Risk Sharing for Extreme Events

Alexander Braun*, Martin Eling †, Marcel Freyschmidt‡

We develop a holistic model framework of insurance markets for extreme event risk. Companies can cede their extreme event risk to insurers, who, in turn, can purchase reinsurance and/or place risk in the capital market. In addition, we introduce a governmental agency that may provide backstops for the highest loss layers. We parametrize the model to reflect the particularities of natural disasters, pandemics, and cyber catastrophes. Our results show that, particularly for the most extreme events, government backstops in the highest loss layers are necessary for a private insurance market to share heavy tail risk in the first place.

Keywords: Extreme Events, Risk Sharing

1 Introduction

The optimal risk transfer for individuals and societies has been a formative topic of insurance economics since the first fundamental contributions of Arrow (1963), Mossin (1968), Smith (1968) and others. Over the last 20 years, especially in light of increasing natural catastrophes and terror risks, the discussion around risk pooling and public-private partnerships has emerged (see, e.g., Kunreuther (2002) and Kunreuther (2015)). Recently, governments undertake huge efforts to curb the coronavirus spread and to avoid the collapse of the whole economy, again leading to the question whether an ex-ante pooling of risk could be better than an ex-post financing via taxpayers (see, e.g., Gründl et al. (2021)). Forward-looking, a large part of the future risks will be generated in the digital economy, with great concerns on potential extreme cyber scenarios with accumulation risk (see, e.g., Biener et al. (2015)), again leading to questions of insurability and optimal risk sharing.

So far, there is no standard design for the various risk transfer mechanisms across different risk categories and a consistent analysis within a model or conceptual framework. We aim to provide this. We look at various forms of risk transfer (insurance and reinsurance, alternative risk transfers (especially cat bonds), public-private partnership) and different risk categories ("normal" cat risks such as Nat Cat and "extreme" cat risks such as pandemic or

*alexander.braun@unisg.ch; University of St. Gallen, Tannenstrasse 19, CH-9000 St. Gallen; Phone: +41 71 224 3653.

†martin.eling@unisg.ch; University of St. Gallen; Girtannerstrasse 6, CH-9010 St.Gallen; Phone: +41 71 224 7980.

‡marcel.freyschmidt@unisg.ch; University of St. Gallen; Girtannerstrasse 6, CH-9010 St.Gallen; Phone +41 71 224 7974.

cyber). The key difference between these categories is mainly the loss distribution and the correlation with the capital market (and thus with the economy's evolution in general).

We consider a microeconomic model of the decision-making problem faced by companies, (re)insurers, investors in the capital market and a sovereign government. Building on various individual pieces from the literature (e.g., Zanjani (2002), Froot and O'Connell (2008)), we develop a consistent conceptual setup to study how different hedging strategies of insurers affect the hedging of companies. We extend the existing literature by analyzing a more realistic model framework; e.g., Froot and O'Connell (2008) look at a risk transfer in the form of a stop loss for simplicity, where we focus on the more realistic case of an excess of loss (XL) contract. Empirically motivated we simulate different risk categories to provide a realistic view of real-world decision making and vary all input parameters to study the sensitivity of our results and to identify the critical parameters. We analyze how (and where) the government can influence an existing market equilibrium. In this context we argue that the finding by Arrow and Lind (1970), where governments should evaluate projects from a risk-neutral perspective, does not necessarily apply to financially constrained countries.

The results show the value of insurance, without and with risk pooling and diversification opportunities. The capital available for risk coverage increases when we incorporate reinsurance and the capital market; however, the extra money is only available if risk adequate cost of capital can be earned. The use of diversification strongly depends on the type of risk (e.g., natural events tend to be regional and can be diversified globally; a pandemic cannot). A key is the correct modelling of the cost of capital - in a consistent way for the market participants we consider.

The structure of the paper is as follows. In section 2, we provide the model framework. We start with a simple excess of loss contract between a company and an insurer. Afterwards we look at the pricing, the cost of capital and diversification opportunities, followed by adapting the model by reinsurance, the capital market, and the government. In section 3, we calibrate the model with economic data and present the results. Section 4 concludes.

2 Model

We first consider the situation that a company can hedge its extreme event risk with an insurer and that no other risk transfer options are available. In the second step, we look at how the availability of further risk transfers (e.g., through reinsurance or insurance-linked securities (ILS)) influences the contract between the company and the insurer. In the third step, we analyze how the strategy of the company and insurer changes when the government enters. To keep the results tractable we consider a single-period model, which can be extended though to a multi-period setup.

