigned the shortest reaction time by the agent. The following procedure applies:

A random figure between 0 and 1 is chosen for every reaction function and, in this way, the reaction time for the relevant reaction function is determined:

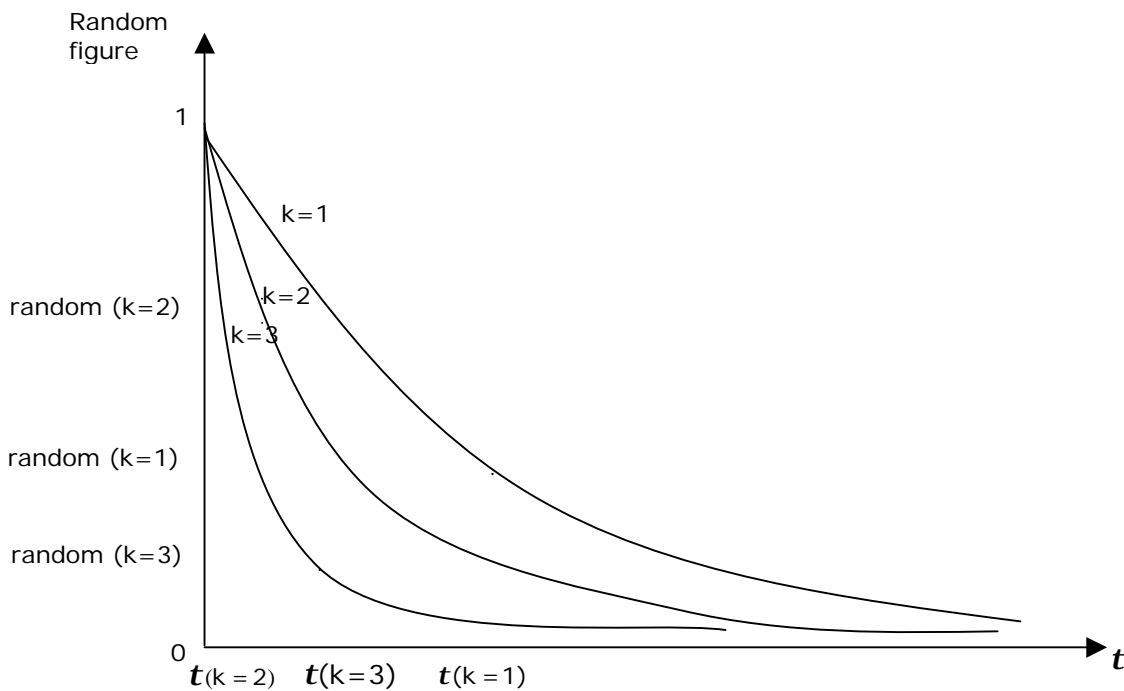


Fig 20 Random Element in Determining shortest ratio time

From this table it emerges that the random figure does not in fact allocate the shortest reaction time to action k=3 – which is in fact rated more highly – but rather to action k=2, which is not rated so highly.

In individual cases, the random experiment can lead to a change in the order of waiting time derived from the action's utility. It is probable that the actions with higher utility values will lead to shorter reaction rates.

2.6.1.2.2.3 Selection of the Action to be Implemented from all Possible Actions A_i of Agent i .

In line with the principle that an agent can only carry out a single action, the share in which share the agent wants to become active now needs to be established. All the shares in which the agent is able to trade must be observed. Every agent i then has to evaluate M sets of actions A_{ij} , $j = 1..M$. The behavior of an agent i with regard to the selection of an action from $A_i = A_{i1} \cup A_{i2} \cup \dots \cup A_{iM}$ will be described below.

The aim is to find whichever alternative actions $a_{ik} \in A_i$ the agent i plans to implement next, as well as the waiting time until implementation.⁶⁴ In line with the notations used thus far, the following variable declarations will be employed:

The random variable \tilde{Y}_{A_i} describes the alternative actions that the agent selects for a given alternative number A_i .

The random variable \tilde{T}_{A_i} describes the agent's planned waiting or reaction time until the implementation of the selected alternative. The distributions \tilde{Y}_{A_i} and \tilde{T}_{A_i} are characterized as follows:

$$P(\tilde{Y}_{A_i} = a_{ik'} / A_i) = \frac{I_{a_{ik'}}}{\sum_{a_{ik} \in A_i} I_{a_{ik}}}$$

The probability that an agent i will select the action $a_{ik'} \in A_i$ from a given number of possible actions A_i is proportional to the sum of the reaction rates of all actions from A_i .

$$P(\tilde{T}_{A_i} \leq t) = 1 - e^{-\sum_{a_{ik} \in A_i} I_{a_{ik}} t}$$

The agent's waiting or reaction time until the implementation of the selected alternative is exponentially distributed with the parameter $\sum_{a_{ik} \in A_i} I_{a_{ik}}$.

⁶⁴ The index j , used earlier to allot an action k by an agent i to a share j , will no longer be referred to here, for reasons of simplicity.

The following relation applies here, too, as before: the probability $P(\tilde{Y}_{A_i} = a_{ik} / A_i)$ is equal to the probability that the action a_{ik} exhibits the shortest reaction time of all of the alternative actions from A_i . This minimum reaction time can also be taken as the minimum $t_{A_i}^{\min}$ (given in column 9) of all *share-specific* reaction times $t_{A_{ij}}^{\min}$, $j = 1..M$ (given in column 8) of agent i :

$$t_{A_i}^{\min} = \min\{t_{A_{ij}}^{\min} \mid j = 1..M\}$$

The reaction time $t_{A_i}^{\min}$, $i \in I = \{1..N\}$ is the realization of the random variables $\tilde{T}_{A_i}^{\min} = \min\{\tilde{T}_{A_{ij}}^{\min} \mid j = 1..M\}$. This random variable is also exponentially distributed. Its distribution parameter is given by $I_i = \sum_{a_{ik} \in A_i} I_{a_{ik}}$ (see column 9, line 1). It corresponds to the reaction rate of agent i .⁶⁵ Once more, the following applies:

The greater the benefit (utility) gained from an agent i 's actions for all shares j , $j = 1..M$, the faster his/her reaction rate I_i , and the shorter his/her expected reaction time for the implementation of an action will be.⁶⁶

2.6.1.2.2.4 Market Decision Behavior for an Amount A

Finally it is necessary to determine the agent who will trade next from the set of all agents i , $i = 1..N$. At this level, too, trading cannot take place simultaneously. This selection is once again made on the basis of the greatest achievable benefit (utility).

Let $A = A_1 \cup A_2 \cup \dots \cup A_N$ be the number of alternative actions by all agents for all shares. The action sought, $a_k \in A$, is the one which will be implemented next. The selection also determines the time of its realization.⁶⁷ The usual notations apply:

The random variable \tilde{Y}_A describes the action that will be realized next.

⁶⁵ See Landes/Loistl (1992) p.223 f.

⁶⁶ Agent i 's expected reaction time is $1/I_i$.

⁶⁷ The index i , used earlier to allot an action k by an agent i is no longer required with this question and is therefore not given.

The random variable \tilde{T}_A describes the waiting time until the implementation of the chosen action. The distributions \tilde{Y}_A and \tilde{T}_A are linked to each other as follows:

$$P(\tilde{Y}_A = a_{k'} / A) = \frac{I_{a_{k'}}}{\sum_{a_k \in A} I_{a_k}}$$

The probability that, for a given set of actions A , the action $a_{k'}$ will be the next to be implemented, is proportional to the sum of the reaction rates of all actions from A .

$$P(\tilde{T}_A \leq t) = 1 - e^{-\sum_{a_k \in A} I_{a_k} t}$$

The waiting time until the implementation of the action to be carried out is exponentially distributed with the parameter $\sum_{a_k \in A} I_{a_k}$.

It is equal to the probability that the action $a_{k'}$ will exhibit the minimum reaction time out of all the alternatives from A . This reaction time can be given as the minimum t_A^{\min} given in column 10 of all *agent*-specific reaction times $t_{A_i}^{\min}$, $i = 1..N$ given in column 9:

$$t_A^{\min} = \min\{t_{A_i}^{\min} \mid i = 1..N\}$$

The reaction time t_A^{\min} is the realization of the random variables $\tilde{T}_A^{\min} = \min\{\tilde{T}_{A_i}^{\min} \mid i = 1..N\}$. It is exponentially distributed with the parameter $I = \sum_{a_k \in A} I_{a_k}$, given in column 10, line 1.

This parameter corresponds to the reaction rate for the entire market. The higher the market reaction rate I , the shorter the average waiting time until the implementation of the next action overall.⁶⁸

The stochastic selection behavior of the market participants, and hence the market dynamics, can now be described as follows:

The number of alternative actions by an agent i , $i \in I = \{1..N\}$, for a share j , $j \in J = \{1..M\}$, is combined in amount A_{ij} . For share 1 the amount A_{i1} applies, for share 2 the amount A_{i2} , ..., and, finally, for share M the amount A_{iM} applies.

In his/her first step, agent i selects an action $a_{ijk} \in A_{ij}$ for every share j that s/he would implement if observing share j in isolation. This action is described as a potential action of agent i with regard to share j and is determined subject to the shortest reaction time $t_{A_{ij}}^{\min} = \min\{t_{a_{ijk}} \mid a_{ijk} \in A_{ij}\}$. It is probable that agent i will choose the action from A_{ij} that offers him the greatest gain.

After this selection step, agent i must select from the total M alternative actions the one which should be realized in the end, from his or her point of view. This action is once again determined by the criterion of the shortest reaction time: $t_{A_i}^{\min} = \min\{t_{A_{ij}}^{\min} \mid j = 1..M\}$. Again, it is probable that the alternative selected will be that which most stimulates the agent, in other words, which promises the greatest gain.

A total of M agents are active on the stock exchange. The agents' decision-making processes are independent of each other; i.e. they take place in parallel. As a result, every agent will determine the action most advantageous for him, and that s/he will want to realize. The reaction times $t_{A_i}^{\min}$, $i = 1..N$

⁶⁸ The mean market reaction time $E(\tilde{T}_A^{\min})$ is equal to $1 / \sum_{a_k \in A} I_{a_k}$. Using this figure, the reaction rates of all actions from A can also be described as follows:

$$I_{a_k} = \frac{I_{a_k}}{\sum_{a_k \in A} I_{a_k}} \sum_{a_k \in A} I_{a_k} = P(\tilde{Y}_A = a_{a_k} / A) \frac{1}{E(\tilde{T}_A^{\min})}$$

This formula can be useful if the reaction rates cannot be measured directly. If the selection probabilities and the mean reaction times are available, then the reaction rates can be calculated from the latter (see Karlin/Taylor (1975) p.150ff and the literature cited).

determine the agent whose turn it will actually now be to act, whose action will be implemented and who will change the observed market conditions. Of all the agents, it is the one with the shortest reaction time who will be the next to act: $t_A^{\min} = \min\{t_{A_i}^{\min} \mid i = 1..N\}$.

2.6.1.2.2.5 Summary

In summary, it can be said that the following applies: The waiting time until the realization of the next action is given by t_A^{\min} , while the action to be realized is the one which will be implemented in this minimum time. The probability that agent i' will implement the next action to be realized corresponds to the probability that one of his/her potential actions exhibits the shortest reaction time:

$$P(\text{agent } i' \text{ has minimum reaction time}) = \frac{I_{i'}}{\sum_{i \in I = \{1..N\}} I_i} = \frac{\sum_{a_{i'k} \in A_{i'}} I_{a_{i'k}}}{\sum_{i \in I = \{1..N\}} \sum_{a_{ik} \in A_i} I_{a_{ik}}}$$

The probability that agent i' will be the next to act is equal to the relation of his/her reaction rate $I_{i'}$ to the sum of the reaction rates of all agents. This relation makes it clear once again that it is probable that the next market participant to act will be the one who has the greatest preference for action and hence the shortest reaction time.

Once t_A^{\min} has been determined, the market state vector z is transformed in accordance with the activity to be implemented and the process time is increased by t_A^{\min} . If this activity involves a buy or sell action observable by all market participants, the process described above for determining the next action is repeated on the basis of the new market conditions.⁶⁹ All agents reappraise the benefit to be gained by their activities in line with the market changes that they notice. On the other hand, if the action involves a revision of the expected price of a market participant and is not generally visible, only the agent i' who takes his expectation change into account will reappraise his/her

⁶⁹ All state variables except the expectation components \hat{p}^{ext} and \hat{p} , are affected by bids and asks, transactions and cancellations. For instance, a trade in a share j induces a change in j 's current market price, j 's order list, excess supply or demand for share j , j 's price trend as well as the cash in hand and shareholdings of the trading partners involved.

alternative actions.⁷⁰ A new reaction time $t_{A_i}^{\min}$ will also be calculated for this agent. In the mean time, decision processes by the other market participants continue as before. The reaction times of the other market participants $t_{A_i}^{\min}$, $i = 1..N$, $i \neq i'$ are reduced accordingly by the time which has elapsed up to that point, t_A^{\min} . The next action to be implemented is then determined on the basis of the shortened reaction times. This process repeats itself until an agent's buy or sell action, which can be explicitly observed by all market participants, is implemented. Market activities then continue as usual, according to the principle described, until a prescribed maximum process time is reached.

2.6.2 List of Formulae Used

The following pages list the formulae used in order to provide the reader with a better overview.

2.6.2.1 Motivation Potential for a Value Adjustment

$$\mathbf{w}^{VA}(z) = W_V^{\pm}(\Phi_j^V(z)) = W_V^{\pm}(h_i^{ext}\Phi_{ij}^{ext} + h_i^{inf}\Phi_{ij}^{inf} + h_i^{pot}\Phi_{ij}^{pot} + h_i^{trd}\Phi_{ij}^{trd})$$

$$\Phi_{ij}^{ext} = \left(\frac{\text{sign}(\hat{p}_{ij}^{ext} - \hat{p}_{ij}) |\hat{p}_{ij}^{ext} - \hat{p}_{ij}|^k}{\hat{p}_{ij}^{ext}} \right)$$

$$\Phi_{ij}^{inf} = \left(\frac{p_j - \hat{p}_{ij}}{p_j} \right)$$

$$\Phi_{ij}^{pot} = \left(r \frac{m_j}{p_j} + (1-r) \frac{1}{M} \sum_{j' \neq j} \frac{m_{j'}}{p_{j'}} \right)$$

$$\Phi_{ij}^{trd} = \left(r \frac{p_j^T}{p_j - p_j^T} + (1-r) \frac{1}{M} \sum_{j' \neq j} \frac{p_{j'}^T}{p_{j'} - p_{j'}^T} \right)$$

⁷⁰ The implementation of an expectation change leads to a transformation of the state variables \hat{p} in one dimension.

2.6.2.2 Motivation Potential for a Bid

$$\begin{aligned}
 \mathbf{w}_{ij}^{bid}(p_{ij}^{pot}) &= W_E^{\Phi_{ij}^{bid}(p_{ij}^{pot})} \\
 \Phi_{ij}^{bid} &= \mathbf{x}_i^{real} \Phi_{ij}^{real} + \mathbf{x}_i^{ip} \Phi_{ij}^{ip} + \mathbf{x}_i^{trade} \Phi_{ij}^{trade} \\
 \Phi_{ij}^{real} &= \begin{cases} 1 - \frac{\hat{p}_{ij}^{cut}}{p_{ij}^{pot}} & \text{if } q_j^0 = 0 \\ 1 - \frac{\hat{p}_{ij}^{cut}}{p_{ij}^{bid}} & \text{if } q_j^0 < 0 \end{cases} \\
 \Phi_{ij}^{ip} &= \frac{\hat{p}_{ij} - p_{ij}^{pot}(1+r)}{\hat{p}_{ij}} = 1 - \frac{p_{ij}^{pot}(1+r)}{\hat{p}_{ij}} \\
 \Phi_{ij}^{trade} &= \begin{cases} 0 & \text{if } q_j^0 = 0 \\ 1 & \text{if } q_j^0 < 0 \end{cases} \\
 \text{with} \\
 \hat{p}_{ij}^{cut} &= \frac{\hat{p}_{ij}}{(1+r)}, \quad \hat{p}_{ij} = p(1 + \hat{r}_{ij}) \\
 p_{ij}^{bid} &= \max\{p \in N \mid p < \hat{p}_{ij}^{cut}\} \\
 p_{ij}^{ask} &= \min\{p \in N \mid p > \hat{p}_{ij}^{cut}\}
 \end{aligned}$$

r is the yield of an alternative investment based on the trader's forecast horizon.

