When is safety a normal good?*

Richard Peter†

June 27, 2022

Abstract

A probability threshold determines whether the demand for safety is normal or inferior for a fixed loss severity (Sweeney and Beard, 1992). The size of this threshold is unknown. We show that it is 0.5 for quadratic utility, less than 0.5 for standard utility, and compute it explicitly for iso-elastic and linex utility. Unless the loss exposure puts a large share of final wealth at risk and risk aversion is high, the demand for safety is inferior, which is puzzling. We then characterize when an increase in loss severity raises the demand for safety. Combining both results, we show that safety is always a normal good for losses that are sufficiently income-sensitive. From a practical standpoint, the levels of income sensitivity needed to ensure the normality of safety are low.

Keywords: Safety · self-protection · risk preferences · income effects · comparative statics

JEL-Classification: D11 · D81

*I would like to thank Maximilian Bär and Vijay Krishna for helpful comments on an earlier draft of this paper. I would also like to thank conference participants at the Southern Risk and Insurance Association Annual Meeting and the American Risk and Insurance Association Annual Meeting for valuable feedback and discussions. The usual disclaimer applies.

†University of Iowa, Department of Finance, E-Mail: richard-peter@uiowa.edu, Phone: 319-335-0944.

"Prepare and prevent, don't repair and repent."

Ezra Taft Benson

1 Introduction

Safety oftentimes refers to the control of recognized hazards with the goal of achieving an acceptable level of risk. Questions of safety percolate many areas of everyday life. In the US, the Food and Drug Administration is responsible for food safety and the safety of drugs and medical devices, the National Highway Traffic Safety Administration is concerned with road safety, and the Occupational Safety and Health Administration regulates workplace safety. Each time we eat something, we go somewhere, or we work, the safety of the activity has been considered at some point in time. Engineers care about the safety of the systems they design, members of an industry form committees to propose safety standards, and homeland security engages in terrorism prevention for the safety of all. To the extent that more and more activities of our lives involve digital spaces, issues of data and cybersecurity are gaining importance as well. In fact, it turns out to be difficult to come up with activities, products or services that do not involve at least some consideration of safety.

Unsurprisingly, issues of safety have also received attention from economists. Peltzman (1975) studies automobile safety regulation and documents the importance of offsetting behavior, Viscusi and Aldy (2003) provide a global survey of value of statistical life estimates, a measure that is used in road safety, Antle (2001) review the economic analysis of food safety, and Lakdawalla and Zanjani (2005) provide an economic analysis of terrorism prevention and the role of government intervention in the terrorism insurance market. The simplest model of safety is Ehrlich and Becker's (1972) framework of self-protection or prevention, a costly investment to reduce the probability of an unfavorable outcome, for example, an accident.

Despite its simplicity, it has been notoriously difficult to derive clear predictions about the demand for safety from Ehrlich and Becker's model. In fact, some of the most basic economic properties of the demand for safety remain poorly understood to date. We tackle one such question in this paper. We analyze under what conditions safety is a normal good. Neoclassical consumer theory distinguishes between normal and inferior goods, depending on whether the demand for them is upward- or downward-sloping in income. It is hard to believe that there is no satisfactory answer to the question whether safety is a normal good more than 50 years after Ehrlich and Becker's pioneering work. Intuitively, one might expect that the demand for safety increases in the decision-maker's income. While a rise in income does indeed lower the marginal cost of safety, it also introduces a conflicting negative effect on its marginal benefit. An increase in income lowers the utility gain from avoiding the loss state because the additional income provides an extra buffer against the loss. This creates the possibility for safety to be inferior and calls for further analysis.

In the insurance literature, a simple criterion determines whether the demand for insurance is normal or inferior. In the presence of a positive premium loading and when loss severity is fixed, insurance is a normal good under increasing absolute risk aversion (IARA) and an inferior good under decreasing absolute risk aversion (DARA). Mossin (1968) shows this result for proportional coinsurance and Schlesinger (1981) for straight deductible contracts.[1] Lee (2005, 2010b) obtains similar results for self-insurance activities. Self-protection behaves quite differently, which is typical in the literature. For example, while increased risk aversion in the sense of Pratt (1964) raises the demand for market insurance and self-insurance, it may lower the demand for investments in self-protection, see Dionne and Eeckhoudt (1985).[2]

Income effects are no exception. DARA and IARA are neither necessary nor sufficient to ensure that safety is a normal good. We are aware of only one paper that has looked at income effects on self-protection before: Sweeney and Beard (1992). They state a probability-threshold result and find that safety is a normal good under DARA if the loss probability is below a threshold value. They provide no results about the size of this threshold, which makes it impossible to make testable predictions. In fact, they even say explicitly that the threshold could be anywhere in the unit interval. A snarky economist might remark that probability-threshold results are just a complicated way of saying *"it depends,"* without specifying what it depends on unless one is able to provide a sense of magnitude of the threshold.

In this paper, we take a fresh look at the question whether safety is a normal good. We find that Sweeney and Beard's probability threshold is 0.5 for quadratic utility, less than 0.5 for standard utility (Kimball, 1993), and calculate it explicitly for iso-elastic and linex utility. Unless preventable losses put a large share of final wealth at risk and risk aversion is high, safety is an inferior good, which seems counterintuitive. In the real world, one would expect that, everything else equal, high-income households invest more in safety than low-income households in order to protect their property and shield themselves against liability. To resolve this puzzle, we also consider how an increase in loss severity affects the demand for safety. The effect depends on a (different) probability threshold. Unless preventable losses put a large share of final wealth at risk and decision-makers are very sensitive to risk, the demand for safety is increasing in loss severity. Our results for iso-elastic and linex utility give practical benchmark values for empirical studies on the issue.

In a third step, we combine the two sets of results and consider safety investments against loss exposures whose severity is increasing in income. For example, high-income households may purchase more valuable assets and may be subject to larger liability awards than low-

[1] The demand for insurance with an upper limit can be normal under DARA, see Cummins and Mahul (2004).

[2] The reason is that actuarially fair self-protection does not reduce risk in the sense of Rothschild and Stiglitz (1970) as shown by Briys and Schlesinger (1990). A probability threshold determines whether an increase in risk aversion raises or lowers effort (see Jullien et al., 1999; Peter, 2021b). This indeterminacy has generated research on the effects of downside risk aversion and prudence on prevention, see Chiu (2000, 2005, 2012), Eeckhoudt and Gollier (2005) and Denuit et al. (2016).

income households. The income sensitivity of the loss severity makes it more likely for safety to be a normal good. We find a critical level of income sensitivity above which the demand for safety is always normal. This result holds for any risk-averse utility function as long as risk aversion is monotonic in final wealth. Sweeney and Beard's probability threshold then becomes dispensable. For example, for iso-elastic utility with relative risk aversion less than two and when the loss puts less than 50% of final wealth at risk, an income sensitivity of 25% or higher ensures that safety is a normal good regardless of the loss probability. For higher levels of risk aversion and a lower share of final wealth at risk, the level of income sensitivity needed to ensure the normality of safety decreases.

Our findings have normative and positive implications. Expected utility is still widely accepted as the predominant normative theory of rational choice under risk. Our results then prescribe how rational decision-makers should adjust their safety investments in response to changes in income. From a descriptive standpoint, our analysis is rich in testable hypotheses. With some liberty in interpretation, one can read our model cross-sectionally, in terms of differences across individuals, households, or firms, or at the unit of the decision-maker whose income may be different at different points in time. To substantiate the descriptive merit of our results, we also consider two modeling extensions, severity risk and rank-dependent utility. These extensions make very similar predictions as our benchmark model. We hope that our results will spur empirical research on the economics of safety investments, foster our understanding of how such decisions are made in practice and how they can be improved.

2 The standard model

2.1 Notation

We use the standard model of self-protection or loss prevention introduced into the literature by Ehrlich and Becker (1972). A decision-maker with income y_0 faces the risk of a potential loss in the amount of $L < y_0$. She can invest in safety to lower the likelihood of loss. Let $s \geq 0$ be the investment in safety and let $p(s)$ be the associated loss probability function with $p' \leq 0$. The prime denotes derivatives of univariate functions. Final wealth levels are $y_\ell = y_0 - s - L$ if the loss occurs and $y_n = y_0 - s$ if the loss does not occur. The subscripts ℓ and n abbreviate the loss state and the no-loss state. To ensure nonnegative final wealth, we assume that safety investments are bounded by $y_0 - L$. Furthermore, the decision-maker would never invest an amount larger than L in safety because she would then be better off remaining inactive. We thus fix an upper bound $\overline{s} \leq \min\{y_0 - L, L\}$ on the safety investment and restrict our attention to $s \in [0, \overline{s}]$.

A three times differentiable von Neumann-Morgenstern utility function u represents the decision-maker's preferences over final wealth. We assume that u is increasing and concave, $u' > 0$ and $u'' < 0$, so that the decision-maker prefers more over less and is risk-averse. Her expected-utility objective is then given by

$$\max_{s\in[0,\bar{s}]} U(s;y_0,L) = p(s)u(y_0-s-L)+(1-p(s))u(y_0-s). \tag{1}$$

We assume that an interior solution $s^* \in (0,\bar{s})$ exists, which is then characterized by the following first-order condition:[3]

$$U_s(s^*;y_0,L) = -p'(s^*)\cdot[u(y_n^*)-u(y_\ell^*)] - \left[p^*u'(y_\ell^*)+(1-p^*)u'(y_n^*)\right] = 0. \tag{2}$$

The subscript s on the objective function denotes the partial derivative with respect to the safety investment and the asterisk indicates optimality. We use $p^* = p(s^*)$ as shorthand for the loss probability at the optimal level of safety s^*. Investing an additional dollar in safety has two effects on the decision-maker. It increases expected utility due to a lower loss probability but also reduces final wealth in either state of the world, which lowers expected utility. Correspondingly, the first term in Equation (2) measures the marginal benefit of safety and the second term its marginal cost. The safety investment is at an optimal level when the marginal benefit equals the marginal cost. In this case, expected utility cannot be increased any further and the decision-maker has no incentive to deviate from s^*.

2.2 An increase in income

At an interior solution to Problem (1), the second-order condition holds for maximality, $U_{ss}(s^*;y_0,L)<0$.[4] We can then derive the directional effect of an increase in income on the optimal safety investment from the implicit function rule. This effect is characterized by the sign of the cross-derivative $U_{sy_0}(s^*;y_0,L)$. We obtain

$$\begin{aligned}
U_{sy_0}(s^*;y_0,L) &= -p'(s^*)\cdot[u'(y_n^*)-u'(y_\ell^*)] - \left[p^*u''(y_\ell^*)+(1-p^*)u''(y_n^*)\right] \\
&= \frac{p^*u'(y_\ell^*)+(1-p^*)u'(y_n^*)}{u(y_n^*)-u(y_\ell^*)}\cdot[u'(y_n^*)-u'(y_\ell^*)] - \left[p^*u''(y_\ell^*)+(1-p^*)u''(y_n^*)\right] \\
&= \left[p^*u'(y_\ell^*)+(1-p^*)u'(y_n^*)\right] \\
&\quad \cdot\left\{-\frac{p^*u''(y_\ell^*)+(1-p^*)u''(y_n^*)}{p^*u'(y_\ell^*)+(1-p^*)u'(y_n^*)} - \left(-\frac{u'(y_n^*)-u'(y_\ell^*)}{u(y_n^*)-u(y_\ell^*)}\right)\right\}.
\end{aligned}$$

---

[3] Rearranging $U_s(0;y_0,L)>0$ and $U_s(\bar{s};y_0,L)<0$ yields sufficient conditions for an interior solution. Loosely speaking, if the safety technology is sufficiently productive (i.e., if $-p'(0)$ is large enough) but its productivity declines sufficiently quickly (i.e., if $-p'(\bar{s})$ is small enough), an interior solution exists.

[4] Risk aversion and convexity of the safety technology are not strong enough to ensure the concavity of the objective function in s, see Jullien et al. (1999). As shown by Fagart and Fluet (2013), jointly sufficient conditions are non-increasing absolute risk aversion and log-convexity of the safety technology.

The second equation holds by substituting for $-p'(s^*)$ from first-order condition (2). For compactness, we define the following two random variables. Let $\widetilde{\varepsilon}$ be a random variable with a binary distribution of either a loss of L with probability p^*, or no loss with probability $(1-p^*)$. Let $\widetilde{\omega}$ be a random variable that is uniformly distributed from $-L$ to zero. Define by $v_{\widetilde{\varepsilon}}(y) = \mathbb{E}u(y+\widetilde{\varepsilon})$ and $v_{\widetilde{\omega}}(y) = \mathbb{E}u(y+\widetilde{\omega})$ the corresponding derived utility functions (see Kihlstrom et al., 1981; Nachman, 1982). This allows us to rewrite the two fractions in the curly bracket, and we obtain the following characterization:

$$\operatorname{sgn}\left(\frac{\mathrm{d}s^*}{\mathrm{d}y_0}\right) = \operatorname{sgn}\left(U_{sy_0}(s^*; y_0, L)\right) = \operatorname{sgn}\left(-\frac{v''_{\widetilde{\varepsilon}}(y_n^*)}{v'_{\widetilde{\varepsilon}}(y_n^*)} - \left(-\frac{v''_{\widetilde{\omega}}(y_n^*)}{v'_{\widetilde{\omega}}(y_n^*)}\right)\right).$$

Let $A(u;y) = -u''(y)/u'(y)$ be the Arrow-Pratt measure of absolute risk aversion for utility function u at final wealth level y. Safety is a normal good if and only if $A(v_{\widetilde{\varepsilon}}; y_n^*) > A(v_{\widetilde{\omega}}; y_n^*)$, so if $v_{\widetilde{\varepsilon}}$ is more risk-averse than $v_{\widetilde{\omega}}$ at y_n^*. Comparative risk aversion between $v_{\widetilde{\varepsilon}}(y_n^*)$ and $v_{\widetilde{\omega}}(y_n^*)$ depends on the size of the loss probability p^*. To formulate this effect, we distinguish between decreasing, constant and increasing absolute risk aversion, in short, DARA, CARA and IARA (see Arrow, 1965; Pratt, 1964). We obtain the following result.

Proposition 1 (Sweeney and Beard, 1992)**.** *Consider the optimal safety investment defined implicitly in Equation* (2)*.*

(i) Under DARA there is a $p_c \in (0,1)$ so that safety is a normal good for $p^* > p_c$ and an inferior good for $p^* < p_c$.

(ii) Under CARA the optimal safety investment does not depend on income.

(iii) Under IARA there is a $p_c \in (0,1)$ so that safety is a normal good for $p^* < p_c$ and an inferior good for $p^* > p_c$.

Appendix A.1 provides the proof. Our proof is simpler and more direct than the proof by Sweeney and Beard (1992). The economic intuition behind Proposition 1 comes from the presence of two conflicting effects. An increase in income lowers the marginal benefit of safety because the gain in utility from avoiding the loss state is lower the higher the decision-maker's income. This discourages investing in safety. At the same time, higher income reduces the marginal cost of safety because wealthy decision-makers can dispense an additional dollar more easily than less wealthy decision-makers. This effect encourages safety. The net effect is thus indeterminate. Proposition 1 shows that either of the two partial effects can prevail at the optimum. For DARA and a high loss probability, an increase in income lowers the marginal cost of safety by more than its marginal benefit. The decision-maker then finds it optimal to increase the safety investment. In other words, safety is a normal good. However, when the loss probability is low, matters are reversed. The increase in income now lowers the marginal benefit of safety by more than its marginal cost, which results in a decrease in the optimal safety investment. In this case, safety is an inferior good.

The possibility that safety can be inferior is perhaps not surprising. Similarly, insurance can be an inferior good (Mossin, 1968; Schlesinger, 2013), and both insurance and safety can even be Giffen goods (Hoy and Robson, 1981; Peter, 2021a). However, unlike insurance whether safety is a normal or an inferior good not only depends on how absolute risk aversion changes in wealth but also on a probability threshold. Threshold results like Proposition 1 are common in the economic analysis of self-protection. They arise for risk aversion (Jullien et al., 1999), downside risk aversion (Eeckhoudt and Gollier, 2005; Peter, 2021b), probability weighting (Baillon et al., 2020), and loss aversion Macé and Peter (2021). Threshold results have little practical value unless we know how large the threshold is. Take the case of DARA; if p_c is close to one, we would conclude that safety is an inferior good for the majority of losses unless they are almost certain to happen. If p_c is close to zero instead, we would rather argue that safety is a normal good for most losses unless they are really rare. Without some idea about the size of the probability threshold, a result like Proposition 1 remains largely uninformative. Sweeney and Beard (1992) offer little help in this regard. To the contrary, they provide conceptual examples of the threshold being close to one and other examples of the threshold being close to zero. Their standpoint is that anything goes because *"[c]onclusions can not be reached from knowledge of general characteristics of a (nonconstant) risk-aversion function; an almost complete specification of the risk-aversion function is required."* This leaves the question when safety is a normal good effectively unanswered.

To provide empirically meaningful results, we quantify the probability threshold p_c in Proposition 1. We derive a general result and results for different functional forms of the utility function. While at the expense of generality, this approach allows us to calculate the probability threshold explicitly, analyze its magnitude, and identify its determinants. In a second step, we revisit the comparative static effects of loss severity on safety investments, and then combine both sets of results in a final step. Whether loss severity is sensitive to changes in income will turn out to play a crucial role for the normality of safety.

3 Quantifying the probability threshold

3.1 Quadratic utility

We start with Result (*iii*) in Proposition 1. A commonly used example of IARA utility is the quadratic form, which is popular for its tractability. The following result holds.

Proposition 2. *The probability threshold for quadratic utility is 0.5.*

Appendix A.2 provides the proof. Already Dionne and Eeckhoudt (1985) note that, when comparing two agents with quadratic utility in terms of optimal self-protection, the more risk-averse agent selects a larger (smaller) level of self-protection if the loss probability is smaller (larger) than 0.5. This simple result holds because quadratic utility mutes downside risk aversion. For quadratic utility, an increase in income raises absolute risk aversion due to

IARA. A probability threshold of 0.5 is consistent with Dionne and Eeckhoudt's result. For decision-makers who are downside risk-neutral, safety is a normal good when losses occur less than 50% of the time and an inferior good when losses occur more than 50% of the time.

Eisenhauer (1997) and Eisenhauer and Halek (1999) find that life insurance purchases increase with wealth. They interpret this finding as evidence of IARA. In Baillon and Placido's (2019) experiment 20% of the subjects exhibited risk behavior that is best classified as IARA. Already Arrow (1971) hypothesized that DARA appears more reasonable than IARA upon introspection. Levy (1994), Holt and Laury (2002) and Huber et al. (2022) find experimental evidence in favor of DARA.[5] We discuss a particular class of DARA utility functions next to strengthen Result (i) in Proposition 1.