Mayers and Smith (1982) and Kunreuther et al. (1993) find that companies and (re)insurers exhibit risk-averse behavior when faced with uncertain and ambiguous risks. Mayers and

Smith (1990) argue for property/liability insurers that the transaction costs associated with bankruptcy make risk-averse behavior rational and explain the demand for reinsurance. Therefore, we model the hedging decision of the company and insurer through a utility function to reflect risk aversion. Also the modern risk management literature argues that decision making in companies can be considered as "quasi"-risk averse because of market frictions such as bankruptcy costs (see Froot et al. (1993)).

2.1 Modeling of payments from company and insurer

A company seeks coverage against a loss L in exchange for a premium π. The company accepts an amount of R as a retention. Any loss below remains by the company (or by other coverage). Additional, there is a detachment point K, above is no more protection possible. R and K are thus the lower and upper protection levels. The following random variable defines the loss of the company:

$$X(R,K) = \begin{cases} L & \text{if } L < R \\ R & \text{if } R \leq L \leq K \\ R + (L - K) & \text{if } K < L \end{cases} \tag{1}$$

or

$$X(R,K) = min(L,R) + max(L-K,0),$$

and the following random variable defines the payoff of the insurer to the company (if the insurer is not insolvent):

$$Y(R,K) = \begin{cases} 0 & \text{if } L \leq R \\ L - R & \text{if } R < L \leq K \\ K - R & \text{if } K < L \end{cases} \tag{2}$$

or

$$Y(R,K) = max(L-R,0) + min(K-L,0).$$

2.2 Company and insurer

2.2.1 Modeling the strategy of companies

We define E_0^{com} as the initial equity of a limited liability company and E_1^{com} as the terminal equity. r_{com} is the company's average yield on equity.[1] The company expects a stochastic loss L at time $t = 1$ and has at $t = 0$ the possibility to buy an insurance contract with payoff Y by paying a premium π. The premium is charged directly. The company is free to choose the retention R and the cap K. In the case of an insurer's insolvency the difference between

[1]We will calculate r_com deterministically. If the insolvency risk of the firm should also be taken into account, r_com can be defined stochastically.

the contractual payoff $Y(R,K)$ and the actual payoff is denoted as $D^{ins}(R,K)$. Section 2.2.3 specifies D^{ins} in more detail. The company's equity in $t=1$ can be written as

$$\begin{aligned} E_1^{com}(R,K) &= (1+r_{com})(E_0^{com}-\pi) - L + Y - D^{ins} \\ &= (1+r_{com})(E_0^{com}-\pi) - X - D^{ins}. \end{aligned} \tag{3}$$

In case $(1+r_{com})(E_0^{com}-\pi) - X - D^{ins} \leq 0$ the company is bankrupt.[2]

To find the optimal hedging policy, we define the company's utility as $U\big(max(E_1^{com},0)\big)$, where $U(\cdot)$ is a company-specific utility function depending on the remaining equity. So, the company finds the optimal hedging decision by maximizing its expected utility $\mathbb{E}[U]$ through the hedging variables R and K by

$$\max_{R,K} \mathbb{E}[U\big(max(E_1^{com}(R,K),0)\big)].$$

2.2.2 Modeling the strategy of insurer

We define E_0^{ins} as the initial equity of the insurer. The insurer receives in $t=0$ the premium π and pays in $t=1$ the loss Y. The equity and premium can be invested at a risk-free rate r_f, since insurance companies hold a large proportion of fixed-income securities in their portfolios, see, e.g., Trottier (2017). Additionally, the insurer has costs $c(E_0^{ins},Y)$. We define the terminal equity as

$$E_1^{ins} = max\big((1+r_f)(E_0^{ins}+\pi(R,K)-c(E_0^{ins},Y)) - Y(R,K), 0\big). \tag{4}$$

In case $(1+r_f)(E_0^{ins}+\pi(R,K)-c(E_0^{ins},Y)) - Y(R,K) \leq 0$, the insurer is bankrupt.