2.6.2.3 Motivation Potential for an Ask

$$\begin{aligned}
 \mathbf{w}_{ij}^{ask}(p_{ij}^{pot}) &= W_E^{\Phi_{ij}^{ask}(p_{ij}^{pot})} \\
 \Phi_{ij}^{ask} &= \mathbf{x}_i^{real} \Phi_{ij}^{real} + \mathbf{x}_i^{ip} \Phi_{ij}^{ip} + \mathbf{x}_i^{trade} \Phi_{ij}^{trade} \\
 \Phi_{ij}^{real} &= \begin{cases} 1 - \frac{p_{ij}^{pot}}{\hat{p}_{ij}^{cut}} & \text{if } q_j^0 = 0 \\ 1 - \frac{p_{ij}^{ask}}{\hat{p}_{ij}^{cut}} & \text{if } q_j^0 > 0 \end{cases} \\
 \Phi_{ij}^{ip} &= \frac{p_{ij}^{pot}(1+r) - \hat{p}_{ij}}{p_{ij}^{pot}(1+r)} = 1 - \frac{\hat{p}_{ij}}{p_{ij}^{pot}(1+r)} \\
 \Phi_{ij}^{trade} &= \begin{cases} 0 & \text{if } q_j^0 = 0 \\ 1 & \text{if } q_j^0 > 0 \end{cases}
 \end{aligned}$$

2.6.2.4 Calculation of quantity q^*

Once p_{ij}^{bid} or p_{ij}^{ask} is determined, the corresponding stock quantity q_{ij}^* belonging to this price is calculated as

$$q_{ij}^* = \frac{1}{2} \left(\frac{y_i}{p_j} - x_{ij} + \frac{w_i}{p_i} \left(\frac{\hat{p}_{ij}}{p_{ij}^{bid \text{ or } ask}} - r \right) \frac{x_i^{risk}}{\quad} \right)$$

whereby the agent i 's total wealth $w_i = y_i + \sum_j x_{ij} p_j$.

This quantity q_{ij}^* will be adjusted to the trader's portfolio in a following step, i.e. the trader cannot buy stocks exceeding his/her cash holdings or sell more stocks than he owns.

2.6.3 The Distribution of Subjective Price Expectations

At the beginning of a trading session every trader has a price expectation regarding the available shares. The question as to whether the discounted price expectation is higher or lower than the current share price has an impact on the trader's selling or buying decision. The sum of all individual traders' price expectations is described by means of a frequency distribution. Currently, the frequency distribution consists of two truncated normal distributions, each described by mean and standard deviation. Each of the means is expressed as percentage deviation from the opening quotation. The parameter constellation, e.g.:

$$\begin{aligned} p^{ext 1} &= 0.995 \\ \sigma 1 &= 0.8 \\ p^{ext 2} &= 1.02 \\ \sigma 2 &= 0.8 \end{aligned}$$

implies that the \hat{p}^{ext} distribution comprises two truncated normal distributions. The lower one is described by the mean = 0.995 times the opening quotation and 0.8 times the opening quotation standard deviation.

The upper one is described by the mean = 1.02 times the opening quotation, and the sigma equals 0.8 times the opening quotation. Different economic scenarios are related to different sets of the \hat{p}^{ext} distribution.

Traders can revise their subjective expectations. They do this when the transition time of their expectation adjustment is shorter than the transition times' minimum of market events. The adjustment of price expectations plays an important role in the development of the share price. This is employed when prototypical KapSyn parameter patterns are drawn up.

The following page shows sample 3D-graphics for each of the seven prototype KapSyn scenarios. They illustrate the fact that price expectations vary over time. The horizontal line indicates the subjective price expectations, while the height shows the number of traders holding these expectations. This means that the higher the peak of the "mountain structure", the more traders share the respective price expectation. The time arrow points from the back of the graphic to the front. This means that the rearmost vertical layer of the mountain structure shows the \hat{p} distribution at the beginning of the simulation. The forward edge of intersection illustrates the \hat{p} distribution at the end of the trading session.

For reasons of perspective, the mountain structure is viewed from the front and to the right. This explains why movement in time to the left, i.e. towards lower price expectations, is over-estimated at first sight. The lines drawn from the back to the front can be helpful when trying to estimate whether the entire distribution of expectations is moving towards higher or lower price expectations over the course of time.

In order to facilitate comparison, the respective counter-scenarios are placed beside each other. These "scenario pairs" depict similar share price developments but have opposing signs.

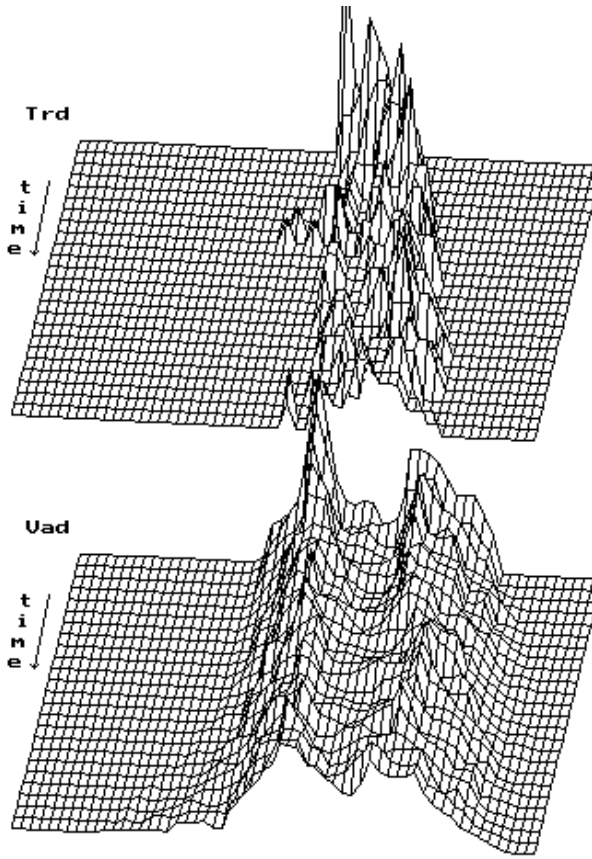


Fig 21 Bullish trend scenario (#1)

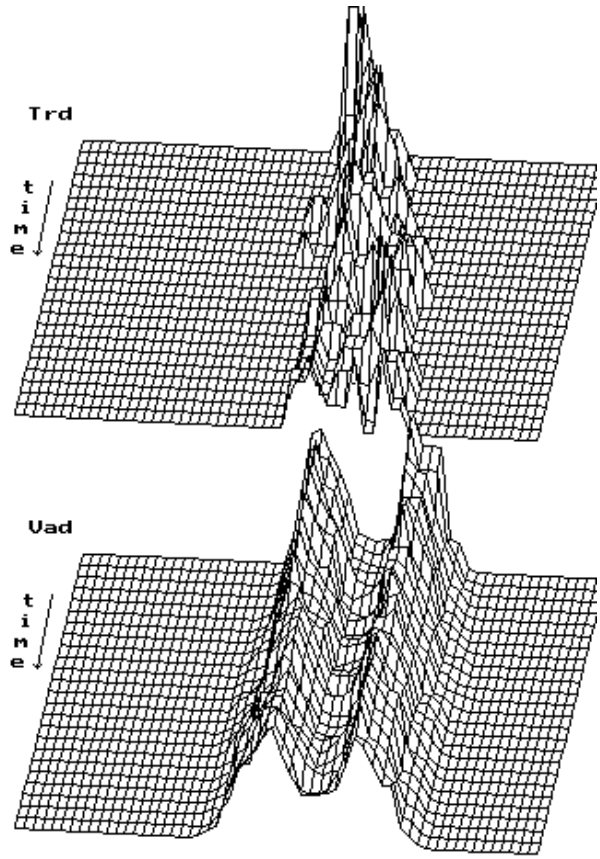


Fig 23 Bullish fundamental scenario (#3)

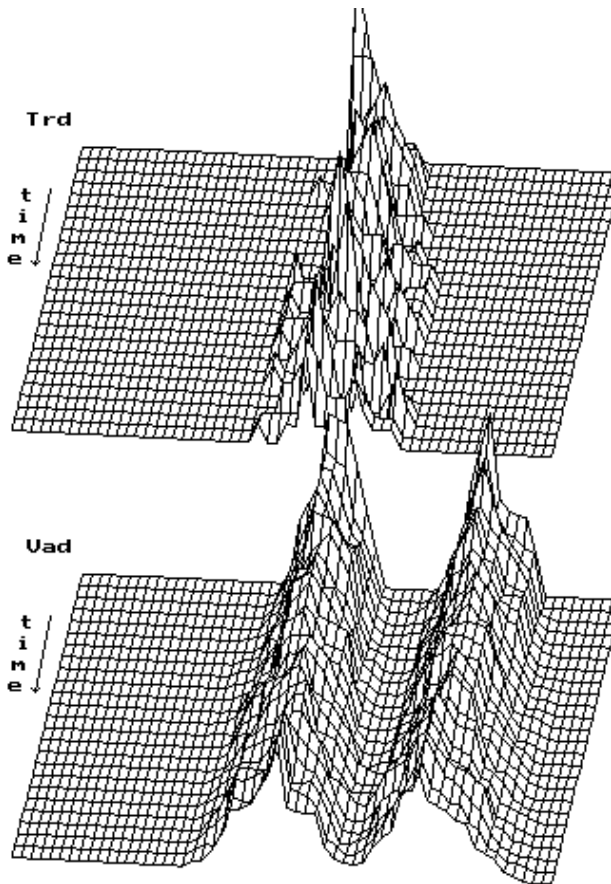


Fig 22 Bullish global scenario (#2)

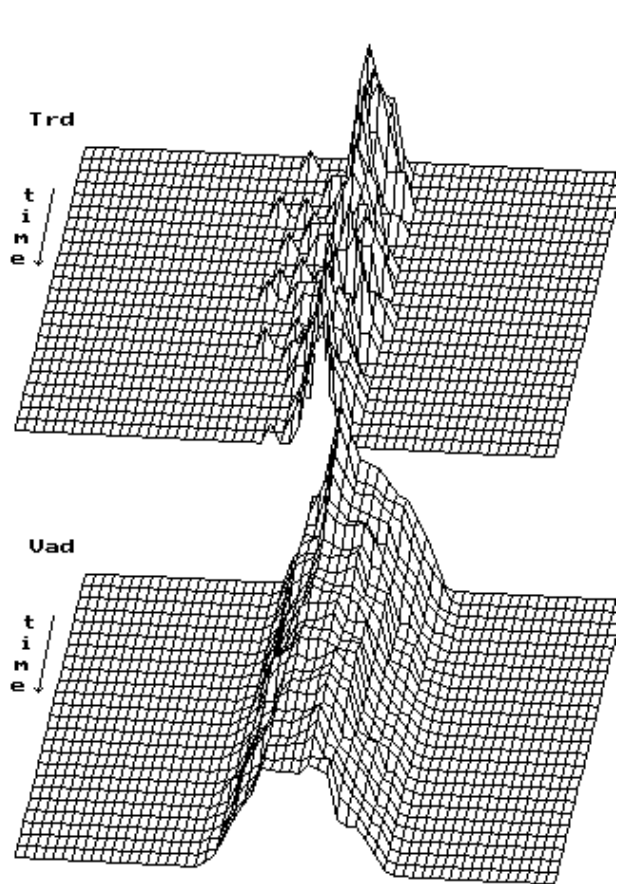


Fig 24 Volatile scenario (#4)

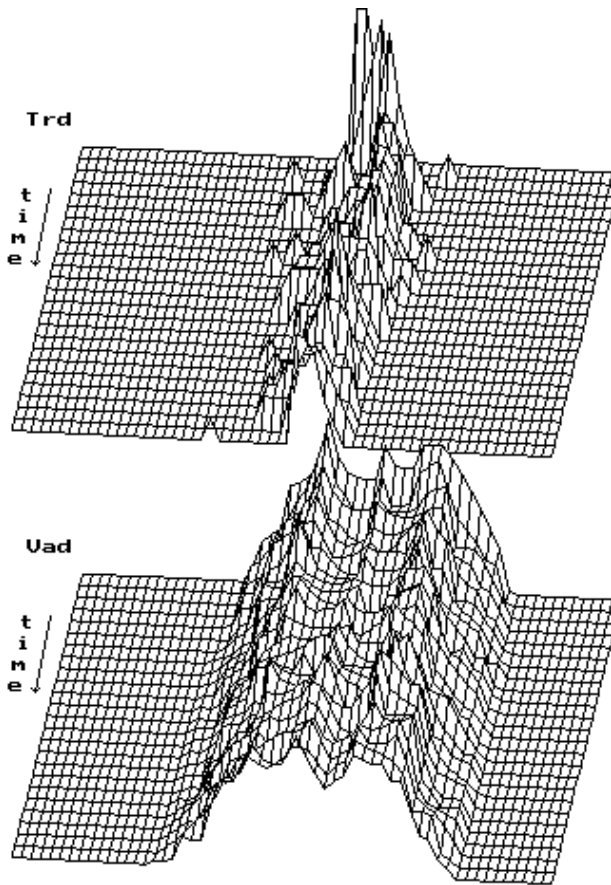


Fig 25 Bearish fundamental scenario (#5)

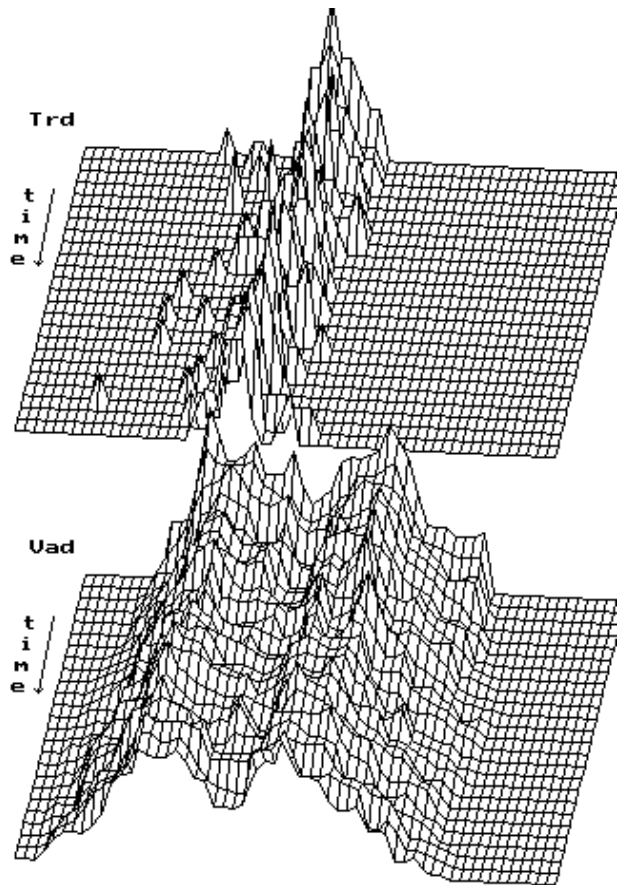


Fig 27 Bearish trend scenario (#7)

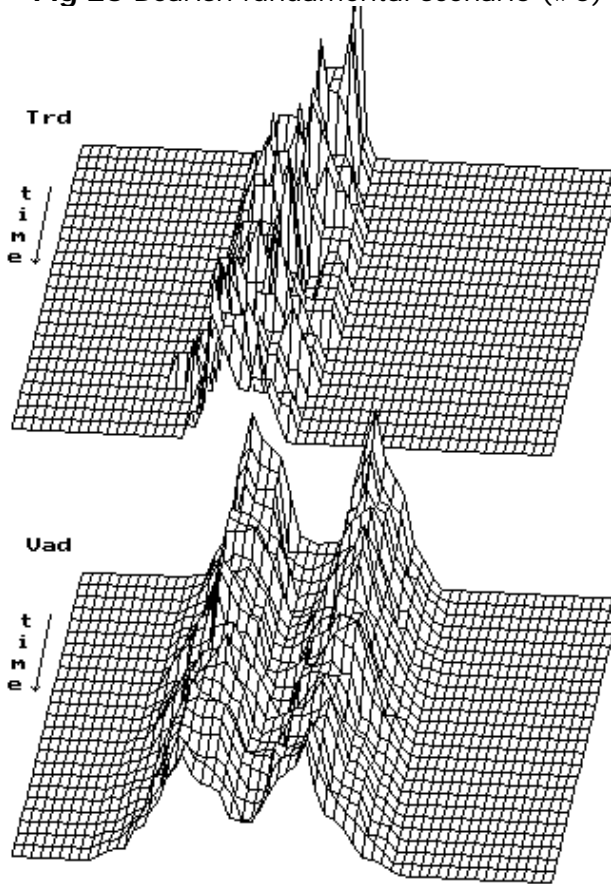


Fig 26 Bearish global scenario (#6)

All figures show the distribution of share prices in the upper half and the distribution of the corresponding \hat{p}^{ext} values in the lower half.