3.2 Standard utility

The notion of standardness in risk theory is due to Kimball (1993). A risk is called undesirable if it reduces the decision-maker's expected utility. A risk is called loss-aggravating if it increases the decision-maker's utility loss from a nonrandom reduction in final wealth. Kimball (1993) defines a utility function to be standard if every loss-aggravating risk aggravates every undesirable risk. Standardness is stronger than the concept of proper risk aversion proposed by Pratt and Zeckhauser (1987). DARA and decreasing absolute prudence (DAP) are necessary and sufficient for standardness. Another characterization is that any loss-aggravating risk reduces the demand for an independent risky asset. Many common utility functions are standard including log-utility, power utility, and linex utility.[6]

For standard utility, we obtain the following refinement of Result (i) in Proposition 1.

Proposition 3. *The probability threshold for standard utility is less than* 0.5.

Appendix A.3 states the proof. No direct evidence of standardness exists in the literature to date. Eisenhauer and Ventura (2003) find DARA and DAP for Italian households based on survey results. Guiso and Paiella (2008) use household survey data to construct a direct measure of absolute risk aversion. They report evidence of DARA and show that income uncertainty raises risk aversion. Both studies use cross-sectional data whereas the claim of standardness is that a given decision-maker's degree of absolute risk aversion and absolute prudence are negatively associated with her income. It is not a comparison across households.[7]

---

[5] In Baillon and Placido (2019), only 8.6% of subjects are classified as DARA. 71.4% of their subjects exhibit risk behavior that is best classified as CARA. Given the wide prevalence of monotonicity assumptions on absolute risk aversion in economics, the issue is surprisingly unsettled from an empirical standpoint.

[6] Gollier (2001) provides an overview of standardness and related concepts in his chapter 9.

[7] The same caveat applies to Eisenhauer's and Eisenhauer and Halek's evidence of IARA, which is based on cross-sectional data. Cohen and Einav (2007) estimate risk aversion from deductible choice in the cross section and make this point clear. Variables that are likely to be correlated with income are positively associated with absolute risk aversion in their study. They caution the reader on page 762 by saying that *"[o]ur results (...) should not be thought of as a test of the DARA property."*

At the individual level, Beaud and Willinger (2015) find evidence of risk vulnerability in an incentivized laboratory experiment. 81% of their subjects choose a less risky portfolio when exposed to a zero-mean background risk. Risk vulnerability is a necessary condition for standardness (see Gollier and Pratt, 1996).

Proposition 3 shows that standardness places an upper bound on the probability threshold. When $p_c < 0.5$, safety is a normal good for losses that occur at least 50% of the time. Many loss distributions in risk management exhibit positive skewness. In the binary-outcome setting considered here, this requires a loss probability below 0.5 (see Chiu, 2010; Ebert, 2015). But if both p^* and p_c are below 0.5, we cannot draw any definitive conclusion about which one is larger. Standardness allows us to strengthen Result (i) in Proposition 1 but an upper bound of 0.5 on the probability threshold is still not good enough from a practical standpoint. For this reason, we will now impose additional assumptions on the utility function to calculate p_c explicitly. We consider iso-elastic and linex utility.

3.3 Iso-elastic utility

Iso-elastic utility is popular in applications for its tractability. The functional form is

$$u(y) = \begin{cases} y^{1-\rho}/(1-\rho), & \text{for } \rho \neq 1, \\ \log(y), & \text{for } \rho = 1, \end{cases}$$

with parameter $\rho > 0$ measuring relative risk aversion. Iso-elastic utility satisfies DARA and DAP and is therefore standard. We thus already know from Proposition 3 that the probability threshold is less than 0.5. This upper bound can be tightened. Iso-elastic utility also satisfies constant relative risk aversion (CRRA). In Baillon and Placido (2019), 42.9% of subjects exhibit behavior that is best classified as CRRA.

For our next result, we define by $\eta \in (0,1)$ the loss size as a percentage of final wealth in the no-loss state, $\eta = L/y_n^*$. For example, if $y_n^* = \$100$ and $L = \$20$, then $\eta = 0.2$ and we can say that the loss puts 20% of final wealth at risk. Parameter η is based on y_n^* and thus already takes the optimal safety investment s^* into account. We obtain the following result.

Proposition 4. *The probability threshold for iso-elastic utility is given by*

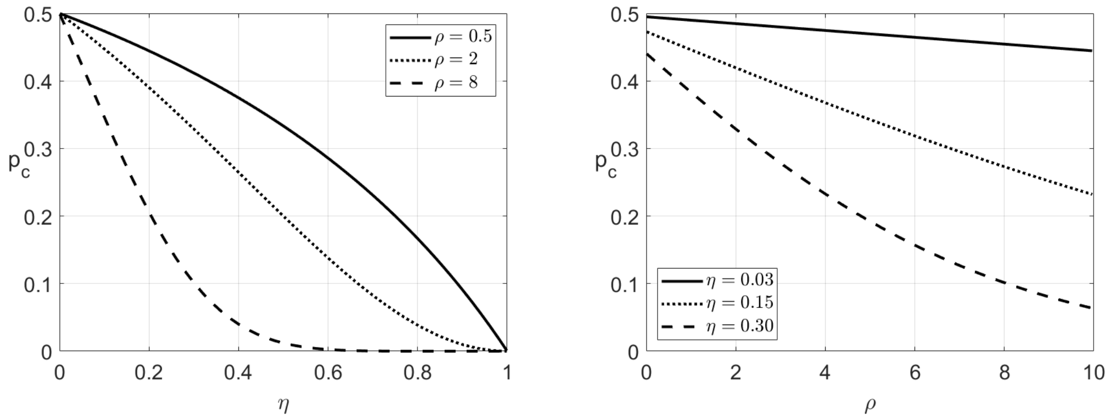
$$p_c = \frac{(1-\eta)\big((1-\eta)^\rho + \rho\eta - 1\big)}{(1-\eta)^{1-\rho} - 2(1-\eta) - \rho\eta^2 + (1-\eta)^{1+\rho}} \qquad \textit{for } \rho \neq 1$$

and by

$$p_c = \frac{(1-\eta)\big((1-\eta)\log(1-\eta) + \eta\big)}{\eta\big((\eta-2)\log(1-\eta) - \eta\big)} \qquad \textit{for } \rho = 1.$$

It is decreasing in ρ and η with

$$\lim_{\rho\to 0} p_c = -\frac{1-\eta}{\eta^2}\big(\eta + \log(1-\eta)\big), \qquad \lim_{\eta\to 0} p_c = 0.5 \qquad \textit{and} \qquad \lim_{\rho\to\infty} p_c = \lim_{\eta\to 1} p_c = 0.$$



(a) Threshold p_c as a function of η (b) Threshold p_c as a function of ρ

Figure 1: Probability threshold p_c for iso-elastic utility, $u(y) = y^{1-\rho}/(1-\rho)$ for $\rho \neq 1$ and $u(y) = \log(y)$ for $\rho = 1$. Panel (a) shows the threshold p_c as a function of the share of final wealth at risk η, Panel (b) shows p_c as a function of relative risk aversion ρ.

Appendix A.4 shows these results. We obtain the threshold for log-utility by taking the limit of p_c for $\rho \to 1$ and applying l'Hôpital's rule. In Figure 1 we plot how the probability threshold depends on relative risk aversion ρ and on the share of final wealth at risk η. Panel (a) shows that p_c decreases in η from 0.5 for $\eta = 0$ to 0 for $\eta = 1$. Its curvature is concave for low levels of risk aversion and convex for high levels of risk aversion. It is closest to linear for $\rho \approx 1.28$. In this case we obtain the rule of thumb that $p_c \approx 0.5 \cdot (1 - \eta)$. We can interpret $(1 - \eta)$ as the share of riskless final wealth. In the above example, if $y_n^* = \$100$ and $L = \$20$, the decision-maker's final wealth in the loss state is $y_\ell^* = \$100 - \$20 = \$80$ or 80% of \$100. So her final wealth is at least 80% of final wealth in the no-loss state if the loss happens despite the safety investment. The rule of thumb then says that safety is a normal good if the loss probability is larger than 40%, which is half of 80%, and inferior otherwise. Panel (b) shows that p_c decreases in relative risk aversion at a decreasing rate. For low values of η, the convexity is only apparent when extending the graph to levels of risk aversion beyond 10. To summarize, safety is more likely to be a normal good the higher the share of final wealth at risk and the higher the decision-maker's relative risk aversion.

3.4 Linex utility

One criticism of iso-elastic utility is that relative risk aversion may not be constant in wealth. Indeed, Holt and Laury (2002) find evidence of increasing relative risk aversion whereas Ogaki and Zhang (2001) and Huber et al. (2022) find decreasing relative risk aversion. Baillon and Placido (2019) classify 35.7% of subjects as increasing and 21.4% of subjects as decreasing relative risk aversion. A utility function that is more flexible in this regard is the so-called class of linear plus exponential, or linex utility functions defined by

$$u(y) = ly - k \cdot \exp(-\gamma y) \qquad \text{for constants } l, k, \gamma > 0.$$

We set $l = 1$ without loss of generality by applying a suitable positive affine transformation. Bell (1988) introduces linex utility and shows that it is one of four classes of utility functions satisfying the one-switch rule and the only such class when adding some other reasonable restrictions.[8] Bell (1995, p. 1148) gives praise to this class of utility functions by saying that it *"deserves consideration as* the *appropriate utility function for generic analyses of financial risk taking."* Denuit et al. (2013) prove that any risk-averse utility function satisfying DARA and DAP in the sense of Ross (1981) necessarily belongs to the linex class. Linex utility thus satisfies a stronger version of Kimball's standardness property, and we know from Proposition 3 that the probability threshold is less than 0.5.

The coefficients of absolute risk aversion and absolute prudence for linex utility are

$$-\frac{u''(y)}{u'(y)} = \frac{k\gamma^2 \cdot \exp(-\gamma y)}{1 + k\gamma \cdot \exp(-\gamma y)} \qquad \text{and} \qquad -\frac{u'''(y)}{u''(y)} = \gamma.$$

Linex utility satisfies DARA and, as noted by Denuit et al. (2013), its absolute prudence is constant in wealth. Unlike iso-elastic utility, it can accommodate non-constant shapes of relative risk aversion.[9] Relative prudence of linex utility is given by $P(u; y) = -yu'''(y)/u''(y) = \gamma y$, and we write P_ℓ as shorthand for relative prudence in the loss state, $P_\ell = P(u; y_\ell^*)$. We then obtain the following result.

Proposition 5. *The probability threshold for linex utility is*

$$p_c = \frac{1}{\eta P_\ell/(1-\eta)} - \frac{1}{\exp(\eta P_\ell/(1-\eta)) - 1}.$$

It is decreasing in P_ℓ and η with

$$\lim_{P_\ell \to 0} p_c = \lim_{\eta \to 0} p_c = 0.5 \qquad \textit{and} \qquad \lim_{P_\ell \to \infty} p_c = \lim_{\eta \to 1} p_c = 0.$$

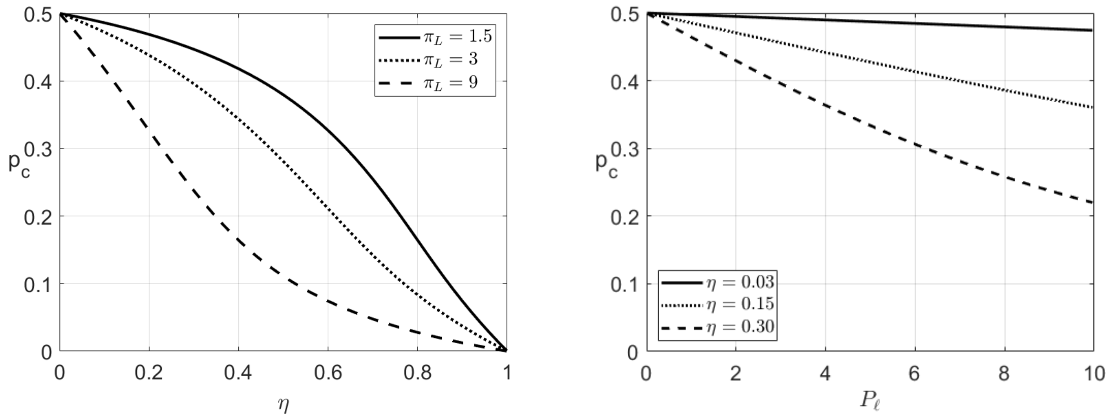
Appendix A.5 provides the proof and Figure 2 illustrates. Panel (a) shows that p_c decreases in η from 0.5 for $\eta = 0$ to 0 for $\eta = 1$. The curves are concave for low and convex for high

8 The one-switch rule requires that there is no more than one critical level of income where the decision-maker's preference over two alternatives switches. Linex utility is the only class of utility functions satisfying the one-switch rule when stipulating risk aversion at all income levels and DARA.

9 Direct computation shows that

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(-\frac{yu''(y)}{u'(y)}\right) = \frac{k\gamma^2 \exp(-\gamma y)}{(1 + k\gamma \cdot \exp(-\gamma y))^2} \cdot [1 - \gamma y + k\gamma \exp(-\gamma y)].$$

The term outside the square bracket is positive. The square bracket is inverse U-shaped in y. There is a critical level $\hat{y}$ so that linex utility has increasing (decreasing) relative risk aversion for $y < (>)\ \hat{y}$.



(a) Threshold p_c as a function of η

(b) Threshold p_c as a function of P_ℓ

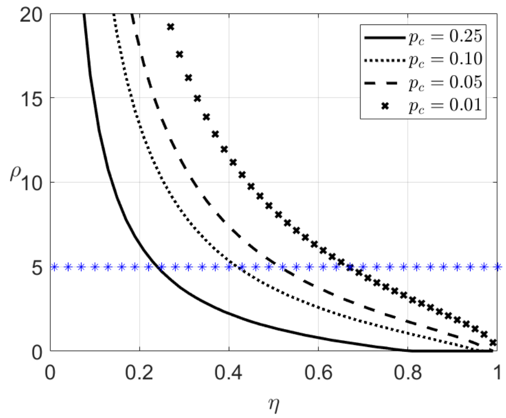
Figure 2: Probability threshold p_c for linex utility, $u(y) = ly - k \cdot \exp(-\gamma y)$ for positive constants l, k, γ. Panel (a) shows the threshold p_c as a function of the share of final wealth at risk η, Panel (b) shows p_c as a function of relative prudence in the loss state P_ℓ.

levels of relative prudence. The probability threshold is closest to linear for $P_\ell \approx 3.55$. In this case, we obtain the same rule of thumb as for iso-elastic utility, that safety is a normal good if the loss probability is larger than half the share of riskless final wealth, and inferior otherwise. Panel (b) shows that p_c decreases in relative prudence at a decreasing rate. The convexity is only visible when extending the graph to a larger range of values for P_ℓ. To summarize, safety is more likely to be a normal good the higher the share of final wealth at risk and the higher the decision-maker's relative prudence.

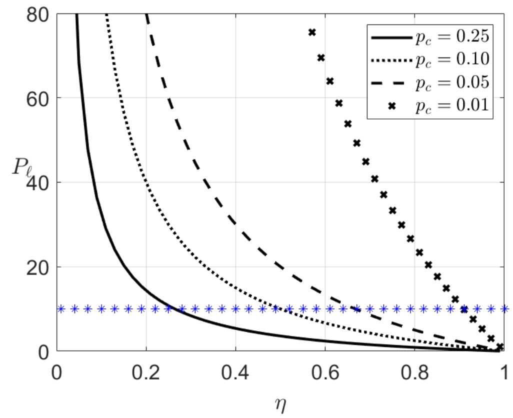
3.5 Comparison

We observe two common patters as we compare the results for iso-elastic and linex utility. The demand for safety is more likely to be normal for losses that put a higher share of final wealth at risk and the more sensitive the decision-maker is towards risk. Despite these commonalities the functional form of the utility function matters for the size of the threshold. The curves for linex utility are uniformly higher than those for iso-elastic utility. Relative prudence for iso-elastic utility is given by $(1+\rho)$, which is why we report the values 0.5, 2 and 8 in Panel (a) of Figure 1 and the values 1.5, 3 and 9 in Panel (a) of Figure 2 for comparability.

To see this more directly, we now look at the iso-threshold curves in the (η, ρ)-plane for iso-elastic utility and in the (η, P_ℓ)-plane for linex utility. These curves collect parameter combinations that produce a specific value of the probability threshold. Say we are looking at a loss with a 25% chance of occurring at the optimal level of safety. We may then wonder for which parameter combinations safety investments against this loss exposure are a normal or an inferior good. The iso-threshold curve for $p_c = 25\%$ partitions the parameter space precisely into those two regions.



(a) Iso-p_c curves for iso-elastic utility



(b) Iso-p_c curves for linex utility

Figure 3: Iso-threshold curves in the (η, ρ)-plane for iso-elastic utility and in the (η, P_ℓ)-plance for linex utility. The threshold p_c takes the values 0.25, 0.10, 0.05 and 0.01. The horizontal line of stars corresponds to $\rho = 5$ in Panel (a) and to $P_\ell = 10$ in Panel (b).

Panel (a) in Figure 3 is for iso-elastic utility. Take the solid curve for $p_c = 0.25$ for a loss that occurs 25% of the time. Safety investments against such risks are normal goods for (η, ρ)-combinations to the right of the curve and inferior goods for (η, ρ)-combinations to the left of the curve. Safety investments against losses that are more frequent than that are more likely to be a normal good because a lower loss severity and a lower level of risk aversion suffice to satisfy the threshold condition. Gollier (2001) argues that a reasonable range for relative risk aversion is from 1 to 4. Gourinchas and Parker (2002) estimate a structural model of lifecycle consumption based on the Consumer Expenditure Survey data, and find relative risk aversion ranging from 0.5 to 1.4. Chetty (2006) uses estimates of the labor supply elasticity to bound relative risk aversion below 2. Meyer and Meyer (2005) consolidate some of the empirical literature on relative risk aversion and adjust reported values by the way the outcome variable is measured (e.g., welath, income, consumption). Most of the adjusted values in their study are between 1 and 5.[10] The horizontal blue line of stars in Panel (a) of Figure 3 represents an upper bound of 5 on relative risk aversion. We can then identify a critical level on η for safety to be an inferior good by intersecting the line for $\rho = 5$ with the iso-p_c curves. Numerically we find $\eta = 0.24$ for $p_c = 0.25$, $\eta = 0.42$ for $p_c = 0.10$, $\eta = 0.51$ for $p_c = 0.05$ and $\eta = 0.67$ for $p_c = 0.01$. For example, safety is an inferior good for losses that occur no more than 10% of the time and put less than 42% of final wealth at risk.

