Do Cryptocurrency Hedge Funds Outperform Stocks and Bonds for Most Investors?

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Abstract

Hedge fund managers worldwide have already increased and plan to substantially increase their holdings of cryptocurrencies. Not only do fund managers care about the performance of their investments in cryptocurrencies, but considerable academic attention has also been drawn to this issue. This paper aims to compare the performance of cryptocurrency hedge funds with that of the U.S. stock and bond markets by adopting the almost stochastic dominance rule, which is non-parametric and utility-based measures. Our results suggest that for most non-satiable and risk-averse investors, cryptocurrency hedge funds dominate the S&P 500 index, short-term bonds, as well as long-term bonds over 1- to 5-year investment horizons. In addition, we also examine the performance of hedge funds which focus on the investment in blockchain companies in the digital asset space. We find that these infrastructure hedge funds outperform the U.S. equity and bond markets over a 3-year investment horizon for all non-satiable investors.

Keywords: Cryptocurrency, hedge funds, almost stochastic dominance, Omega

JEL classification: G11, G23

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1 Introduction

The size of the hedge fund industry has continued to increase worldwide. In the U.S., the market amounted to 61.49 billion USD in 2016 and it increased to 103.09 billion USD in 2021. Among the different investment strategies involving hedge funds, investment in cryptocurrencies has attracted significant attention on the part of fund managers, as well as high-net worth individuals, family offices and academia. According to a survey conducted by the fund administrator, Intertrust, hedge fund managers expect to hold an average of 7.2 per cent of their assets in cryptocurrencies by 2026, which is around USD$313 billion in cryptocurrency holdings around the world. However, investment in cryptocurrencies is very risky, e.g., the standard deviation of the daily returns of Bitcoin was about 6 times that of the S&P 500 index from 2011 to 2020. Thus, whether or not risk-averse investors should invest in cryptocurrency hedge funds is an important and worthwhile research question.

This paper seeks to examine whether cryptocurrency hedge funds outperform stocks and bonds for most risk-averse investors. To evaluate such performance, we adopt the almost stochastic dominance (ASD) rule. A merit of ASD rule is that it takes into account investors' preferences without assuming specific utility functions. Moreover, the ASD rule can rank distributions in certain cases where most investors can have clear preferences but where the traditional stochastic dominance fails to reach a conclusion. For example, assume that the return of a risky asset $\tilde{x}$ could be either 200% with a 99% probability or -1% with a 1% probability. Let the return on a risk-free asset be 1%. Apparently, most investors would prefer $\tilde{x}$ to the risk-free asset. However, the traditional stochastic dominance rule, e.g., first-degree stochastic dominance (FSD), cannot rank these two assets. This is because FSD and all of the higher orders require that the maximum loss of the dominant asset not be greater than that of the dominated asset.

ASD overcomes this drawback by excluding some pathological decision makers. Leshno and Levy (2002) argue that for most economically important decision makers, the ratio of their maximum marginal utility to the minimum marginal utility is not too large. The decision makers with a large ratio are considered to be pathological. They propose almost first-degree stochastic dominance (AFSD) as a distribution ranking criterion by excluding the pathological decision makers in the set of decision makers in FSD. When these pathological decision makers are excluded, the requirement in the FSD rule that the maximum loss of the dominant asset cannot be greater than that of the dominated asset is no longer sustained. Thus, it is possible to reach an unambiguous conclusion regarding the performance in the above example by using AFSD.

In our paper, we adopt the necessary and sufficient distribution conditions in the AFSD proposed by Leshno and Levy (2002) to compare the performance of cryptocurrency hedge funds with that of stocks and bonds. In addition, we further focus on risk-averse decision makers in the set of decision makers considered in AFSD. This new set of decision makers is a special case of generalized almost second-degree stochastic dominance (GASSD) proposed by Tsetlin et al. (2015). Thus, we can answer the question as to whether investing in the cryptocurrency hedge funds dominates the investment in stocks and bonds for most risk-averse investors.

A study closely related to ours is Bali et al. (2013) who compare the performance of hedge funds with that of stocks and bonds by using AFSD. Although they have examined the performance of different types of investment strategies for hedge funds, cryptocurrency hedge funds are not included in their study. In addition, they adopt an inaccurate condition for non-pathological risk-averse investors proposed by Leshno and Levy (2002) to examine the performance of hedge funds. In other words, our paper can correctly answer the question as to whether most risk-averse investors would prefer hedge funds to stocks and bonds.

In recent years, the finance literature has paid extensive attention to investment in cryptocurrencies. The theoretical literature on cryptocurrencies builds models to evaluate the equilibrium price (e.g., Pagnotta and Buraschi 2018; Biais et al. 2020; Cong, Li, and Wang 2021). Liu and Tsyvinski (2021) empirically test the factor models for cryptocurrencies and find that the cryptocurrency returns can be explained by cryptocurrency network factors but not cryptocurrency production factors. They also find that the returns of cryptocurrencies can be predicted by momentum and investor attention. The blooming empirical literature examines the performance of cryptocurrencies from different angles, e.g., the short- and long-term performance after initial coin offerings (Fisch and Momtaz 2020; Benedetti and Kostovetsky 2021; Momtaz 2021), and the value added to a portfolio without cryptocurrencies (Kajtazi and Moro 2019; Mroua et al. 2020). Instead of the cryptocurrencies themselves, our paper investigates the performance of cryptocurrency hedge funds. We not only examine the overall performance of these hedge funds, but also examine the performance of infrastructure hedge funds, which mainly invest in blockchain companies, as well as some investable cryptocurrency indices.

Our research data are obtained from the Hedge Fund Research Database (HFR) and the major data period is from January 2005 to December 2020. Both dead and alive funds are included in our data set. We find that most non-pathological risk-averse investors prefer the cryptocurrency hedge funds to the S&P 500 index over 9-month to 5-year investment horizons. As for the comparison with bond markets, most non-pathological risk-averse investors consider that the performance of cryptocurrency hedge funds surpasses that of bonds in the long run. Regarding the infrastructure hedge funds, we find that these type of hedge funds dominates the S&P 500 index and the bond markets in terms of FSD for a 3-year investment horizon, which is the longest investment horizon in our data based on these type of investment strategy. The above results are robust for the investable indices.

The remainder of this paper is organized as follows. Section 2 reviews the AFSD and GASSD rules and investors' preferences. Section 3 evaluates the performance of cryptocurrency and infrastructure hedge funds based on these rules. Section 4 presents the results for other measures, including the Omega, economic performance measures, downside risk-adjusted performance measures and manipulation-proof performance measure. Section 5 concludes the paper.

2 Decision Rules

This section introduces the notation and reviews the rules for AFSD and GASSD. To exclude the pathological preferences, Leshno and Levy (2002) defined the following set of decision makers:

$$U_{\varepsilon}^{(1)} = \left\{ u \middle| u'(x) \geq 0, \sup\{u'(x)\} \leq \inf\{u'(x)\} \times (\frac{1}{\varepsilon} - 1) \right\},$$

where $u'(x)$ denotes the first order derivative of the utility function u. The parameter ε is a constant that lies between 0 and 0.5. If ε approaches 0, then $U_{\varepsilon}^{(1)}$ includes all decision makers with $u'(x) \geq 0$, which is the set of decision makers in the first-degree stochastic dominance rule. If ε equals 0.5, then the condition $\sup\{u'(x)\} \leq \inf\{u'(x)\} \times (\frac{1}{\varepsilon} - 1)$ becomes $\sup\{u'(x)\} \leq \inf\{u'(x)\}$. The only utility satisfying this condition is that $u'(x)$ is a constant for all x. In other words, $U_{\varepsilon}^{(1)}$ only contains risk-neutral decision makers.

According to Muller et al. (2017), the condition $\sup\{u'(x)\} \leq \inf\{u'(x)\} \times (\frac{1}{\varepsilon} - 1)$ can be rewritten as

$$\frac{1}{(\frac{1}{\varepsilon} - 1)} \leq \frac{u'(x)}{u'(y)} \leq (\frac{1}{\varepsilon} - 1) \text{ for all } x \leq y.$$

Furthermore, we know that

$$-\int_{x}^{y} \frac{u''(t)}{u'(t)} dt = \ln \frac{u'(x)}{u'(y)}.$$

Thus, this condition can be rewritten as

$$-\ln(\frac{1}{\varepsilon} - 1) \leq \int_{x}^{y} \left[-\frac{u''(t)}{u'(t)} \right] dt \leq \ln(\frac{1}{\varepsilon} - 1) \text{ for all } x \leq y. \tag{1}$$

In other words, the set of decision makers in AFSD includes all non-satiable decision makers and the one whose aggregate degree of absolute risk aversion, $-\frac{u''(t)}{u'(t)}$, for any wealth range is bounded by $-\ln(\frac{1}{\varepsilon} - 1)$ and $\ln(\frac{1}{\varepsilon} - 1)$.