Each figure shows the results of 16 simulations.

Trd Trade: Stands for frequency distributions of price quotations

Vad Value adjustment: Stands for individually adjusted price expectations.

2.7 The Current Stage of the Model's Development

The KapSyn model is based on the approach developed in *Complexity Models in Financial Markets* (Landes and Loistl, 1992). However, it also fully incorporates further developments from Chapter 7.3 of *Capital Market Theory (Kapitalmarkttheorie)*, Loistl, 1994).

In addition, the following modifications and additions to the model were performed:

- The calculation methods for some of the simulation process-dependent variables were modified in order to increase the information value. This is dealt with more closely in the next chapter.
- New stock exchange modules have been developed to accommodate recent exchange regulations and have been integrated with the existing environment. For details see chapters 2.7.2 and 2.7.3.
- Models for the computation of transaction costs were implemented. Chapter 2.7.4 deals with this topic.

2.7.1 Modifications to the Previous Model

The individual ask or bid decisions by traders were considerably simplified and are therefore much more transparent: if the trader i 's discounted price estimation \hat{p}_{ij}^{cut} for stock j ,

$$\hat{p}_{ij}^{cut} = \frac{\hat{p}_{ij}}{(1+r)},$$

(where $\hat{p}_{i,j}$ is the price expectation and r is the yield of an alternative investment) is above the actual price for stock j , p_j , this represents an incentive to buy. If \hat{p}_{ij}^{cut} is below the current stock price, this represents an incentive to sell the stock:

$$\begin{aligned}\hat{p}_{ij}^{cut} > p_j &\Rightarrow ask? \\ \hat{p}_{ij}^{cut} < p_j &\Rightarrow bid?\end{aligned}$$

In this context '?' indicates that, given appropriate general conditions, such as a sufficient amount of cash or shares, the corresponding transition rate for an ask or bid is calculated.

Tick size

An important modification to the model relates to the variable δ_j , referring to the tick which defines the smallest possible change in value of the trader's price expectations and the share price p_j . Where δ_j was previously an integer number, it can now be less than 1 currency unit. This mainly comes in useful in the case of low-priced shares. Currently share prices between 10 and 5,000 c.u. can be traded without any significant price-based distortion. As a result of this modification, all amounts which change the share price must be based on the next multiple of δ_j . In addition, a stock exchange-based minimum stock price change $d_{Stock\ Exchange}$ is introduced.

$$d = d_{Stock\ Exchange} \text{ round} \left(\frac{0.005 p_i}{d_{Stock\ Exchange}} \right)$$

stock price	tick size	XETRA minimum stock price change
2.00	0.10	0.10
10.00	0.10	0.10
33.00	0.20	0.10
55.00	0.30	0.10
5000.00	25.00	0.10

Tab 5 Stock prices and tick size

A further modification relates to the minimum volume of a newly-placed order. A stock exchange-specific minimum limit was implemented. New orders with a lower volume are raised to the minimum share limit, or are cancelled, if the peripheral specifications (such as the portfolio) do not allow it. If an order has already been placed it can be traded even if the volume is below the limit. This can sometimes occur in the case of partially settled orders.

- For every market event c_e , the market power m has until now been calculated as follows:

$$m'_j = (1 - \mathbf{I}_j) m_j + \begin{cases} \text{sign}(c_e) & \text{if } c_e \neq a \\ -\mathbf{e} \text{sign}(c_e) & \text{if } c_e = a \end{cases}$$

The revised formula reads:

$$m'_j = (1 - \mathbf{1}_j)m_j + 1 \quad (\text{ask})$$

$$m'_j = (1 - \mathbf{1}_j)m_j - 1 \quad (\text{bid})$$

with the explicit visible events $e = (i_e, c_e, p_e)$, $c_e \in \{d, s, a\}$, and i_e being the current activity of the operating trader i ; c_e , the type of activity; d , the offer to buy, s , the offer to sell; a , the acceptance of an offer, and p_e the price assigned to the activity.

This modification ensures that m can have a positive or negative value during the course of trading, independently of λ and ε , and thus give an accurate picture of market activity.

- The price trend p^T is cleared when

$$|\Delta_j^T| > \Delta_j^{\max} \frac{p_j}{200}$$

that is

$$\begin{aligned} p_j^T &= \Delta_j^T \\ \Delta_j^T &= 0 \end{aligned}$$

where $p_j/200$ is in the order of \mathbf{d} but, in contrast to \mathbf{d} , which was defined at the start of the simulation, takes the exact current share price into account.

2.7.2 The Price-Finding Process in the XETRA Module

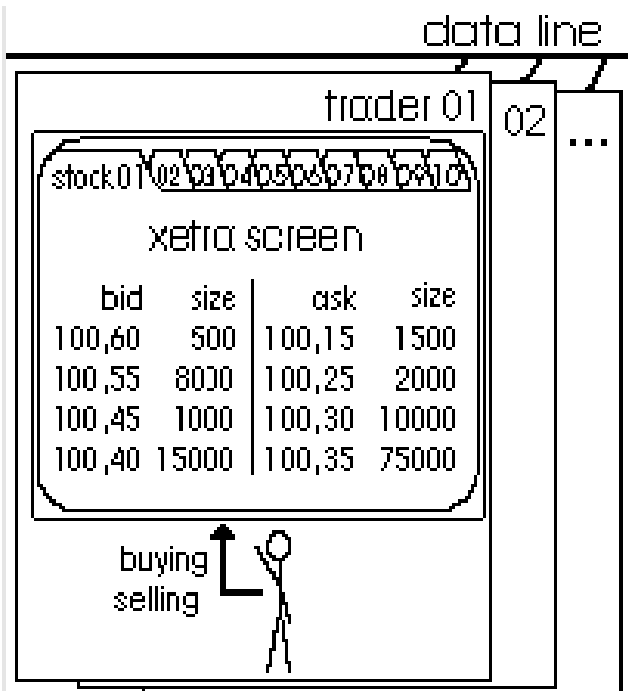


Fig 28 XETRA: German computerized trading system.

Traders sit isolated in front of their trading screen. They trade directly with each other, no market maker or specialist is involved: They place unsolicited quotes to buy or sell, or hit and take quotes, generating a trade and a price.

The KapSyn XETRA module simulates the electronic trading system of the Frankfurt Stock Exchange (release 3). The XETRA system was introduced in June 1997 and has been consistently updated. Release 3.1 is currently in operation and is the basis for the KapSyn XETRA module.

The XETRA traders sit in front of their online trading screen, isolated from each other. During normal trading they can view all existing orders (open order book), and they can enter their own market or limit orders in any size at any time during XETRA trading, that is between 8:25 and 17:00 MEST. Orders are prioritized according to time and price.

Every XETRA trading day starts and ends with an auction, and is interrupted by at least one. When an auction is taking place the order book is partially closed: only the best ask and bid quotes are visible, and normal trading is stopped. Every auction starts with an invitation to place orders, and, after a certain time, ends with a *market matching*. Auction price matching is conducted according to the principle, where the price with the highest trading volume is determined as the new stock price. Further auctions can be triggered by a stock price change which exceeds a defined limit, as compared to the previous price.

Example of market clearing: At the end of the invitation to place order time period, the situation as shown in Tab 6 can be found. The old price of the share is 100.00. All the asks -beginning with low prices - are now accumulated and set against the bids that have cumulated from high to low prices. The primary idea behind this procedure is that an ask at 99.40 can also be settled at a higher p and, likewise, a bid at 100.60 can also be transacted at a lower p .

The lowest number of the accumulated bid/ask pairs with which the highest sales volume can be achieved must now be found. All orders are subsequently settled at this price, according to their time of entry. Orders that have not, or that have only partly been serviced, are upheld. The new share price in this example is 99.80.

p	volume ask	volume bid	accumulated asks	accumulated bids	tradable volume
99.20	0	0	0	19'300	0
99.40	2'000	0	2'000	19'300	2'000
99.60	6'500	1'000	8'500	19'300	8'500
99.80	8'000	2'000	16'500	18'300	16'500
<i>100.00</i>	<i>10'000</i>	<i>3'000</i>	<i>26'500</i>	<i>16'300</i>	<i>16'300</i>
100.20	2'500	7'000	29'000	13'300	13'300
100.40	0	4'000	29'000	6'300	6'300
100.60	0	2'300	29'000	2'300	2'300
100.80	0	0	29'000	0	0

Tab 6 Example of a price-finding process according to XETRA auction. Italics indicate the old share price; bold indicates the new share price

A designated sponsor was introduced to increase liquidity for second-line stocks. Each designated sponsor is responsible for a certain stock, and quote requests can be made to him/her by any trader. The designated sponsors should then set an ask and bid quote, though they are not obliged to. The *reaction time*, the minimum bid/ask volume, and the maximum spread are all defined and monitored.

The following is a description of the KapSyn XETRA designated sponsor. The sponsor's start portfolio w_0 is

$$w_0 = \frac{\text{total wealth of all agents}}{10 \cdot \text{number of stocks}},$$

with half of w_0 invested in the designated sponsor's stock.

A quote is requested for the designated sponsor's stock whenever an ask (bid) exists without any counter-bid (counter-ask) from other traders. When the designated sponsor's *reaction time* has elapsed, the stock's designated sponsor decides whether or not to answer the quote request. If a *call for auction* occurs, an immediate quote is requested.

If the sponsor's new bid (ask) price would be below (above) the stock's opening quotation, the sponsor will always answer the quote request. If not,

they make up their mind according to the calculation as follows: They calculate a performance index since the opening of the trading day ($t=0$) with

$$performance = 1 + \left(\left[\frac{total\ wealth(t)}{total\ wealth(t=0)} - 1 \right] * 100 \right)$$

They then use

$$random(0..1) < performance ?$$

to determine their reaction. If the performance is better than 1, they will answer the request. If it is below 1, they will compare performance with a random number between 0 and 1. In other words, the worse the performance, the more unlikely the sponsor will be to answer the quote request.

The quote spread is calculated with

$$spread = \frac{0.75 \cdot maximum\ spread}{performance},$$

and has the limitation:

$$0.5 \cdot maximum\ spread \leq spread \leq maximum\ spread$$

The designated sponsor's quote volume is 5% of their cash for every bid and 5% of their stock portfolio for every ask.

The designated sponsor statistics are used to create a rating code. The rating is composed of two letters between A and D, eg. AA, AB, DC etc. Tab 7 shows the details.

part 1		part 2	
$\frac{\text{quote request answers}}{\text{quote requests}}$	Rating ⁷¹	$\frac{\text{average spread}}{\text{max. spread}}$	rating
> 0.99	A	>0.75	A
>0.85	B	>0.80	B
>0.65	C	>0.85	C
all other	D	all other	D

Tab 7 Designated sponsor rating

The traders i determine their bid price p_{ij}^{bid} within the range:

$$\begin{aligned} \tilde{p}_j - 2\mathbf{d}_j < p_{ij}^{bid} < \tilde{p}_j & \quad \forall p_{ij}^{cut} \leq \tilde{p}_j - 2\mathbf{d}_j \\ p_{ij}^{cut} < p_{ij}^{bid} < \tilde{p}_j + \mathbf{d}_j & \quad \forall p_{ij}^{cut} > \tilde{p}_j - 2\mathbf{d}_j \end{aligned}$$

with

$$\begin{aligned} \tilde{p}_j &= p_j && \text{(closed order book)} \\ \tilde{p}_j &= p_j^{best\ ask} && \text{(open order book)} \end{aligned}$$

and their ask price p_{ij}^{ask} within the range:

$$\begin{aligned} \tilde{p}_j < p_{ij}^{ask} < \tilde{p}_j + 2\mathbf{d}_j & \quad \forall p_{ij}^{cut} \geq \tilde{p}_j + 2\mathbf{d}_j \\ \tilde{p}_j - \mathbf{d}_j < p_{ij}^{ask} < p_{ij}^{cut} & \quad \forall p_{ij}^{cut} < \tilde{p}_j + 2\mathbf{d}_j \end{aligned}$$

with

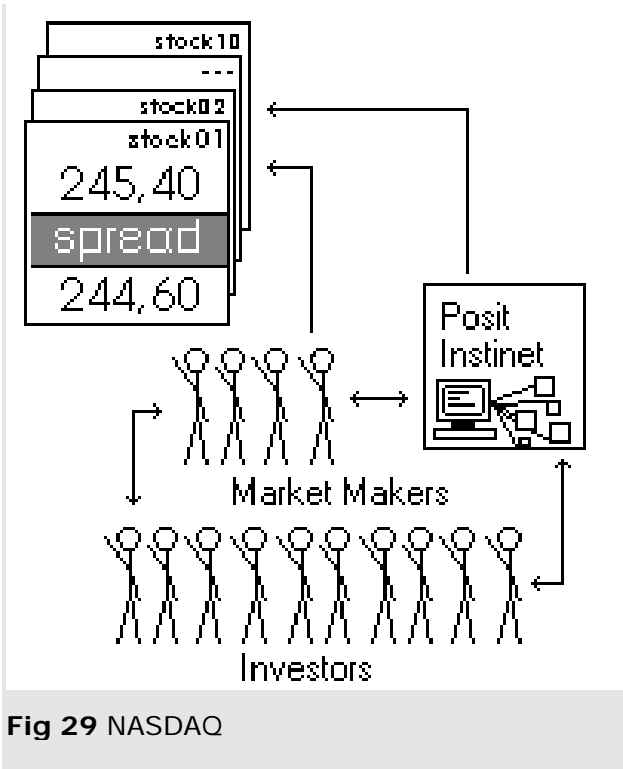
$$\begin{aligned} \tilde{p}_j &= p_j && \text{(closed order book)} \\ \tilde{p}_j &= p_j^{best\ bid} && \text{(open order book)} \end{aligned}$$

with p_j as the actual share quotation. The smallest amount allowed for a new order is 100 stocks.

With a closed order book, asks and bids are not observable by other traders, thus the market power m is 0.

⁷¹ The real XETRA rating *part 1* is defined as 'average quote duration'. In KapSyn XETRA, we cannot measure a quote duration since designated sponsor quotes expire after order book matching. Instead, the ratio between quote requests and quote request answers is used as substitute.

2.7.3 The Price-Finding Process in the NASDAQ Module⁷²



The KapSyn NASDAQ module simulates the electronic trading system of the NASDAQ. The NASDAQ, also called “market of markets”, is the market with the highest trading volume of any trading place in the US. More than 1000 trading firms, each with many traders, are using this order driven trading system, and around 6000 different stocks are available for trading.

The NASDAQ stock market is constructed as a dealer market where investors buy at the dealers ask price and sell at the dealers bid price.

Incoming market orders are executed against the inside dealer quote - the best bid and the best ask among all the dealers, often called inside spread or best displayed price. Limit orders are not exposed to the rest of the market and are not executed against the dealer’s quote.