[10] Relative risk aversion in laboratory experiments is often lower. In Holt and Laury (2002), for example, it is centered around the 0.3 to 0.5 range. Studies using deductible choice find implausibly high levels of risk aversion in the triple or quadruple digits (Sydnor, 2010). A potential explanation is that ordinary consumers may find it difficult to assess the costs and benefits of cost-sharing provisions in insurance contracts, which raises doubts about the usefulness of insurance choice data to estimate risk aversion (Bhargava et al., 2017).

Panel (b) in Figure 3 shows the iso-p_c curves for linex utility. Empirical measures of relative prudence vary in the literature. Dynan (1993) uses the Consumer Expenditure Survey data and concludes that *"[w]e cannot reject the hypothesis that the coefficient of relative prudence is zero."* Her largest point estimate is 0.312.[11] Merrigan and Normandin (1996) obtain estimates for relative prudence ranging from 1.78 to 2.33 based on longitudinal expenditure data from the UK. Using survey data on life insurance holdings, Eisenhauer (2000) finds a range between 1.51 and 5.15. Eisenhauer and Ventura (2003) find higher values ranging from 7.32 to 8.65, on average, based on a hypothetical survey question about the willingness to take risk. Noussair et al. (2014) measure relative prudence directly for a demographically representative sample of the Dutch population. Their estimates range between 1.68 and 2.24. The horizontal blue line of stars in Panel (b) of Figure 3 corresponds to relative prudence of 10. The critical level on η for safety to be an inferior good is then $\eta = 0.26$ for $p_c = 0.25$, $\eta = 0.5$ for $p_c = 0.1$, $\eta = 0.67$ for $p_c = 0.05$ and $\eta = 0.91$ for $p_c = 0.01$. Using the same example as before, safety is an inferior good for losses that occur no more than 10% of the time and put less than 50% of final wealth at risk.

Both classes of utility functions suggest that safety investments are inferior goods for the vast majority of practical applications unless preventable losses put a large share of final wealth at risk or the decision-maker is very sensitive to risk. Our focus on iso-elastic and linex utility, while less general than an arbitrary risk-averse utility function, allows us to confront the model with empirical estimates of preference parameters, which then yields a definitive answer to the question whether safety is a normal good. The main caveat is that we assumed the loss severity L to be unaffected by income shocks. To relax this assumption, we will first develop results on the comparative static effects of loss severity on the demand for safety. This sets the stage for extending the model to income-sensitive losses.

4 Loss severity

4.1 General results

We first return to the setting with a general risk-averse utility function. Intuition suggests that more severe losses should lead to higher investments in safety because there is more to gain from avoiding the loss state. But if the loss occurs, having made a high investment in safety seems unfortunate in hindsight because the money could have been rather used to better cope with the loss. Therefore, it is not clear *a priori* how loss severity affects the optimal safety investment. According to the implicit function rule, the sign of $U_{sL}(s^*; y_0, L)$

[11] Skinner (1988), Kuehlwein (1991), Guiso et al. (1992) and Parker (1999) also find little to no evidence of precautionary saving. By implication, this suggests little to no evidence of prudence. Due to substitution effects, this reasoning relies on saving being the only channel for households to express precautionary motives, see Heinzel and Peter (2021).

informs us about the direction of this effect. We obtain

$$\begin{aligned} U_{sL}(s^*; y_0, L) &= -p'(s^*)u'(y_\ell^*) + p^* u''(y_\ell^*) \qquad (3) \\ &= [p^* u'(y_\ell^*) + (1-p^*)u'(y_n^*)] \cdot \left\{ \frac{u'(y_\ell^*)}{u(y_n^*) - u(y_\ell^*)} + \frac{p^* u''(y_\ell^*)}{p^* u'(y_\ell^*) + (1-p^*)u'(y_n^*)} \right\}. \end{aligned}$$

The first line reflects the trade-off between a higher marginal benefit and a higher marginal cost of safety from the increase in loss severity. The equality holds by substituting for $-p'(s^*)$ from first-order condition (2) and factoring out expected marginal utility. Let $R(u; y) = -yu''(y)/u'(y)$ denote the Arrow-Pratt measure of relative risk aversion for utility function u at final wealth level y, and let R_ℓ be shorthand for relative risk aversion in the loss state, $R_\ell = R(u; y_\ell^*)$. We then obtain the following results.

Proposition 6. *Consider the optimal safety investment defined implicitly in Equation* (2). *An increase in loss severity raises the optimal safety investment:*

(i) If and only if the decay rate of the loss probability exceeds absolute risk aversion in the loss state.

(ii) If the utility function exhibits non-decreasing absolute risk aversion.

(iii) If the utility function exhibits DARA and $p^* < p_\#$ for a threshold $p_\# > 0$.

The probability threshold $p_\#$ for DARA:

(iv) Exceeds unity when $\eta \leq 1/(1 + R_\ell)$.

(v) Exceeds 0.5 *when $\eta \leq 1/(1 + 0.5 \cdot R_\ell)$.*

Appendix A.6 provides the proof. Result (*i*) also plays a role in the comparative statics of multivariate prevention decisions, see Hofmann and Peter (2015). The decay rate of the loss probability, defined as $-p'(s)/p(s)$, measures how quickly the safety investment reduces the loss probability. When the decay rate is non-increasing in s and the utility function has non-increasing absolute risk aversion, Problem (1) is globally concave in s and the first-order approach is valid (see Fagart and Fluet, 2013). Courbage et al. (2017) use this measure to analyze optimal prevention for correlated risks. Result (*ii*) is Sweeney and Beard's Result B. When the utility function has non-decreasing absolute risk aversion, the increase in the marginal benefit due to a more severe loss always outweighs the accompanying increase in the marginal cost. Result (*iii*) states that the net effect for DARA utility may be indeterminate when the probability threshold $p_\#$ is less than one. Results (*iv*) and (*v*) provide simple conditions that allow us to bound the probability threshold from below. These conditions involve relative risk aversion in the loss state and the share of final wealth at risk. For example, when relative risk aversion is less than 2, the loss puts less than 50% of final wealth at risk and occurs less than 50% of the time, we expect increased safety investments as loss severity rises.

Sweeney and Beard (1992) provide two additional results for DARA utility. Their Result C compares the expected loss to a function that depends on absolute risk aversion in the loss state and absolute risk aversion in the no-loss state. It is unclear how to evaluate their condition from a practical standpoint. Their Result D holds for increasing relative risk aversion. In our notation, they find a probability threshold of $p_{\#}^{SB} = (1-\eta)^{1+R_n}/\eta$, where superscript SB abbreviates Sweeney and Beard and R_n is shorthand for relative risk aversion in the no-loss state, $R_n = R(u; y_n^*)$.[12] Their threshold can also be bounded from below because $p_{\#}^{SB} \geq 1$ for $(1-\eta)^{1+R_n} \geq \eta$ and $p_{\#}^{SB} \geq 0.5$ for $(1-\eta)^{1+R_n} \geq 0.5\eta$. The difficulty in comparing $p_{\#}^{SB}$ to $p_{\#}$ lies in the fact that we use R_ℓ while they use R_n. One way to compare the two thresholds is by assuming $R_n \approx R_\ell$. Alternatively, we can adjust Sweeney and Beard's threshold for the difference in relative risk aversion across states. Holt and Laury (2002) find increasing relative risk aversion so that $R_n > R_\ell$. They estimate Saha's (1993) expo-power utility function, defined as $u(y) = (1-\exp(-\theta y^{1-\xi}))/\theta$, with relative risk aversion of $R(u; y) = \xi + \theta(1-\xi)y^{1-\xi}$. This provides the following relationship between relative risk aversion in the loss state and relative risk aversion in the no-loss state:

$$R_n = \xi + (R_\ell - \xi)(1-\eta)^{\xi-1}.$$

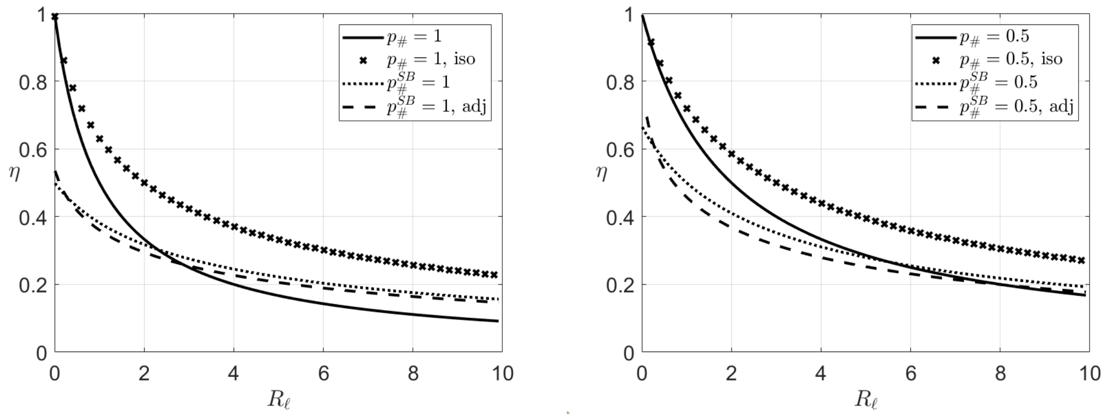
We can then use Holt and Laury's estimate for ξ, given by 0.269, to rewrite Sweeney and Beard's threshold in terms of R_ℓ.

Figure 4 illustrates Results (*iv*) and (*v*) in Proposition 6, and contrasts them with the corresponding conditions based on Sweeney and Beard's threshold. Adjusting for differences in relative risk aversion across states slightly lowers the curves for $p_{\#}^{SB}$ for most values of R_ℓ.[13] Each curve in Panel (a) partitions the (R_ℓ, η)-plane into two regions. The region below the curve corresponds to $p_{\#} > 1$ so that an increase in loss severity always raises the investment in safety regardless of the loss probability. The region above the curve represents $p_{\#} < 1$ so that the magnitude of the loss probability relative to the threshold matters. When $R_\ell \leq 2.29$ for the unadjusted curve or $R_\ell \leq 2.85$ for the adjusted curve, our criterion is less restrictive than Sweeney and Beard's. As long as $R_\ell \leq 5$ and $\eta \leq 16.7\%$, the optimal safety investment is always increasing in loss severity regardless of which criterion is used.

Panel (b) looks at a lower bound of 0.5 on the probability thresholds $p_{\#}$ and $p_{\#}^{SB}$. Now the curves partition the (R_ℓ, η)-plane into regions below the curve where $p_{\#} > 0.5$ and regions above the curve where $p_{\#} < 0.5$. Most losses in risk management exhibit positive skewness, which implies $p^* < 0.5$ when outcomes are binary (see Chiu, 2010; Ebert, 2015). In this case, a probability threshold larger than 0.5 ensures that the optimal safety investment is increasing

[12] Their Result D states that $\mathrm{d}s^*/\mathrm{d}L > 0$ when $\mathrm{d}R(u; y)/\mathrm{d}y > 0$ and $(y_\ell^*/y_n^*)^{R_n} > p^* L/y_\ell^*$. Use the definition of η and solve for p^* to obtain $p_{\#}^{SB}$ as stated in the text.

[13] The exact location of the adjusted curves is relatively insensitive to the estimate of ξ. Changes within three standard errors based on Holt and Laury's estimates do not lead to perceptible differences.



(a) Comparison of $p_{\#} = 1$ versus $p_{\#}^{SB} = 1$

(b) Comparison of $p_{\#} = 0.5$ versus $p_{\#}^{SB} = 0.5$

Figure 4: Locus of (R_ℓ, η)-combinations so that $p_{\#} = 1$ and $p_{\#} = 0.5$. The solid lines illustrate Results (iv) and (v) in Proposition 3, the dotted and dashed lines illustrate the corresponding conditions for Sweeney and Beard's threshold without adjustment (i.e., $R_n \approx R_\ell$) and with adjustment (i.e., $R_n > R_\ell$).

in loss severity. As evident from Panel (b), our criterion is less restrictive than Sweeney and Beard's when $R_\ell \leq 5.51$ for the unajdusted curve and when $R_\ell \leq 8.09$ for the adjusted curve. In other words, for the plausible range of relative risk aversion between 1 and 5, our criterion yields a larger region of (R_ℓ, η)-combinations for which the intuitive result prevails. As long as $R_\ell \leq 5$ and $\eta \leq 28.6\%$, skewed loss exposures will induce larger safety investments as their severity increases.

Overall, the results in Proposition 6 suggest that, for the relevant case of DARA utility, safety investments are increasing in loss severity unless preventable losses put a large share of final wealth at risk or decision-makers are very sensitive to risk. We emphasize that we can quantify what we mean by a large share of final wealth or high levels of risk aversion without specifying the utility function. Results (iv) and (v) in Proposition 6 provide easy-to-verify rules of thumb for practical applications. As we will show in the next section, these results can be strengthened by focusing on particular classes of utility functions.

4.2 Results for iso-elastic and linex utility

We revisit Proposition 6 for iso-elastic and linex utility. In both cases, we can calculate the threshold $p_{\#}$ in Result (iii) explicitly. We start with iso-elastic utility.

Proposition 7. *The probability threshold $p_{\#}$ for iso-elastic utility is given by*

$$p_{\#} = \frac{(1-\rho)(1-\eta)^{\rho}}{\rho(1-\eta)^{\rho-1} + (1-\rho)(1-\eta)^{\rho} - 1} \qquad \textit{for } \rho \neq 1$$

and by

$$p_{\#} = -\frac{1-\eta}{\eta + \log(1-\eta)} \qquad \textit{for } \rho = 1.$$

It is decreasing in ρ *and* η *with*

$$\lim_{\rho\to 0} p_{\#} = \lim_{\eta\to 0} p_{\#} = \infty \qquad \textit{and} \qquad \lim_{\rho\to\infty} p_{\#} = \lim_{\eta\to 1} p_{\#} = 0.$$

Appendix A.7 gives the proof. The threshold for log-utility can either be obtained by direct computation or via l'Hôpital's rule by taking the limit of $p_{\#}$ for $\rho \to 1$. We state the conditions for $p_{\#} \geq 1$ and for $p_{\#} \geq 0.5$ in Appendix A.7, thus specifying Results (*iv*) and (*v*) in Proposition 6 to iso-elastic utility. Figure 4 illustrates the threshold $p_{\#}$ in Proposition 7 as the line marked with crosses. The locus of (ρ, η)-combinations for which $p_{\#} = 1$ in Panel (a) lies above the corresponding locus for a general DARA utility function in Result (*iv*) of Proposition 6. It also lies above the locus based on Sweeney and Beard's criterion, regardless of whether we adjust for differences in relative risk aversion across states or not. Moving to iso-elastic utility expands the set of parameter combinations for which an increase in loss severity always leads to a larger safety investment. When looking at Panel (b) of Figure 4, we see that the locus of (ρ, η)-combinations for which $p_{\#} = 0.5$ for iso-elastic utility also lies above all the other curves. Moving to iso-elastic utility also expands the set of parameter combinations for which an increase in loss severity always leads to a larger safety investment against skewed loss exposures with $p^* < 0.5$.

To give a sense of magnitude, we use the expression in Proposition 7 to provide bounds on the share of final wealth at risk η. Let us bound relative risk aversion and assume $\rho \leq 5$. We can then wonder about the share of final wealth at risk so that $p_{\#}$ exceeds a certain cutoff. The first row of Table 1 provides these thresholds. Safety investments are increasing in loss severity for losses that put less than 33.1% of final wealth at risk regardless of their loss probability because we have $p_{\#} \geq 1$. Safety investments against losses that occur less than 10% of the time are increasing in loss severity as long as they do not put more than 53.6% of final wealth at risk, etc. Table 1 reveals a trade-off. The lower the likelihood of loss, the higher the share of final wealth that it may put at risk to still obtain the intuitive result that an increase in loss severity raises the optimal safety investment.

Linex utility also allows us to determine the threshold $p_{\#}$ in Proposition 6(*iii*) explicitly. The following proposition provides this result, and Appendix A.8 gives the proof.

Proposition 8. *The probability threshold* $p_{\#}$ *for linex utility is given by*

$$p_{\#} = \frac{P_\ell \left[P_\ell - R_\ell \left(1 - \exp\left(-\frac{\eta P_\ell}{1-\eta}\right)\right)\right]}{R_\ell (P_\ell - R_\ell)\left[\frac{\eta P_\ell}{1-\eta} - \left(1 - \exp\left(-\frac{\eta P_\ell}{1-\eta}\right)\right)\right]}.$$

	$p_{\#} \geq 1$	$p_{\#} \geq 0.5$	$p_{\#} \geq 0.25$	$p_{\#} \geq 0.1$	$p_{\#} \geq 0.05$	$p_{\#} \geq 0.01$
Iso-elastic utility $\rho \leq 5$	33.1%	39.4%	45.7%	53.6%	59.1%	70.0%
Linex utility $P_\ell \leq 10$	20.3%	26.8%	35.8%	52.5%	67.7%	91.0%
Linex utility $P_\ell \leq 10, R_\ell \leq 5$	23.3%	33.4%	47.3%	67.7%	80.4%	95.2%

Table 1: Thresholds on the share of final wealth at risk so that safety investments are increasing in loss severity. The first row is for iso-elastic utility with $\rho \leq 5$, the second row for linex utility with $P_\ell \leq 10$, and the third row for linex utility with $P_\ell \leq 10$ and $R_\ell \leq 5$.

It is decreasing in P_ℓ and η with

$$\lim_{P_\ell \to 0} p_{\#} = \lim_{\eta \to 0} p_{\#} = \infty, \qquad \lim_{P_\ell \to \infty} p_{\#} = \frac{1-\eta}{2\eta R_\ell} \qquad \textit{and} \qquad \lim_{\eta \to 1} p_{\#} = 0.$$

It is U-shaped in R_ℓ with

$$\lim_{R_\ell \to 0} p_{\#} = \lim_{R_\ell \to P_\ell} p_{\#} = \infty.$$

Rows two and three of Table 1 provide bounds on the share of final wealth at risk for linex utility. We first assume relative prudence less than 10 and do not fix relative risk aversion (row two). Safety investments are increasing in loss severity for preventable losses not exceeding 20% of income at risk, irrespective of their loss probability. For losses that occur less than 50% of the time, we obtain a positive effect of loss severity on safety if losses put no more than 27% of income at risk. For losses that occur less than 10% of the time, the threshold is even higher at 53% of income at risk, etc. Taking the U-shape of $p_{\#}$ into account, the reported thresholds are those obtained for the value of R_ℓ that minimizes $p_{\#}$ over $(0, P_\ell)$. If we assume relative prudence less than 10 and relative risk aversion less than 5 (row three), safety investments are increasing in loss severity when losses do not exceed 23% of income at risk regardless of their probability, for losses not exceeding 33% of income at risk for a probability below 50%, and for losses not exceeding 68% of income at risk for a probability below 10%, etc. We observe the same trade-off as for iso-elastic utility that, the lower the loss probability, the higher the share of final wealth at risk that is possible while still finding the intuitive effect that loss severity increases the demand for safety.