Let F and G respectively denote two cumulative distribution functions (CDFs) of the return on two risky assets $\tilde{x}$ and $\tilde{y}$ with support $[a, b]$. Leshno and Levy (2002) define AFSD as follows:

Definition 1. *The risky asset $\tilde{x}$ dominates another risky asset $\tilde{y}$ in terms of ε-AFSD, denoted as $\tilde{x}$ ε-AFSD $\tilde{y}$ if all $u \in U_{\varepsilon}^{(1)}$ prefer $\tilde{x}$ to $\tilde{y}$.*

The necessary and sufficient distribution condition for AFSD is as follows:

Theorem 1. *All $u \in U_{\varepsilon}^{(1)}$ prefer $\tilde{x}$ to $\tilde{y}$ if and only if*

$$\int_{F \geq G} (F(x) - G(x))dx \leq \varepsilon \int_{a}^{b} |F(x) - G(x)|\, dx.$$

In the example provided in the Introduction, the CDFs of $\tilde{x}$ and the risk-free asset are presented in Figure 1. The solid blue line in Figure 1 represents the CDF of $\tilde{x}$ and the dash red line in Figure 1 represents the CDF of the risk-free asset. Thus, $\tilde{x}$ dominates the risk-free asset in terms of ε-AFSD if and only if

$$(1\% - (-1\%)) \times 1\% \leq \varepsilon \left[(1\% - (-1\%)) \times 1\% + (200\% - 1\%) \times 99\%\right],$$

or,

$$0.01015\% \leq \varepsilon.$$

That is, all agents with $u'(x) \geq 0$ and

$$-9.1953 \leq \int_{x}^{y} \left[-\frac{u''(t)}{u'(t)}\right] dt \leq 9.1953 \text{ for all } x \leq y,$$

would prefer $\tilde{x}$ to the risk-free asset.

Our paper will examine whether all investors in the set $U_{\varepsilon}^{(1)}$ would prefer the cryptocurrency hedge funds to stocks and bonds. We are also interested in whether the cryptocurrency hedge funds are preferred for all risk-averse investors who are in the set $U_{\varepsilon}^{(1)}$, i.e.,

$$U_{\varepsilon}^{(2)} = \left\{ u \,\middle|\, u(x) \in U_{\varepsilon}^{(1)} \text{ and } u''(x) \leq 0 \right\},$$

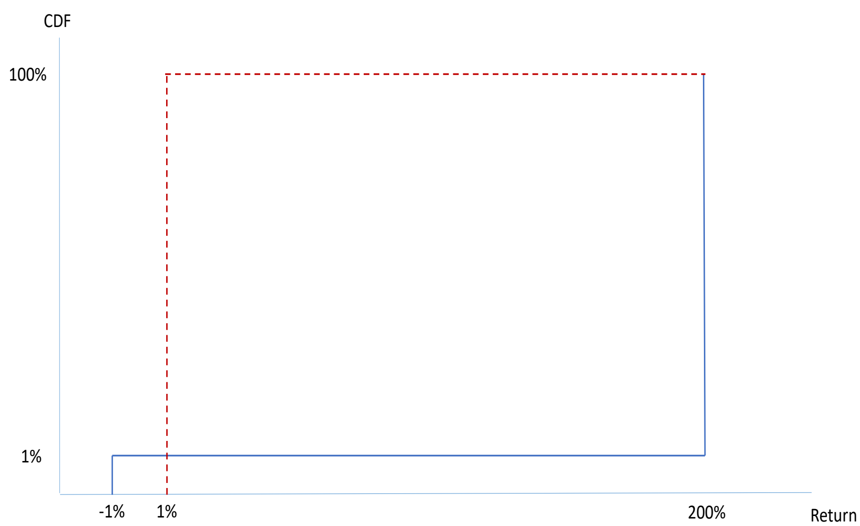

Figure 1: The CDFs of the example assets.

where $u''(x)$ denotes the second order derivative of the utility function u. This set of decision makers is a special case of the generalized almost second-degree stochastic dominance (GASSD) proposed by Tsetlin et al. (2015). Based on Tsetlin et al. (2015), we can define ε-GASSD as follows:

Definition 2. *For $0 < \varepsilon \leq 0.5$, the risky asset $\tilde{x}$ dominates another risky asset $\tilde{y}$ in terms of ε-GASSD, denoted as $\tilde{x}$ ε-GASSD $\tilde{y}$ if all $u \in U_\varepsilon^{(2)}$ prefer $\tilde{x}$ to $\tilde{y}$.*

Denote $F^{(2)}(x) = \int_a^x F(t)dt$ and $G^{(2)}(x) = \int_a^x G(t)dt$. The necessary and sufficient distribution condition for ε-GASSD is as follows:

Theorem 2. *For $0 < \varepsilon \leq 0.5$, all $u \in U_\varepsilon^{(2)}$ prefer $\tilde{x}$ to $\tilde{y}$ if and only if $E(\tilde{x}) \geq E(\tilde{y})$ and*

$$\max_{x \in [a,b]} F^{(2)}(x) - G^{(2)}(x) \leq \frac{\varepsilon}{1 - 2\varepsilon} \left[E(\tilde{x}) - E(\tilde{y}) \right].$$

Note that in the case where $F(x)$ single crosses $G(x)$, the condition in ε-GASSD is the same as that in ε-AFSD. To see this, let us assume that $F(x)$intersects $G(x)$ from above only once at x_0. In this case, we have

$$\max_{x\in[a,b]} F^{(2)}(x) - G^{(2)}(x) = \int_a^{x_0} (F(x) - G(x))dx.$$

Furthermore, $F^{(2)}(b) = \int_a^b F(t)dt = b - E(\tilde{x})$ and $G^{(2)}(b) = b - E(\tilde{y})$. Thus, we have

$$E(\tilde{x}) - E(\tilde{y}) = G^{(2)}(b) - F^{(2)}(b) = -\int_a^{x_0} (F(x) - G(x))dx + \int_{x_0}^b (G(x) - F(x))dx.$$

Therefore, the condition in Theorem 2 can be rewritten as

$$\int_a^{x_0} (F(x) - G(x))dx \leq \frac{\varepsilon}{1-2\varepsilon}\left[-\int_a^{x_0} (F(x) - G(x))dx + \int_{x_0}^b (G(x) - F(x))dx\right],$$

or,

$$\int_a^{x_0} (F(x) - G(x))dx \leq \varepsilon\left[\int_a^{x_0} (F(x) - G(x))dx + \int_{x_0}^b (G(x) - F(x))dx\right],$$

which is the same condition in ε-AFSD.

To compare the performance of hedge funds with that of stocks and bonds, we use Theorems 1 and 2 by defining the cumulative distribution of the hedge fund as F and that of stocks or bonds as G. Let us define $\hat{\varepsilon}_F$ as:

$$\hat{\varepsilon}_F = \frac{\int_{F\geq G}(F(x) - G(x))dx}{\int_a^b |F(x) - G(x)|\, dx}. \tag{2}$$

From Theorem 1, we know that hedge funds dominate stocks or bonds in terms of ε-AFSD if and only if $\hat{\varepsilon}_F \leq \varepsilon$. Similarly, we define $\hat{\varepsilon}_S$ as:

$$\hat{\varepsilon}_S = \frac{\max_{x\in[a,b]} F^{(2)}(x) - G^{(2)}(x)}{[E(\tilde{x}) - E(\tilde{y})] + 2\left[\max_{x\in[a,b]} F^{(2)}(x) - G^{(2)}(x)\right]}. \tag{3}$$

Theorem 2 indicates that hedge funds dominate stocks or bonds in terms of ε-GASSD if and only if $\hat{\varepsilon}_S \leq \varepsilon$.

The literature has provided estimates of ε. Levy et al. (2010) conducted experiments to estimate the parameter ε in AFSD. Based on the lottery choices of 400 subjects, their results suggest that $\varepsilon = 5.9\%$. Huang et al. (2020) employed real world data for a choice of deductibles in automobile theft insurance to estimate ε by using the ε-GASSD rule. Their data included more than 0.9 million observations. They suggest that in order to have all of the policyholders in their data having a consensus on the choice of the insurance contract, the estimated ε is 0.14%. For 99% (95%) of the policyholders, the estimated ε is 4.05% (7.32%). Since their data satisfies the single cross property between contract choices, their estimation can be used for the AFSD rule. In our empirical analyses, we adopt the estimations in Huang et al. (2020) to check whether $\hat{\varepsilon}_F$ and $\hat{\varepsilon}_G$ are smaller than 4.05% and 7.32% to respectively conclude the dominance relation for 99% and 95% of decision makers.

3 Empirical Analyses

3.1 Data

Our data are acquired from the Hedge Fund Research (HFR) database, which is one of the most commonly adopted hedge fund databases in the literature. The HFR database classifies hedge funds according to different investment strategies.[1] One of the main strategy types is the blockchain strategy, which includes two substrategies: cryptocurrency and infrastructure.[2] In our empirical analyses, we examine not only the performance of cryptocurrency and infrastructure hedge funds separately, but also the overall performance of the blockchain strategy. The sample periods for the cryptocurrency and blockchain hedge funds are January 2005 to December 2020, and for the

[1] There are seven distinct main strategy styles reported in the HFR database, including the equity hedge, event driven, relative value, macro, fund of funds, risk parity, and blockchain styles. From January 1994 to December 2020, about 27,200 hedge funds provided information to the HFR database with total assets under management being close to $3.42 trillion. Hence, this database is one of the most used hedge fund databases in the literature, for example, as Jagannathan et al. (2010), and Denuit et al. (2014).