The market maker determines his initial estimate for the spread he is offering by finding the price expectations of the pessimists and the optimists by negotiating with the brokers

$$spread_{MM,j} = \left| p_j - \bar{\hat{p}}_{ij} \right|_{i \in opt} + \left| p_j - \bar{\hat{p}}_{ij} \right|_{i \in pess}$$

with

$$\bar{\hat{p}}_j^{opt,pess} = \frac{\sum_{i \in opt,pess}^{n/2} \hat{p}_{ij}}{n/2}.$$

According to the revenue and the volume of trades the market maker has achieved in his trading history the market maker will modify his spread to either encourage more trades by decreasing his spread or to increase his margin by increasing his spread:

⁷² Contributed by M. Schossmann

$$\begin{pmatrix} p_{MM,j}^{buy} \\ p_{MM,j}^{sell} \end{pmatrix} = p_j + \mathbf{d} \begin{pmatrix} -range_{MM,j} \\ range_{MM,j} \end{pmatrix}$$

with

$$range_{MM,j} = \begin{cases} rev_{MM,j} > \frac{\sum rev_{MM,j}}{j,MM} : range_{MM,j} + 3 \\ rev_{MM,j} > 0 : range_{MM,j} + 1 \\ rev_{MM,j} < 0 : range_{MM,j} - 1 \end{cases}$$

where the revenue $rev_{MM,j}$ of the market maker for any given stock j has been calculated from the previous trade history and the initial range value has been calculated from

$$range_{MM,j} = \frac{|p_j - \hat{p}_{ij}|_{i \in opt, pess}}{d_j}$$

The market maker has to modify the resulting spread with respect to the limit orders which are placed with him for execution. NASD rules state that the dealer has to honor the prices of his limit orders and the market maker is not allowed to quote worse prices than the quotes of the limit orders in his order book:

$$\begin{pmatrix} p_{MM,j}^{buy} \\ p_{MM,j}^{sell} \end{pmatrix}_{quote} = \begin{pmatrix} \min p_{MM,j}^{buy,orderbook} \\ \max p_{MM,j}^{sell,orderbook} \end{pmatrix}$$

For trades amounting equal or less 1000 shares the SOES of the NASDAQ market is used, the minimum trade size for this system is 100 shares. The current implementation of the NASDAQ module assumes that all orders sized up to 1000 shares are routed to the SOES and therefore are executed immediately by moving the shares and the payments from the broker to the dealer and vice versa. All other orders are treated as limit orders and are stored with reference to a specific market maker. After adjusting their spreads for their limit orders the market makers execute their limit orders.

Currently, the lowest price increment for any stock in the NASDAQ market is \$1/16.

The bid prices p_{ij}^{bid} fall into the interval

$$\begin{aligned} \tilde{p}_j - range_{MM,j} \mathbf{d}_j < p_{ij}^{bid} < \tilde{p}_j & \quad \forall p_{ij}^{cut} \leq \tilde{p}_j - range_{MM,j} \mathbf{d}_j \\ p_{ij}^{cut} < p_{ij}^{bid} < \tilde{p}_j + \mathbf{d}_j & \quad \forall p_{ij}^{cut} > \tilde{p}_j - range_{MM,j} \mathbf{d}_j \end{aligned}$$

with $\tilde{p}_j = p_j$

and the ask prices p_{ij}^{ask} are in the interval

$$\begin{aligned} \tilde{p}_j < p_{ij}^{ask} < \tilde{p}_j + range_{MM,j} \mathbf{d}_j & \quad \forall p_{ij}^{cut} \geq \tilde{p}_j + range_{MM,j} \mathbf{d}_j \\ \tilde{p}_j - \mathbf{d}_j < p_{ij}^{ask} < p_{ij}^{cut} & \quad \forall p_{ij}^{cut} < \tilde{p}_j + range_{MM,j} \mathbf{d}_j \end{aligned}$$

with $\tilde{p}_j = p_j$

with p_j as actual share quotation.

2.7.3.1 KapSyn NASDAQ ATS Trading

Trading over ATS like Instinet or Posit allows agents to trade stocks while bypassing the market makers. Only a portion of all agents participate in ATS trading, the number can be set in the settings dialog box. The margin for trading over an ATS can also be set here. This is a simplification of the real trading environment since agents will set their ATS-quotes dynamically, resulting in changing margins.

Prices for KapSyn NASDAQ ATS-trading are set by matching the price expectations of the agents and accounting for a small ATS margin (parameter, set in the dialog box). For agent I and share j the following conditions apply for a ask:

$$\hat{p}_{ij} > \max_k \hat{p}_{kj} + mrgn_{ATS}, \quad k \in ATS$$

and

$$p_{MM,j}^{ask} > \max_k \hat{p}_{kj} + mrgn_{ATS}, \quad k \in ATS .$$

The ATS trade will only occur if both agents participate in ATS trading and the price expectation of one agent is higher than the price expectation of all other agents participating in the ATS, including margin. The second condition states that the price has to be better than the best quote from the market makers. The ATS trade is actually executed inside the NASDAQ spread. The same conditions apply to bid prices with $>$ replaced by $<$, max by min and $+mrgn$ by $-mrgn$.

2.7.3.2 KapSyn NASDAQ Preferencing Agreements

Market maker will establish preferencing agreements with certain agents, thus guaranteeing a continuous order flow. The market maker will pay the agent for orders placed with him, typically between \$.01 and \$.02 per share. This payment is an incentive for the agent to trade with a specific market maker. The KapSyn implementation of the NASDAQ market allows for fixed preferencing agreements between a given set of market makers and agents at a given payment rate. Since it takes a long time to establish preferencing agreements the preferencing agreements remain constant for a full simulation.

2.7.4 Transaction Cost Statistics

The KapSyn transaction cost statistics are calculated as follows:

- Calculation of $tc^{asset\ price-previous\ price}$

$$tc^{asset\ price-previous\ price} = \frac{\sum_{all\ transactions} |p^{trade} - p^{last\ price\ before\ trade}| q^{trade}}{\sum_{all\ transactions} p^{opening\ price} q^{trade}}$$

- Calculation of $tc^{asset\ price-price\ at\ offer}$

$$tc^{asset\ price-previous\ price} = \frac{\sum_{all\ transactions} |p^{trade} - p^{price\ at\ offer}| q^{trade}}{\sum_{all\ transactions} p^{opening\ price} q^{trade}}$$

where $p_{price\ expectation\ at\ offer} = p_{\max}(\Phi_{ask,bid}(p_{pot}))$

for $p_{trade} < p_{pot} < p_{cut}$ (bid)

or $p_{cut} < p_{pot} < p_{trade}$ (ask)

For each transaction, costs for buyers and sellers are calculated separately. When trading with a market maker or designated sponsor, only the trader's transaction costs are monitored.

3 The KapSyn computer program

The KapSyn program has quite a long history. The first program was written in 1992 as a DOS application. The current code was completely redesigned and written as a pure 32 bit Windows application, thus the program must be run under Windows 95/98/NT. One of the main goals was to hide the complexity of the KapSyn model without taking away the possibility to change any parameter as needed. Furthermore, the program is written object oriented and provides a defined interface to integrate new stock exchange modules into the existing environment. The KapSyn program is entirely written in C++.

This chapter will explain installation, program handling, error messages and the result files produced by a single simulation or a simulation series.

3.1 Installation and system requirements

The KapSyn program can be installed simply by extracting the `KapSyn.zip` file into an empty directory. No Windows registry entries are made and no system files are overwritten, so the program can be removed completely by deleting the installation directory.

Everytime the program starts it tries to load a parameter input file named `standard.pat`. If this file is missing, the program produces an error message stating the missing file and quits immediately. This means that, without `standard.pat`, a program start is impossible.

The minimum system requirements are

- a 486 processor (Pentium with 166 MHz or faster is recommended)
- Windows 95/98 or NT
- a monitor with 800x600 resolution (1024x768 is recommended)
- RAM as recommended for the operating system
- 2 Mbytes free space on the hard disk, plus around 80 kbytes for every simulation result of a simulation series.

3.2 Terminology

Bid A trader's offer to buy

Ask A trader's offer to sell

Parameter settings

A set of parameters needed to describe a KapSyn scenario (eg. bearish, volatile etc.). These include the opening share prices of all stocks, the KapSyn parameters η , ρ and ξ in their various forms, and stock exchange specific settings. Parameter settings are saved in files with `.pat` extension.

Simulation series

A set of individual simulations with the same KapSyn parameter settings. The average will be calculated from the individual results to reduce the deviation of stochastic processes.

3.3 Choosing a stock exchange module

The *modules* parameter box can be used to activate a stock exchange (currently XETRA, NASDAQ and Market Maker are available) and to choose settings that are stock exchange specific. When starting a simulation, the chosen stock exchange will be simulated.

3.3.1 The XETRA module

The KapSyn XETRA module simulates the electronic trading system of the Frankfurt Stock Exchange, release 3. The XETRA system was introduced in June 1997 and has been extended several times. Currently, release 3.1 is in operation.

The XETRA traders sit in front of their online trading screen, isolated from each other. During normal trading they can view all existing orders (open order book), and they can give in their own market or limit orders in any size at any time during XETRA trading, that is between 8:25 and 17:00 MEST. Orders are treated with time/price-priority. Normal trading is interrupted by at least three auctions, as explained below.

There are three features that are specific for this KapSyn module: Order book, designated sponsor and auctions. As real XETRA trading rules can be very complex, some simplifications had to be made.

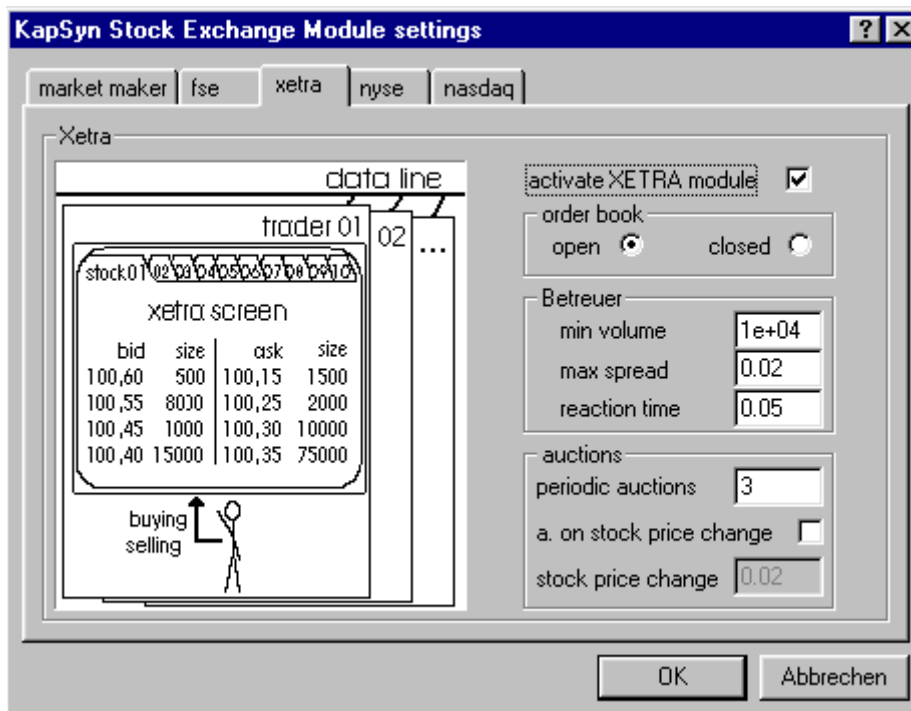


Fig 30 KapSyn stock exchange module settings: XETRA page

3.3.1.1 Order book

Whereas real XETRA has an open order book during normal trading time, the KapSyn XETRA module can simulate a closed order book as well. With an open order book, during the offer price finding process, the traders take the best ask and bid prices into account. With a closed order book, the last stock price is taken for determination of the KapSyn traders offer price. The closed order book is provided for transaction cost statistic purposes.

3.3.1.2 Designated sponsor

To increase liquidity for second-line stocks, a "Betreuer" (German for designated sponsor) has been introduced. Each designated sponsor is responsible for a certain stock, and "quote requests" can be made to him by any trader. The designated sponsor then should, but is not obliged to, set a limited ask and bid quote. The maximum reaction time, the minimum ask/bid volume, and the maximum spread are defined and controlled. These three values can be set with the KapSyn XETRA module dialog box. The minimum volume is given in currency units, the maximum spread as fraction of the current stock price, and the reaction time as fraction of the total trading day time. The designated sponsor can be disabled by setting his reaction time to the value of 2.0. Designated sponsor actions and statistics are shown in the general graphics window (cf. chapter 3.5.1.1).

3.3.1.3 Auctions

Every XETRA trading day starts with, is interrupted by at least one, and ends with an auction. During auction time the order book is partially closed: only the best ask and bid quotes are visible, and normal trading is stopped. Every auction starts with a *call*, and, after a certain time, ends with a *market matching*. The order book is matched according to the highest trading volume principle, where the new stock price is determined as the price with the highest trading volume. Additional auctions can be triggered by a stock price change exceeding a defined limit.

In the KapSyn XETRA dialog box, the number of periodic auctions can be set to any value between 2 and 10. If set to 0, the periodic auctions are disabled. For "real XETRA" simulation this value is set to 3. If *auction on stock price change* is marked, a stock price change trigger in percent can be entered.

The exact course of all auctions can be viewed in the *price/volume* graphics window (cf. chapter 3.5.1.2).

3.3.2 The NASDAQ module

The KapSyn NASDAQ module simulates the electronic trading system of the NASDAQ. The NASDAQ, also called "market of markets", is the market with the highest trading volume of the United States of America. More than 1000 trading firms, each with many traders, are using this order driven trading system, and around 6000 different stocks are available for trading. The daily trade volume exceeds 900 millions dollar at an average of 1 million trades per day [NASDAQ Performance Overview June 1999].

The NASDAQ stock market is constructed as a dealer market where investors buy at the dealers ask price and sell at the dealers bid price. The bid-ask quotes of competing dealers are electronically disseminated to brokers offices. The number of dealers in an active stock often exceeds 30 and can approach 60 for the most active issues. NASDAQ rules require at least two market makers per share to guarantee a competition for investor orders by displaying their buy and sell interests in NASDAQ-listed stocks. Each dealer in an active stock is obligated to trade 1000 shares at the quote. Incoming market orders are executed against the inside dealer quote - the best bid and the best ask among all the dealers, often called inside spread or best displayed price. Limit orders are not exposed to the rest of the market and are not executed against the dealer's quote. NASDAQ rules introduce some competition from limit orders

into the market. Dealers are not allowed to trade ahead of any customer limit order. Thus a dealer cannot buy at a bid price lower than a customer's limit order buy price. The customer's order has to be satisfied first.

Two special market makers, alternative trading systems (ATS), are also part of the NASDAQ. Instinet and SelectNet are not accessible to the general investing public and allow to close out positions inside the NASDAQ spread. Trades on these networks are still reported to NASDAQ and NASD rules state that a market maker is not allowed to simultaneously post a quote for a specific stock on the NASDAQ and a better quote on any other network. The current implementation of the NASDAQ module features one ATS.

Much of the order flow on NASDAQ is internalized or preferenced to specific dealers. A broker-dealer internalizes order flow by trading for his own account with the customer. The Rules of best execution apply, which means the trade takes place at the inside quote of the NASDAQ. Preferenced order flow is sent by the broker to a particular market maker who guarantees execution at the best inside quote. Under a preferencing agreement the market maker will often pay for the orders routed to him from a specific broker (between \$.01 and \$.02 a share for stocks trading at least 1/8 - 1/4 point spreads [NASD 1991]) and guarantee the broker execution at the best price. As stated by Kandel and Marx (1999) these preferencing agreements are costly to establish and maintain and therefore only brokers with intermediate to high trade volumes will choose to establish these agreements.

The market maker determines his initial estimate for the spread he is offering by finding the price expectations of the pessimists and the optimists by negotiating with the brokers.

Market makers often have a sense for large orders other market makers are dealing with since they also receive calls from other dealers [JFE 50, p. 41 (1998)]. Therefore it is fairly easy for a market maker to determine the varying price expectations of the dealers and the customers.

According to the revenue and the volume of trades the market maker has achieved in his trading history the market maker will modify his spread to either encourage more trades by decreasing his spread or to increase his margin by increasing his spread. The market maker closely observes the quotes and the executed trades which are reported through the NASDAQ. By comparing his trading history with other trades the market maker can easily derive his performance against the performance of the other dealers. The

market maker has to modify the resulting spread with respect to the limit orders which are placed with him for execution. NASD rules state that the dealer has to honor the prices of his limit orders and the market maker is not allowed to quote worse prices than the quotes of the limit orders in his order book.

For trades amounting equal or less 1000 shares the SOES of the NASDAQ market is used. This electronic system allows for immediate trades at the posted quotes at very low execution costs. The minimum trade size is 100 shares.

The current implementation of the NASDAQ module assumes that all orders sized up to 1000 shares are routed to the SOES and therefore are executed immediately by moving the shares and the payments from the broker to the dealer and vice versa. All other orders are treated as limit orders and are stored with reference to a specific market maker. After adjusting their spreads for their limit orders the market makers execute their limit orders.

Currently, the lowest price increment for any stock in the NASDAQ market is \$1/16. The NASDAQ market will switch to a decimal increment as soon as the various systems are changed to account for the new quoting basis. The corresponding legal framework has already been established.