We define the insurer's utility as $U^{ins}\big(E_1^{ins}(R,K)\big)$, where $U^{ins}(\cdot)$ is an insurer-specific utility function depending on the remaining equity. The insurer maximize its utility through

$$\max_{R,K} \mathbb{E}[U^{ins}\big(E_1^{ins}(R,K)\big)].$$

2.2.3 Modeling the pricing of insurance policies

Zanjani (2002) considers three key assumptions for pricing in catastrophe insurance. First, because of the uncertainty in average loss, insurers may default. Second, it is costly for the insurer to hold capital and third, the risk of insolvency matters for the costumer. The first point is based on the heavy tail distribution of extreme events. The second point refers to the amount of the insurer's initial capital E_0^{ins}. Therefore, the premium includes a share of the capital cost. The last point is the probability that the insurer gets bankrupt. In this case, the company does not receive the total amount of the insured loss. Furthermore, the premium includes operational costs k. We define the cost function as

[2] For conceptual reasons we omit the maximum operator $max(E_1^{com}(R,K),0)$ here.

$c(E_0^{ins}, Y) = capital\ cost(E_0^{ins}, Y) + k.$

The expected payoff of the insurer to the company can be written as

$$\mathbb{E}[Y] = \big(\mathbb{P}(L > R) - \mathbb{P}(L > K)\big)\mathbb{E}[L - R|R < L \leq K] + \mathbb{P}(L > K)(K - R).$$

In the case of insurer's insolvency, the company might not receive the total contractual payoff Y. Thus, the difference between the contractual payoff and the realized payoff is

$$\begin{aligned} D^{ins} &= max\big(0, Y - \mathbb{E}[Y] - \mathbb{E}[E_1^{ins}]\big) \\ &= max\Big(0, Y - \big((1+r)(E_0^{ins} + \pi(R,K) - c(E_0^{ins}, Y)\big)\Big). \end{aligned} \tag{5}$$

Following Zanjani (2002), we search for a given R and K the premium π and initial equity E_0^{ins} which solve

$$\pi = B_0\big(\mathbb{E}[Y] - \mathbb{E}[D^{ins}]\big) + c(E_0^{ins}, Y), \tag{6}$$

where $B_0 = \frac{1}{1+r_f}$ is the price of a one-year zero-coupon bond with principal 1. In the following, we use B_0 as a discount factor. The first term in the bracket describes the expected payoff of the insurance contract; the second term in the bracket is the adjustment in case of insurers' insolvency; the last term outside the bracket is the costs. We show in Appendix A that equation 6 has no closed-form solution since π also occurs in D^{ins}.

We define with

$$E_{1-}^{ins} = (1 + r_f)\big(E_0^{ins} + \pi(R,K) - c(E_0^{ins}, Y)\big)$$

the equity of the insurer directly before the loss occurs. Insurance companies are forced to hold a minimum capital, so we consider M_q as a statistic of regulatory risk measurement depending on Y. Following, $RCR(Y)$ denotes the regulatory capital requirement for risk Y.[3] In the analysis, we consider both the case of a mono-line insurer as well as the case that the insurer holds a portfolio $\mathcal{P}$ of other insurance contracts (multi-line insurer). The insurer's portfolio may affect the regulatory capital requirement needed for the new risk.[4] In general $E_{1-}^{ins} \geq RCR(Y)$ must hold.

Finally we find the premium π and the initial equity E_0^{ins} through[5]

$$\begin{aligned} (\pi, E_0^{ins}) = \text{argmin}_{\pi, E_0^{ins}} &|\pi - B_0\big(\mathbb{E}[Y] - \mathbb{E}[D^{ins}]\big) - c(E_0^{ins}, Y)| \\ \text{subject to } &E_{1-}^{ins} \geq RCR(Y) \\ &\pi, E_0^{ins} \geq 0. \end{aligned} \tag{7}$$

[3]In the context of European Union insurance companies, it could be the value at risk at the 99.5% level of Y; in Swiss insurance companies, it could be the expected shortfall at the 99% level of Y.

[4]In case of a mono-line insurer $RCR^{mono}(Y) = M_q(Y)$. In case of a multi-line insurer $RCR^{multi}(Y) = M_q(Y + \mathcal{P}) - M_q(\mathcal{P})$ as the regulatory capital requirement already exists for the existing portfolio. When $\mathcal{P} = 0$, $RCR^{multi} = RCR^{mono}$ holds. Should an existing portfolio and the new loss (partially) diversify, the regulatory capital requirement should decrease.