[2] According to the HFR database, these two strategies are defined as follows: "Fund managers focusing on cryptocurrency utilize a variety of trading strategies to profit from exposure to Bitcoin, Ethereum and other digital currencies. Managers focusing on infrastructure invest in companies developing blockchain and other distributed ledger technologies which are fundamentally disintermediating payments, banking, market trading structure, the Internet of Things (IoT), healthcare, remittances, supply chains, digital identity and more."

infrastructure hedge funds are January 2017 to December 2020. For these hedge funds, we construct indices for the comparisons with U.S. equities and bonds. In addition to the indices, we also employ the investable indices directly provided by the HFR database. There are two indices: the HFR Blockchain Composite Index (blockchain index) and HFR Cryptocurrency Index (cryptocurrency index). The return information for these two indices are recorded from January 2016. Thus, the data period for these two indices is from January 2016 to December 2020.

In our sample period, there exist 70 cryptocurrency hedge funds in the HFR database, in which managers invest in cryptocurrencies directly by using a variety of trading strategies in order to generate profit. Furthermore, for 7 hedge funds with the infrastructure strategy, managers focus on investing in companies which are developing blockchain and other distributed ledger technologies. A commonly used measure of a fund size is the average monthly total assets under management over the life of the fund. Both research and empirical results indicate that there exist obvious differences in the sizes of hedge funds (e.g., Liang 2003; Bali et al. 2007; Bali et al. 2013; Denuit et al. 2014). As for the cryptocurrency and infrastructure hedge funds, we find a similar property, in that the mean hedge fund size is \$113.04 million, and the median hedge fund size is only \$11.93 million, which indicates that the winner takes all, reflecting a key feature of the hedge funds industry.

In order to construct quantitative indices of hedge funds, we follow Bali et al. (2013) to calculate the equal-weighted average rates of return on hedge funds for the cryptocurrency, infrastructure, and blockchain strategies. In addition, since the prior literature indicates that there are sample measurement bias problems that emerge in hedge fund research and the database (Fung and Hsieh 2004; Kosowski et al. 2007; Fung et al. 2008), we also follow the procedures in the literature to mitigate these problems.

To ease the survivorship bias, both alive and dead hedge funds are included. We incorporate 31 funds in the defunct database and 46 funds in the live hedge fund database in our sample. If the dead hedge funds are not included, there will be a survivorship bias of around 4.67% in the average annual rate of return for hedge funds.

Backfill bias is also taken into consideration. Hedge funds generally amplify a track record before reporting their monthly returns to the database due to the nature of the voluntary reporting scheme. Backfill bias refers to the situation where the database obtains the historical performance and backfills the monthly returns of a new hedge fund, which gives rise to an instant history in the database. Since the difference between the inception of the performance date and the date on which the fund is added to the database is around 1 year in our sample, we remove the first 12 months of returns for all individual hedge funds to reduce the backfill bias. We find that the average annual return of the hedge funds over the first year is 31.6% higher than the average annual returns in subsequent years.

Finally, multiperiod sampling bias is caused by most studies requiring that individual hedge funds conform to minimum survival criteria to be included in the sample, while funds with a short life are excluded. When trading off between the multiperiod sampling bias and the rational measures of hedge fund risk, we follow the literature to require that only hedge funds with at least 24 months of return can be included in our sample. Ultimately, we have 23 hedge funds in our sample (3 dead funds and 20 active funds) after applying the above filters, of which 19 hedge funds are for the cryptocurrency strategy and 4 hedge funds are for the infrastructure strategy.

As in Bali et al. (2013), we employ the S&P 500 index as a proxy for the U.S. stock market. With regard to the bond market, we employ the one-month T-bill and ten-year Treasury. One-month T-bill returns are obtained from Kenneth French's online data library. Ten-year Treasury returns are obtained from Federal Reserve Economic Data (FRED).

The summary statistics for the monthly returns of the hedge funds, the S&P 500 index, the one-month T-bills, and ten-year Treasuries are presented in Table 1. We find that the average returns of all of the hedge funds are higher than those of the S&P 500 index, one-month T-bill and ten-year Treasury bond in the corresponding sample period. However, the standard deviations of these hedge funds are much larger than those of the equities and bonds. Thus, based on the mean-variance rule, we cannot conclude that the hedge fund portfolios outperform the U.S. stock

Table 1: Summary Statistics

Assets	Mean (%)	Median (%)	Std. Dev. (%)	Skewness	Kurtosis	Min (%)	Max (%)	JB	p-value
Panel A: Hedge Funds									
Cryptocurrency	2.47	0.81	11.47	1.58	7.57	-25.42	49.16	246.89	0.0000
Infrastructure	6.16	4.57	17.64	0.59	3.44	-26.26	59.35	3.14	0.2078
Blockchain	2.43	0.81	11.21	1.54	7.58	-25.51	49.71	243.76	0.0000
Cryptocurrency Index	10.13	5.68	27.38	1.84	8.26	-33.65	130.73	103.17	0.0000
Blockchain Index	9.98	5.47	27.42	1.87	8.34	-34.23	130.73	106.16	0.0000
Panel B: Stocks and Bonds									
S&P 500 Index									
2005/01-2020/12	0.68	1.19	4.26	-0.65	4.79	-16.94	12.68	39.14	0.0000
2016/01-2020/12	1.12	1.75	4.40	-0.49	4.51	-12.51	12.68	8.15	0.0170
2017/01-2020/12	1.19	1.79	4.72	-0.54	4.23	-12.51	12.68	5.37	0.0683
One-month T-bill									
2005/01-2020/12	0.10	0.01	0.13	1.22	3.23	0.00	0.44	47.84	0.0000
2016/01-2020/12	0.09	0.09	0.07	0.24	1.58	0.00	0.21	5.61	0.0604
2017/01-2020/12	0.11	0.12	0.07	-0.17	1.74	0.00	0.21	3.38	0.1844
Ten-year Treasury									
2005/01-2020/12	0.24	0.22	0.09	0.27	2.35	0.05	0.43	5.74	0.0567
2016/01-2020/12	0.17	0.18	0.06	-0.52	2.33	0.05	0.26	3.78	0.1507
2017/01-2020/12	0.17	0.19	0.07	-0.66	2.16	0.05	0.26	4.85	0.0886

This table shows the summary statistics of monthly returns for the hedge fund portfolios, the S&P 500 index, the one-month T-bill, and the ten-year Treasury. The sample periods for the cryptocurrency and blockchain hedge funds are from January 2005 to December 2020, for the infrastructure hedge funds from January 2017 to December 2020, and for the cryptocurrency index and blockchain index from January 2016 to December 2020. We compute the equal-weighted average monthly returns for each of the hedge funds. The Jarque–Bera statistic, JB $=n[S^2/6+(K-3)^2/24]$, is a formal measure for testing whether the returns are normally distributed, where n is the number of observations, S is skewness and K is kurtosis. The JB statistics follow a Chi-square distribution with two degrees of freedom.

or bond markets, and vice versa.

Most of the empirical monthly return distributions for hedge funds are skewed and leptokurtic with a fat tail. In addition, when we conduct the formal test of normality, the Jarque-Bera (JB) statistics indicate that the return distributions of most of the hedge fund portfolios are not normal. These findings support the view that a performance measure which considers the whole of the return distributions should be used to evaluate the performance of hedge funds compared with the U.S stock or bond markets.

3.2 Performance Evaluation Using AFSD

This subsection provides an evaluation of the performance of the hedge funds, S&P 500 index and bonds. To do so, we first use the monthly returns to obtain the returns of these assets for 1-month, 3-month, 6-month, 9-month, 1-year, 2-year, 3-year, 4-year and 5-year investment horizons. The

Table 2: Almost first-order stochastic dominance

	Investment horizons								
Hedge fund portfolio	1-month	3-month	6-month	9-month	1-year	2-year	3-year	4-year	5-year
Panel A: Hedge funds vs. S&P 500 index									
Cryptocurrency	0.2878	0.1753	0.1168	0.0634	0.0354	0.0100	0.0002	0.0048	0.0356
Blockchain	0.2874	0.1768	0.1200	0.0656	0.0378	0.0113	0.0002	0.0048	0.0356
Infrastructure	0.2744	0.2103	0.1819	0.1505	0.1436	0.3152	FSD		
Cryptocurrency Index	0.2189	0.1296	0.0724	0.0387	0.0216	0.0098	FSD	FSD	
Blockchain Index	0.2212	0.1320	0.0758	0.0409	0.0230	0.0110	0.0001	FSD	
Panel B: Hedge funds vs. one-month T-bills									
Cryptocurrency	0.3367	0.2345	0.1604	0.1038	0.0679	0.0235	0.0146	0.0113	0.0073
Blockchain	0.3372	0.2361	0.1632	0.1062	0.0700	0.0245	0.0147	0.0113	0.0073
Infrastructure	0.2883	0.2138	0.1747	0.1399	0.1265	0.2130	FSD		
Cryptocurrency Index	0.2390	0.1375	0.0736	0.0382	0.0206	0.0075	FSD	FSD	
Blockchain Index	0.2412	0.1397	0.0766	0.0403	0.0219	0.0082	FSD	FSD	
Panel C: Hedge funds vs. ten-year T-bonds									
Cryptocurrency	0.3458	0.2473	0.1732	0.1146	0.0776	0.0310	0.0214	0.0203	0.0161
Blockchain	0.3464	0.2490	0.1762	0.1172	0.0801	0.0322	0.0216	0.0203	0.0161
Infrastructure	0.2905	0.2163	0.1772	0.1421	0.1292	0.2222	FSD		
Cryptocurrency Index	0.2410	0.1394	0.0749	0.0390	0.0212	0.0080	FSD	FSD	
Blockchain Index	0.2431	0.1417	0.0780	0.0412	0.0225	0.0088	0.0000	FSD	

This table shows the empirical estimates of ε-AFSD for 1-month to 5-year investment horizons. Due to the sample size limitation for the infrastructure hedge funds and the indices, we can only require returns up to 3-year and 4-year investment horizons, respectively. All of the results are based on comparisons of actual empirical return distributions. Panels A, B, and C show the results of comparing the hedge fund portfolios with the S&P 500 index, one-month T-bill, and ten-year Treasury bond, respectively.

empirical distributions are conducted by using the overlapping return intervals. The probability of each observation is assigned a value of 1/N, where N is the total number of returns observed for the corresponding investment periods.