From a practical standpoint, we conclude that safety investments are increasing in loss severity unless preventable losses put a large share of final wealth at risk or decision-makers are very sensitive to risk. This prediction holds for a general risk-averse utility function and is strengthened for the classes of iso-elastic and linex utility.

5 Safety investments for income-sensitive losses

5.1 General results

In a final step, we combine the two sets of results to analyze income effects on safety investments for income-sensitive losses. As income increases, decision-makers are likely to acquire more valuable assets that are subject to preventable losses, which may stimulate an additional demand for safety. From an empirical standpoint, income is often not observed at the individual level. However, Cohen and Einav (2007) use the value of the car as a proxy for income in the context of auto insurance, and Barseghyan et al. (2013) assume that wealth is proportional to home value in the context of homeowners insurance. While no specific estimates exist on the size of the income-sensitivity of preventable losses, and while there is certainly heterogeneity in this parameter across households, it is fair to say that allowing for loss severity to depend on income will only make the model more realistic.

Specifically, we let $L = L(y_0)$ and assume that loss severity is differentiable in income with $\chi = L'(y_0)$. We refer to χ as the income sensitivity of the loss severity and let $\chi \in (0, 1)$. The limiting case of $\chi \to 0$ corresponds to the analysis in Section 3 because then loss severity is unaffected by income shocks. For $\chi \to 1$, decision-makers spend each additional dollar in income on assets that are subject to preventable losses. Given the lack of empirical evidence on the size and distribution of χ in a representative cross-section of households, we consider the entire unit interval in our analysis.

We adjust objective function (1) by replacing L by $L(y_0)$. First-order condition (2) also requires us to adjust final wealth in the loss state accordingly. For income-sensitive losses, the chain rule then yields the following:

$$\frac{\mathrm{d}s^*}{\mathrm{d}y_0} = \frac{\partial s^*}{\partial y_0} + L'(y_0) \cdot \frac{\partial s^*}{\partial L} = \frac{\partial s^*}{\partial y_0} + \chi \cdot \frac{\partial s^*}{\partial L}.$$

The sign of $\mathrm{d}s^*/\mathrm{d}y_0$ coincides with the sign of $U_{sy_0}(s^*; y_0, L(y_0)) + \chi U_{sL}(s^*; y_0, L(y_0))$ due to the implicit function rule. We can use the previous analysis to rewrite this term as follows:

$$\begin{aligned} & \left[p^* u'(y_\ell^*) + (1-p^*) u'(y_n^*)\right] \cdot \left\{ -\frac{p^* u''(y_\ell^*) + (1-p^*) u''(y_n^*)}{p^* u'(y_\ell^*) + (1-p^*) u'(y_n^*)} - \left(-\frac{u'(y_n^*) - u'(y_\ell^*)}{u(y_n^*) - u(y_\ell^*)} \right) \right\} \\ + \quad & \left[p^* u'(y_\ell^*) + (1-p^*) u'(y_n^*)\right] \cdot \chi \cdot \left\{ \frac{u'(y_\ell^*)}{u(y_n^*) - u(y_\ell^*)} + \frac{p^* u''(y_\ell^*)}{p^* u'(y_\ell^*) + (1-p^*) u'(y_n^*)} \right\}. \qquad (4) \end{aligned}$$

We factor out expected marginal utility and set the remainder equal to zero. We thus obtain the probability threshold that determines income effects on safety investments for income-sensitive losses. The following proposition summarizes our findings.

Proposition 9. *Consider the optimal safety investment defined implicitly in Equation* (2) *for an income-sensitive loss exposure with* $L'(y_0) = \chi$.

(i) Under DARA there is a $p_\$ < 1$ *so that safety is a normal good for* $p^* > p_\$$ *and an inferior good for* $p^* < p_\$$*. The probability threshold* $p_\$$ *is decreasing in* χ*. Furthermore, there is a critical level of income sensitivity, given by*

$$\hat{\chi} = \frac{u(y_n^*) - u(y_\ell^*)}{u'(y_\ell^*)} \cdot [A(v_{\tilde{\omega}}; y_n^*) - A(u; y_n^*)] \in (0, 1). \tag{5}$$

For $\chi \geq \hat{\chi}$*, we have* $p_\$ \leq 0$ *and safety is always a normal good.*

(ii) Under CARA safety is always a normal good.

(iii) Under IARA there is a $p_\$ > 0$ *so that safety is a normal good for* $p^* < p_\$$ *and an inferior good for* $p^* > p_\$$*. The probability threshold* $p_\$$ *is increasing in* χ*. Furthermore, there is a critical level of income sensitivity, given by*

$$\hat{\chi} = [A(v_{\tilde{\omega}}; y_n^*) - A(u; y_\ell^*)] \cdot \left[\frac{u(y_n^*) - u(y_\ell^*)}{u'(y_\ell^*)} - A(u; y_\ell^*)\right]^{-1} \in (0, 1). \tag{6}$$

For $\chi \geq \hat{\chi}$*, we have* $p_\$ \geq 1$ *and safety is always normal.*

Appendix A.9 provides the proof. Proposition 9 shows that the income sensitivity of the loss severity plays a key role in determining the income effects on the demand for safety. Take the case of DARA. For a fixed loss severity, our main conclusion at the end of Section 3 was that safety investments are inferior for the majority of applications unless preventable losses put a large share of final wealth at risk or decision-makers are very sensitive to risk. Allowing for income-sensitive losses has the potential to turn this conclusion upside down. While we still find a probability threshold, the degree of income sensitivity reduces the magnitude of this threshold, which makes it more likely for the loss probability to exceed it. What's more, if the loss severity is sufficiently income-sensitive in a precise technical sense, the probability threshold turns negative and safety investments are always a normal good, consistent with intuition. As explained in Section 4, an increase in loss severity tends to raise the demand for safety, which then contributes to the normality of safety investments for income-sensitive losses. In the knife-edge case of CARA utility, the critical level of income sensitivity is zero, $\hat{\chi} = 0$, so that safety is always normal for *any* level of income sensitivity $\chi > 0$.

For IARA utility, the results are flipped in the sense that the loss probability now needs to fall below the probability threshold for safety to be a normal good and because the probability threshold is now increasing in the degree of income sensitivity. However, the same general conclusion arises as in the DARA case. Income sensitivity increases the range of loss probabilities for which safety is a normal good, and as soon as the income sensitivity is large enough in a precise technical sense, safety investments will always be normal. As such, Proposition 9 contains a strong general message. Regardless of whether absolute risk aversion is decreasing, constant or increasing, the degree of income sensitivity of the loss exposure always makes it more likely for safety to be a normal good.

5.2 Results for quadratic, iso-elastic and linex utility

Proposition 9 begs the question how large the critical level of income sensitivity $\hat{\chi}$ is. We started our analysis by noting that threshold results are of little practical use without some idea about the size of the threshold. This qualification applies to $\hat{\chi}$ as well. Take the case of DARA; if $p_\$$ declines very slowly in χ and $\hat{\chi}$ is close to one, our previous results stand and we have little reason to deviate from our conclusion that, from an applied standpoint, the demand for safety is an inferior good in most situations. If, however, $p_\$$ declines rapidly in χ and $\hat{\chi}$ is close to zero, we have all reason to conclude that safety investments will generally be normal goods precisely because most losses are income-sensitive in practice. As earlier, the magnitude of the threshold makes all the difference.

The following proposition provides explicit formulas for $\hat{\chi}$ in case of quadratic, iso-elastic and linex utility. Of course, we could also calculate $p_\$$ from Proposition 9 explicitly for those functional forms, or determine other levels of income sensitivity, for example, such that $p_\$ \leq 0.01$ in case of DARA. For clarity and compactness, we focus on $\hat{\chi}$ because in this case safety is a normal good regardless of the size of the loss probability.

Proposition 10. *Consider the critical level of income sensitivity $\hat{\chi}$ defined in Proposition 9.*

(i) For quadratic utility, we obtain

$$\hat{\chi} = \frac{\eta^2 R_n^2}{1 + (1 + \eta R_n)^2}.$$

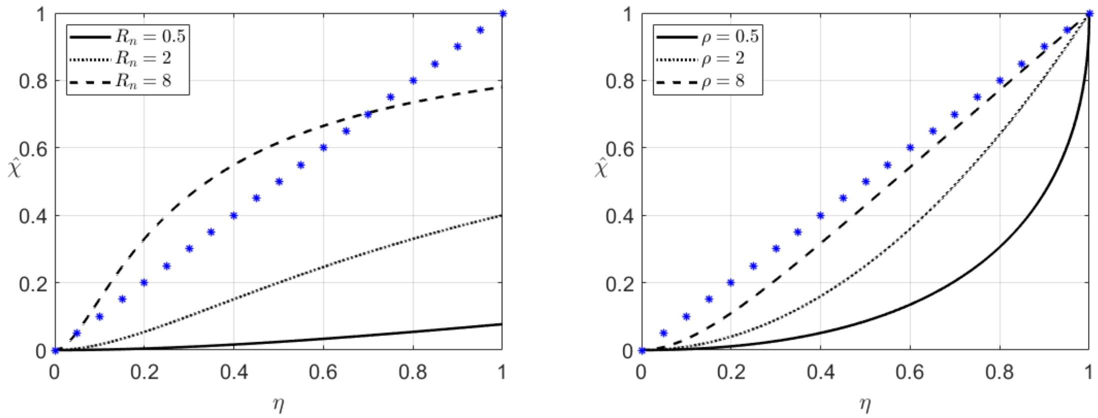
(ii) For iso-elastic utility, we obtain

$$\hat{\chi} = \frac{(1-\eta)^\rho + \rho\eta - 1}{\rho - 1} \text{ for } \rho \neq 1, \text{ and } \hat{\chi} = (1-\eta)\log(1-\eta) + \eta \text{ for } \rho = 1.$$

(iii) For linex utility, we obtain

$$\hat{\chi} = R_\ell \left(1 - \frac{R_\ell}{P_\ell}\right) \cdot \frac{1 - \exp\left(-\frac{\eta P_\ell}{1-\eta}\right)\left(1 + \frac{\eta P_\ell}{1-\eta}\right)}{P_\ell - R_\ell\left(1 - \exp\left(-\frac{\eta P_\ell}{1-\eta}\right)\right)}.$$

Appendix A.10 provides the proof. Figure 5 illustrates Results (i) and (ii), and Figure 6 in Appendix B.1 illustrates Result (iii). The critical threshold $\hat{\chi}$ is increasing in η so the higher the share of final wealth at risk, the higher the required level of income sensitivity of the loss severity to ensure that safety is a normal good. Similarly, with a higher degree of risk aversion, losses need to be more income-sensitive to guarantee that safety is always normal regardless of the loss probability. Both of these observations are intuitive and in line with our previous findings. The reason why safety might fail to be normal in the first place is that an increase in income lowers its marginal benefit. This effect occurs because the decision-maker is risk-averse. Furthermore, the marginal benefit of safety is larger the higher the share of



(a) Threshold $\hat{\chi}$ for quadratic utility

(b) Threshold $\hat{\chi}$ for iso-elastic utility

Figure 5: Threshold $\hat{\chi}$ for quadratic utility, see Proposition 10(i), and for iso-elastic utility, see Proposition 10(ii). The blue line of stars indicates the identity line where $\hat{\chi} = \eta$.

final wealth at risk. To ensure that the decrease in the marginal benefit does not dominate, a higher level of income sensitivity is thus needed.

Figure 5 also provides a sense of magnitude. For example, when relative risk aversion is 2 and the loss puts 50% of final wealth at risk, we obtain $\hat{\chi} = 0.2$ for quadratic utility and $\hat{\chi} = 0.25$ for iso-elastic utility. In other words, for relative risk aversion below 2 and when the loss puts less than 50% of final wealth at risk, an income sensitivity of 0.2 (0.25) or higher ensures that the demand for safety is always normal for quadratic (iso-elastic) utility. Taking these numbers on their face, such levels of income sensitivity do not seem implausible.

More generally, the critical level of income sensitivity $\hat{\chi}$ is less than η for quadratic utility unless risk aversion is high, and always less than η for iso-elastic utility.[14] In Figure 5, the case of $\hat{\chi} = \eta$ is marked with a blue line of stars, and it is evident that all black lines are below it the only exception being Panel (a) for $R_n = 8$. In the special case in which loss severity is simply a linear function of income, $L(y_0) = \chi y_0$ for a $\chi \in (0, 1)$, we obtain that $\eta = L/y_n^* = \chi \cdot y_0/(y_0 - s^*)$. Then it follows that $\eta > \chi$ because $y_0 > y_0 - s^*$. So if $\hat{\chi}$ was always larger than η, the income sensitivity of the loss severity would never ensure that safety is always normal, and we would be back to a probability-threshold situation. As shown in Figure 5, this is not the case because $\hat{\chi} < \eta$ is quite plausible. Furthermore, when loss severity is linear in income and the decision-maker's safety investment is small relative to income, then η and χ will be relatively close in magnitude. As shown in Figure 5, the critical level $\hat{\chi}$ is often substantially smaller than η so that $\chi > \hat{\chi}$ should then be easily satisfied.

[14] For quadratic utility, $\hat{\chi} < \eta$ is equivalent to $R_n^2\eta^2 - (R_n^2 - 2R_n)\eta + 2 > 0$, which is a quadratic function in η. Its discriminant is $R_n^2(R_n^2 - 4R_n - 4)$, which is negative for R_n between zero and $2 + 2\sqrt{2} \approx 4.828$. So for $R_n < 2+2\sqrt{2}$, we obtain that $\hat{\chi} < \eta$ always holds. For iso-elastic utility, $\hat{\chi} < \eta$ rearranges to $(1-\eta)^\rho > 1-\eta$ when $\rho < 1$, to $(1-\eta)\log(1-\eta) < 0$ when $\rho = 1$, and to $(1-\eta)^\rho < 1-\eta$ when $\rho > 1$. Due to $\eta \in (0, 1)$, all three inequalities are always satisfied.

We conclude this section by noting that the income sensitivity of the loss severity plays a crucial role for the normality of safety investments. As shown in Proposition 9, income sensitivity expands the range of loss probabilities for which safety is a normal good. What's more, we find a critical level of income sensitivity above which the loss probability becomes irrelevant because the demand for safety is always normal. These results hold for any risk-averse utility function as long as the degree of risk aversion is monotonic in final wealth. If we make parametric assumptions about the utility function, we can determine the critical level of income sensitivity explicitly. It is often low, especially when the level of risk aversion is not too high and the loss does not put too much final wealth at risk. In these cases, safety is always a normal good even for losses that are only moderately income-sensitive.

6 Extensions

6.1 Severity risk

One limitation of Ehrlich and Becker's self-protection model is the assumption of a binary risk with only two states of the world, loss or no loss. Lee (2010a) shows in the context of self-insurance that more risk-averse decision-makers may no longer invest more in self-insurance as soon as multiple loss states are possible (see also Hiebert, 1989; Li and Peter, 2021). One may thus wonder how our results hold up in a model that allows for multiple loss states. We consider severity risk in this section to answer this question.

Let $\widetilde{L}$ denote the random loss with values in $(0, L]$. We assume $\mathbb{P}(\widetilde{L} \leq l) < 1$ for all $l < L$ and $\mathbb{P}(\widetilde{L} \leq L) = 1$ so that we can interpret L as the maximum possible loss. The decision-maker's expected-utility objective is then given by

$$\max_{s \in [0,\bar{s}]} U(s; y_0, \widetilde{L}) = p(s)\mathbb{E}u(y_0 - s - \widetilde{L}) + (1 - p(s))u(y_0 - s), \tag{7}$$

and the first-order condition for an interior solution s^* is

$$U_s(s^*; y_0, \widetilde{L}) = -p'(s^*) \cdot [u(y_n^*) - \mathbb{E}u(\widetilde{y}_\ell^*)] - \left[p^*\mathbb{E}u'(\widetilde{y}_\ell^*) + (1 - p^*)u'(y_n^*)\right] = 0. \tag{8}$$

In the extended model with severity risk, non-increasing absolute risk aversion and log-convexity of the safety technology continue to ensure the concavity of the objective function in s, see Footnote 4. We abbreviate random final wealth in the loss state by $\widetilde{y}_\ell = y_0 - s - \widetilde{L}$. The asterisk indicates the optimal safety level s^*. While we choose the same notation as before, severity risk affects the choice of s compared to a situation with deterministic loss severity.[15]

[15] Crainich et al. (2016) and Peter (2017) show that severity risk raises the optimal safety investment for imprudent decision-makers and has conflicting effects under prudence. We discuss this briefly in Appendix B.2.

The effect of a change in income on the optimal safety investment follows from the implicit function rule. We obtain

$$U_{sy_0}(s^*; y_0, \widetilde{L}) = -p'(s^*) \cdot [u'(y_n^*) - \mathbb{E}u'(\widetilde{y}_\ell^*)] - \left[p^* \mathbb{E}u''(\widetilde{y}_\ell^*) + (1-p^*)u''(y_n^*)\right].$$

The presence of severity risk does not alter the main economic trade-off. As income increases, the marginal benefit of safety decreases because the decision-maker has an additional buffer against losses. At the same time, the marginal cost of safety decreases as well because higher income reduces the decision-maker's pain from spending an additional dollar on safety. As in the case without severity risk, the net effect is indeterminate *a priori*.

Appendix B.3 shows that Proposition 1 continues to hold under severity risk. A threshold p_c on the loss probability determines whether an increase in income raises or lowers the optimal safety investment. For DARA utility, the loss probability p^* needs to exceed the threshold for safety to be normal, for IARA utility, it needs to fall below the threshold. Under CARA income effects are absent. Qualitatively nothing has changed although, of course, the size of the threshold p_c depends on the level of severity risk.

For changes in loss severity, let $\widetilde{L}_2$ and $\widetilde{L}_1$ be two loss random variables such that $\widetilde{L}_2$ dominates $\widetilde{L}_1$ in the sense of first-order stochastic dominance (FSD). We can then interpret $\widetilde{L}_2$ as the loss with stochastically higher severity because losses are, on average, higher under $\widetilde{L}_2$ than under $\widetilde{L}_1$. We show in Appendix B.4 that a stochastic increase in loss severity always raises the optimal safety investment for CARA utility. In general, a probability threshold $p_\# > 0$ arises, and the positive effect prevails if $p^* < p_\#$, just like in Proposition 6(*iii*). While we lose the definitive result for IARA, the other results are qualitatively unchanged.