There are two features that are specific for this KapSyn module: ATS trades and preferencing agreements. The implementation of the module somewhat simplifies the very complex NASDAQ trading, but it does cover the main features of this stock market.

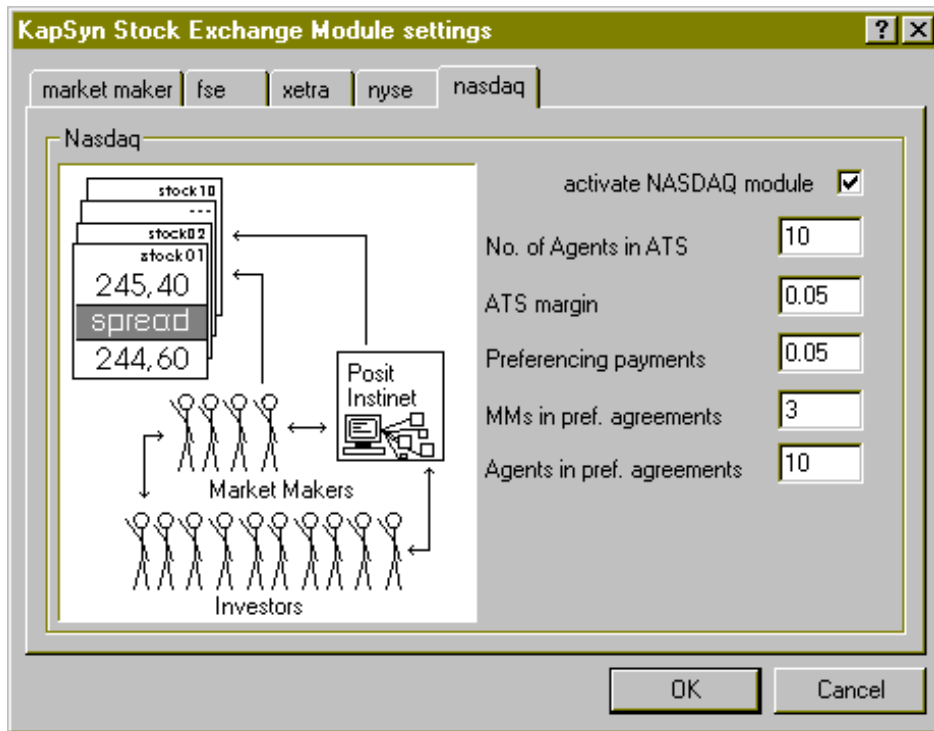


Fig 31 KapSyn stock exchange module settings: NASDAQ page

ATS Trading

Trading over ATS like Instinet or Posit allows agents to trade stocks while bypassing the market makers. Only a portion of all agents participate in ATS trading, the number can be set in the settings dialog box. The margin for trading over an ATS can also be set here. This is a simplification of the real trading environment since agents will set their ATS-quotes dynamically, resulting in changing margins.

Preferencing Agreements

Market maker will establish preferencing agreements with certain agents, thus guaranteeing a continuous order flow. The market maker will pay the agent for orders placed with him, typically between \$.01 and \$.02 per share. This payment is an incentive for the agent to trade with a specific market maker. The KapSyn implementation of the NASDAQ market allows for fixed preferencing agreements between a given set of market makers and agents at a given payment rate. Since it takes a long time to establish preferencing agreements the preferencing agreements remain constant for a full simulation.

3.4 Setting the KapSyn parameters

There are numerous KapSyn parameters, and some effort was made to make their handling as easy and straightforward as possible.

First, the parameters are grouped by themes, resulting in

- general parameters
- scenario dependant parameters (including traders expectations)
- stock names, start prices and stock/scenario associations
- futures trader parameters
- KapSyn module adjustment factors
- transaction cost statistics parameters
- loading and saving all of the above parameter settings

Second, some parameters are split up into two different trader groups, so their behaviour can be defined independently.

For most of the parameters, a brief explanation is given in the bottom of the dialog box, and suggested values are shown in the "value limits" section.

It is important to note that extreme combinations of parameter values, although all in the allowed range, can lead to a unrealistic simulation behaviour, produce errors and can lead to a simulation stop or even a program crash.

The internal reason for an error is displayed on the KapSyn log screen and can be read before the program shuts down. However, the range of possible simulation paths is wide enough, even to simulate exotic runs.

All parameter groups will be explained in the next chapters.

3.4.1 General parameters

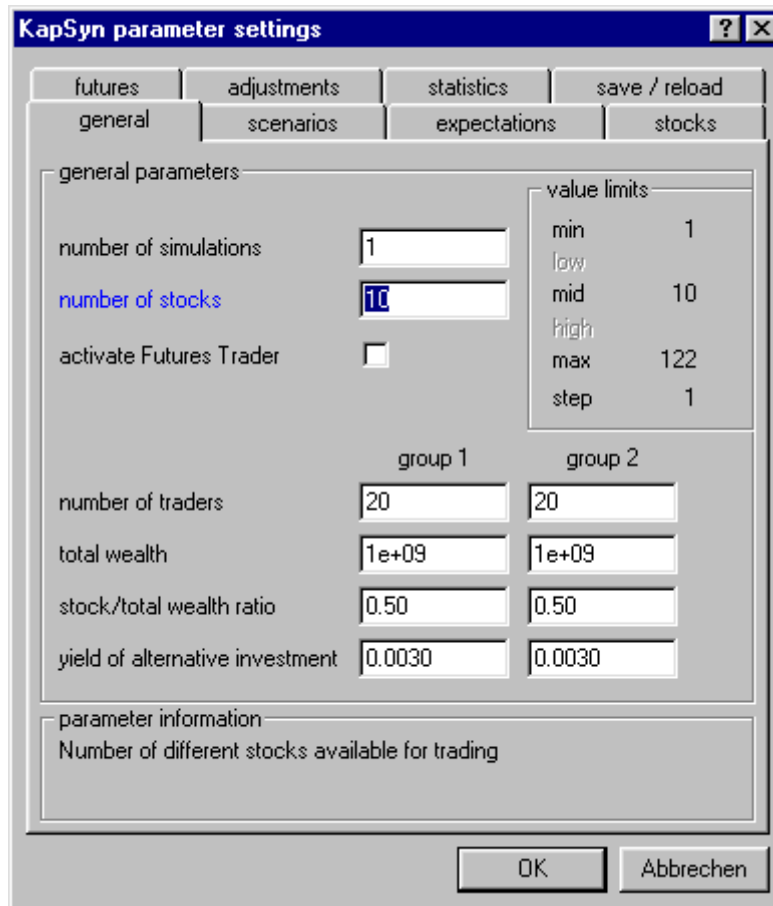


Fig 32 KapSyn parameter settings: General page

Number of simulations Number of subsequent simulations to run. When choosing a value higher than 1, the same parameters are used to run the specified number of simulations, and the average of all simulations is calculated. This is useful to eliminate stochastic influences.

Number of stocks Number of different stocks available for trading. Increasing this value increases the time needed to calculate a simulation.

Activate futures trader Apart from the regular traders, a futures trader can be activated. The futures traders variables can be found on the *futures trader* dialog page.

Number of traders Number of traders for each of the two trader groups. The number of traders has to be even, because one half is belonging to optimists, the other half to pessimists (see chapter 2.4.1.8 and 3.4.3). Increasing the number of traders increases the time needed to calculate a simulation, but reduces stochastic fluctuations.

Total wealth Total wealth in currency units of each trader at the beginning of a simulation.

Stock/total wealth ratio The part of the traders total wealth invested in stocks. With a ratio of 0.5 and 10 stocks available for trading, the traders start portfolio holds around 5% of the traders total wealth in each stock.

Yield of an alternative investment, based on the traders planning horizon.

3.4.2 Scenarios

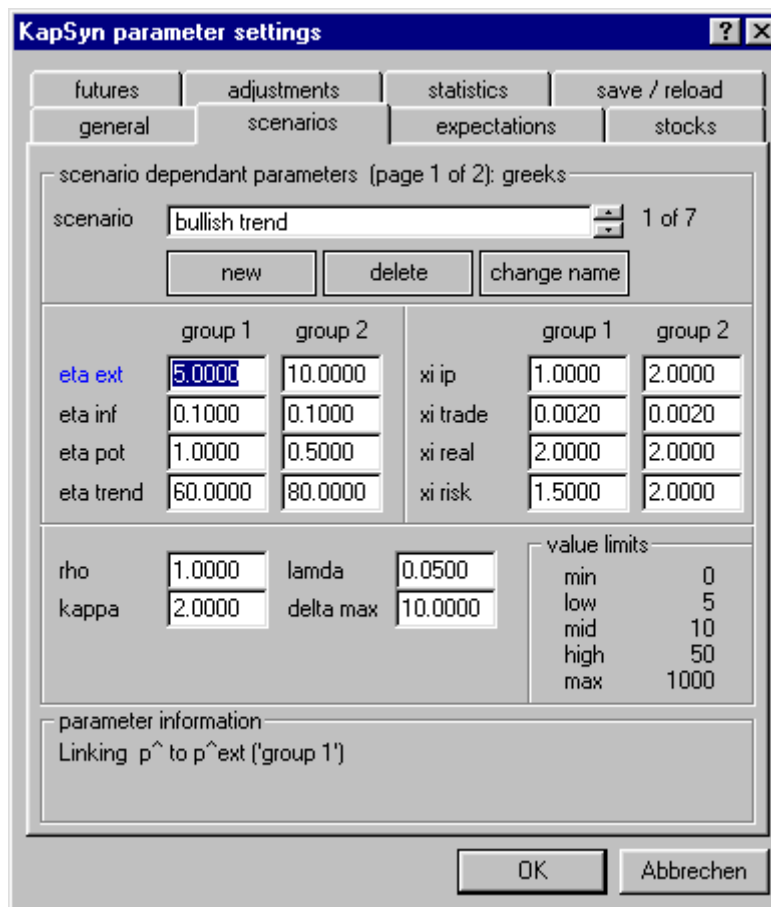


Fig 33 KapSyn parameter settings: Scenarios page

All parameters that influence the traders behaviour are grouped to scenarios. Every KapSyn trader can buy and sell the number of different stocks given in the general section (ch. 3.4.1), and for each stock the traders have a certain opinion – determined by a scenario. It is possible to create as much as 40 different scenarios, although a smaller number is usually sufficient. When installing the KapSyn program, seven predefined scenarios are available: three *bullish*, one *volatile* and three *bearish* scenarios. Scenarios can be deleted, new ones can be created, or just the scenario name can be changed. Two arrow buttons are provided to select a scenario, and the corresponding parameter values are shown. Because of the large number of parameters, the scenario

dependant \hat{p}_{ext} parameters can be found in a separate *expectations* dialog page. All parameters are explained in detail in chapter 2.

The decision which scenarios are used for a simulation can be made using the *stocks* dialog page (ch. 3.4.4).

3.4.3 Expectations

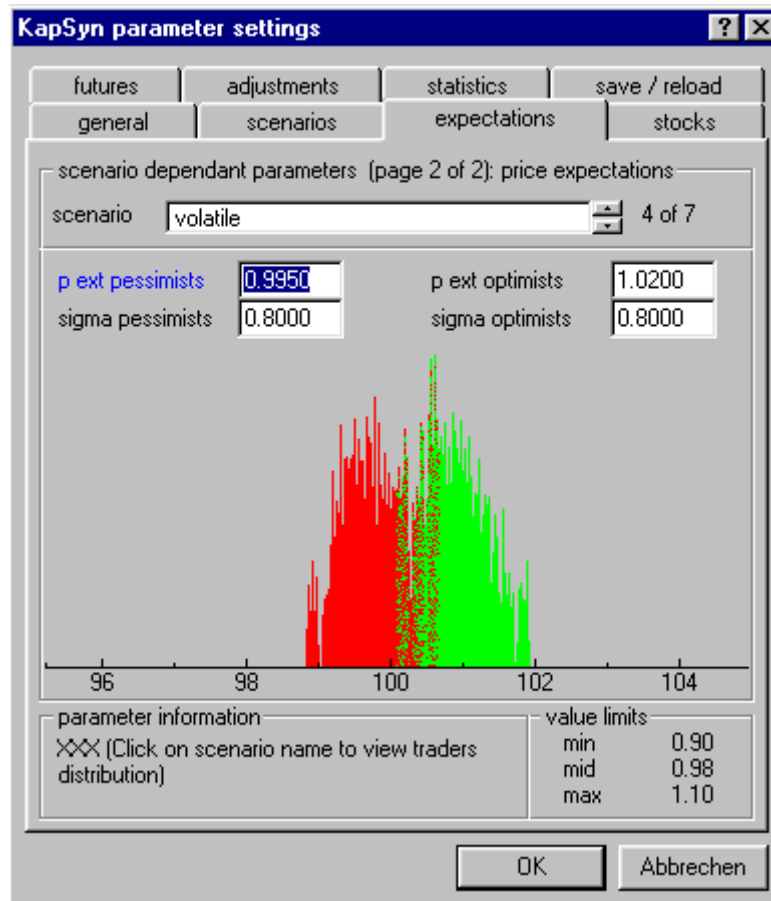


Fig 34 KapSyn parameter settings: Expectations page

The second page belonging to scenario dependent parameters is showing the traders' expectations. One half of the traders is assigned to optimists, the other half to pessimists. This assignment is independent from the trader groups that are used in the *general* and *scenarios* dialog page: Each group is split up into optimists and pessimists.

The higher the p_{ext} (\hat{p}_{ext}) values, the more likely the stock price will rise, and vice versa. If both optimists and pessimists p_{ext} values are above (below) 1, trading becomes very rare because all traders want to buy (sell), but no one likes to sell (buy). The σ values describe the standard deviation of the distributions (See chapter 2 for a in-depth description). Clicking on the scenario name produces a \hat{p}_{ext} -distribution graphic.

3.4.4 Stocks

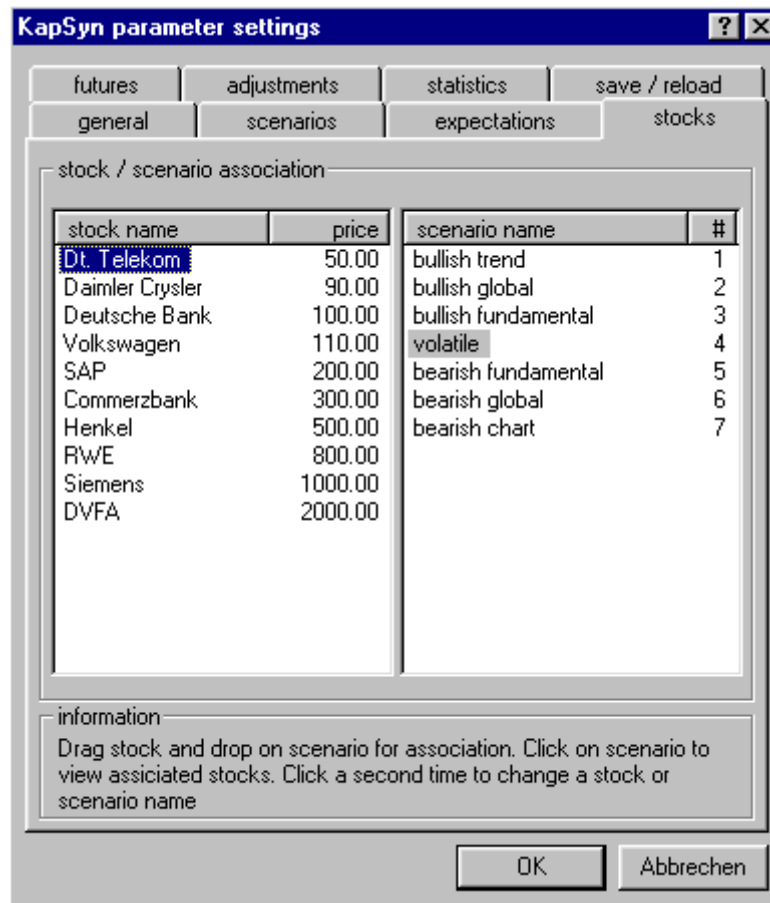


Fig 35 KapSyn parameter settings: Stock/scenario association page

The *stock / scenario association* dialog page has two list boxes. The left box displays all stocks and their start prices, the right box lists all available scenarios. This dialog page is designed to handle four different tasks.