Table 2 reports $\hat{\varepsilon}_F$ as shown in Equation (2) for the comparisons between hedge fund portfolios and stocks or bonds for different investment horizons. For the infrastructure hedge funds, we only have 4 years of data. Thus, we only evaluate the performance up to a 3-year investment horizon. By the same token, we evaluate the performance of both the cryptocurrency index and blockchain index up to a 4-year investment horizon since the data period covers only 5 years. To draw a conclusion regarding a dominant relationship, we adopt the findings in Huang et al. (2020), i.e., we use values of ε of 4.05% and 7.32% as the benchmark values to conclude that 99% and 95% of non-pathological investors would prefer hedge funds to stocks or bonds, respectively.

Panel A of Table 2 shows the values of $\hat{\varepsilon}_F$ for the comparison between hedge funds and the S&P 500 index. We find that for the cryptocurrency hedge funds, $\hat{\varepsilon}_F$ is less than 4.05% for 1- to 5-year investment horizons. According to Huang et al. (2020), our results suggest that 99% of non-pathological investors would agree that the cryptocurrency hedge funds outperform the U.S. stocks when the investment horizon is more than 1 year. In addition, we find that $\hat{\varepsilon}_F$ is less than 7.32% for 9-month to 5-year investment horizons. In other words, our results suggest that 95% of non-pathological investors would agree that the cryptocurrency hedge funds outperform the U.S. stocks when the investment horizon exceeds 9 months.

As for the infrastructure hedge funds, we find that they dominate the S&P 500 index in terms of FSD over a 3-year investment horizon. The result suggests that all non-satiable investors would prefer to invest in infrastructure hedge funds than in the S&P 500 index over a 3-year investment horizon. However, this result might be biased due to the limited number of observations. For a 1-year investment horizon, the value of $\hat{\varepsilon}_F$ is 14.36%. According to Equation (1), this means that all investors with an aggregate degree of absolute risk aversion in between -1.7857 and 1.7857 would agree that infrastructure hedge funds outperform the S&P 500 index.

Although blockchain hedge funds include both cryptocurrency and infrastructure hedge funds, the return distributions are mainly determined by the cryptocurrency hedge funds due to the number of such hedge funds in the portfolio. Thus, the performance of blockchain hedge funds is similar to that of cryptocurrency hedge funds. We find that the values of $\hat{\varepsilon}_F$ are between 0.02% and 3.78%, indicating that 99% of non-pathological investors prefer blockchain hedge funds to the U.S stock market over a 1- to 5-year investment horizon.

Besides the hedge fund portfolios that we construct ourselves, we also examine the cryptocurrency index provided by the HFR database. The value of $\hat{\varepsilon}_F$ is 7.24% for the cryptocurrency index for a 6-month investment period, which is lower than 7.32%. This result suggests that the investable cryptocurrency index is preferred by 95% of non-pathological investors. In addition, we find that $\hat{\varepsilon}_F$ decreases in the investment period. This means that more and more non-pathological investors prefer the cryptocurrency index as the investment horizon increases. Furthermore, when the investment horizon is 3 or 4 years, there is even a FSD relationship. Similar results are found for the blockchain index.

Next, we compare each of the hedge fund strategies with one-month T-bills over 1-month to 5-year investment horizons. The results of $\hat{\varepsilon}_F$ are presented in Panel B of Table 2. As for the cryptocurrency hedge funds, the value of $\hat{\varepsilon}_F$ gradually decreases as the investment horizon become longer. This result suggests that, in the long run, cryptocurrency hedge funds outperform short-term bonds. Specifically, when the investment horizon exceeds 1 year, the $\hat{\varepsilon}_F$ is less than 7.32%. This means that the cryptocurrency hedge funds outperform the one-month T-bills for 95% of non-pathological investors in terms of AFSD. Moreover, for 2- to 5-year investment periods, the cryptocurrency hedge funds are found to have persistently superior performance relative to the one-month T-bills for 99% of non-pathological investors based on AFSD.

We also explore the dominance of cryptocurrency index. The values of $\hat{\varepsilon}_F$ are in the range of 0% to 3.82%. This result indicates that the cryptocurrency index outperforms the one-month T-bills for 99% of non-pathological investors over 9-month to 4-year investment horizons. Furthermore, there even exists a FSD relationship if the investors hold these hedge fund index over 3-year or 4-year investment periods. This suggests that all of the investors should take part in the cryptocurrency index rather than the one-month T-bills.

As for the infrastructure hedge funds, we find that they dominate the one-month T-bills in terms of FSD when the investment periods are 3 years. In addition, for the blockchain hedge funds and their index, we have similar results compared to the cryptocurrency hedge funds and their index. Specifically, the values of $\hat{\varepsilon}_F$ are between 0.73% to 7% over 1- to 5-year investment horizons. The results reveal that at least 95% of non-pathological investors should invest in the blockchain hedge funds relative to the short-term bonds in those periods.

Finally, Panel C of Table 2 represents our analysis for the ten-year Treasury bonds. We find similar results to those for the one-month T-bills. Generally speaking, the cryptocurrency hedge

Table 3: ε-Generalized almost second-order stochastic dominance

	Investment horizons								
Hedge fund portfolio	1-month	3-month	6-month	9-month	1-year	2-year	3-year	4-year	5-year
Panel A: Hedge funds vs. S&P 500 index									
Cryptocurrency	0.2878	0.1753	0.1168	0.0634	0.0319	0.0003	SSD	0.0002	0.0338
Blockchain	0.2874	0.1768	0.1200	0.0656	0.0344	0.0019	SSD	0.0002	0.0338
Infrastructure	0.2744	0.2103	0.1819	0.1505	0.1436	0.3152	FSD		
Cryptocurrency Index	0.2189	0.1296	0.0724	0.0387	0.0216	0.0098	FSD	FSD	
Blockchain Index	0.2212	0.1320	0.0758	0.0409	0.0230	0.0110	0.0001	FSD	
Panel B: Hedge funds vs. one-month T-bills									
Cryptocurrency	0.3367	0.2345	0.1604	0.1038	0.0679	0.0235	0.0146	0.0113	0.0073
Blockchain	0.3372	0.2361	0.1632	0.1062	0.0700	0.0245	0.0147	0.0113	0.0073
Infrastructure	0.2883	0.2138	0.1747	0.1399	0.1265	0.2130	FSD		
Cryptocurrency Index	0.2390	0.1375	0.0736	0.0382	0.0206	0.0075	FSD	FSD	
Blockchain Index	0.2412	0.1397	0.0766	0.0403	0.0219	0.0082	FSD	FSD	
Panel C: Hedge funds vs. ten-year T-bonds									
Cryptocurrency	0.3458	0.2473	0.1732	0.1146	0.0776	0.0310	0.0214	0.0203	0.0161
Blockchain	0.3464	0.2490	0.1762	0.1172	0.0801	0.0322	0.0216	0.0203	0.0161
Infrastructure	0.2905	0.2163	0.1772	0.1421	0.1292	0.2222	FSD		
Cryptocurrency Index	0.2410	0.1394	0.0749	0.0390	0.0212	0.0080	FSD	FSD	
Blockchain Index	0.2431	0.1417	0.0780	0.0412	0.0225	0.0088	0.0000	FSD	

This table shows the empirical estimates of ε-GASSD over 1-month to 5-year investment horizons. Due to the sample size limitation for the infrastructure hedge funds and the indices, we can only require returns up to 3-year and 4-year investment horizons, respectively. All of the results are based on comparisons of actual empirical return distributions. Panels A, B, and C show the results for comparing the hedge fund portfolios with the S&P 500 index, one-month Treasury bills, and ten-year Treasury bonds, respectively.

funds surpass the ten-year Treasuries over 2- to 5-year investment horizons based on the AFSD rule. This means that 99% of non-pathological investors should acquire cryptocurrency hedge funds relative to the long-term bonds in those periods.

3.3 Performance Evaluation Using GASSD

In this section, we further use ε-GASSD to evaluate the performance of hedge funds over 1-month to 5-year investment horizons. The values $\hat{\varepsilon}_S$, as shown in Equation (3) based on the ε-GASSD rule for the comparison between hedge fund strategies and the S&P 500 index, are exhibited in Panel A of Table 3. We observe that as for the cryptocurrency hedge funds, $\hat{\varepsilon}_S$ is equal to 6.34% for a 9-month investment period, which is less than 7.32%. This result indicates that 95% of non-pathological risk-averse investors would prefer the cryptocurrency hedge funds to the S&P 500 index. For 1- to 5-year investment periods, the values of $\hat{\varepsilon}_S$ are lower than 4.05%. The results indicate that the cryptocurrency hedge funds outperform the equities for 99% of non-pathological risk-averse investors. Up to the 3-year investment horizons, there exists a SSD relationship between the cryptocurrency hedge funds and the S&P 500 index. This implies that all risk-averse investors should invest in the cryptocurrency hedge funds as opposed to the stocks.