Now consider an income-sensitive random loss, $\widetilde{L}(y_0)$. Assume that the income sensitivity of the loss severity is a deterministic quantity χ. This is the case if income affects the size of the average loss but not the randomness surrounding it. If we write $\widetilde{L}(y_0) = \overline{L}(y_0) + \widetilde{\nu}$ with $\mathbb{E}\widetilde{\nu} = 0$, then $\partial\widetilde{L}(y_0)/\partial y_0 = \overline{L}'(y_0)$ is not random. The standard arguments then show that the sign of $\mathrm{d}s^*/\mathrm{d}y_0$ coincides with the sign of expression (4) except that we need to replace $u(y_\ell^*)$ by $\mathbb{E}u(\widetilde{y}_\ell^*)$, and likewise for $u'(y_\ell^*)$ and $u''(y_\ell^*)$. It then follows with the same arguments as in Appendix A.9 that income sensitivity makes it more likely for safety to be a normal good except for the fact that we now need to assume

$$-\frac{\mathbb{E}u''(\widetilde{y}_\ell^*)}{\mathbb{E}u'(\widetilde{y}_\ell^*)} > (<) -\frac{u'(y_n^*) - \mathbb{E}u'(\widetilde{y}_\ell^*)}{u(y_n^*) - \mathbb{E}u(\widetilde{y}_\ell^*)}$$

in case of DARA (IARA). To see why this inequality may fail to hold in the presence of severity risk, take the case of DARA and let $v(y) = \mathbb{E}u(y + (\overline{L} - \widetilde{L}))$ denote the derived utility function for the noise associated with severity risk. DARA is preserved under background risk, see Corollary 3 in Gollier (2001). We thus obtain

$$-\frac{\mathbb{E}u''(\widetilde{y}_\ell^*)}{\mathbb{E}u'(\widetilde{y}_\ell^*)} = -\frac{v''(y_n^* - \overline{L})}{v'(y_n^* - \overline{L})} > -\frac{v'(y_n^*) - v'(y_n^* - \overline{L})}{v(y_n^*) - v(y_n^* - \overline{L})}$$

following the arguments in Appendix A.1. Risk aversion implies $v(y_n^*) < u(y_n^*)$, which lowers the ratio further, but prudence implies $v'(y_n^*) > u'(y_n^*)$, which increases the ratio. Therefore, if the decision-maker's degree of prudence and/or the level of severity risk are bounded in a technical sense, our main result stands, and the income sensitivity of the random loss severity makes it more likely for safety to be a normal good.

6.2 Rank-dependent utility

We now return to the binary risk assumption. Our main analysis is based on expected utility, which is known to have some descriptive shortcomings (Starmer, 2000). Quiggin's (1982) rank-dependent utility allows for probability distortions and is thus more flexible. By now there is abundant evidence that people's preferences over risky prospects are not linear in probabilities. Among many others, Bleichrodt and Pinto (2000) provide evidence from the laboratory and compare estimates of parametric probability weighting functions. Barseghyan et al. (2013) find probability distortions in the field for household insurance decisions.

Let $w : [0, 1] \to [0, 1]$ be a probability weighting function that is increasing with $w(0) = 0$ and $w(1) = 1$. We follow the approach in Baillon et al. (2020) and formulate the decision-makers rank-dependent utility objective as

$$\max_{s\in[0,\bar{s}]} V(s; y_0, L) = w(p(s))u(y_0 - s - L) + (1 - w(p(s)))u(y_0 - s).$$

Log-convexity of the safety technology p is not strong enough to ensure log-convexity of $w \circ p$. If we assume in addition that the elasticity of the probability weighting function is non-decreasing in the loss probability, $w \circ p$ is log-convex in s, see Appendix B.5. In this case, objective function V is globally concave in s as long as absolute risk aversion is non-increasing (Fagart and Fluet, 2013). A non-decreasing elasticity of the probability weighting function is equivalent to subproportionality (Segal, 1987), which, in turn, represents a parsimonious characterization of common-ratio violations (Allais, 1953; Prelec, 1998). Many probability weighting functions are subproportional over the entire unit interval including Prelec's (1998) one- and two-parameter forms, the one proposed by Abdellaoui et al. (2010), the one implied by Gul's (1991) disappointment aversion for binary risks, the quadratic form proposed by Safra and Segal (1998), and the neo-additive form (Chateauneuf et al., 2007).

For rank-dependent utility, all our results continue to hold except that we now need to compare $w(p^*)$ instead of p^* against any threshold values. For example, Proposition 1 remains valid by comparing p^* against $w^{-1}(p_c)$, where p_c denotes the threshold analyzed in Sections 2 and 3 and w^{-1} the inverse of the probability weighting function. Descriptive decision theory often finds an inverse S-shape of the probability weighting function (Abdellaoui et al., 2011). This shape involves overweighting of small probabilities and underweighting of large probabilities with a unique fixed point somewhere between 0.3 to 0.4. The inverse of the probability weighting function is then S-shaped so that $w^{-1}(p_c) < (>)\ p_c$ if the threshold p_c is below (above) the fixed point. One may thus be inclined to conclude that smaller shares of final wealth at risk and lower levels of utility curvature suffice to ensure that safety is a normal good even for fixed loss exposures.

This rationale is incomplete for two reasons. First, the optimal safety investment under rank-dependent utility is different from the optimal safety investment under expected utility. Baillon et al. (2020) show that likelihood insensitivity leads to underprevention in the sense that decision-makers invest less in safety. Ceteris paribus, we thus expect the loss probability under rank-dependent utility to be higher than under expected utility. This effect makes it indeed more likely for safety to be a normal good even if losses are not income-sensitive. However, empirical research finds that utility functions are less concave when allowing for probability distortions because, loosely speaking, the probability weighting function absorbs some risk aversion (Selten et al., 1999; Fox et al., 1996; Diecidue and Wakker, 2002). As we showed in Figures 1(a) and 2(a), the expected-utility threshold p_c increases as utility curvature declines, which then also increases $w^{-1}(p_c)$. This makes it less likely for safety to be a normal good when the loss severity is fixed. So while we find the same qualitative threshold result under rank-dependent utility compared to expected utility, it is unclear whether the condition for safety to be normal is more or less likely satisfied when probabilities are distorted.

Nevertheless the fact that the income sensitivity of losses contributes to the normality of safety investments is unaffected by the presence of probability weighting even if we reduce utility curvature accordingly. What's more, the critical levels of income sensitivity in Eq. (5) and (6) are derived by setting $p_\$ = 0$ for DARA and $p_\$ = 1$ for IARA. Given that $w(0) = 0$ and $w(1) = 1$, the resulting expressions for $\hat{\chi}$ are the same under rank-dependent utility as under expected utility. Figure 5 informs that $\hat{\chi}$ is decreasing in utility curvature. Furthermore, to the extent that likelihood insensitivity reduces safety investments, y_n^* increases, which lowers η. Both effects lower the critical level of income sensitivity that is needed for safety to always be a normal good. Rank-dependent utility reinforces our main conclusion.

7 Conclusion

In this paper, we studied income effects on the demand for safety to answer the question whether safety is a normal or an inferior good. When loss severity is fixed, we found that safety is an inferior good unless preventable losses put a large share of final wealth at risk and risk aversion is high. This is puzzling because we expected to find the opposite. To resolve this puzzle, we derived comparative statics of safety with respect to loss severity and then integrated those results with the income effects on safety. When losses are sufficiently income sensitive, safety is always a normal good. In practice, little income sensitivity suffices to achieve this. The more realistic model thus delivers a more intuitive result. Our main conclusion is robust to the presence of severity risk and is strengthened when allowing for probability distortions in the rank-dependent utility model. We hope that our results will prove useful for much-needed empirical work on the demand for safety.

References

Abdellaoui, M., Baillon, A., Placido, L., and Wakker, P. P. (2011). The rich domain of uncertainty: Source functions and their experimental implementation. *American Economic Review*, 101(2): 695–723.

Abdellaoui, M., l'Haridon, O., and Zank, H. (2010). Separating curvature and elevation: A parametric probability weighting function. *Journal of Risk and Uncertainty*, 41(1): 39–65.

Allais, M. (1953). Le comportement de l'homme rationnel devant le riskque: Critique des postulats et axiomes de l'ecole americaine. *Econometrica*, 21(4): 503–546.

Antle, J. M. (2001). Economic analysis of food safety. *Handbook of Agricultural Economics*, 1: 1083–1136.

Arrow, K. J. (1965). Aspects of the theory of risk-bearing. *Helsinki: Yrjö Jahnsson Foundation.*

Arrow, K. J. (1971). The theory of risk aversion. *Essays in the theory of risk-bearing*, pages 90–120.

Baillon, A., Bleichrodt, H., Emirmahmutoglu, A., Jaspersen, J. G., and Peter, R. (2020). When risk perception gets in the way: Probability weighting and underprevention. *Operations Research (forthcoming).*

Baillon, A. and Placido, L. (2019). Testing constant absolute and relative ambiguity aversion. *Journal of Economic Theory*, 181: 309–332.

Barseghyan, L., Molinari, F., O'Donoghue, T., and Teitelbaum, J. C. (2013). The nature of risk preferences: Evidence from insurance choices. *American Economic Review*, 103(6): 2499–2529.

Beaud, M. and Willinger, M. (2015). Are people risk vulnerable? *Management Science*, 61(3): 624–636.

Bell, D. E. (1988). One-switch utility functions and a measure of risk. *Management Science*, 34(12): 1416–1424.

Bell, D. E. (1995). A contextual uncertainty condition for behavior under risk. *Management Science*, 41(7): 1145–1150.

Bhargava, S., Loewenstein, G., and Sydnor, J. (2017). Choose to lose: Health plan choices from a menu with dominated option. *The Quarterly Journal of Economics*, 132(3): 1319–1372.

Bleichrodt, H. and Pinto, J. L. (2000). A parameter-free elicitation of the probability weighting function in medical decision analysis. *Management Science*, 46(11): 1485–1496.

Briys, E. and Schlesinger, H. (1990). Risk aversion and the propensities for self-insurance and self-protection. *Southern Economic Journal*, 57(2): 458–467.

Chateauneuf, A., Eichberger, J., and Grant, S. (2007). Choice under uncertainty with the best and worst in mind: Neo-additive capacities. *Journal of Economic Theory*, 137(1): 538–567.

Chetty, R. (2006). A new method of estimating risk aversion. *American Economic Review*, 96(5): 1821–1834.

Chiu, W. H. (2000). On the propensity to self-protect. *Journal of Risk and Insurance*, 67(4): 555–577.

Chiu, W. H. (2005). Degree of downside risk aversion and self-protection. *Insurance: Mathematics and Economics*, 36(1): 93–101.

Chiu, W. H. (2010). Skewness preference, risk taking and expected utility maximisation. *The Geneva Risk and Insurance Review*, 35(2): 108–129.

Chiu, W. H. (2012). Risk aversion, downside risk aversion and paying for stochastic improvements. *The Geneva Risk and Insurance Review*, 37(1): 1–26.

Cohen, A. and Einav, L. (2007). Estimating risk preferences from deductible choice. *American Economic Review*, 97(3): 745–788.

Courbage, C., Loubergé, H., and Peter, R. (2017). Optimal prevention for multiple risks. *Journal of Risk and Insurance*, 84(3): 899–922.

Crainich, D., Eeckhoudt, L., and Menegatti, M. (2016). Changing risks and optimal effort. *Journal of Economic Behavior & Organization*, 125: 97–106.

Cummins, J. D. and Mahul, O. (2004). The demand for insurance with an upper limit on coverage. *Journal of Risk and Insurance*, 71(2): 253–264.

Denuit, M., Eeckhoudt, L., Liu, L., and Meyer, J. (2016). Tradeoffs for downside risk-averse decision-makers and the self-protection decision. *The Geneva Risk and Insurance Review*, 41(1): 19–47.

Denuit, M. M., Eeckhoudt, L., and Schlesinger, H. (2013). When Ross meets Bell: The linex utility function. *Journal of Mathematical Economics*, 49(2): 177–182.

Diecidue, E. and Wakker, P. (2002). Dutch books: Avoiding strategic and dynamic complications, and a comonotonic extension. *Mathematical Social Sciences*, 43(2): 135–149.

Dionne, G. and Eeckhoudt, L. (1985). Self-insurance, self-protection and increased risk aversion. *Economics Letters*, 17(1-2): 39–42.

Dynan, K. E. (1993). How prudent are consumers? *Journal of Political Economy*, 101(6): 1104–1113.

Ebert, S. (2015). On skewed risks in economic models and experiments. *Journal of Economic Behavior & Organization*, 112: 85–97.

Eeckhoudt, L. and Gollier, C. (2005). The impact of prudence on optimal prevention. *Economic Theory*, 26(4): 989–994.

Ehrlich, I. and Becker, G. S. (1972). Market insurance, self-insurance, and self-protection. *Journal of Political Economy*, 80(4): 623–648.

Eisenhauer, J. G. (1997). Risk aversion, wealth, and the DARA hypothesis: A new test. *International Advances in Economic Research*, 3(1): 46–53.

Eisenhauer, J. G. (2000). Estimating prudence. *Eastern Economic Journal*, 26(4): 379–392.

Eisenhauer, J. G. and Halek, M. (1999). Prudence, risk aversion, and the demand for life insurance. *Applied Economics Letters*, 6(4): 239–242.

Eisenhauer, J. G. and Ventura, L. (2003). Survey measures of risk aversion and prudence. *Applied Economics*, 35(13): 1477–1484.

Fagart, M.-C. and Fluet, C. (2013). The first-order approach when the cost of effort is money. *Journal of Mathematical Economics*, 49(1): 7–16.

Fox, C., Rogers, B., and Tversky, A. (1996). Options traders exhibit subadditive decision weights. *Journal of Risk and Uncertainty*, 13(1): 5–17.

Gollier, C. (2001). *The Economics of Risk and Time*. MIT Press, Cambridge, MA.

Gollier, C. and Pratt, J. W. (1996). Risk vulnerability and the tempering effect of background risk. *Econometrica*, 64(5): 1109–1123.

Gourinchas, P.-O. and Parker, J. A. (2002). Consumption over the life cycle. *Econometrica*, 70(1): 47–89.

Guiso, L., Jappelli, T., and Terlizzese, D. (1992). Earnings uncertainty and precautionary saving. *Journal of Monetary Economics*, 30(2): 307–337.

Guiso, L. and Paiella, M. (2008). Risk aversion, wealth, and background risk. *Journal of the European Economic Association*, 6(6): 1109–1150.

Gul, F. (1991). A theory of disappointment aversion. *Econometrica*, 59(3): 667–686.

Heinzel, C. and Peter, R. (2021). Precautionary motives with multiple instruments. *Working Paper, available at SSRN 3889836*.

Hiebert, L. D. (1989). Optimal loss reduction and increases in risk aversion. *Journal of Risk and Insurance*, 56(2): 300–305.

Hofmann, A. and Peter, R. (2015). Multivariate prevention decisions: Safe today or sorry tomorrow? *Economics Letters*, 128: 51–53.

Holt, C. A. and Laury, S. K. (2002). Risk aversion and incentive effects. *American Economic Review*, 92(5): 1644–1655.

Hoy, M. and Robson, A. J. (1981). Insurance as a Giffen good. *Economics Letters*, 8(1): 47–51.

Huber, T., Jaspersen, J. G., Richter, A., and Strümpel, D. (2022). On the change of risk aversion in wealth: Evidence from a closed economic system. *Experimental Economics (forthcoming)*.

Jullien, B., Salanié, B., and Salanié, F. (1999). Should more risk-averse agents exert more effort? *The Geneva Risk and Insurance Review*, 24(1): 19–28.

Kihlstrom, R. E., Romer, D., and Williams, S. (1981). Risk aversion with random initial wealth. *Econometrica*, 49(4): 911–920.

Kimball, M. S. (1993). Standard risk aversion. *Econometrica*, 61(3): 589–611.

Kuehlwein, M. (1991). A test for the presence of precautionary saving. *Economics Letters*, 37(4): 471–475.

Lakdawalla, D. and Zanjani, G. (2005). Insurance, self-protection, and the economics of terrorism. *Journal of Public Economics*, 89(9-10): 1891–1905.

Lee, K. (2005). Wealth effects on self-insurance and self-protection against monetary and nonmonetary losses. *The Geneva Risk and Insurance Review*, 30(2): 147–159.

Lee, K. (2010a). Risk aversion and self-insurance. *Journal of Economics*, 101(3): 277–282.

Lee, K. (2010b). Wealth effects on self-insurance. *The Geneva Risk and Insurance Review*, 35(2): 160–171.

Levy, H. (1994). Absolute and relative risk aversion: An experimental study. *Journal of Risk and Uncertainty*, 8(3): 289–307.

Li, L. and Peter, R. (2021). Should we do more when we know less? The effect of technology risk on optimal effort. *Journal of Risk and Insurance*, 88(3): 695–725.

Macé, S. and Peter, R. (2021). The impact of loss aversion on optimal prevention. *Working Paper, available at SSRN 3882200*.

Merrigan, P. and Normandin, M. (1996). Precautionary saving motives: an assessment from UK time series of cross-sections. *The Economic Journal*, 106(438): 1193–1208.

Meyer, D. J. and Meyer, J. (2005). Relative risk aversion: What do we know? *Journal of Risk and Uncertainty*, 31(3): 243–262.

Mossin, J. (1968). Aspects of rational insurance purchasing. *Journal of Political Economy*, 46(4): 553–568.

Nachman, D. C. (1982). Preservation of "more risk averse" under expectations. *Journal of Economic Theory*, 28(2): 361–368.

Noussair, C. N., Trautmann, S. T., and Van de Kuilen, G. (2014). Higher order risk attitudes, demographics, and financial decisions. *Review of Economic Studies*, 81(1): 325–355.

Ogaki, M. and Zhang, Q. (2001). Decreasing relative risk aversion and tests of risk sharing. *Econometrica*, 69(2): 515–526.

Parker, J. A. (1999). The reaction of household consumption to predictable changes in social security taxes. *American Economic Review*, 89(4): 959–973.

Peltzman, S. (1975). The effects of automobile safety regulation. *Journal of Political Economy*, 83(4): 677–725.

Peter, R. (2017). Optimal self-protection in two periods: On the role of endogenous saving. *Journal of Economic Behavior & Organization*, 137: 19–36.