[5]The objective function can also be written down as $\big(\pi - B_0\mathbb{E}[Y]\big) + \mathbb{E}[D^{ins}] - c(E_0^{ins}, Y)$. The premium and the equity have to be chosen so that the tradeoff between risk premium, default and cost of capital balance out. Thus, the premium resp. the equity cannot reach unrealistic values.

2.2.4 Capital costs

Following Zanjani (2002), the total cost of capital is a decomposition of a frictional component and a risk component. The frictional component combines taxes and other frictions like additional monitoring, agency, or liquidity costs associated with the insurer's investment. These costs are independent of the risk and, according to Zanjani (2002), linear increasing with the amount of initial equity E_0^{ins}.[6] The frictional cost per unit of capital we denote following with α^{ins}. Frictional costs have a large impact on the risk management of (re)insurers and should therefore be adequately quantified, see, e.g. Yow and Sherris (2008). The risk component results from the relationship between insurance liabilities and the capital market. It represents the compensation demanded by capital holders for the coverage, according to the relationship between the covered risk and the capital market. Therefore, this component does not depend directly on the amount of initial equity, but on the risk and thus affects the capital required. We define $\mu^{risk}(R)$ as risk cost per unit of capital, depending on the retention R. Braun et al. (2021) show that only the probability of the first loss is of importance for risk costs. Therefore μ^{risk} does not depend on K. However, the cap is included in the calculation of the capital E_0^{ins}. We define the cost of capital through

$$\begin{aligned} capital\ cost =& frictional\ costs + risk\ costs \\ :=& r_f \cdot E_0^{ins} + \alpha^{ins} \cdot E_0^{ins} + \delta \cdot \mu^{risk}(R) \cdot E_0^{ins}. \end{aligned}$$

where delta is a weighting factor for diversification and is calculated by $\delta = \frac{RCR^{multi}(Y)}{RCR^{mono}(Y)} = \frac{M_q(Y+\mathcal{P})-M_q(\mathcal{P})}{M_q(Y)}$. This means that only the portion of the risk not already present in the portfolio is considered. We follow the idea of Hann et al. (2013), which defines the cost of capital as the weighted sum of all company segments. Diversification is, in general, an important point of discussion. The literature shows that insurers operating in more volatile business areas do not necessary diversify more, see, e.g., Berry-Stölzle et al. (2012), and non-diversified insurer consistently outperform diversified insurers, see e.g., Lamont and Polk (2001). The corporate finance literature shows that there is a loss of value for companies through diversification, see, e.g., Lang and Stulz (1994), Berger and Ofek (1995), Graham et al. (2002), Mansi and Reeb (2002),Liebenberg and Sommer (2008). On the other side, Yan (2006) shows higher valuations for diversified companies when external capital is more costly and Hann et al. (2013) examine that diversified firms have a lower cost of capital and additional, diversified firms with less correlated segment cash flows have a lower cost of capital.

For catastrophe losses, Froot et al. (1995) and others argued that this kind of loss is uncorrelated with the capital market. Moreover, empirical studies like Cummins and Harrington (1985) found estimates close to zero for more general insurance-related betas. We conclude that for these types of risks the cost of capital is not driven by covariation with capital market returns, so $\mu^{risk} \approx 0$, and taxation and other constraints are the main cost drivers, so $capital\ cost = frictional\ costs := (r_f + \alpha^{ins})E^{ins}$.

[6]Fixed costs are not considered here as they are included in the operational costs k.

However, with extreme events, we can no longer assume uncorrelatedness to the capital market, and thus higher capital costs must be expected. Also, the diversification effect within the insurance portfolio is significantly limited, which directly impacts the insurer and reinsurer.[7]

2.2.5 Interaction Company and Insurer

Based on the models in the previous sections, an optimal tradeoff between the company and the insurer would maximize the company value of both. We show in appendix A that this is the case when we find a R which solve

$$\begin{aligned} \mathbb{E}[U_{E_1^{com}}] &= \frac{-\mathbb{E}[U_{E_1^{com}} \mathbb{1}_{(L \geq R)}] - \mathbb{E}[U_{E_1^{com}} \frac{dD^{ins}}{dR}]}{(1+r_{com})\frac{d\pi}{dR}}, \\ \mathbb{E}[U^{ins}_{E_1^{ins}}] &= \frac{-\mathbb{E}[U^{ins}_{E_1^{ins}} \mathbb{1}_{(L \geq R)}]}{(1+r_f)(\frac{d\pi}{dR} - \frac{dc}{dR})}, \end{aligned} \tag{8}$$