As for the infrastructure hedge funds, we find that they outperform the S&P 500 index only over the 3-year investment period.[3] This result shows that all risk-averse investors would prefer the infrastructure hedge funds relative to the U.S stocks.[4] However, due to the limitation of observations, the result might be biased. Furthermore, the blockchain hedge funds are in a similar situation to the cryptocurrency hedge funds. In general, 99% of non-pathological risk-averse investors would prefer the blockchain hedge funds over 1- to 5-year investment horizons since the values of $\hat{\varepsilon}_S$ are smaller than 4.05%.

In addition, in the case of the cryptocurrency index, the values of $\hat{\varepsilon}_S$ are smaller than 7.32% for the 6-month to 4-year investment horizons and displays decreasing pattern. These results suggest that as least 95% of non-pathological risk-averse investors agree that the cryptocurrency index outperforms the equities when the investment periods extend. Moreover, the finding reveals the FSD relationship between the cryptocurrency index and the S&P 500 index when the investment horizon is 3 or 4 years, and suggests that all risk-averse investors should get involved in the cryptocurrency index. Also, the blockchain index has similar results.

Panel B of Table 3 presents the values of $\hat{\varepsilon}_S$ for the comparison between hedge funds and one-month T-bills. We find that as for the cryptocurrency hedge funds, the values of $\hat{\varepsilon}_S$ range from 0.73% to 6.79% over 1- to 5-year investment horizons and show a decreasing pattern. The results suggest that as least 95% of non-pathological risk-averse investors should invest in cryptocurrency hedge funds rather than in one-month T-bills in those periods. We obtain even stronger results when

[3] When the cumulative distributions of hedge funds and the cumulative distributions of the S&P 500 index intersect once, $\hat{\varepsilon}_S$ will be the same as $\hat{\varepsilon}_F$, but they have different implications.

[4] If the hedge funds dominate the S&P 500 index in terms of FSD, by implication the hedge funds dominate the S&P 500 index in terms of SSD.

analyzing the cryptocurrency index. It is found that 99% of non-pathological risk-averse investors would prefer the cryptocurrency index over 9-month to 4-year investment horizons. Even when the holding periods are 3 years or 4 years, a FSD relationship exists between the cryptocurrency index and one-month T-bills. This means that all risk-averse investors should get involved in the cryptocurrency index.

Only over the 3-year investment horizon does the FSD relationship exist between the infrastructure hedge funds and short-term bonds. These result suggests that all risk-averse investors would prefer the infrastructure hedge funds when the investment periods are 3 years. Besides, we discover that the blockchain hedge funds and their index have comparable outcomes to the cryptocurrency hedge funds and their index. The values of $\hat{\varepsilon}_S$ are less than 7.32% and decrease over time. The results indicate that at least 95% of non-pathological risk-averse individuals consider that the performance of blockchain hedge funds superior than that of one-month T-bills over 1- to 5-year investment periods.

Finally, Panel C of Table 3 exhibits similar outcomes to the one-month T-bills in the case of the ten-year Treasury bonds. The $\hat{\varepsilon}_S$ for the cryptocurrency hedge funds is of smaller magnitudes than 4.05% over 2- to 5-year investment periods. These results reveal that 99% of non-pathological risk-averse investors should prefer the cryptocurrency hedge funds rather than the long-term bonds.

4 Robustness Checks: Other Performance Measures

Since both AFSD and GASSD are partial ranking criteria, we may have results regarding the performance ranking that are indeterminate. In this section, we employ some important performance measures which are full ranking criteria to compare the performance.

4.1 Omega and second-order Omega

We use first-order Omega (Ω^1) and second-order Omega (Ω^2) in this subsection to examine the performance of each hedge fund and the S&P 500 index. These two performance indices, Ω^1 and

Ω^2, satisfy the property of monotonicity with respect to the FSD and SSD rules, respectively, as shown by Bi et al. (2019). That is, if portfolio F is preferred by all non-satiable (and risk-averse) investors to portfolio G, then the Ω^1 (Ω^2) ratio of portfolio F will be greater than that of G.

Omega (Ω^1) is proposed by Keating and Shadwick (2002) and is defined by the following equation:

$$\Omega^1_F(L) = \frac{\int_{F \le H}(H(x) - F(x))dx}{\int_{F>H}(F(x) - H(x))dx} = \frac{\int_L^b(1 - F(x))dx}{\int_a^L(F(x))dx} = \frac{E_F(\tilde{x}) - L}{E_F[(L - \tilde{x})^+]} + 1, \tag{4}$$

where H is a CDF of obtaining L with support $(a, b]$ for sure and F is a CDF of the returns of the risky asset $\tilde{x}$. H and F intersect once from above at L. L can serve as a specific target payoff in financial analysis and can differ for different investors. We set L to zero in our paper, which means that risky assets are evaluated by both positive returns and negative returns. Based on the FSD rule, the numerator can be viewed as the area that satisfies the condition where F dominates H in terms of FSD and the denominator is the part that violates the FSD condition. Therefore, Omega can be viewed as "the win part" of F compared with H divided by "the loss part" of F compared with H. For the risky asset $\tilde{y}$, we calculate Ω^1_G by using a similar definition.

Second-order Omega (Ω^2) is proposed by Bi et al. (2019), and is defined as the following equation[5] :

$$\Omega^2_F(L) = \frac{\int_{F^{(2)} \le H^{(2)}}(H^{(2)}(x) - F^{(2)}(x))dx}{\int_{F^{(2)}>H^{(2)}}(F^{(2)}(x) - H^{(2)}(x))dx} = \frac{(b - L)^2 - E_F(b - \tilde{x})^2}{E_F[(h_2 - \tilde{x})^+]^2 - (h_2 - L)^2} + 1, \tag{5}$$

where $H^{(2)}$ and $F^{(2)}$ is the integration of H and F, respectively. h_2 is the intersection point of $F^{(2)}(x)$ and $H^{(2)}(x)$. $L \le h_2$ if $F^{(2)}(b) \le H^{(2)}(b)$ because both F and H are non-decreasing functions. For the risky asset $\tilde{y}$, we calculate Ω^2_G by using a similar definition.

As we know, the risky asset $\tilde{x}$ has better performance than the risky asset $\tilde{y}$ when $\tilde{x}$ has higher Omega than $\tilde{y}$ for a specific payoff. The results of Omega are presented in Panel A of Table 4. As for the cryptocurrency hedge funds, the values of Omega are all larger than that of the S&P 500

[5] Bi et al. (2019) extend the traditional Omega proposed by Keating and Shadwick (2002) to higher orders referred to as N-th order Omega. Omega is a special case of N-th order Omega when N=1.

Table 4: Omega and Second-order Omega

Assets	Panel A: Omega 1-month	6-month	1-year	2-year	Panel B: Second-order Omega 1-month	6-month	1-year	2-year
Hedge funds								
Cryptocurrency	2.0115	5.3776	14.0369	42.3338	3.6220	63.2923	760.7804	4961.6880
Blockchain	2.0080	5.2686	13.5830	40.5763	3.8288	61.6891	692.6175	4672.1160
Infrastructure	2.5010	4.8345	7.2117	4.2610	5.2076	26.6141	88.3608	12.4878
Cryptocurrency Index	3.2057	12.6809	48.2076	137.2871	16.6345	300.2130	3092.6920	22396.7500
Blockchain Index	3.1689	12.1481	45.2917	125.0649	16.1600	277.0797	2728.5240	18678.7600
S&P 500 Index								
2005/01-2020/12	1.5335	2.7891	3.7098	4.9559	0.8957	4.3129	6.2936	13.0754
2016/01-2020/12	1.9727	6.8561	18.5828	213.7654	1.6060	30.2601	102.4392	7713.8250
2017/01-2020/12	2.0050	8.9554	28.1388	393.2194	1.8003	44.5175	166.7456	13339.8400

Panels A and B show Omega and second-order Omega for each of hedge fund portfolios and the S&P 500 index, respectively. The sample period for the cryptocurrency and blockchain is from January 2005 to December 2020, for the infrastructure from January 2017 to December 2020, and for the cryptocurrency index and blockchain index from January 2016 to December 2020.

index over 1-month to 2-year investment horizons. Although the loss parts of the cryptocurrency hedge funds compared with the benchmark are bigger than that for the S&P 500 index, the win parts of the cryptocurrency hedge funds compared with the benchmark are sufficiently large to cause the cryptocurrency hedge funds to have a greater Omega. Specifically, the Omega for the cryptocurrency hedge funds is 42.33, which is much higher than that for the S&P 500 index of only 4.96 when the investment horizons are 2 years. This means that the cryptocurrency hedge funds have better performance than the equities, in which case the investors should prefer the cryptocurrency hedge funds. We also examine the cryptocurrency index, and the results are similar to those we have obtained for the cryptocurrency hedge funds with the exception of the 2-year investment period.