Peter, R. (2021a). Prevention as a Giffen good. *Economics Letters*, 208: 110052.

Peter, R. (2021b). Who should exert more effort? Risk aversion, downside risk aversion and optimal prevention. *Economic Theory*, 71(4): 1259–1281.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica*, 32(1–2): 122–136.

Pratt, J. W. and Zeckhauser, R. J. (1987). Proper risk aversion. *Econometrica*, 55(1): 143–154.

Prelec, D. (1998). The probability weighting function. *Econometrica*, 66(3): 497–527.

Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior & Organization*, 3(4): 323–343.

Ross, S. A. (1981). Some stronger measures of risk aversion in the small and the large with applications. *Econometrica*, 49(3): 621–638.

Rothschild, M. and Stiglitz, J. E. (1970). Increasing risk: I. A definition. *Journal of Economic Theory*, 2(3): 225–243.

Safra, Z. and Segal, U. (1998). Constant risk aversion. *Journal of Economic Theory*, 83(1): 19–42.

Saha, A. (1993). Expo-power utility: a 'flexible' form for absolute and relative risk aversion. *American Journal of Agricultural Economics*, 75(4): 905–913.

Schlesinger, H. (1981). The optimal level of deductibility in insurance contracts. *Journal of Risk and Insurance*, (3): 465–481.

Schlesinger, H. (2013). The theory of insurance demand. In Dionne, G., editor, *Handbook of Insurance*, chapter 7, pages 167–184. Springer, 2 edition.

Segal, U. (1987). Some remarks on Quiggin's anticipated utility. *Journal of Economic Behavior & Organization*, 8(1): 145–154.

Selten, R., Sadrieh, A., and Abbink, K. (1999). Money does not induce risk neutral behavior, but binary lotteries do even worse. *Theory and Decision*, 46(3): 213–252.

Skinner, J. (1988). Risky income, life cycle consumption, and precautionary savings. *Journal of Monetary Economics*, 22(2): 237–255.

Starmer, C. (2000). Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk. *Journal of Economic Literature*, 38(2): 332–382.

Sweeney, G. H. and Beard, T. R. (1992). The comparative statics of self-protection. *Journal of Risk and Insurance*, 59(2): 301–309.

Sydnor, J. (2010). (Over) insuring modest risks. *American Economic Journal: Applied Economics*, 2(4): 177–99.

Viscusi, W. K. and Aldy, J. E. (2003). The value of a statistical life: A critical review of market estimates throughout the world. *Journal of Risk and Uncertainty*, 27(1): 5–76.

A Mathematical proofs

A.1 Proof of Proposition 1

Let $A(u;y) = -u''(y)/u'(y)$ denote the Arrow-Pratt measure of absolute risk aversion for utility function u at final wealth level y. We find that

$$\frac{\mathrm{d}A(v_{\widetilde{\varepsilon}};y_n^*)}{\mathrm{d}p^*} = \frac{u'(y_\ell^*)u'(y_n^*)}{(p^*u'(y_\ell^*) + (1-p^*)u'(y_n^*))^2} \cdot [A(u;y_\ell^*) - A(u;y_n^*)]\,.$$

When u has DARA, $A(v_{\widetilde{\varepsilon}};y_n^*)$ increases in p^* from $A(u;y_n^*)$ to $A(u;y_\ell^*)$. When u has IARA, $A(v_{\widetilde{\varepsilon}};y_n^*)$ decreases in p^* from $A(u;y_n^*)$ to $A(u;y_\ell^*)$. Furthermore, when $A(u;y)$ is decreasing or increasing in y, $A(v_{\widetilde{\omega}};y_n^*)$ lies strictly between $A(u;y_n^*)$ and $A(u;y_\ell^*)$. To see this, take the case of DARA; we obtain

$$u''(y_n^*)u'(y_n^*+t) - u'(y_n^*)u''(y_n^*+t) \geq 0$$

for all $t \in [-L,0]$ with a strict inequality for $t \neq 0$. Integration over $[-L,0]$ yields

$$u''(y_n^*)\int_{-L}^{0} u'(y_n^*+t)\,\mathrm{d}t - u'(y_n^*)\int_{-L}^{0} u''(y_n^*+t)\,\mathrm{d}t > 0$$

or equivalently $A(v_{\widetilde{\omega}};y_n^*) > A(u;y_n^*)$. The proof is analogous for $A(v_{\widetilde{\omega}};y_n^*) < A(u;y_\ell^*)$, and a similar argument establishes $A(u;y_\ell^*) < A(v_{\widetilde{\omega}};y_n^*) < A(u;y_n^*)$ when u has IARA. If $A(v_{\widetilde{\omega}};y_n^*)$ lies strictly between $A(u;y_n^*)$ and $A(u;y_\ell^*)$, and if $A(v_{\widetilde{\varepsilon}};y_n^*)$ is strictly monotonic in p^* with endpoints $A(u;y_n^*)$ and $A(u;y_\ell^*)$, then there is a unique $p^* \in (0,1)$ for which $A(v_{\widetilde{\omega}};y_n^*) = A(v_{\widetilde{\varepsilon}};y_n^*)$. We denote this threshold by p_c.

Our arguments so far also show that, when u has DARA, we have $A(v_{\widetilde{\varepsilon}};y_n^*) > A(v_{\widetilde{\omega}};y_n^*)$ when $p^* > p_c$ and $A(v_{\widetilde{\varepsilon}};y_n^*) < A(v_{\widetilde{\omega}};y_n^*)$ when $p^* < p_c$. This proves $\mathrm{d}s^*/\mathrm{d}y_0 > 0$ in the first case and $\mathrm{d}s^*/\mathrm{d}y_0 < 0$ in the second case, which is Result (*i*). When going to IARA, the monotonicity of $A(v_{\widetilde{\varepsilon}};y_n^*)$ in p^* flips, which shows Result (*iii*). When u has CARA, then

$$A(v_{\widetilde{\varepsilon}};y_n^*) = -\frac{\mathbb{E}u''(y_n^*+\widetilde{\varepsilon})}{\mathbb{E}u'(y_n^*+\widetilde{\varepsilon})} = -\frac{u''(y_n^*)\cdot\mathbb{E}u'(\widetilde{\varepsilon})}{u'(y_n^*)\cdot\mathbb{E}u'(\widetilde{\varepsilon})} = -\frac{u''(y_n^*)}{u'(y_n^*)},$$

and likewise for $A(v_{\widetilde{\omega}};y_n^*)$. Then $U_{sy_0}(s^*;y_0,L)$ is uniformly zero, which is Result (*ii*).

A.2 Proof of Proposition 2

Let $u(y) = y - \alpha y^2$ with $\alpha > 0$ small enough to ensure positive marginal utility. We then find

$$A(v_{\widetilde{\varepsilon}};y_n^*) = \frac{2\alpha}{1-2\alpha\cdot(y_n^*+\mathbb{E}\widetilde{\varepsilon})} \qquad \text{and} \qquad A(v_{\widetilde{\omega}};y_n^*) = \frac{2\alpha}{1-2\alpha\cdot(y_n^*+\mathbb{E}\widetilde{\omega})}.$$

Therefore, $\mathrm{d}s^*/\mathrm{d}y_0$ is positive if and only if $\mathbb{E}\widetilde{\varepsilon} > \mathbb{E}\widetilde{\omega}$, which is equivalent to $p^* < 0.5$.

A.3 Proof of Proposition 3

By verifying the integral condition, it is easy to see that $\widetilde{\varepsilon}$ is an increase in risk over $\widetilde{\omega}$ in the sense of Rothschild and Stiglitz (1970) when $p^* = 0.5$. Their Theorem 2 then implies that $\widetilde{\varepsilon}$ is equal in distribution to $\widetilde{\omega}$ plus noise,

$$\widetilde{\varepsilon} \overset{d}{=} \widetilde{\omega} + \widetilde{\kappa} \qquad \text{with} \quad \mathbb{E}(\widetilde{\kappa}|t) = 0 \quad \text{for all } t \in [-L, 0].$$

Utility function u is standard if and only if it satisfies DARA and DAP. Both properties are preserved by the introduction of an independent background risk (see Kihlstrom et al., 1981; Gollier, 2001, Proposition 23). As a result, the derived utility function $v_{\widetilde{\omega}}(y)$ satisfies DARA and DAP and is therefore risk vulnerable (see Gollier and Pratt, 1996). Consequently,

$$A(v_{\widetilde{\varepsilon}}; y_n^*) = -\frac{\mathbb{E}u''(y_n^* + \widetilde{\varepsilon})}{\mathbb{E}u'(y_n^* + \widetilde{\varepsilon})} = -\frac{\mathbb{E}u''(y_n^* + \widetilde{\omega} + \widetilde{\kappa})}{\mathbb{E}u'(y_n^* + \widetilde{\omega} + \widetilde{\kappa})} = -\frac{\mathbb{E}v''_{\widetilde{\omega}}(y_n^* + \widetilde{\kappa})}{\mathbb{E}v'_{\widetilde{\omega}}(y_n^* + \widetilde{\kappa})} > -\frac{\mathbb{E}v''_{\widetilde{\omega}}(y_n^*)}{\mathbb{E}v'_{\widetilde{\omega}}(y_n^*)} = A(v_{\widetilde{\omega}}; y_n^*).$$

This implies $\mathrm{d}s^*/\mathrm{d}y_0 > 0$ when $p^* = 0.5$. Result (i) in Proposition 1 then yields $p_c < 0.5$

A.4 Proof of Proposition 4

We compute

$$A(v_{\widetilde{\omega}}; y_n^*) = -\frac{1-\rho}{y_n^*} \cdot \frac{(1-\eta)^{-\rho} - 1}{(1-\eta)^{1-\rho} - 1} \qquad \text{for } \rho \neq 1,$$

$$A(v_{\widetilde{\omega}}; y_n^*) = -\frac{1}{y_n^*} \cdot \frac{1}{(1-\eta)\log(1-\eta)} \qquad \text{for } \rho = 1,$$

and

$$A(v_{\widetilde{\varepsilon}}; y_n^*) = \frac{\rho}{y_n^*} \cdot \frac{p^*(1-\eta)^{-\rho-1} + (1-p^*)}{p^*(1-\eta)^{-\rho} + (1-p^*)}.$$

We then obtain p_c by setting $A(v_{\widetilde{\omega}}; y_n^*) = A(v_{\widetilde{\varepsilon}}; y_n^*)$ and solving for p^*. This renders the terms stated in Proposition 4. If we take the expression for p_c for $\rho \neq 1$ and take the limit $\rho \to 1$, we can apply l'Hôpital's rule to obtain

$$\begin{aligned} \lim_{\rho \to 1} p_c &= \lim_{\rho \to 1} \frac{(1-\eta)\big((1-\eta)^{\rho} \cdot \log(1-\eta) + \eta\big)}{(1-\eta)^{(1-\rho)} \cdot \log(1-\eta) - \eta^2 + (1-\eta)^{1+\rho} \cdot \log(1-\eta)} \\ &= \frac{(1-\eta)\big((1-\eta)\log(1-\eta) + \eta\big)}{\eta\big((\eta-2)\log(1-\eta) - \eta\big)}. \end{aligned}$$

This shows that the expression for p_c in case of $\rho = 1$ is consistent with the expression for p_c in case of $\rho \neq 1$. The limits for $\rho \to 0$, $\eta \to 0$, $\rho \to \infty$ and $\eta \to 1$ are obtained similarly.

A.5 Proof of Proposition 5

For linex utility we obtain

$$A(v_{\widetilde{\varepsilon}}; y_n^*) = \frac{k\gamma^2 \cdot \mathbb{E}\exp\big(-\gamma(y_n^* + \widetilde{\varepsilon})\big)}{l + k\gamma \cdot \mathbb{E}\exp\big(-\gamma(y_n^* + \widetilde{\varepsilon})\big)} \quad \text{and} \quad A(v_{\widetilde{\omega}}; y_n^*) = \frac{k\gamma^2 \cdot \mathbb{E}\exp\big(-\gamma(y_n^* + \widetilde{\omega})\big)}{l + k\gamma \cdot \mathbb{E}\exp\big(-\gamma(y_n^* + \widetilde{\omega})\big)}.$$

Therefore, $A(v_{\widetilde{\varepsilon}}; y_n^*) = A(v_{\widetilde{\omega}}; y_n^*)$ if and only if $\mathbb{E}\exp(-\gamma(y_n^* + \widetilde{\varepsilon})) = \mathbb{E}\exp(-\gamma(y_n^* + \widetilde{\omega}))$. Using the definitions of $\widetilde{\varepsilon}$ and $\widetilde{\omega}$, we obtain:

$$\begin{aligned} \mathbb{E}\exp(-\gamma(y_n^* + \widetilde{\varepsilon})) &= p^* \exp(-\gamma(y_n^* - L)) + (1 - p^*) \exp(-\gamma y_n^*), \\ \mathbb{E}\exp(-\gamma(y_n^* + \widetilde{\omega})) &= \frac{1}{\gamma L} \left[\exp(-\gamma(y_n^* - L)) - \exp(-\gamma y_n^*)\right]. \end{aligned}$$

Setting terms equal and solving for p^* renders $\frac{1}{\gamma L} - \frac{1}{\exp(\gamma L) - 1}$. We use $L = \eta y_n^*$ and $P_\ell = \gamma y_\ell^* = \gamma(1-\eta) y_n^*$ to show that $\gamma L = \eta P_\ell / (1-\eta)$, which then yields the expression for p_c stated in Proposition 5. The limits for $P_\ell \to 0$, $\eta \to 0$, $P_\ell \to \infty$ and $\eta \to 1$ follow easily per direct computation.

A.6 Proof of Proposition 6

Result (i) follows from Equation 3 because $U_{sL}(s^*; y_0, L) > 0$ if and only if

$$-\frac{p'(s^*)}{p(s^*)} > -\frac{u''(y_\ell^*)}{u'(y_\ell^*)} = A(u; y_\ell^*).$$

The left-hand side is the decay rate of the loss probability, the right-hand side is absolute risk aversion in the loss state. To show Result (ii), rearrange first-order condition (2) as follows:

$$-p'(s^*) \cdot [u(y_n^*) - u(y_\ell^*)] - p(s^*) \cdot [u'(y_\ell^*) - u'(y_n^*)] = u'(y_n^*).$$

Positive marginal utility then bounds the decay rate of the loss probability from below because $u'(y_n^*) > 0$ implies

$$-\frac{p'(s^*)}{p(s^*)} > -\frac{u'(y_n^*) - u'(y_n^* - L)}{u(y_n^*) - u(y_n^* - L)} = -\frac{v''_{\widetilde{\omega}}(y_n^*)}{v'_{\widetilde{\omega}}(y_n^*)} = A(v_{\widetilde{\omega}}; y_n^*).$$

Under non-decreasing absolute risk aversion, we have $A(v_{\widetilde{\omega}}; y_n^*) \geq A(y_\ell^*)$ following the arguments in Appendix A.1. In other words, non-decreasing absolute risk aversion implies the necessary and sufficient condition for an increase in the safety investment. This is Result (ii). Result (iii) is obtained by rearranging the curly bracket in Equation (3). When solving for p^*, we find that $U_{sL}(s^*; y_0, L) > 0$ if and only if

$$p^* < \frac{u'(y_\ell^*) u'(y_n^*)}{-u''(y_\ell^*)(u(y_n^*) - u(y_\ell^*)) - u'(y_\ell^*)(u'(y_\ell^*) - u'(y_n^*))}.$$

The right-hand side is the threshold $p_{\#}$. Under DARA, its denominator is positive so that $p_{\#} > 0$. We obtain $p_{\#} \geq 1$ if and only if $(u'(y_\ell^*))^2 \geq -u''(y_\ell^*)(u(y_n^*) - u(y_\ell^*))$. Rearrange to

$$\frac{u'(y_\ell^*)}{u(y_n^*) - u(y_\ell^*)} \geq -\frac{u''(y_\ell^*)}{u'(y_\ell^*)} \qquad \Leftrightarrow \qquad \frac{y_\ell^*}{L} \cdot \frac{u'(y_\ell^*)}{(u(y_n^*) - u(y_\ell^*))/L} \geq -y_\ell^* \cdot \frac{u''(y_\ell^*)}{u'(y_\ell^*)},$$

where the right-hand side is relative risk aversion in the loss state, R_ℓ. The first factor on the left-hand side can be rewritten as $(1-\eta)/\eta$ from the definition of η. The second factor on the left-hand side exceeds unity due to risk aversion, $u'(y_\ell^*) > (u(y_n^*) - u(y_\ell^*))/L$. So a sufficient condition for $p_{\#} \geq 1$ is that $(1-\eta)/\eta \geq R_\ell$, which rearranges to $\eta \leq 1/(1+R_\ell)$. This proves Result (*iv*). For Result (*v*), rearrange $p_{\#} \geq 0.5$ as follows:

$$\frac{y_\ell^*}{L} \cdot \frac{u'(y_\ell^*) + u'(y_n^*)}{(u(y_n^*) - u(y_\ell^*))/L} \geq -y_\ell^* \cdot \frac{u''(y_\ell^*)}{u'(y_\ell^*)}.$$

DARA implies prudence, and under prudence we have $0.5(u'(y_\ell^*)+u'(y_n^*)) \geq (u(y_n^*)-u(y_\ell^*))/L$ from Eeckhoudt and Gollier's Lemma 1. Therefore, $2(1-\eta)/\eta \geq R_\ell$ is a sufficient condition for $p_{\#} \geq 1/2$. We rearrange this inequality to $\eta \leq 1/(1+0.5 \cdot R_\ell)$ as stated in the text.

A.7 Proof of Proposition 7

For iso-elastic utility, the curly bracket in Equation (3) is given by

$$\frac{(y_n^*)^{-\rho}(1-\eta)^{-\rho}(1-\rho)}{(y_n^*)^{1-\rho} - (y_n^*)^{1-\rho}(1-\eta)^{1-\rho}} + \frac{-p^*\rho(y_n^*)^{-\rho-1}(1-\eta)^{\rho-1}}{(1-p^*)(y_n^*)^{-\rho} + p^*(y_n^*)^{-\rho}(1-\eta)^{-\rho}} \qquad \text{for } \rho \neq 1$$

and by

$$\frac{1}{y_n^*(1-\eta)(\log(y_n^*) - \log(y_n^*(1-\eta)))} - \frac{p^*}{(y_n^*)^2(1-\eta)^2\left(\frac{1-p^*}{y_n^*} + \frac{p^*}{y_n^*(1-\eta)}\right)} \qquad \text{for } \rho = 1.$$

The threshold $p_{\#}$ is then obtained by setting the expressions equal to zero and solving for p^*, which yields the expressions stated in the text. We obtain $p_{\#} \geq 1$ if and only if $\eta \leq 1-\rho^{1/(1-\rho)}$, and $p_{\#} \geq 0.5$ if and only if $(1-\rho)(1-\eta)^{\rho} + 1 \geq \rho(1-\eta)^{\rho-1}$.