- Stock and scenario names can be changed by clicking on an already selected name.
- Stock prices can be changed by double clicking on the stock name
- Stocks can be associated with scenarios by dragging a stock name and dropping on a scenario name.
- Associations can be reviewed by clicking on a stock or scenario name.

3.4.5 Futures

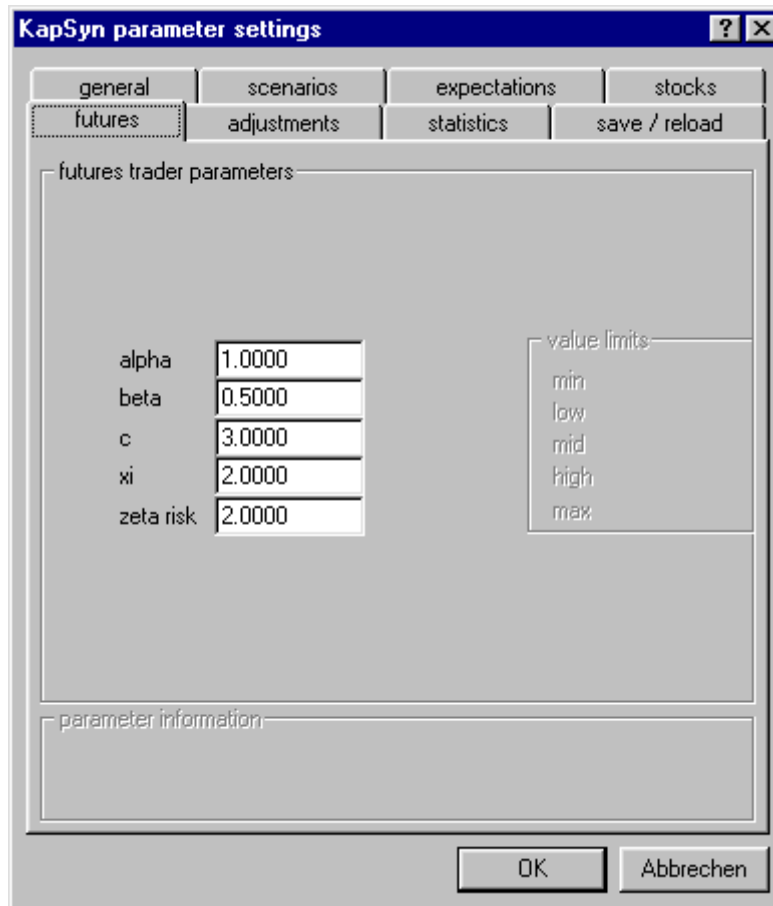


Fig 36 KapSyn parameter settings: Futures trader page

All futures trader parameters can be found on the *futures* dialog page. Activation of the futures trader is on the *general dialog* page. The parameters are explained in chapter 2.

3.4.6 Adjustments

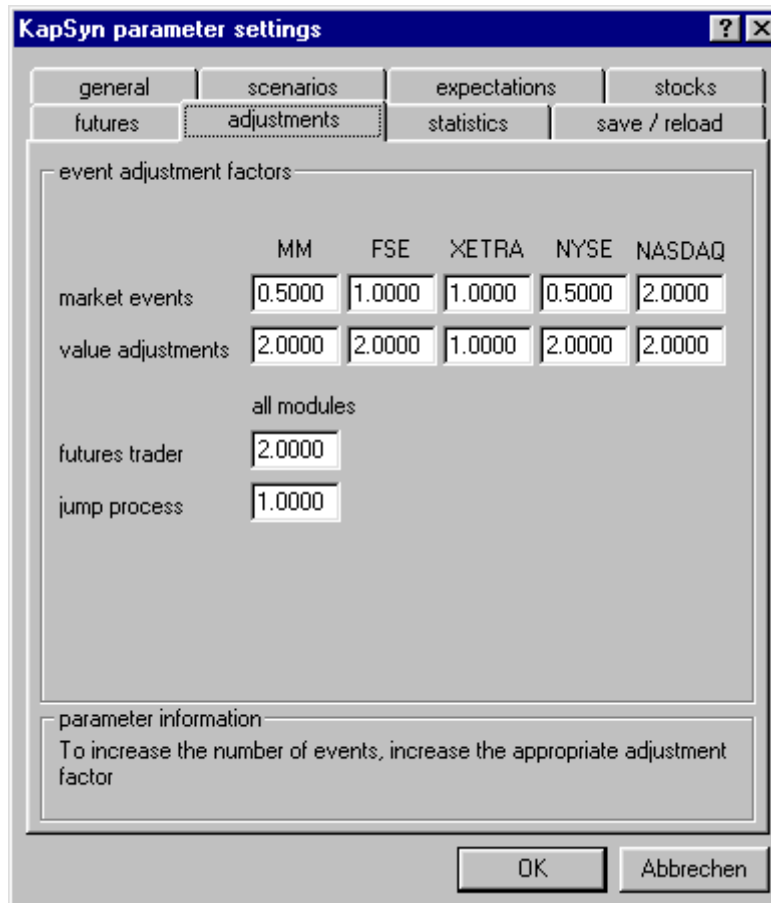


Fig 37 KapSyn parameter settings: Event adjustment factors page

The *event adjustment factors* are used to adjust the number of value adjustments and market events that are happening during a simulation run. To obtain a realistic simulation, these values should produce an amount of trades in the order of the real stock exchange trading day.

3.4.7 Statistics

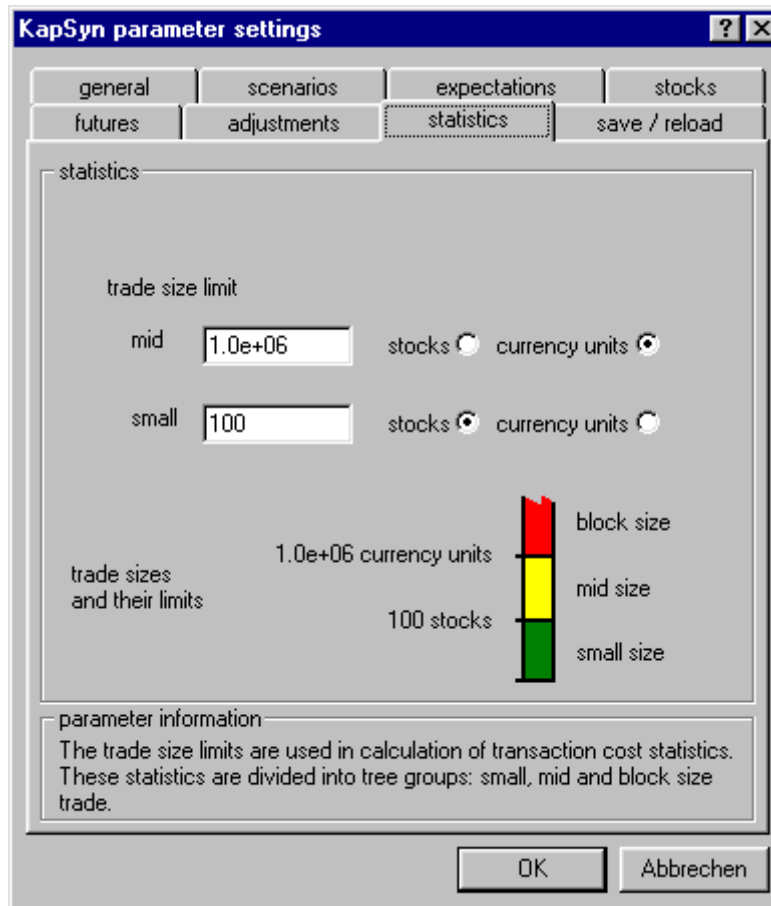


Fig 38 KapSyn parameter settings: Statistics page

The KapSyn transaction cost statistics are collected for each trade. There are three transaction size categories, and the limits between these categories can be changed. With the values given in the figure the *small size* class collects statistics for each trade up to 100 stocks in size. The *mid size* class collects statistics for all trades with a size between 101 stocks and 1 billion currency units, and *block size* all trades above 1 billion currency units.

3.4.8 Save/reload

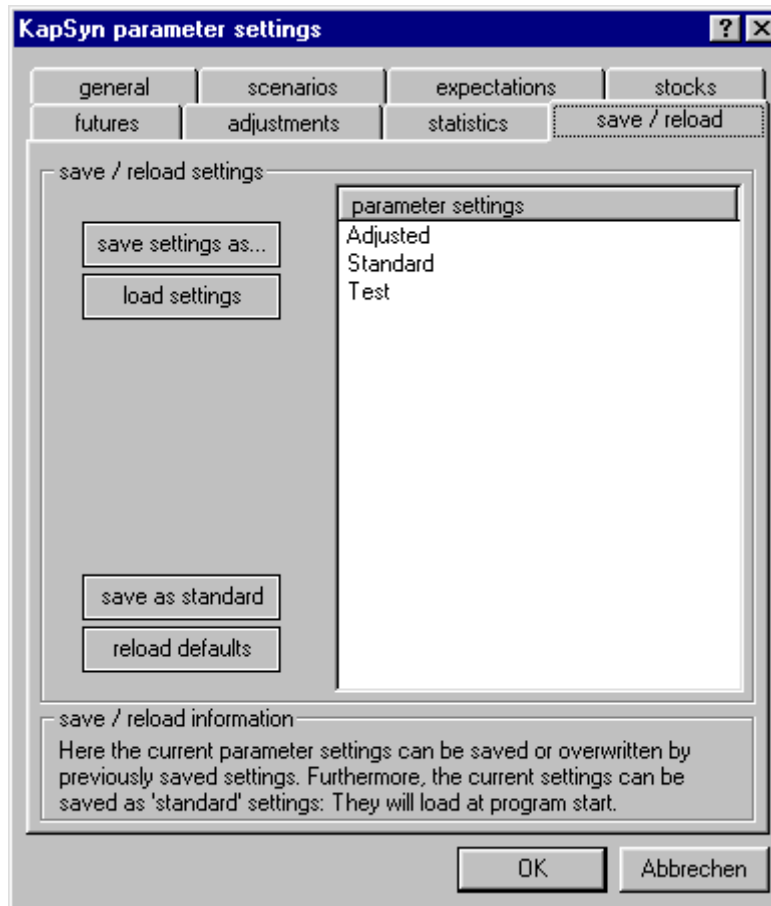


Fig 39 KapSyn parameter settings: Save/reload settings page

All settings – parameter as well as module settings – can be saved to a parameter settings file. Everytime the KapSyn program is started it loads the standard settings file `standard.pat`. With the *save as standard* option this file can be changed to the current parameter settings. Alternative settings can be saved with *save setting as...* and can be reloaded with the *load settings* option.

**All parameter changes that are not saved
are lost after the program is closed.**

The parameter input files are plain text format with a `.pat` extension, and they can be reviewed, changed or created with any text processing program. It is possible, for instance, to create a whole settings series with a serial print procedure. When creating new parameter input files, care must be taken to replicate the original data structure found in `standard.pat`.

With the *reload defaults* option, the standard settings file can be reset to the original "factory" settings.

3.5 Program output and simulation results

During a simulation run, a lot of information can be obtained about what happens. First, general information, warnings and errors are logged to the KapSyn log screen. The status bar shows the progress for the current simulation and the simulation series. The graphics window presents a real-time view of the simulation evolution, and the ticker window outputs every ask, bid or trade that is made while the simulation runs.

Previous simulation results can be reviewed with the graphics window, and all simulation statistics are appended to a text file.

3.5.1 The graphics window

The graphics window is opened and closed with the *Graphics* command found in the *Simulation* menu, or with a shortcut on the symbol bar. There are three views available, the *general*, *price/volume* and *parameters* view (found in the *Display* menu of the graphics screen).

All data shown in the general view is permanently stored, so this view can be recreated when necessary. All other views are temporary: they are created only during a simulation run and are not permanently stored due to the large amount of data necessary. As soon as the window view is changed, these graphics disappear and their historic parts can not be recreated.

The graphics window can be opened or closed at any time, even during a simulation run. It needs at least a screen resolution of 800x600 pixels (1024x768 is recommended).

3.5.1.1 The general view

The *general view* graphics window is divided into different sections. Results can be viewed during or after a simulation. The simulation data needed to create the general view is available until a new simulation is started. If not available, the last simulation results can be reloaded with *Simulations / Load results* (File type set to *Last Simulation Results*). Data from automated simulation runs (ch. 3.6) can be loaded in the same way.

Whenever a result file is loaded, all current parameter settings are overwritten by the settings that were used during the loaded simulation.

The graphics and statistics are displayed for a single stock, which can be chosen with the `stock` menu items.

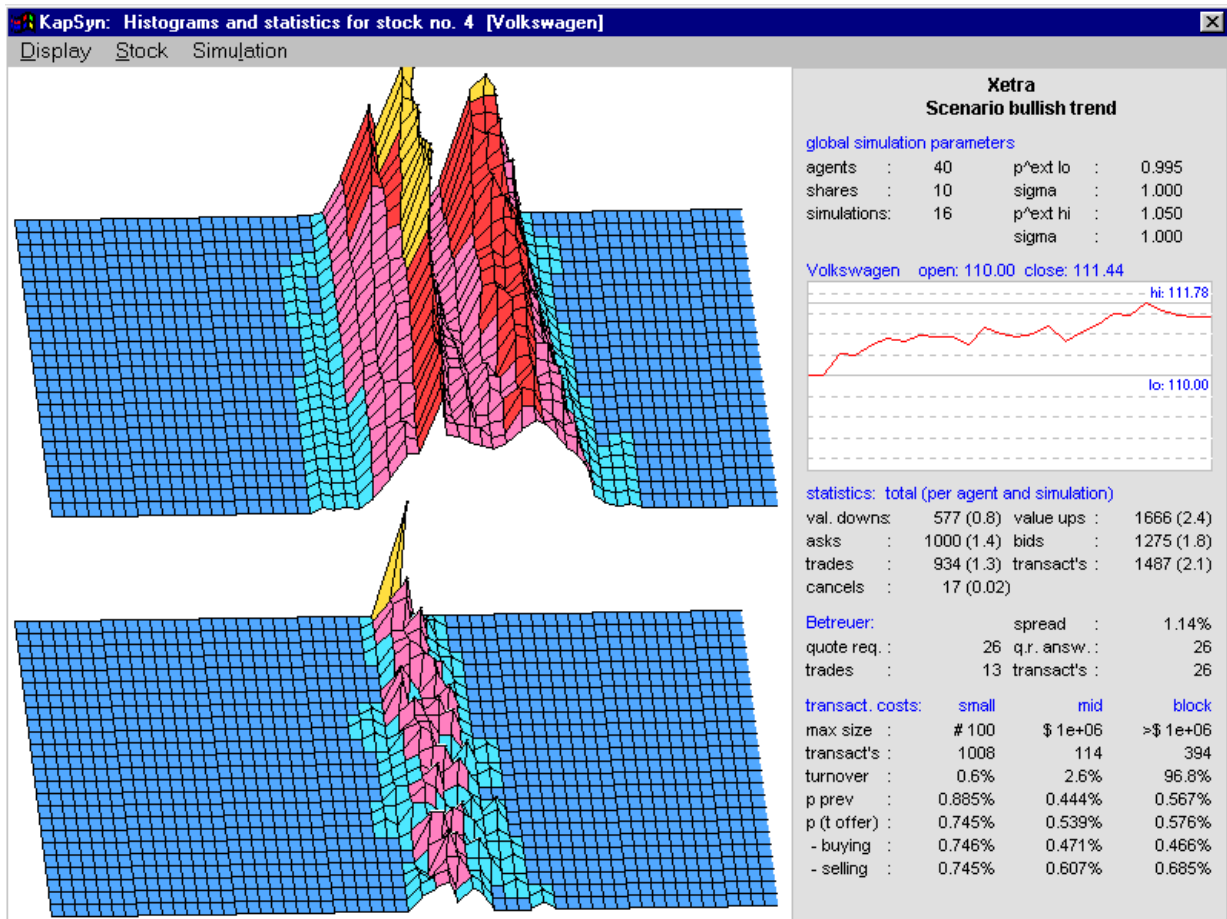


Fig 40 Graphics window: General view

The upper left 3D-graphic illustrates the variation of the traders price expectations in time. The horizontal lines represent the traders subjective price expectations, going from low (left) to high (right) values. Time leads from the back to the front of the graphic: The backmost horizontal layer of the mountain structure shows the \hat{p} -distribution at the beginning, the frontmost layer illustrates the \hat{p} -distribution at the end of the trading session. The altitude indicates the number of traders (frequency) that had this price expectation at that point in time: The higher the peak of the "mountain", the more traders shared the respective price expectation. Values from a simulation series are cumulated.