However, the infrastructure hedge funds have a lower Omega than the S&P 500 index for most of the investment horizons except for the 1-month investment period. For example, the Omega for the S&P 500 index is 213.77, whereas the Omega for the infrastructure hedge funds is only 4.26 for the 2-year investment horizon. The result indicates that the performance of the infrastructure hedge funds is worse than that of the S&P 500 index for the 2-year investment period, in which case investors should hold on to the stocks, and not the infrastructure hedge funds. The loss parts of the infrastructure hedge funds compared with the benchmark are bigger than that for the S&P 500 index, but the win parts of the infrastructure hedge funds compared with the benchmark are not large enough, which will cause the infrastructure hedge funds to have a smaller Omega.

In addition, as for the blockchain hedge funds and their index, the results are similar to those for the cryptocurrency hedge funds and their index. Specifically, the Omegas for the blockchain hedge funds are in the range of 2.01 to 40.58, whereas the Omegas for the S&P 500 index range from 1.53 to 4.96. The results reveal that the performance of the blockchain hedge funds is superior to that of the S&P 500 index, the investors should invest in the blockchain hedge funds, and not the stock market.

With regard to the second-order Omega, which also has the property that the higher the value, the better the performance, the results are presented in Panel B of Table 4. As for the cryptocurrency hedge funds, the second-order Omegas are all higher than that for the S&P 500 index over 1-month to 2-year investment periods. To be specific, the second-order Omegas are in the range of 3.62 to 4961.69 for the cryptocurrency hedge funds while they range from only 0.90 to 13.08 for the S&P 500 index over 1-month to 2-year investment horizons. The outcomes imply that the cryptocurrency hedge funds' performance surpasses that of the S&P 500 index, in which case all risk-averse investors should get involved in the cryptocurrency hedge funds rather than the stocks. In addition, the cryptocurrency index has analogous results to what we have observed for the cryptocurrency hedge funds. The second-order Omegas of the cryptocurrency index are superior to those for the S&P 500 index over 1-month to 2-year investment periods. This demonstrates that risk-averse investors investing in the cryptocurrency index can have superior performance to those investing in U.S stocks.

Nevertheless, the infrastructure hedge funds have smaller second-order Omegas than in the case of the S&P 500 index except for the 1-month investment period, showing that the risk-averse investors should not invest in the infrastructure hedge funds over 6-month to 2-year investment periods. Finally, the second-order Omegas of the blockchain hedge funds and their index have comparable outcomes to those for the cryptocurrency hedge funds. The outcomes suggest that risk-averse investors hold the blockchain hedge funds or index can result in superior returns than for the S&P 500 index. Overall, the outcomes based on the Omega and second-order Omega are roughly analogous to the results derived from the AFSD and ε-GASSD rules.

4.2 Sharpe Ratio and Generalized Sharpe Ratio

Although the Sharpe ratio is the most commonly used performance measure, it uses the standard deviation as the proxy for the riskiness, which may be inappropriate when the return distributions follow the non-normalities.[6] The Sharpe ratio is defined as the ratio of the expected return that exceeds the risk-free rate (the excess return) to the standard deviation, which tells us how much of the return for a portfolio can compensate the investor for one unit of risk. The results of the Sharpe ratio for each hedge fund and the S&P 500 index are displayed in Panel A of Table 5. As for the cryptocurrency hedge funds, the Sharpe ratios are lower than those for the S&P 500 index over 1- and 2-year investment horizons. The results reveal that the outcomes of the cryptocurrency hedge funds are worse than those of the S&P 500 index for 1- and 2-year investment periods. This indicates that investors should not invest in the cryptocurrency hedge funds during those times, a finding that is not consistent with the ASD rule. Furthermore, we also have analogous outcomes when investigating other hedge funds and their index. Generally speaking, we may have an inaccurate answer as to whether investors should participate in the cryptocurrency hedge funds when employing the Sharpe ratio as the performance measure.

It will be of interest to know to what extent the investors' choice changes when considering the non-normality. To this end, Homm and Pigorsch (2012) proposed a performance index that generalizes the Sharpe ratio by replacing the standard deviation with the AS index. More importantly, this performance index is monotonic with respect to the SSD rule and considers the mean, variance, and higher momoents of the return distributions. Homm and Pigorsch (2012) defined the

[6]As we mentioned above, the return distributions of the hedge funds are characterized by skewness, fatter tails, and higher peaks, which may undermine the usefulness of the Sharpe ratio.

Table 5: Sharpe ratios and generalized Sharpe ratios

Assets	1-month	6-month	1-year	2-year	1-month	6-month	1-year	2-year
	Panel A: Sharpe ratios				Panel B: Mean-AS ratios			
Hedge funds								
Cryptocurrency	0.207	0.368	0.336	0.414	0.1107	0.8182	2.5836	10.2836
Blockchain	0.208	0.365	0.336	0.411	0.1114	0.7916	2.4699	9.9655
Infrastructure	0.343	0.489	0.385	0.462	0.2831	0.9726	1.3224	0.7777
Cryptocurrency Index	0.367	0.533	0.553	0.642	0.4431	3.2364	11.4298	39.1054
Blockchain Index	0.360	0.531	0.555	0.635	0.4281	3.0803	10.6722	35.3626
S&P 500 Index								
2005/01-2020/12	0.136	0.322	0.448	0.612	0.0407	0.1850	0.3361	0.6917
2016/01-2020/12	0.230	0.629	1.000	1.635	0.1081	1.0495	2.7510	22.1911
2017/01-2020/12	0.233	0.759	1.286	1.909	0.1091	1.3327	3.8176	33.4216
	Panel C: Mean-FH ratios				Panel D: Mean-BCCY ratios			
Hedge funds								
Cryptocurrency	0.0933	0.4351	0.9738	3.9462	0.0359	0.2599	0.7239	3.0071
Blockchain	0.0915	0.4086	0.9365	3.9229	0.0360	0.2506	0.6917	2.9404
Infrastructure	0.2303	0.7392	1.1495	0.7053	0.0924	0.2108	0.4733	0.2784
Cryptocurrency Index	0.2984	2.0458	5.5973	18.0831	0.1459	0.9991	3.6036	12.6509
Blockchain Index	0.2888	1.9446	5.2834	16.6977	0.1410	0.9497	3.3761	11.5303
S&P 500 Index								
2005/01-2020/12	0.0343	0.0823	0.1495	0.2905	0.0130	0.0512	0.0927	0.1944
2016/01-2020/12	0.0868	0.3963	0.9241	6.8940	0.0336	0.2619	0.7060	5.5019
2017/01-2020/12	0.0820	0.4336	1.1381	9.2557	0.0338	0.3203	0.9208	7.7235

Panels A, B, C, and D present the Sharpe ratios, the Mean-AS ratios, the Mean-FH ratios, and the Mean-BCCY ratios for each hedge fund portfolio and the S&P 500 index, respectively. The sample period for the cryptocurrency and blockchain is from January 2005 to December 2020, for the infrastructure from January 2017 to December 2020, and for the cryptocurrency index and blockchain index from January 2016 to December 2020.

Mean-AS index as follows:

$$Mean - AS\ index = \frac{E(\tilde{r_x}) - r_f}{AS\ index}, \tag{6}$$

where $\tilde{r_x}$ is the return of risky asset $\tilde{x}$, and r_f is the risk-free rate. If portfolio F is preferred by all non-satiable and risk averse investors to portfolio G, then the Mean-AS index of portfolio F is higher than that of portfolio G.

Because most of the traditional risk measures are not monotonic with respect to the SSD rule, it is necessary to develop riskiness indices which can satisfy the monotonicity condition. For example, Aumann and Serrano (2008) used the reciprocal of the absolute risk aversion (ARA) of an agent with constant ARA who is indifferent between acceptance and rejection of an uncertain portfolio as the riskiness of a portfolio, and referred to it as the AS index, which is defined as in the following equation:

$$E[e^{-\frac{\tilde{r_x}}{R^{AS}}}] = 1, \tag{7}$$

where $\tilde{r_x}$ is the return of the risky asset $\tilde{x}$, and where it is required that the values of the return have some positive and negative values as well as the expected return of the risky asset $\tilde{x}$ being positive. By plugging the values of $\tilde{r_x}$ and solving Equation (7), $R^{AS} > 0$ is obtained, which is referred to as the AS index of risky asset $\tilde{x}$. When an agent is less risk-averse, he will accept a more risky portfolio, and the AS index will be larger. Thus, these index can help decision makers to evaluate which portfolio is riskier.

Following the concept of Aumann and Serrano (2008), some researchers have also proposed other risk indices with the property that monotonic with respect to the SSD rule. Specifically, Foster and Hart (2009) employed the critical wealth level of an agent with a log utility function who is indifferent between acceptance and rejection of an uncertain portfolio as the riskiness of the portfolio, and referred to it as the FH index, which is defined by the following equation:

$$E[ln(\frac{\tilde{r_x}}{R^{FH}} + 1)] = 0, \tag{8}$$

where $\tilde{r_x}$ is the return of the risky asset $\tilde{x}$ and where the same requirements for the AS index apply. Solving Equation (8) to acquire $R^{FH} > 0$ is referred to as the FH index of risky asset $\tilde{x}$. This index is based on the criterion whereby for a portfolio the critical wealth level is lower than that for which it becomes risky to accept the portfolio. In extending the FH index by using an iso-power utility function, Bali et al. (2011) proposed the BCCY index, which is defined by the following equation:

$$\frac{1}{\gamma}[E(\frac{\tilde{r_x}}{R^{BCCY}} + 1)^{\gamma} - 1] = 0, \tag{9}$$

where $\tilde{r_x}$ is the return of the risky asset $\tilde{x}$ and the same requirements apply as for the AS index.