A.8 Proof of Proposition 8

For linex utility, direct computation shows that

$$p_{\#} = \frac{(1 + k\gamma \cdot \exp(-\gamma y_\ell^*))) \cdot (1 + k\gamma \cdot \exp(-\gamma y_n^*))}{k\gamma \cdot (\gamma L \cdot \exp(-\gamma y_\ell^*) - \exp(-\gamma y_\ell^*) + \exp(-\gamma y_n^*))}.$$

We know from Section 3 that $\gamma y_\ell^* = P_\ell$ because absolute prudence is constant for linex utility and given by γ. Likewise, we obtain $\gamma y_n^* = \gamma y_\ell^*/(1-\eta) = P_\ell/(1-\eta)$. We can then determine relative risk aversion in the loss state as follows:

$$R_\ell = \frac{k\gamma^2 \cdot y_\ell^* \cdot \exp(-\gamma y_\ell^*)}{1 + k\gamma \cdot \exp(-\gamma y_\ell^*)} = \frac{k\gamma \cdot P_\ell \exp(-P_\ell)}{1 + k\gamma \cdot \exp(-P_\ell)}.$$

This allows us to express $k\gamma$ as a function of P_ℓ and R_ℓ,

$$k\gamma = \frac{R_\ell}{(P_\ell - R_\ell)\exp(-P_\ell)} \qquad \text{for } R_\ell < P_\ell.$$

Furthermore, we obtain $\gamma L = \gamma y_n^* \cdot \frac{L}{y_n^*} = \frac{\gamma y_\ell^*}{1-\eta} \cdot \eta = \frac{\eta P_\ell}{1-\eta}$. Substitute terms accordingly and rearrange to obtain the threshold $p_\#$ stated in the text.

A.9 Proof of Proposition 9

Recall from Appendix A.1 that DARA implies

$$A(u; y_\ell^*) > A(v_{\tilde{\omega}}; y_n^*) > A(u; y_n^*). \tag{9}$$

As explained in the main text, we obtain $\mathrm{d}s^*/\mathrm{d}y_0 > 0$ if and only if

$$\begin{aligned} &\left\{ -\frac{p^* u''(y_\ell^*) + (1-p^*)u''(y_n^*)}{p^* u'(y_\ell^*) + (1-p^*)u'(y_n^*)} - \left(-\frac{u'(y_n^*) - u'(y_\ell^*)}{u(y_n^*) - u(y_\ell^*)} \right) \right\} \\ + \quad &\chi \cdot \left\{ \frac{u'(y_\ell^*)}{u(y_n^*) - u(y_\ell^*)} + \frac{p^* u''(y_\ell^*)}{p^* u'(y_\ell^*) + (1-p^*)u'(y_n^*)} \right\} > 0. \end{aligned}$$

We rearrange this inequality as follows:

$$\begin{aligned} &p^* \{ \overbrace{(u(y_n^*) - u(y_\ell^*))(u''(y_n^*) - (1-\chi)u''(y_\ell^*)) - (u'(y_\ell^*) - u'(y_n^*))((1-\chi)u'(y_\ell^*) - u'(y_n^*))}^{=f(\chi)} \} \\ > \quad &u''(y_n^*)(u(y_n^*) - u(y_\ell^*)) + u'(y_n^*)((1-\chi)u'(y_\ell^*) - u'(y_n^*)). \end{aligned} \tag{10}$$

Define the multiplier on p^* as auxiliary function $f(\chi)$ for $\chi \in (0,1)$. We then obtain

$$f'(\chi) = (u(y_n^*) - u(y_\ell^*))u''(y_\ell^*) + (u'(y_\ell^*) - u'(y_n^*))u'(y_\ell^*),$$

which is negative under DARA due to (9). Furthermore,

$$\lim_{\chi \to 1} f(\chi) = (u(y_n^*) - u(y_\ell^*))u''(y_n^*) + (u'(y_\ell^*) - u'(y_n^*))u'(y_n^*),$$

which is positive under DARA also due to (9). Therefore, $f(\chi) > 0$ for all $\chi \in (0,1)$. Define

$$p_\$ = \frac{u''(y_n^*)(u(y_n^*) - u(y_\ell^*)) + u'(y_n^*)((1-\chi)u'(y_\ell^*) - u'(y_n^*))}{(u(y_n^*) - u(y_\ell^*))(u''(y_n^*) - (1-\chi)u''(y_\ell^*)) - (u'(y_\ell^*) - u'(y_n^*))((1-\chi)u'(y_\ell^*) - u'(y_n^*))};$$

we then obtain that $\mathrm{d}s^*/\mathrm{d}y_0 > 0$ if $p^* > p_\$$ and $\mathrm{d}s^*/\mathrm{d}y_0 < 0$ if $p^* < p_\$$, as claimed in the text. To assess the effect of an increase in χ on $p_\$$, we calculate the derivative $\mathrm{d}p_\$/\mathrm{d}\chi$. The numerator of $\mathrm{d}p_\$/\mathrm{d}\chi$ can be simplified to

$$u'(y_\ell^*)u'(y_n^*) \cdot \left\{u'(y_\ell^*)A(u;y_n^*) - (u(y_n^*) - u(y_\ell^*))A(u;y_\ell^*)A(u;y_n^*) - u'(y_n^*)A(u;y_\ell^*)\right\}.$$

Under DARA, we can use (9) to bound the curly bracket from above by

$$A(u;y_n^*)(u(y_n^*) - u(y_\ell^*)) \cdot [A(v_{\tilde{\omega}};y_n^*) - A(u;y_\ell^*)]\,, \tag{11}$$

which is negative due to (9). Hence, $\mathrm{d}p_\$/\mathrm{d}\chi < 0$. Finally, we obtain the critical level $\hat{\chi}$ by setting $p_\$$ equal to zero and solving for χ. Inequality (9) implies $\hat{\chi} > 0$, and $\hat{\chi} < 1$ holds because $u''(y_n^*)(u(y_n^*) - u(y_\ell^*)) < 0 < (u'(y_n^*))^2$. This proves Result (i).

Result (ii) follows directly from combining Proposition 1(ii) with Proposition 6(ii). For Result (iii), recall from Appendix A.1 that IARA yields

$$A(u;y_\ell^*) < A(v_{\tilde{\omega}};y_n^*) < A(u;y_n^*). \tag{12}$$

This implies $f'(\chi) > 0$ and $\lim_{\chi\to 1} f(\chi) < 0$ under IARA for auxiliary function $f(\chi)$. Consequently, $f(\chi) < 0$ for all $\chi \in (0,1)$, and dividing by the multiplier on p^* now flips inequality (10). We thus have $\mathrm{d}s^*/\mathrm{d}y_0 > 0$ if $p^* < p_\$$ and $\mathrm{d}s^*/\mathrm{d}y_0 < 0$ if $p^* > p_\$$, as claimed in the text. Under IARA, the curly bracket in the numerator of $\mathrm{d}p_\$/\mathrm{d}\chi$ is bounded from below by (11), which is positive. Therefore, $\mathrm{d}p_\$/\mathrm{d}\chi > 0$ under IARA. We obtain the critical level $\hat{\chi}$ by setting $p_\$$ equal to one and solving for χ. Due to (12) and by direct calculation we have

$$0 < A(v_{\tilde{\omega}};y_n^*) - A(u;y_\ell^*) < \frac{u'(y_\ell^*)}{u(y_n^*) - u(y_\ell^*)} - A(u;y_\ell^*)$$

so that $\hat{\chi} \in (0,1)$. This completes the proof of Result (iii).

A.10 Proof of Proposition 10

To show (i), let $u(y) = y - \alpha y^2$ with $\alpha > 0$ small enough to ensure positive marginal utility on the relevant domain. We then have $u'(y) = 1 - 2\alpha y$ and $u''(y) = -2\alpha$. Threshold $\hat{\chi}$ in Equation 6 can be rewritten as follows:

$$\hat{\chi} = \frac{u''(y_\ell^*)(u(y_n^*) - u(y_\ell^*)) + u'(y_\ell^*)(u'(y_\ell^*) - u'(y_n^*))}{u''(y_\ell^*)(u(y_n^*) - u(y_\ell^*)) + (u'(y_\ell^*))^2}.$$

For quadratic utility, we have $u(y_n^*) - u(y_\ell^*) = L(y_0) \cdot u'((y_n^* + y_\ell^*)/2) = L(y_0) \cdot u'(y_{\ell/2}^*)$, where $y_{\ell/2}^*$ is defined as $y_0 - s^* - L(y_0)/2$, and $u'(y_\ell^*) - u'(y_n^*) = 2\alpha L(y_0)$. Threshold $\hat{\chi}$ becomes

$$\hat{\chi} = \frac{-2\alpha L(y_0)(1 - 2\alpha y_{\ell/2}^*) + (1 - 2\alpha y_\ell^*)2\alpha L(y_0)}{-2\alpha L(y_0)(1 - 2\alpha y_{\ell/2}^*) + (1 - 2\alpha y_\ell^*)^2}.$$

The numerator simplifies to $2\alpha^2 L(y_0)^2$, the denominator to $1 - 4\alpha y^*_{\ell/2} + 2\alpha^2\left[(y^*_n)^2 + (y^*_\ell)^2\right]$. Relative risk aversion for quadratic utility is given by $R(y) = 2\alpha y/(1-2\alpha y)$, and hence we have $2\alpha y^*_n = R_n/(1+R_n)$. Recalling that $L(y_0) = \eta y^*_n$, we then obtain

$$\hat{\chi} = \frac{\frac{1}{2}\eta^2 R_n^2/(1+R_n)^2}{1 - 2(1-\frac{1}{2}\eta)R_n/(1+R_n) + \frac{1}{2}(1+(1-\eta)^2)R_n^2/(1+R_n)^2},$$

which can be simplified to the expression in the text by expanding the fraction by $2(1+R_n)^2$ and combining terms accordingly.

Result (*ii*) follows by noting that threshold $\hat{\chi}$ in Equation 5 can be rewritten as follows:

$$\hat{\chi} = \frac{u''(y^*_n)(u(y^*_n) - u(y^*_\ell)) + u'(y^*_n)(u'(y^*_\ell) - u'(y^*_n))}{u'(y^*_n)u'(y^*_\ell)}.$$

For iso-elastic utility with $u(y) = y^{1-\rho}/(1-\rho)$ for $\rho \neq 1$, we then find

$$\hat{\chi} = \frac{-\rho(y^*_n)^{-\rho-1}((y^*_n)^{1-\rho} - (y^*_\ell)^{1-\rho})/(1-\rho) + (y^*_n)^{-\rho}((y^*_\ell)^{-\rho} - (y^*_n)^{-\rho})}{(y^*_n)^{-\rho}(y^*_\ell)^{-\rho}}.$$

Using $y^*_\ell = (1-\eta)y^*_n$, we reduce the fraction by $(y^*_n)^{-2\rho}$ and expand it by $(1-\rho)$, which yields

$$\hat{\chi} = \frac{-\rho(1-(1-\eta)^{1-\rho}) + (1-\rho)((1-\eta)^{-\rho} - 1)}{(1-\rho)(1-\eta)^{-\rho}}.$$

Expanding the fraction by $-(1-\eta)^\rho$ then gives the expression in the text. When $\rho = 1$, utility is logarithmic, that is, $u(y) = \log(y)$ with $u'(y) = 1/y$ and $u''(y) = -1/y^2$. In this case,

$$\hat{\chi} = \frac{-(\log(y^*_n) - \log(y^*_\ell))/(y^*_n)^2 + (1/y^*_\ell - 1/y^*_n)/y^*_n}{1/(y^*_n y^*_\ell)}.$$

Using $y^*_\ell = (1-\eta)y^*_n$, we expand the fraction by $(y^*_n)^2$ so that

$$\hat{\chi} = \frac{\log(1-\eta) + (1/(1-\eta) - 1)}{1/(1-\eta)} = (1-\eta)\log(1-\eta) + \eta.$$

As a consistency check, threshold $\hat{\chi}$ in case of logarithmic utility can also be obtained via l'Hôpital's rule from the expression for $\rho \neq 1$ because

$$\frac{\mathrm{d}}{\mathrm{d}\rho}\left[(1-\eta)^\rho + \rho\eta - 1\right] = (1-\eta)^\rho \log(1-\eta) + \eta.$$

To show Result (*iii*), let the utility function take the linex form, $u(y) = ly - k\exp(-\gamma y)$ for positive constants k and γ, and $l = 1$. Per direct computation, we then find

$$u''(y^*_n)(u(y^*_n) - u(y^*_\ell)) + u'(y^*_n)(u'(y^*_\ell) - u'(y^*_n)) = k\gamma\exp(-\gamma y^*_n)\cdot\left[\exp(\gamma y^*_n\eta) - 1 - \gamma y^*_n\eta\right].$$

Using the fact that

$$k\gamma = \frac{R_\ell}{(P_\ell - R_\ell)\exp(-P_\ell)} \qquad \text{and} \qquad \gamma y_n^* = \frac{P_\ell}{1-\eta},$$

see also Appendix A.8, we obtain the following for the numerator of $\hat{\chi}$:

$$\begin{aligned}
& u''(y_n^*)(u(y_n^*) - u(y_\ell^*)) + u'(y_n^*)(u'(y_\ell^*) - u'(y_n^*)) \\
= \; & \frac{R_\ell}{(P_\ell - R_\ell)\exp(-P_\ell)} \exp\left(-\frac{P_\ell}{1-\eta}\right)\left[\exp\left(\frac{\eta P_\ell}{1-\eta}\right) - 1 - \frac{\eta P_\ell}{1-\eta}\right] \\
= \; & \frac{R_\ell}{(P_\ell - R_\ell)}\left(1 - \exp\left(-\frac{\eta P_\ell}{1-\eta}\right)\left(1 + \frac{\eta P_\ell}{1-\eta}\right)\right).
\end{aligned}$$

The denominator of $\hat{\chi}$ is given by

$$\begin{aligned}
u'(y_n^*)u'(y_\ell^*) &= (1 + k\gamma\exp(-\gamma y_n^*))\,(1 + k\gamma\exp(-\gamma y_n^*(1-\eta))) \\
&= \left(1 + \frac{R_\ell}{(P_\ell - R_\ell)}\exp\left(-\frac{\eta P_\ell}{1-\eta}\right)\right)\left(1 + \frac{R_\ell}{(P_\ell - R_\ell)}\right)
\end{aligned}$$

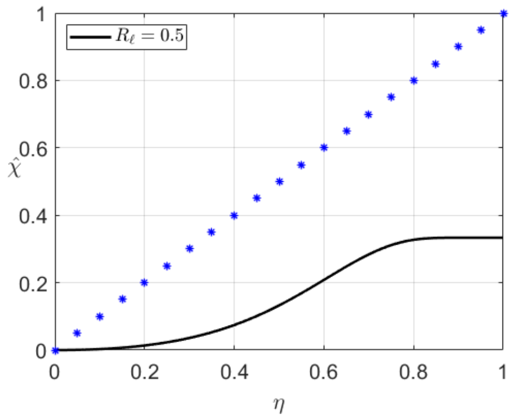
when the utility function is linex. We then obtain the expression for $\hat{\chi}$ in the text by expanding the fraction by $(P_\ell - R_\ell)$ and rearranging terms.

B Supplementary material

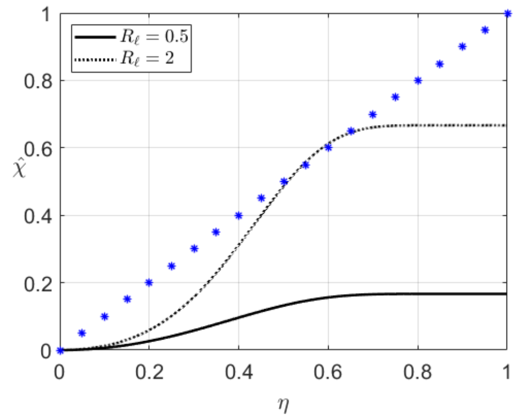
B.1 Illustration of Result (*iii*) in Proposition 10

Figure 6 illustrates the critical threshold $\hat{\chi}$ for linex utility. If the income sensitivity of the loss severity exceeds $\hat{\chi}$, the demand for safety is always normal regardless of the loss probability. Linex utility has two degrees of freedom, relative prudence (in the loss state) and relative risk aversion (in the loss state). Consistent with Results (*i*) and (*ii*) in Proposition 10, we observe that $\hat{\chi}$ is increasing in η and increasing in risk aversion. By comparing the lines across panels, we find in addition that $\hat{\chi}$ is decreasing in relative prudence. The black solid line is lower in Panel (c) than in Panel (b), and lower in Panel (b) than in Panel (a). The black dotted line is lower in Panel (c) than in Panel (b).

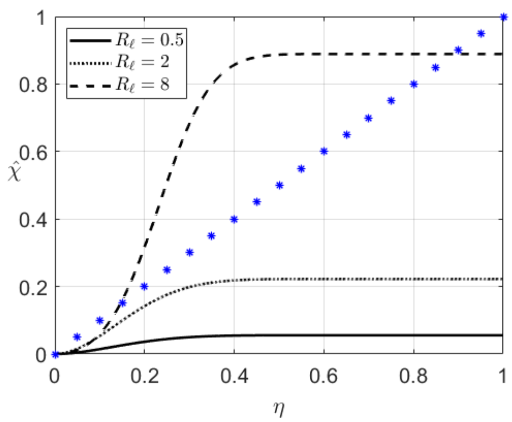
We observe that $\hat{\chi}$ can be larger than η in those cases in which the black line lies above the blue line of stars. This can also happen for quadratic utility when risk aversion is larger than a threshold while it cannot happen for iso-elastic utility (see Footnote 14). To shed more light on this, Panel (d) illustrates the combinations of relative prudence and relative risk aversion that lead to $\hat{\chi} \geq \eta$ for linex utility. The lower the share of final wealth at risk, the smaller the size of the relevant region in the (P_ℓ, R_ℓ)-plane. For example, when the loss puts 10% of final wealth at risk, relative prudence needs to exceed 8.9 and relative risk aversion needs to



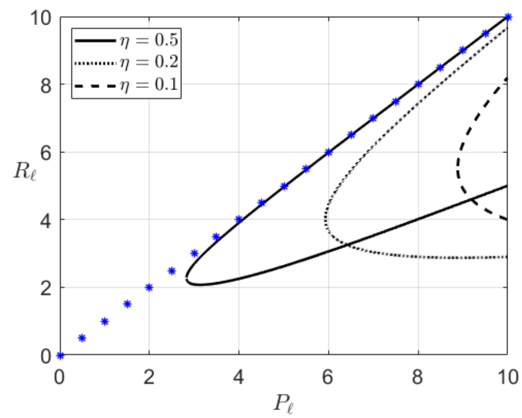
(a) Threshold $\hat{\chi}$ for linex utility with $P_\ell = 1.5$ and $R_\ell = 0.5$



(b) Threshold $\hat{\chi}$ for linex utility with $P_\ell = 3$ and $R_\ell \in \{0.5, 2\}$



(c) Threshold $\hat{\chi}$ for linex utility with $P_\ell = 9$ and $R_\ell \in \{0.5, 2, 8\}$



(d) Combinations of P_ℓ and R_ℓ where $\hat{\chi} \geq \eta$ for linex utility.