The lower left 3D-graphic has the same structure than the above, but illustrates the variation of stock prices in time. Since every new simulation of a series creates a new stock price path, the frequency of all the paths can be shown. The higher the "mountain", the more simulations had the same stock price at that point in time.

For calculation purposes, the frequencies are measured by an equidistant grid. However, the individual path quotations during simulation are asynchronous, not equidistant. In the general view graphic, the quotation at a grid point is the last real quotation preceding the grid's point in time.

For perspective-related reasons the 3D-view is slightly turned to the right. This is why the movement in time to the left, i.e. towards lower price expectations, is over-estimated at first sight. The lines drawn from the back to the front can be helpful when trying to estimate whether the distribution moves towards higher or lower prices.

The *global simulation parameters* on the right of Fig 40 present the basic parameters of the simulation. The \hat{p}_{ext} distribution is characterized by the lower mean of $\hat{p}_{ext} = 0.995$ and the upper mean of $\hat{p}_{ext} = 1.05$, with each standard deviation set to 1.0. The chart below shows the average of the 16 simulated price paths.

The lower right corner displays market statistics: The traders adjusted their \hat{p} -estimations 577 times downwards, 1666 times upwards, placed 1000 offers to sell and 1275 offers to buy, resulting in 934 trades with 1487 transactions⁷³. 17 times an offer (ask or bid) was cancelled. The values in brackets show the statistics per agent and simulation.

Depending on the simulated stock exchange, some specific statistics are given in the next section. In Fig 40 the statistics display the KapSyn XETRA designated sponsor. There were 26 quote requests and the same amount of quote request answers (ch. 3.3.1.2) with an average quote spread of 1.14%, resulting in 13 trades with 26 transactions¹. For NASDAQ this section exhibits ATS statistics.

The calculation of transaction costs is split into three categories according to *small*, *mid* and *block* trades. The maximum size is given for each category. In Fig 40 *small* means all trades up to 100 shares (#), *block* all trades with more than 1 billion currency units, and *mid* anything in between. The next lines display transaction statistics for each category.

First, the number of transactions and their relative turnover is calculated. Here, for instance, 1008 small transactions are making 0.6% of the total turnover.

⁷³ The number of transactions may be higher than the number of asks, bids or trades. This can happen, if a large volume ask is met by numerous smaller bids, and vice versa. Only one trade, but each individual transaction is counted.

Second, the transaction costs are given as *previous price* and *price at offer*. All transaction cost statistics are based on the complete execution of offers. In most cases transaction costs calculated on *prices at offer* (i.e. prices valid at the moment the order is placed on the market) are higher than those calculated on *previous price*. (The corresponding formulae are given in chapter 2.7.5) This is the case because it takes some time to settle an entire order, so the *price at offer* can differ – sometimes considerably – from the actual price. In a market maker module, there is no time delay between order and settlement, therefore transaction costs calculated on *prices at offer* are zero. A more detailed explanation and the formula to provide an appropriate transaction cost figure is given in chapter 2.7.4.

3.5.1.2 The price/volume view

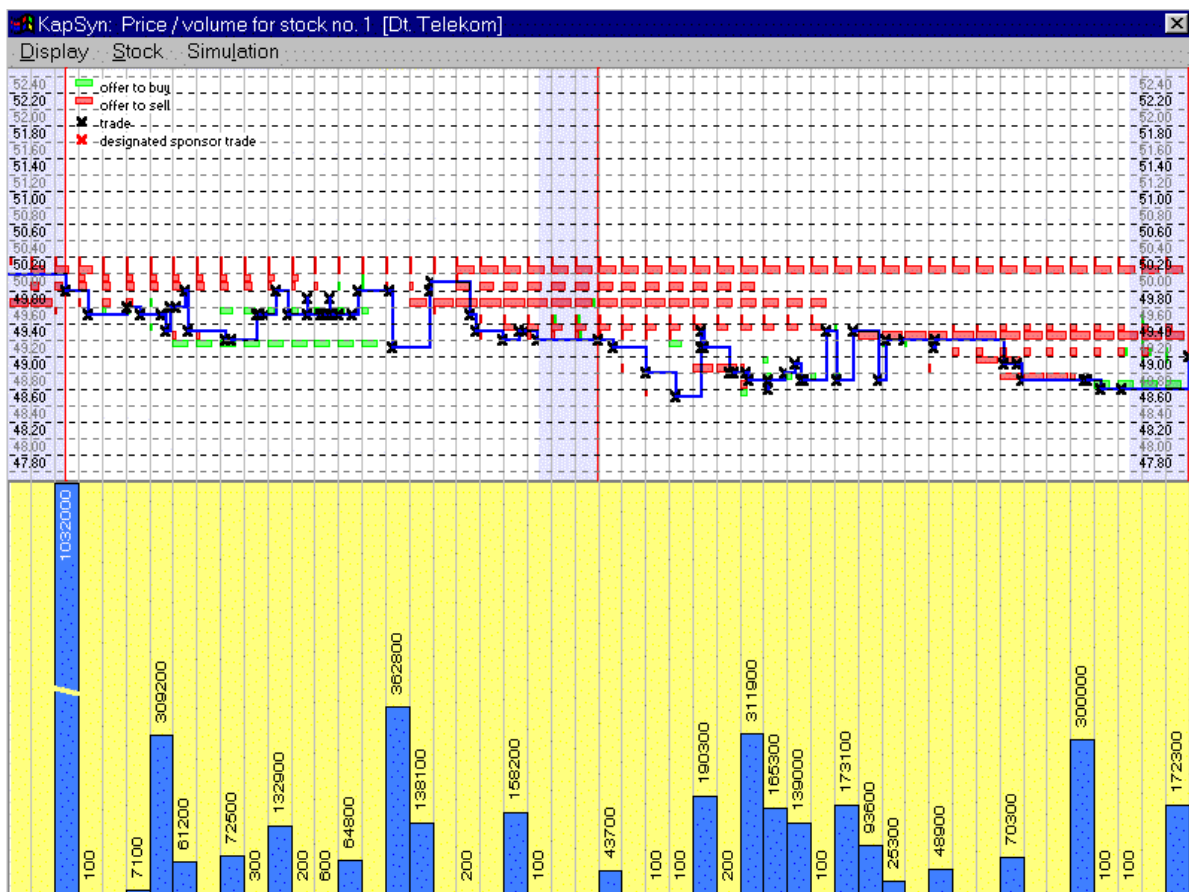


Fig 41 Graphics window: Price/volume view

The *price/volume* view is drawn only during a simulation run. The upper half shows the exact stock price evolution (blue line), and every trade is marked by a black x, whereas XETRA designated sponsor trades are marked with a red x. The light blue areas in the upper half represent XETRA *call for auction* periods (ch. 3.3.1.3), while the red lines indicate the auction itself. The green and red rectangles are order book histograms, drawn in 48 equidistant time steps on

the price (y-) axis. The higher (in x-axis direction) the rectangle, the more supply (red) or demand (green) for the given price is in the order book.

The lower half of the window produces a trade volume histogram in 48 equidistant time intervals.

3.5.1.3 The variables view

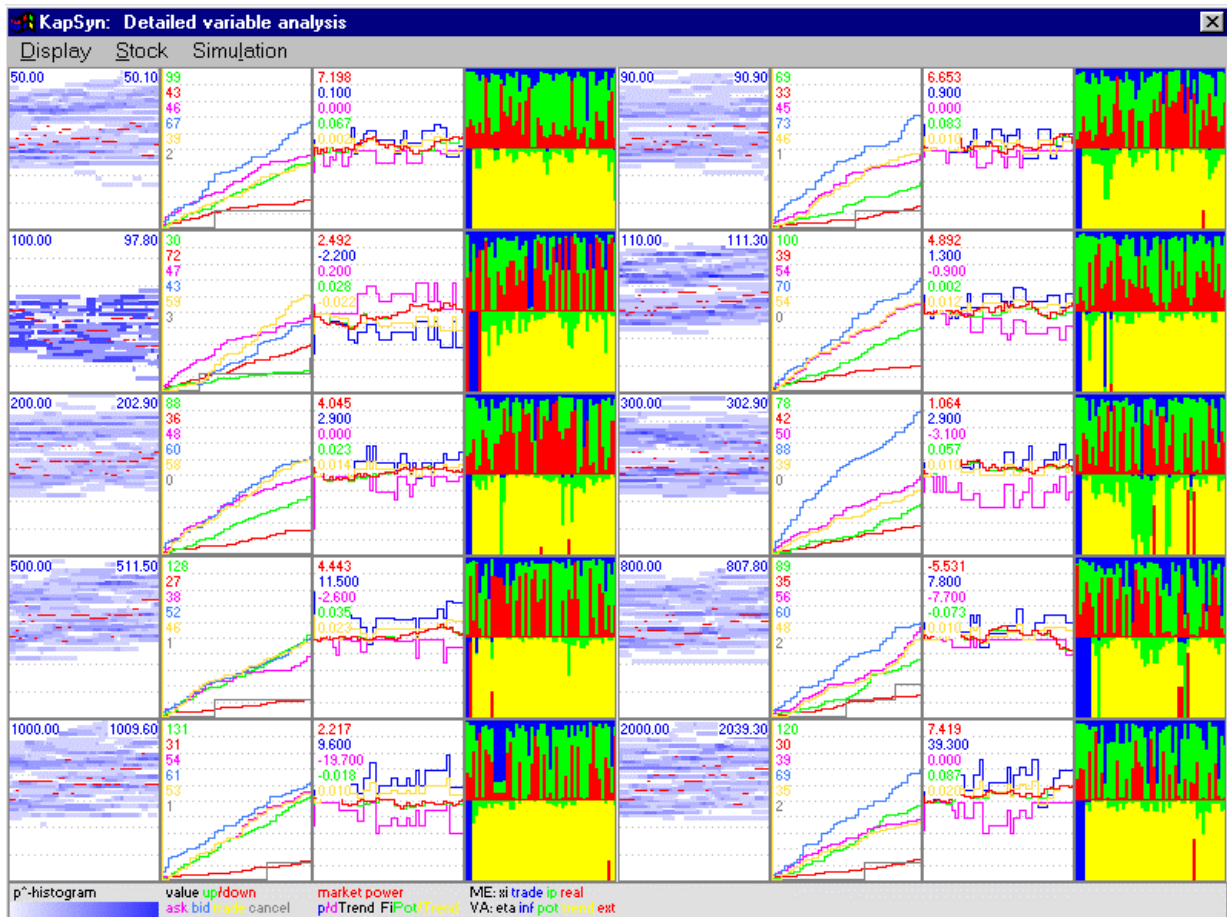


Fig 42 Graphics window: Variables view

The *variables* view is a tool that graphically represents all KapSyn parameter values for all stocks at 48 intervals. This view is useful to study the effects of KapSyn parameter variations on the stock exchange performance.

Four small windows are provided for each stock, the first showing the traders expectations distribution in blue. A deeper color symbolizes more traders with the same stock price expectation in a certain price range (y-axis) at a point in time (x-axis). The stock price itself is given in red, open and close price is given on top.

The second window shows the evaluation of event counters for price expectation *value ups* and *value downs*, for *asks*, *bids*, *trades* and *cancel*s. The final values are given in the line colour.

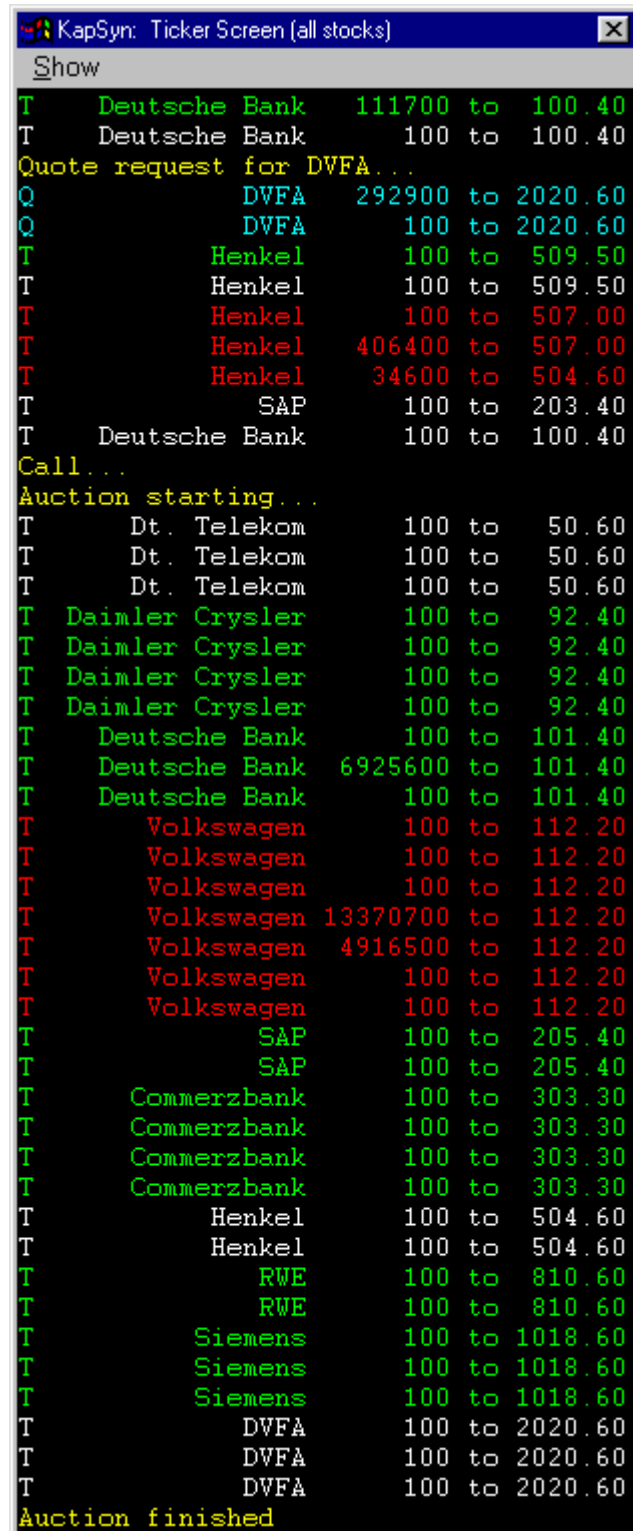
In the third window market power with related $pTrend$ and $dTrend$ as well as fi pot and fi trend are drawn, and final values are given in the variable colors.

The fourth window is divided in two sections. The upper half shows the relative weight of all summands that make the market event transition rate. The lower half is used to show the relative weight of all value adjustment summands. In Fig 42 for instance the lower half of the windows show that the value adjustment transition rates are dominated by *eta trend* most of the time (the yellow colour).

3.5.2 The ticker window

The ticker window provides information about all trades that are made during a simulation run. The type of information is depicted by colours. In the *all stocks* view (the default) white depicts a trade without stock price change, whereas red means a lower, green a higher stock price. XETRA designated sponsor trades are given in cyan, whilst informational messages are yellow. For each trade, the stock name, quantity, and trading price is given.