Then, there exists a unique number $R^{BCCY} > 0$ that is determined by Equation (9), which is the BCCY index of the risky asset $\tilde{x}$.

The results of the Mean-AS index are shown in Panel B of Table 5. All of the Mean-AS ratios are superior to those of the S&P 500 index for the cryptocurrency hedge funds over 1-month to 2-year investment periods. Specifically, the Mean-AS ratios of the cryptocurrency hedge funds are in the range of 0.11 to 10.28 whereas the Mean-AS ratios for the S&P 500 index range from 0.04 to 0.69 over 1-month to 2-year investment horizons. The results indicate that the performance of the cryptocurrency hedge funds is better than that of the S&P 500 index, in which case investors should engage in the cryptocurrency hedge funds over those horizons. Besides, we discover that the results for the cryptocurrency index are similar to what we have observed for the cryptocurrency hedge funds, meaning that investors should participate in the cryptocurrency index. However, the Mean-AS ratios appear to be lower than the S&P 500 index for the infrastructure hedge funds except for the 1-month investment period. The outcomes imply that investors may not prefer the infrastructure hedge funds over 6-month to 2-year investment horizons. Finally, we obtain similar results for the blockchain hedge funds and their index to those for the cryptocurrency hedge funds. Furthermore, notice that we have different choices compared with the Sharpe ratios, a similar finding to that of Homm and Pigorsch (2012).

Inspired by Homm and Pigorsch (2012), we also calculate the Mean-AS index by replacing the AS index with the FH and BCCY indices, respectively. The results of the Mean-FH index and the Mean-BCCY index are exhibited in Panels C and D of Table 5, respectively. For the Mean-FH ratios and the Mean-BCCY ratios, we obtain consequences analogous to those for the Mean-AS ratios. The outcomes indicate that the cryptocurrency hedge funds are preferred by risk-averse individuals. A notable point in Panel C of Table 5 is that the investors appear to participate in the infrastructure hedge funds over 1-month to 1-year investment periods. In general, the cryptocurrency hedge funds are preferred by risk-averse investors relative to the stock market based on the results for the Mean-AS index, Mean-FH index and Mean-BCCY index, which is consistent with the ASD rule.

Table 6: Downside risk-adjusted performance measures

	Panel A: Mean-VaR ratios				Panel B: Calmar ratios				Panel C: Sortino ratios			
Assets	1-month	6-month	1-year	2-year	1-month	6-month	1-year	2-year	1-month	6-month	1-year	2-year
Hedge funds												
Cryptocurrency	0.1018	0.4828	1.1042	4.2988	0.0361	0.1926	0.5428	1.3633	0.2896	1.1403	2.7291	7.8347
Blockchain	0.1019	0.4857	1.0514	4.2624	0.0356	0.1883	0.5317	1.3553	0.2890	1.1095	2.6876	7.6131
Infrastructure	0.2332	0.7868	1.1614	0.7449	0.0939	0.4209	0.7688	0.3834	0.5136	1.3497	2.0290	1.1780
Cryptocurrency Index	0.3430	2.0593	5.6573	20.0196	0.1384	1.0419	4.0115	11.5059	0.6983	2.8677	9.4394	27.8132
Blockchain Index	0.3398	1.9692	5.3316	18.2869	0.1346	1.0113	3.8262	10.7695	0.6844	2.7428	8.8709	25.3478
S&P 500 Index												
2005/01-2020/12	0.0522	0.1004	0.1683	0.3227	0.0111	0.0362	0.0671	0.1387	0.1176	0.2563	0.3224	0.4920
2016/01-2020/12	0.0993	0.4786	1.0328	na	0.0543	0.2313	0.7975	6.8939	0.1771	0.9278	1.3976	6.8939
2017/01-2020/12	0.0973	0.5566	1.3238	na	0.0513	0.2531	0.9822	9.2558	0.1928	1.0151	1.7213	9.2558

Panels A, B, and C present the Mean-VaR ratios, the Calmar ratios, and the Sortino ratios for each hedge fund portfolio and the S&P 500 index, respectively. The sample period for the cryptocurrency and blockchain is from January 2005 to December 2020, for the infrastructure from January 2017 to December 2020, and for the cryptocurrency index and blockchain index from January 2016 to December 2020.

4.3 Downside Risk-adjusted Performance Measures

The hedge funds have return distributions with higher peaks, meaning that the possibility of extreme events taking place is much higher than normal. Although the standard deviation attaches equal weights to both downside and upside returns, people care more about the downside risk, which is the probability of making a loss. In the literature, a safety-first investor will try to maximize the expected return under a downside risk constraint. Therefore, in this subsection we employ three downside risk-adjusted performance measures, namely, the Mean-VaR ratio, the Calmar ratio, and the Sortino ratio, to evaluate the performance of the hedge funds and the S&P 500 index.

First, when we adopt VaR to evaluate the downside risk, the Mean-VaR ratio is defined by the following equation:

$$Mean - VaR = \frac{E(\tilde{r_x}) - r_f}{VaR}, \tag{10}$$

where $\tilde{r_x}$ is the return of the risky asset $\tilde{x}$, r_f is the risk-free rate, and the 1% VaR is calculated by the left tail of the empirical return distribution.

Panel A of Table 6 displays the outcomes of the Mean-VaR ratios. We observe that as for the cryptocurrency hedge funds, the Mean-VaR ratios range from 0.10 to 4.30 while the S&P 500 index range from 0.05 to 0.32. These results reveal that the cryptocurrency hedge funds outperform the equities, in which case safety-first investors should participate in the cryptocurrency hedge funds.

As for the cryptocurrency index, the Mean-VaR ratios are superior to those for the S&P 500 index for 1-month to 1-year investment periods. The outcomes point that safety-first investors had better invest in the cryptocurrency index over 1-month to 1-year investment horizons.[7]

The results for the infrastructure hedge funds indicate that safety-first investors should participate in infrastructure hedge funds over 1-month to 1-year investment horizons. In addition, the blockchain hedge funds and their index give rise to comparable consequences to those for the cryptocurrency hedge funds and their index. Overall, the results obtained from the Mean-VaR ratios slightly differ from the ASD rule.

Next, we alter the downside risk measurement by using the maximum drawdown (MDD), which is the greatest loss that an investor would suffer if he/she buys at the highest price and sells at the lowest price. An alternative downside risk adjusted performance measure referred to as the Calmar ratio is defined by the following equation:

$$Calmar\ ratio = \frac{E(\tilde{r_x}) - r_f}{MDD}, \tag{11}$$

where $\tilde{r_x}$ is the return of the risky asset $\tilde{x}$, r_f is the risk-free rate, and the MDD is the largest cumulative loss from a market peak to the following trough.

Panel B of Table 6 points that the Calmar ratios for the cryptocurrency hedge funds are all higher than that for the S&P500 index. Specifically, the Calmar ratios range from 0.04 to 1.36 for the cryptocurrency hedge funds, while they are only between 0.01 and 0.14 for the S&P500 index. The outcomes discover that the cryptocurrency hedge funds have superior performance to the stocks, and that safety-first investors should participate in the cryptocurrency hedge funds. Similarly, the magnitudes of the cryptocurrency index and blockchain hedge funds and their index also suggest that these hedge funds outperform the equities, in which case safety-first investors had better hold on to the cryptocurrency and/or blockchain hedge funds. However, the Calmar

[7]Both of the 1% VaR for the S&P 500 index from January 2016 to December 2020 and from January 2017 to December 2020 are zero, and there is no number for the Mean-VaR ratios. Therefore, if we compare the hedge funds with the S&P 500 index, the stocks have better performance than the hedge funds.

ratios worsen for the infrastructure hedge funds compared to the S&P500 index over 1- and 2-year investment horizons. The results imply that investors should not get involved in infrastructure hedge funds in those periods. Generally speaking, investors should invest in the cryptocurrency hedge funds since the performance of the cryptocurrency hedge funds surpasses that for the equities, which is analogous to the ASD rule.

Finally, we employ the Sortino ratios only when considering returns that fall below a specific target payoff or a benchmark. These ratio is defined by the following equation:

$$Sortino\ ratio = \frac{E(\tilde{r_x}) - r_f}{LPM},$$

$$LPM = \int_{-\infty}^{q} (q - r) f(r) dr, \tag{12}$$

where q refers to the target returns or the required rates of return, and $f(r)$ is the probability density function of returns. Here we use the LPM to measure losses compared to the benchmark, which we set to zero in our empirical analysis. Risk-averse investors should choose portfolios with higher Sortino ratios. The magnitudes of the Sortino ratios are presented in Panel C of Table 6.