Figure 6: Panels (a), (b) and (c) display the threshold $\hat{\chi}$ for linex utility, see Proposition 10(iii). The blue line of stars indicates the identity line where $\hat{\chi} = \eta$. Panel (d) shows the combinations of P_ℓ and R_ℓ where $\hat{\chi} \geq \eta$ for linex utility. We consider three values for the share of final wealth at risk, $\eta \in \{0.1, 0.2, 0.5\}$.

be roughly around 6. So while possible, the more realistic case is $\hat{\chi} < \eta$. For example, when we focus on $P_\ell \leq 5$, then $\hat{\chi}$ is always below η regardless of the value of relative risk aversion as long as $\eta \leq 0.25$.

B.2 The effect of severity risk on the optimal level of safety

Let $\overline{L} = \mathbb{E}\widetilde{L}$ denote the expected severity level and s^0 the safety investment in the absence of severity risk. Superscript 0 indicates that severity risk is set to zero. The safety level s^0 is characterized by first-order condition

$$U_s(s^0; y_0, \overline{L}) = -p'(s^0) \cdot \left[u(y_n^0) - u(\overline{y}_\ell^0)\right] - \left[p^0 u'(\overline{y}_\ell^0) + (1 - p^0) u'(y_n^0)\right] = 0,$$

with $y_n^0 = y_0 - s^0$, $\overline{y}_\ell^0 = y_0 - s^0 - \overline{L}$ and $p^0 = p(s^0)$. Inserting s^0 into the first-order expression in the presence of severity risk yields

$$U_s(s^0; y_0, \widetilde{L}) = -p'(s^0) \cdot \left[u(y_n^0) - \mathbb{E}u(\widetilde{y}_\ell^0)\right] - \left[p^0 \mathbb{E}u'(\widetilde{y}_\ell^0) + (1-p^0)u'(y_n^0)\right],$$

with $\widetilde{y}_\ell^0 = y_0 - s^0 - \widetilde{L}$. Risk aversion implies $\mathbb{E}u(\widetilde{y}_\ell^0) < u(\mathbb{E}(\widetilde{y}_\ell^0)) = u(\overline{y}_\ell^0)$, and (weak) imprudence yields $\mathbb{E}u'(\widetilde{y}_\ell^0) \leq u'(\mathbb{E}(\widetilde{y}_\ell^0)) = u'(\overline{y}_\ell^0)$. In this case, $U_s(s^0; y_0, \widetilde{L}) > U_s(s^0; y_0, \overline{L}) = 0$, and the optimal safety investment increases. Crainich et al. (2016) derive this result.

The more relevant case of prudence introduces conflicting effects. While severity risk raises the marginal benefit of safety due to risk aversion, it also raises the marginal cost of safety when the decision-maker is prudent. To resolve this indeterminacy, we solve $U_s(s^0; y_0, \overline{L}) = 0$ for $p'(s^0)$, insert it into $U_s(s^0; y_0, \widetilde{L})$, and rearrange to obtain the following:

$$U_s(s^0; y_0, \widetilde{L}) = \left[p^0 u'(\overline{y}_\ell^0) + (1-p^0)u'(y_n^0)\right] \cdot \left\{ \frac{u(y_n^0) - \mathbb{E}u(\widetilde{y}_\ell^0)}{u(y_n^0) - u(\overline{y}_\ell^0)} - \underbrace{\frac{p^0 \mathbb{E}u'(\widetilde{y}_\ell^0) + (1-p^0)u'(y_n^0)}{p^0 u'(\overline{y}_\ell^0) + (1-p^0)u'(y_n^0)}}_{=f(p^0)} \right\}.$$

Auxiliary function $f(p^0)$ is the ratio of the marginal cost in the presence of severity risk to the marginal cost when severity risk is absent. Under imprudence, this ratio is always below one and the curly bracket is positive regardless of the size of p^0. Under prudence, the ratio exceeds one. We find $f(0) = 1$, so when the loss probability is small enough, severity risk always raises the optimal level of safety even for prudent decision-makers. Furthermore,

$$f'(p^0) = \frac{u'(y_n^0) \cdot \left[\mathbb{E}u'(\widetilde{y}_\ell^0) - u'(\overline{y}_\ell^0)\right]}{\left[p^0 u'(\overline{y}_\ell^0) + (1-p^0)u'(y_n^0)\right]^2},$$

which is positive under prudence. The higher the loss probability p^0, the more likely it is for the negative effect to prevail. For $p^0 = 1$ we obtain $f(1) = \mathbb{E}u'(\widetilde{y}_\ell^0)/u'(\overline{y}_\ell^0)$, which may be smaller or larger than the ratio of the marginal benefits. If it is smaller, the curly bracket is always positive regardless of the size of p^0 and severity risk increases the optimal safety level. If $f(1)$ is larger than the ratio of the marginal benefits, there is a probability threshold $p_\diamond$ so that severity risk increases safety for $p^0 < p_\diamond$ and reduces safety for $p^0 > p_\diamond$.

Assume a small severity risk. Using second-order Taylor approximations for $\mathbb{E}u(\widetilde{y}_\ell^0)$ and $\mathbb{E}u'(\widetilde{y}_\ell^0)$, we find that

$$f(1) > \frac{u(y_n^0) - \mathbb{E}u(\widetilde{y}_\ell^0)}{u(y_n^0) - u(\overline{y}_\ell^0)} \qquad \text{if} \qquad -\frac{\overline{y}_\ell^0 u'''(\overline{y}_\ell^0)}{u''(\overline{y}_\ell^0)} > \frac{\overline{y}_\ell^0}{\overline{L}} \cdot \frac{u'(\overline{y}_\ell^0)}{(u(y_n^0) - u(\overline{y}_\ell^0))/\overline{L}}.$$

The left-hand side is relative prudence. The right-hand side is an inverse measure of the share of final wealth at risk and a ratio related to the decision-maker's degree of risk aversion. The higher the degree of prudence, the higher the share of final wealth at risk, and the lower the degree of risk aversion, the more likely it is for there to be a probability threshold at which the effect of severity risk switches from positive to negative.

We can also turn to CARA utility and consider a severity risk of arbitrary size. Let the utiltiy function be $u(y) = -\exp(-ay)$ for $a > 0$. We then have $\mathbb{E}u(\widetilde{y}_\ell^0) = u(y_n^0) \cdot \mathbb{E}\exp(a\widetilde{L})$, $u(\overline{y}_\ell^0) = u(y_n^0) \cdot \exp(a\overline{L})$, $\mathbb{E}u'(\widetilde{y}_\ell^0) = u'(y_n^0) \cdot \mathbb{E}\exp(a\widetilde{L})$, and $u'(\overline{y}_\ell^0) = u'(y_n^0) \cdot \exp(a\overline{L})$. These relations allow us to simplify the curly bracket in $U_s(s^0; y_0, \widetilde{L})$ as follows:

$$\frac{\mathbb{E}\exp(a\widetilde{L}) - \exp(a\overline{L})}{\big(\exp(a\overline{L}) - 1\big)\big(1 + p^0\big(\exp(a\overline{L}) - 1\big)\big)}.$$

This term is always positive regardless of the size of p^0 because the exponential is convex and $a\overline{L} > 0$. For CARA utility, severity risk always increases the optimal safety level.

B.3 Proposition 1 in the presence of severity risk

We solve for $-p'(s^*)$ from first-order condition (8) and substitute. This yields:

$$\begin{aligned} U_{sy_0}(s^*; y_0, \widetilde{L}) &= \big[p^*\mathbb{E}u'(\widetilde{y}_\ell^*) + (1-p^*)u'(y_n^*)\big] \\ &\cdot \Bigg\{ \underbrace{-\frac{p^*\mathbb{E}u''(\widetilde{y}_\ell^*) + (1-p^*)u''(y_n^*)}{p^*\mathbb{E}u'(\widetilde{y}_\ell^*) + (1-p^*)u'(y_n^*)}}_{=f(p^*)} - \left(-\frac{u'(y_n^*) - \mathbb{E}u'(\widetilde{y}_\ell^*)}{u(y_n^*) - \mathbb{E}u(\widetilde{y}_\ell^*)}\right) \Bigg\}. \end{aligned}$$

Auxiliary function $f(p^*)$ is a normalized measure of the impact of an increase in income on the marginal cost of safety. We find that

$$f'(p^*) = \frac{u''(y_n^*)\mathbb{E}u'(\widetilde{y}_\ell^*) - u'(y_n^*)\mathbb{E}u''(\widetilde{y}_\ell^*)}{\big[p^*\mathbb{E}u'(\widetilde{y}_\ell^*) + (1-p^*)u'(y_n^*)\big]^2}.$$

The sign of $f'(p^*)$ coincides with the sign of the numerator. It is positive (zero, negative) when the utility function has DARA (CARA, IARA) because of

$$\mathbb{E}\left[u'(\widetilde{y}_\ell^*)u''(y_n^*) - u'(y_n^*)u''(\widetilde{y}_\ell^*)\right] = \mathbb{E}\Bigg[\underbrace{u'(\widetilde{y}_\ell^*)u'(y_n^*)}_{>0} \cdot \left(-\frac{u''(\widetilde{y}_\ell^*)}{u'(\widetilde{y}_\ell^*)} - \left(-\frac{u''(y_n^*)}{u'(y_n^*)}\right)\right)\Bigg].$$

The endpoints of $f(p^*)$ are

$$f(0) = -\frac{u''(y_n^*)}{u'(y_n^*)} \qquad \text{and} \qquad f(1) = -\frac{\mathbb{E}u''(\widetilde{y}_\ell^*)}{\mathbb{E}u'(\widetilde{y}_\ell^*)}.$$

Take the case of DARA utility. For any $l \in (0, L]$, the arguments in Appendix A.1 show that

$$-\frac{u''(y_n^*)}{u'(y_n^*)} < -\frac{u'(y_n^*) - u'(y_n^* - l)}{u(y_n^*) - u(y_n^* - l)},$$

which is equivalent to

$$-u''(y_n^*)(u(y_n^*) - u(y_n^* - l)) < -u'(y_n^*)(u'(y_n^*) - u'(y_n^* - l)).$$

The expectation respects monotonicity, and therefore

$$-u''(y_n^*)(u(y_n^*) - \mathbb{E}u(\widetilde{y}_\ell^*)) < -u'(y_n^*)(u'(y_n^*) - \mathbb{E}u'(\widetilde{y}_\ell^*)),$$

or equivalently

$$f(0) = -\frac{u''(y_n^*)}{u'(y_n^*)} < -\frac{u'(y_n^*) - \mathbb{E}u'(\widetilde{y}_\ell^*)}{u(y_n^*) - \mathbb{E}u(\widetilde{y}_\ell^*)}.$$

This shows that there is a probability threshold $p_c > 0$, at which the sign of $U_{sy_0}(s^*; y_0, \widetilde{L})$ switches from negative to positive. However, the arguments in Appendix A.1 no longer allow us to compare $f(1)$ and $-(u'(y_n^*) - \mathbb{E}u'(\widetilde{y}_\ell^*))/(u(y_n^*) - \mathbb{E}u(\widetilde{y}_\ell^*))$. As a result, when severity risk is large, we cannot rule out the possibility that the probability threshold p_c exceeds one. The arguments for CARA and IARA utility are analogous.

B.4 A stochastic increase in loss severity

Let s^* denote the optimal level of safety in the presence of severity risk $\widetilde{L}_1$ and let $U(s; y_0, \widetilde{L}_2)$ be the decision-maker's objective function in the presence of the stochastically higher random loss $\widetilde{L}_2$. Let $\widetilde{y}_{\ell_i}^* = y_0 - s^* - \widetilde{L}_i$ for $i = 1, 2$ be shorthand for random final wealth in the loss state for random loss $\widetilde{L}_i$. Inserting s^* into the first-order expression for $\widetilde{L}_2$ yields

$$\begin{aligned} U_{sy_0}(s^*; y_0, \widetilde{L}_2) &= -p'(s^*) \cdot \left[u(y_n^*) - \mathbb{E}u(\widetilde{y}_{\ell_2}^*)\right] - \left[p^*\mathbb{E}u'(\widetilde{y}_{\ell_2}) + (1-p^*)u'(y_n^*)\right] \\ &= \left[p^*\mathbb{E}u'(\widetilde{y}_\ell^*) + (1-p^*)u'(y_n^*)\right] \\ &\quad \cdot \left\{ \frac{u(y_n^*) - \mathbb{E}u(\widetilde{y}_{\ell_2}^*)}{u(y_n^*) - \mathbb{E}u(\widetilde{y}_{\ell_1}^*)} - \underbrace{\frac{p^*\mathbb{E}u'(\widetilde{y}_{\ell_2}^*) + (1-p^*)u''(y_n^*)}{p^*\mathbb{E}u'(\widetilde{y}_{\ell_1}^*) + (1-p^*)u'(y_n^*)}}_{=f(p^*)} \right\}. \end{aligned}$$

Loss $\widetilde{L}_2$ has FSD over loss $\widetilde{L}_1$ so that $\widetilde{y}_{\ell_1}^*$ has FSD over $\widetilde{y}_{\ell_2}^*$. Hence, $\mathbb{E}u(\widetilde{y}_{\ell_1}^*) > \mathbb{E}u(\widetilde{y}_{\ell_2}^*)$ and

$$\frac{u(y_n^*) - \mathbb{E}u(\widetilde{y}_{\ell_2}^*)}{u(y_n^*) - \mathbb{E}u(\widetilde{y}_{\ell_1}^*)} > 1.$$

Furthermore, $u'' < 0$ implies $\mathbb{E}u'(\widetilde{y}_{\ell_2}^*) > \mathbb{E}u'(\widetilde{y}_{\ell_1}^*)$ so that $f(p^*) > 1$ for $p^* > 0$ and $f(0) = 1$. The numerator of $f'(p^*)$ simplifies to $u'(y_n^*) \cdot \left[\mathbb{E}u'(\widetilde{y}_{\ell_2}^*) - \mathbb{E}u'(\widetilde{y}_{\ell_1}^*)\right]$, which is positive so that f is increasing in p^*. This implies the existence of a threshold $p_\# > 0$ so that $U_{sy_0}(s^*; y_0, \widetilde{L}_2) > 0$ for $p^* < p_\#$ and $U_{sy_0}(s^*; y_0, \widetilde{L}_2) < 0$ for $p^* > p_\#$.

The threshold $p_{\#}$ exceeds unity if and only if

$$f(1) = \frac{\mathbb{E}u'(\widetilde{y}^*_{\ell_2})}{\mathbb{E}u'(\widetilde{y}^*_{\ell_1})} < \frac{u(y^*_n) - \mathbb{E}u(\widetilde{y}^*_{\ell_2})}{u(y^*_n) - \mathbb{E}u(\widetilde{y}^*_{\ell_1})}.$$

This is always the case for CARA utility. If $u(y) = -\exp(-ay)$ for $a > 0$, the inequality above is equivalent to

$$\frac{\exp(-ay^*_n) \cdot \mathbb{E}\exp(a\widetilde{L}_2)}{\exp(-ay^*_n) \cdot \mathbb{E}\exp(a\widetilde{L}_1)} < \frac{-\exp(-ay^*_n) + \exp(-ay^*_n) \cdot \mathbb{E}\exp(a\widetilde{L}_2)}{-\exp(-ay^*_n) + \exp(-ay^*_n) \cdot \mathbb{E}\exp(a\widetilde{L}_1)}.$$

Cancel $\exp(-ay^*_n)$ on each side, cross-multiply and simplify to obtain $\mathbb{E}\exp(a\widetilde{L}_2) > \mathbb{E}\exp(a\widetilde{L}_1)$. This inequality is satisfied because $\exp(aL)$ is increasing in L and $\widetilde{L}_2$ has FSD over $\widetilde{L}_1$. Hence, $p_{\#} > 1$ under CARA so that $U_{sy_0}(s^*; y_0, \widetilde{L}_2) > 0$ always holds regardless of the size of p^*.

B.5 Log-convexity of $w \circ p$

We obtain

$$\begin{aligned}(\log(w(p(s))))'' &= \left(\frac{w'(p(s))p'(s)}{w(p(s))}\right)' \\ &= \frac{w(p(s)) \cdot \left[w''(p(s))p'(s)^2 + w'(p(s))p''(s)\right] - w'(p(s))^2 p'(s)^2}{w(p(s))^2},\end{aligned}$$

which is nonnegative if and only if the numerator is nonnegative. We rewrite the numerator as follows:

$$\underbrace{w(p(s))w'(p(s))}_{\geq 0} \cdot \underbrace{(-p'(s))}_{>0} \cdot \left[-\frac{p''(s)}{p'(s)} + p(s)\frac{w'(p(s))}{w(p(s))} \cdot \frac{p'(s)}{p(s)} - p(s)\frac{w''(p(s))}{w'(p(s))} \cdot \frac{p'(s)}{p(s)}\right]. \quad (13)$$

Log-convexity of p is equivalent to $-p''(s)/p'(s) \geq -p'(s)/p(s)$, and the elasticity of the probability weighting function is non-decreasing in p if and only if

$$\frac{\partial}{\partial p}\left(p\frac{w'(p)}{w(p)}\right) = \frac{w'(p)}{w(p)} + p\frac{w(p)w''(p) - w'(p)^2}{w(p)^2} = \frac{w'(p)}{w(p)} \cdot \left[1 - p\frac{w'(p)}{w(p)} + p\frac{w''(p)}{w'(p)}\right] \geq 0.$$

Given that w is increasing, this implies for the square bracket to be nonnegative. It follows that the square bracket in (13) exceeds

$$\left(-\frac{p'(s)}{p(s)}\right) \cdot \left[1 - p(s)\frac{w'(p(s))}{w(p(s))} + p(s)\frac{w''(p(s))}{w'(p(s))}\right] \geq 0,$$

so that $(\log(w(p(s))))'' \geq 0$ as desired.