Additional, the information can be filtered for a specific stock. In this mode, asks (red), bids (green) and trades (white) are shown.



```
KapSyn: Ticker Screen (all stocks)
Show
T Deutsche Bank 111700 to 100.40
T Deutsche Bank 100 to 100.40
Quote request for DVFA...
Q DVFA 292900 to 2020.60
Q DVFA 100 to 2020.60
T Henkel 100 to 509.50
T Henkel 100 to 509.50
T Henkel 100 to 507.00
T Henkel 406400 to 507.00
T Henkel 34600 to 504.60
T SAP 100 to 203.40
T Deutsche Bank 100 to 100.40
Call...
Auction starting...
T Dt. Telekom 100 to 50.60
T Dt. Telekom 100 to 50.60
T Dt. Telekom 100 to 50.60
T Daimler Crysler 100 to 92.40
T Daimler Crysler 100 to 92.40
T Daimler Crysler 100 to 92.40
T Daimler Crysler 100 to 92.40
T Deutsche Bank 100 to 101.40
T Deutsche Bank 6925600 to 101.40
T Deutsche Bank 100 to 101.40
T Volkswagen 100 to 112.20
T Volkswagen 100 to 112.20
T Volkswagen 100 to 112.20
T Volkswagen 13370700 to 112.20
T Volkswagen 4916500 to 112.20
T Volkswagen 100 to 112.20
T Volkswagen 100 to 112.20
T SAP 100 to 205.40
T SAP 100 to 205.40
T Commerzbank 100 to 303.30
T Commerzbank 100 to 303.30
T Commerzbank 100 to 303.30
T Commerzbank 100 to 303.30
T Henkel 100 to 504.60
T Henkel 100 to 504.60
T RWE 100 to 810.60
T RWE 100 to 810.60
T Siemens 100 to 1018.60
T Siemens 100 to 1018.60
T Siemens 100 to 1018.60
T DVFA 100 to 2020.60
T DVFA 100 to 2020.60
T DVFA 100 to 2020.60
Auction finished
```

Fig 43 The ticker window

3.5.3 The simulation results files

Whenever a simulation is finished, a simulation results file named `last.sim` is saved. Result files can be reloaded using the graphics window (ch. 3.5.1.1).

Whenever a result file is loaded, all current parameter settings are overwritten by the settings that were used during the loaded simulation.

When simulation automatisation is used (ch. Tab 8), the result file names are given respective to the simulation date and time. A file named `19990730123719.out` represents a simulation that was finished on July 30, 1999 at 12:37:19 hours. Since the KapSyn log screen holds the finished simulation times too, it can be used to identify which scenario was simulated.

To save disk space, result files are written in binary mode.

3.5.4 The statistics file

A file named `stats.txt` holds statistics from all previous simulations. Every time a new simulation finishes, it appends a line with all statistics to the file. During time, the statistics file can grow very large. It can be deleted, if the statistics it holds are not needed any more, or it can be renamed to preserve its contents.

106 different entries are collected for each stock and each simulation - a much wider collection of data than the overview given in the graphics window. Tab 8 lists all variables with a brief explanation.

The file is written in text format with semicolons as separators and can be imported to a spread sheet program for further calculations.

info line 1	info line 2	sample value	description
	result file	19990816134503.out	result file name
	stock exchange	XETRA	name of stock exchange
	simulations	1	number of simulations
	agents	40	number of agents
	shares	10	number of stocks
stock	#	0	statistics for stock #0: all following values refer to stock #0
	name	MM #1 Corp.	name of stock
scenario	#	0	associated scenario num.
	name	bullish trend	associated scenario name
pessimists	p^ext	0.995	p^ext-value pessimists
	sigma	1	sigma-value pessimists
optimists	p^ext	1.05	p^ext-value optimists
	sigma	1	sigma-value optimists
price	open	50	opening quotation
	hi	52.4	highest quotation
	lo	50	lowest quotation
	close	51.8	closing quotation
	value ups	345	number of value ups
	value downs	106	number of value downs
	asks	66	number of asks
	bids	93	number of bids
	trades	62	number of trades
	transactions	99	number of transactions
	Cancels	3	number of cancels
volume	sum	3.83E+06	sold volume sum
	average	3.72E+04	...average
	std dev	6.92E+03	...standard deviation
turnover	sum	1.94E+08	turnover sum
	average	1.88E+06	...average
	std dev	3.50E+05	...standard deviation
small size:	units	#	statistics - small transaction size units: # = number of stocks \$ = currency units the following values belong to small sized transactions:
	max. size	1.00E+02	maximum size
	transactions	55	number of transactions
turnover	sum	2.80E+05	turnover sum
	average	5.09E+03	...average
	std dev	6.86E+02	...standard deviation
tc previous price	sum	2.99E+03	transaction costs sum - previous price
	average	5.43E+01	...average
	std dev	8.09E+00	...standard deviation
tc price at offer	sum	2.80E+03	transaction costs sum - price at offer
...selling only	sum	2.47E+03	t.c. sum for selling stocks only
	average	4.48E+01	...average
	std dev	6.32E+00	...standard deviation
...buying only	sum	3.12E+03	t.c. sum for buying stocks only
	average	5.68E+01	...average

	std dev	8.27E+00	...standard deviation
time units per offer	sum	9.97E-01	sum of time units needed to match whole offer relative value, based on one whole trading day
...selling only	sum	5.29E-01	...for selling only
	average	9.63E-03	...average
	std dev	3.34E-03	...standard deviation
...buying only	sum	1.46E+00	...for buying only
	average	2.66E-02	...average
	std dev	4.87E-03	...standard deviation
mid size:	units	\$	statistics - small transaction size units: the following values belong to mid sized transactions:
	max. size	1.00E+06	maximum size
	transactions	8	number of transactions
turnover	sum	5.47E+06	turnover sum
	average	6.83E+05	...average
	std dev	2.53E+05	...standard deviation
tc previous price	sum	1.54E+04	transaction costs sum - previous price
	average	1.92E+03	...average
	std dev	1.05E+03	...standard deviation
tc price at offer	sum	2.93E+04	transaction costs sum - price at offer
...selling only	sum	1.86E+04	t.c. sum for selling stocks only
	average	2.33E+03	...average
	std dev	1.13E+03	...standard deviation
...buying only	sum	3.99E+04	t.c. sum for buying stocks only
	average	4.99E+03	...average
	std dev	2.61E+03	...standard deviation
time units per offer	sum	1.08E-01	sum of time units needed to match whole offer relative value, based on trading day
...selling only	sum	7.76E-02	...for selling only
	average	9.70E-03	...average
	std dev	5.98E-03	...standard deviation
...buying only	sum	1.37E-01	...for buying only
	average	1.72E-02	...average
	std dev	9.88E-03	...standard deviation
block size:	transactions	40	the following values belong to block size transactions:
turnover	sum	1.88E+08	turnover sum
	average	4.71E+06	...average
	std dev	9.01E+05	...standard deviation
tc previous price	sum	2.02E+06	transaction costs sum - previous price
	average	5.05E+04	...average
	std dev	1.46E+04	...standard deviation
tc price at offer	sum	1.17E+06	transaction costs sum - price at offer
...selling only	sum	1.19E+06	t.c. sum for selling stocks only
	average	3.31E+04	...average
	std dev	1.12E+04	...standard deviation
...buying only	sum	1.16E+06	t.c. sum for buying stocks only

	average	2.89E+04	...average
	std dev	6.84E+03	...standard deviation
time units per offer	sum	1.23E+00	sum of time units needed to match whole offer relative value, based on trading day
...selling only	sum	1.33E-01	...for selling only
	average	3.69E-03	...average
	std dev	1.65E-03	...standard deviation
...buying only	sum	2.33E+00	...for buying only
	average	5.82E-02	...average
	std dev	1.71E-02	...standard deviation
XETRA des. sp.	rating	AA	XETRA designated sponsor: rating
	spread	0.0109	designated sponsor spread
	quote. req.	5	number of quote requests
	q.r. answers	5	quote request answers
	trades	2	number of trades
	transactions	4	number of transactions
NASDAQ ATS	trades	[void]	NASDAQ ATS: number of trades
	asks	[void]	number of asks
	bids	[void]	number of bids

Tab 8 Explanation of statistic variables used in stats.txt.
The data is given for each individual stock.

3.6 Simulation automatisation

With the KapSyn program description made so far it is only possible to run a single simulation or simulation series. Every time a new simulation is finished, the result file of the previous simulation is overwritten.

With simulation automatisation, it is possible to run a set of simulation series with different stock prices, stock exchange modules, scenarios, or even different scenario files (ch. 3.4.8) that are loaded automatically when needed. All simulation result files are preserved for a later review (ch. 3.5.3), and the statistics are collected (ch. 3.5.4).

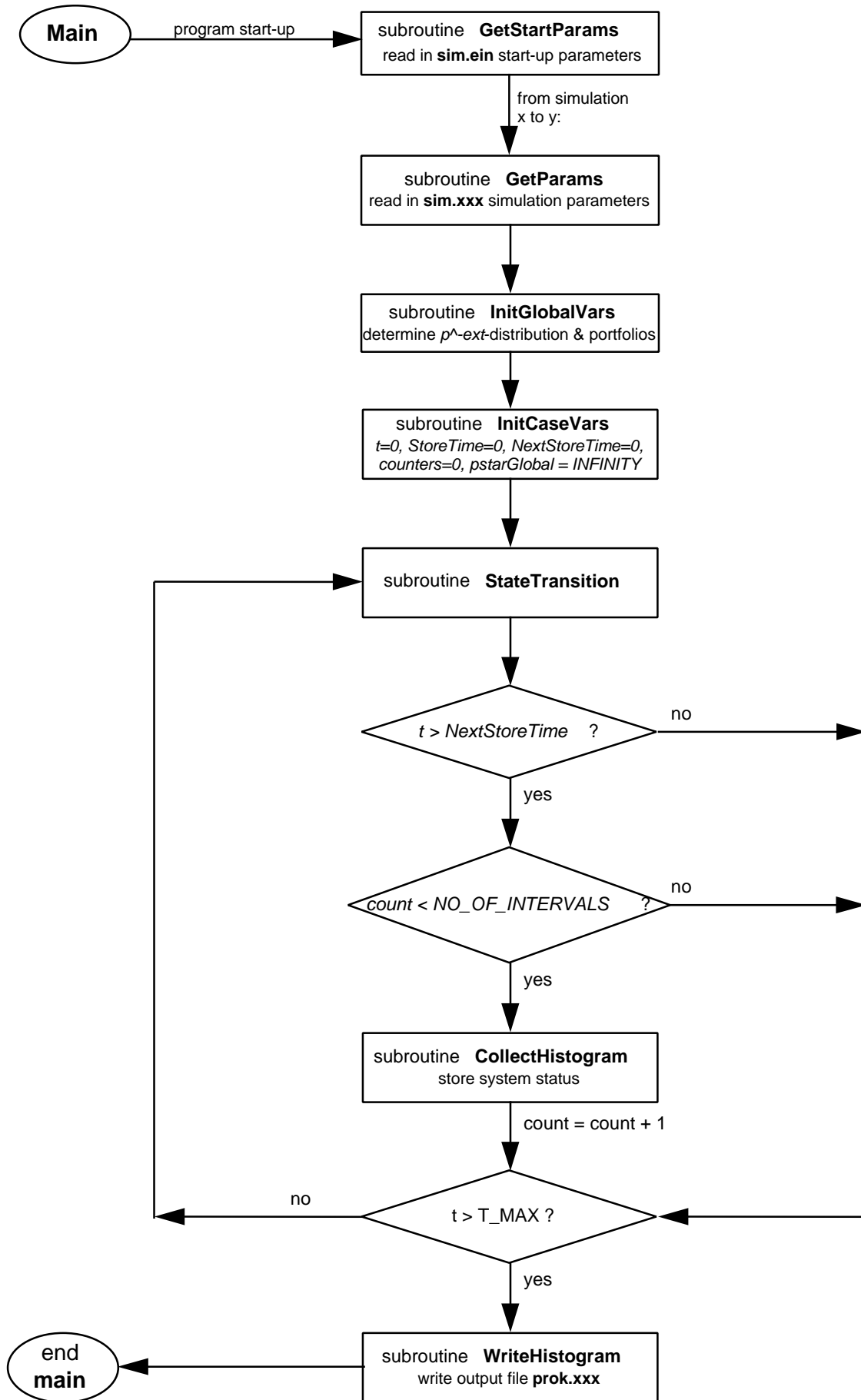
#	DefName	NoMod	NoSim	1	pat	2	pat	...	10	pat
Standard	3	16	97.5	1	20.2	1	...	745.0	5	
Test	1	4	144.0	4	22.5	2	...	750.5	4	
Standard	1	64	108.9		19.7	7	...	733.4	5	
Adjusted	4	3	100.0	1	20.0	4	...	750.0	3	
...	

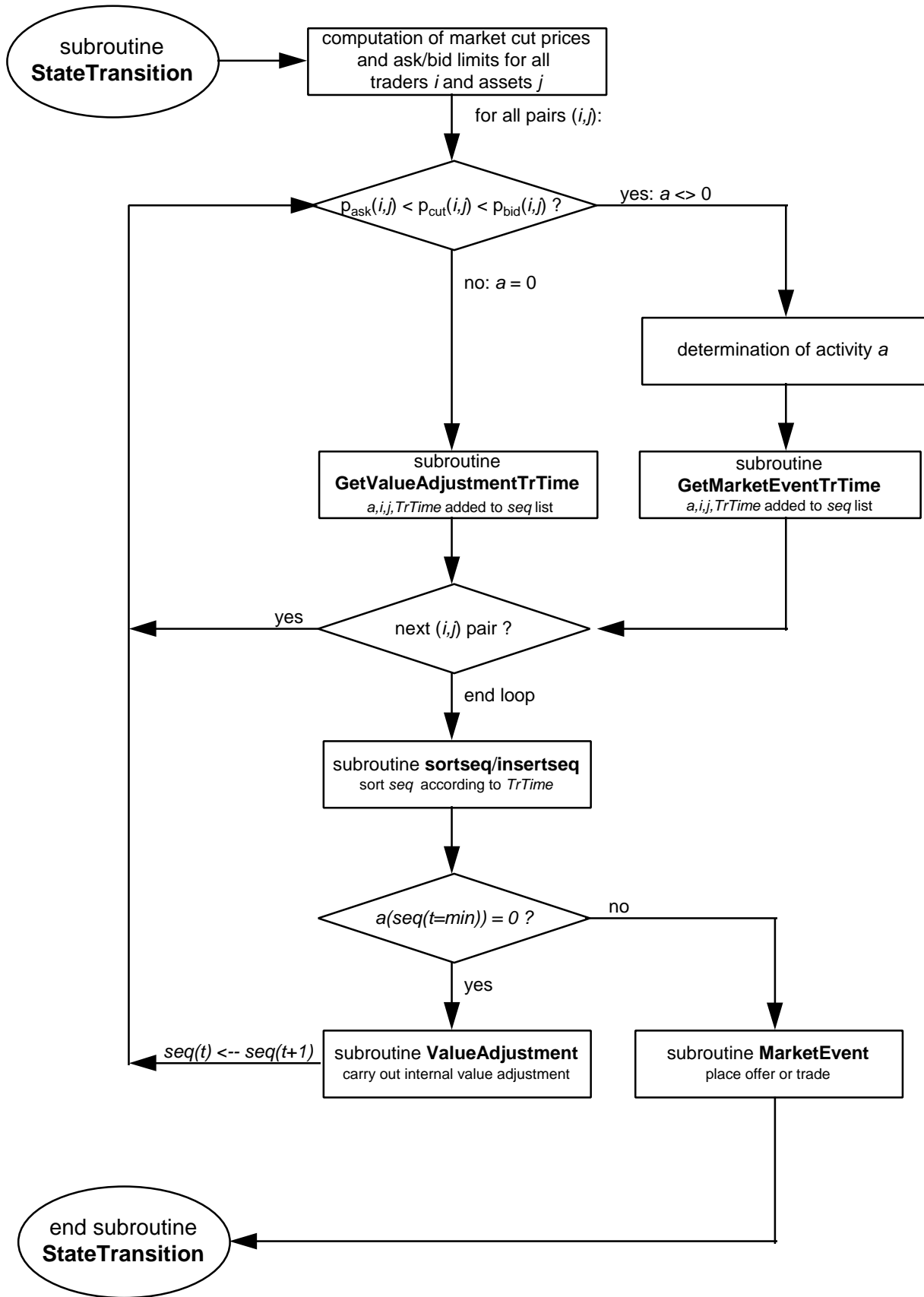
Tab 9 Simulation automatisisation file structure

Automatisisation is controlled by text files that must have a `.dat` extension. The file structure is shown in Tab 9. Comment lines must begin with a pound sign (`#`). When executed, every other line will be interpreted as a simulation instruction. First, the parameter settings file name is given (without file extension), then the stock exchange module number, the number of simulations for the simulation series, and stock prices as well as the associated scenario pattern numbers are given for each stock. The values must be separated by tabs. Because the number of stocks given in parameter settings files can differ, care must be taken to define prices and pattern associations for all stocks. If some values are wrong or missing, or a definition file can not be found, the instruction line will be skipped. It is important to use dots – not commas – as decimal sign.

To use an automatisisation file, it must be loaded with the `open` command in the `file` menu. When starting a simulation, the complete file will be executed.

3.7 Schematic Program Run





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