We discover that as for the cryptocurrency hedge funds, the Sortino ratios lie between 0.29 to 7.83 relative to those for the S&P 500 index of 0.12 to 0.49. This implies that the cryptocurrency hedge funds have better outcomes than the S&P 500 index, in which case investors should prefer the cryptocurrency hedge funds to the stocks. Meanwhile, we observe a similar situation for the cryptocurrency index, blockchain hedge funds and their index. The outcomes suggest that the cryptocurrency or blockchain hedge funds are favored by risk-averse investors. Nevertheless, the Sortino ratios for the infrastructure hedge funds drop down to lower levels than those for the S&P500 index when the investment period is 2 year. The situation indicates that investors may only participate in infrastructure hedge funds over 1-month to 1-year investment horizons. Generally speaking, most of the downside risk-adjusted performance measures suggest that safety-first investors should

Table 7: Other commonly used performance measures

	Panel A: Monthly alphas		Panel B: Treynor ratios				Panel C: MPPM			
Assets	7-factor alpha	t-stat.	1-month	6-month	1-year	2-year	1-month	6-month	1-year	2-year
Hedge funds										
Cryptocurrency	0.0182	2.0450	0.0509	0.2543	0.3469	0.5196	0.0222	0.0247	0.0320	0.0886
Blockchain	0.0179	2.0395	0.0510	0.2570	0.3481	0.5188	0.0268	0.0213	0.0266	0.0860
Infrastructure	0.0481	1.5563	0.0634	0.1064	0.0645	0.0820	-0.1166	-0.7748	-0.8327	-0.2803
Cryptocurrency Index	0.0910	2.0621	0.0816	0.1730	0.0945	0.1048	-0.3216	-0.8519	-1.0209	-0.3644
Blockchain Index	0.0899	2.0387	0.0844	0.1719	0.0945	0.1039	-0.3413	-0.8835	-1.0510	-0.3784
S&P 500 Index										
2005/01-2020/12	-0.0020	-11.4054	0.0058	0.0352	0.0669	0.1387	0.0229	0.0129	0.0076	0.0186
2016/01-2020/12	-0.0019	-7.2191	0.0109	0.0522	0.0814	0.1469	0.0558	0.0561	0.0408	0.0478
2017/01-2020/12	-0.0020	-6.6271	0.0103	0.0571	0.1003	0.1973	0.0516	0.0585	0.0452	0.0619

Panels A, B, and C present the monthly alphas, the Treynor ratios, and the MPPMs for each hedge fund portfolio and the S&P 500 index, respectively. The sample period for the cryptocurrency and blockchain is from January 2005 to December 2020, for the infrastructure from January 2017 to December 2020, and for the cryptocurrency index and blockchain index from January 2016 to December 2020.

participate in the cryptocurrency hedge funds. However, in the case of the infrastructure hedge funds, there exists a slight difference, which is inconsistent with the ASD rule.

4.4 Other Commonly Used Performance Measures

In this subsection, we examine whether some other commonly used performance measures provide different investment suggestions for individuals. First, we employ the seven-factor model proposed by Fung and Hsieh (2004) to evaluate the monthly alphas of each hedge fund and the S&P 500 index. Specifically, the seven factors consist of the equity market factor and the size factor, which is measured by the market index and the SMB (small minus big) factor of Fama and French (1993), respectively, as well as the five trend following factors of Fung and Hsieh (2001) to proxy for the nonlinear return features among the hedge funds.[8]

We perform a regression of the excess monthly returns of hedge funds on a constant and the seven factors. The alphas are presented in Panel A of Table 7. We observe that the alpha of the cryptocurrency hedge funds and their index are 1.82% and 9.10%, respectively, whereas the alphas

[8]The market index and the SMB factor are obtained from Kenneth French's online data library. Fung and Hsieh (2001) proposed five trend-following factors, including the return of the stock index lookback straddle, the return of the bond lookback straddle, the return of the short-term interest rate lookback straddle, the return of the currency lookback straddle, and the return of the commodity lookback straddle. These trend-following factors are provided by David Hsieh at http://faculty.fuqua.duke.edu/ dah7/HFRFData.htm (accessed February 2022).

of the S&P 500 index are -0.2%, with all of the alphas being statistically significant. The results disclose that the cryptocurrency hedge funds and their index have excess returns that exceed those of the S&P 500 index, in which case investors should attach more weight to the cryptocurrency hedge funds. Although the infrastructure hedge funds have an alpha of 4.81%, it is statistically insignificant. There are also positive statistically significant alphas for both blockchain hedge funds and their index with 1.79% and 8.99%, respectively, while the alphas for the S&P 500 index are only -0.2%. Generally speaking, investors should participate in the cryptocurrency hedge funds as opposed to the stocks, which is consistent with the ASD rule.

The second commonly used performance measure that we investigate is the Treynor ratio, which is defined as the excess return divided by the market beta. In contrast to the Sharpe ratio, the Treynor ratio only considers the systematic risk of a portfolio. Panel B of Table 7 exhibits the magnitudes of the Treynor ratio. We find that the Treynor ratios for the cryptocurrency hedge funds range from 0.05 to 0.52 compared with those for the S&P 500 index which range from 0.01 to 0.14. The outcomes indicate that if the investors take part in the cryptocurrency hedge funds, they will earn more profit than if they invest in the stocks. However, the Treynor ratios for the cryptocurrency index are only larger than those for the S&P 500 index for the 1-month and 6-month investment periods. The Treynor ratios for the infrastructure hedge funds are also better than those for the S&P 500 index for the 1-month and 6-month investment periods. In addition, as for the blockchain hedge funds and their index, we acquire comparable outcomes to those for the cryptocurrency hedge funds. Overall, the results derived from the Treynor ratios exhibit some differences from those based on the ASD rule.

Finally, we employ another utility-based performance measure proposed by Goetzmann et al. (2007) to examine whether the rankings for the relative performance of hedge funds and the S&P 500 index are different from those based on the ASD rule. This measure is referred to as the manipulation-proof performance measure and is defined by the following equation:[9]

$$MPPM = \frac{1}{(1-\rho)\Delta t} ln(\frac{1}{T}\sum_{t=1}^{T}[(1+r_{x,t})/(1+r_{f,t})]^{1-\rho}), \tag{13}$$

where $r_{x,t}$ is the return of the risky asset $\tilde{x}$ at time t, $r_{f,t}$ is the risk-free rate at time t, T is the total number of observations, and Δt is the length of time between observations. Goetzmann et al. (2007) use T and Δt to annualize the measure, so that the MPPM can be viewed as the annualized continuously compounded excess return certainty equivalent of the portfolio. They connect the MPPM with some benchmark portfolio which is typically the market index. If the benchmark market portfolio has a lognormal return, $1 + x_{m,t}$, then the parameter ρ should be selected so that

$$\rho = \frac{ln[E(1+r_m)] - ln[E(1+r_f)]}{Var[1+x_m]}, \tag{14}$$

where r_m is the return of the market, and r_f is the risk-free rate. In our paper, we employ the market index of Fama and French (1993) as a proxy for r_m and the one-month T-bill as a proxy for r_f. In addition, ρ can be altered for different data frequencies.

The MPPMs for the hedge fund portfolios and the S&P 500 index are presented in Panel C of Table 7. We find that the cryptocurrency hedge funds have MPPMs ranging from 0.02 to 0.09 compared with those for the S&P 500 index that range from 0.01 to 0.02 for 1-month to 2-year investment periods. The magnitudes show that the cryptocurrency hedge funds outperform the S&P 500 index, and that the individuals should invest in the cryptocurrency hedge funds. In addition, the blockchain hedge funds give rise to similar results. Nevertheless, for the infrastructure hedge funds and the hedge fund index, it is found that all of them have inferior MPPMs to those for the S&P 500 index. The results suggest that the performance of these portfolios is worse than that for the S&P 500 index over 1-month to 2-year investment horizons, in which case investors should not hold on to the hedge funds. Generally speaking, the outcomes obtained from the MPPMs have

[9] Although the MPPM and ASD rules are both utility-based measures, the former adopts a parametric approach and the latter uses a non-parametric approach.

some inconsistencies with the ASD rule.

5 Conclusion

Hedge fund managers have increased their holdings of cryptocurrencies in recent years and the weight attached to such investments is expected to increase in the future. Thus, the performance of cryptocurrency hedge funds has attracted a significant degree of attention from both industry and academia. Due to the fact that the return distributions of these hedge funds are different from normal distributions, the traditional measures cannot fully reflect their performance. In this paper, we have adopted the AFSD and ε-GASSD rules, which consider the whole of the distributions of these evaluated assets and have an economic foundation, in order to evaluate their performance.

Our results suggest that 95% of non-pathological risk-averse investors would prefer the cryptocurrency hedge funds to the S&P 500 index over 9-month to 5-year investment periods. Compared with the bond markets, our findings suggest that 95% (99%) of non-pathological risk-averse investors consider that the cryptocurrency hedge funds outperform one-month T-bills (ten-year T-bonds) over 1- to 5-year (2- to 5-year) investment horizons.

In addition, we employ some performance measures that are monotonic with respect to the SD rule and other performance measures do not satisfy these conditions. As for the performance indices that have the monotonicity property, such as Omega, second-order Omega, and the Mean-AS ratio, it is observed that the performance of the cryptocurrency hedge funds is superior than that of the S&P 500 index over 1-month to 2-year investment horizons, in which case investors should participate in the cryptocurrency hedge funds rather than the equities in those periods. By contrast, the performance indices, which are not characterized by the monotonicity property, give rise to slightly different suggestions in terms of the investors' choices.

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