and a K which solve

$$\begin{aligned} \mathbb{E}[U_{E_1^{com}}] &= \frac{\mathbb{E}[U_{E_1^{com}} \mathbb{1}_{(L > K)}] - \mathbb{E}[U_{E_1^{com}} \frac{dD^{ins}}{dK}]}{(1+r_{com})\frac{d\pi}{dK}}, \\ \mathbb{E}[U^{ins}_{E_1^{ins}}] &= \frac{\mathbb{E}[U^{ins}_{E_1^{ins}} \mathbb{1}_{(L > K)}]}{(1+r_f)(\frac{d\pi}{dK} - \frac{dc}{dK})}. \end{aligned} \tag{9}$$

Lemma 1. *The system of equations 8 and 9 has no common solution. There is no R^* and K^* that optimizes the utility of the company and the insurer.*

Proof. From an economic point of view, a R^* and K^* that optimizes the utility of the company and the insurer would imply that the highest possible utility for the company is also the profit-maximizing policy for the insurer. This is controversial since the insurer finds its optimum where the ratio of premium to payout (i.e., the risk premium) is highest, while the company has the highest utility when the ratio is low. From a mathematical point of view, we have two unknowns for four equations. R and K are linear in our model and a linear system of equations has an unique solution only if the number of unknowns and equations is identical, it follows that the system is not solvable per se. A detailed mathematical proof is in the appendix A. □

Given lemma 1, we reconsider the insurer's strategy following Froot and O'Connell (2008) and we assume that the insurer acts in a competitive market. Thus, the insurer creates an area of supply where the company chooses the utility-maximizing product. Two things are essential for the insurer:

[7] Note that cat losses are probably uncorrelated to the capital market, but general non-cat insurance losses could be assumed to be correlated to the capital market since there is a link between economic activity and activity in the insurance market.

1. The insurer only offers products with a positive expected return,

$$\mathbb{E}[E_1^{ins}] \geq (1+r_f)E_0^{ins} := E_{bound}^{ins}.$$

2. The insurer prices its products subject to regulatory measures and its portfolio,

$$E_{1-}^{ins} \geq RCR(Y).$$

We define the area of supply as

$$G^{ins} := \Big\{E_0^{ins}, \pi : (E_0^{ins}, \pi) \in \big(E_{1-}^{ins}(R,K) - RCR(Y(R,K)) \geq 0 \wedge \\ \mathbb{E}[E_1^{ins}(R,K)] \geq E_{bound}^{ins}\big)\Big\}.$$

Even if the insurer prices its products actuarial fair under the points above, e.g., $\pi = B_0\mathbb{E}[Y - D^{ins}]$, according to Doherty and Schlesinger (1990) the company would not choose full coverage. However, a look at equation 6 shows that the pricing is not actuarial fair, e.g., $\pi > \mathbb{E}[Y - D^{ins}]$. The company searches in the supply space for the retention R^* and the cap K^*, which has the best tradeoff between price and coverage by maximizing the utility. The optimization problem

$$\begin{aligned} (R^*, K^*) := \text{argmax}_{R,K} \quad & U(R,K) \\ \text{subject to} \qquad & (E_0^{ins}, \pi) \in G^{ins} \\ & R \geq 0 \\ & K > R \end{aligned} \tag{10}$$

expresses this situation, where (E_0^{ins}, π) solves problem 7.

2.3 Reinsurer, capital market and government

2.3.1 Modeling the strategy of reinsurer and capital market

We define the stochastic loss of the insurer as $L^{ins} := Y(R,K)$ and include the opportunity of sharing (a part) of the loss with a reinsurer or with the capital market. In general, there are several possibilities for trading between these parties. To avoid overloading the model, we concentrate on an excess of loss reinsurance contract and an indemnity Catastrophe Bond. However, this does not limit our model. Other products can be added or removed.

For sharing risk with a reinsurance company, we define an excess of loss reinsurance contract with a retention R^{re} and a cap K^{re}. The payoff $Y^{re}(R^{re}, K^{re})$ to the insurer is analogous to equation 2. The premium is denoted by $\Pi^{re}(R^{re}, K^{re})$.
We write the terminal equity of the reinsurer as

$$E_1^{re} = max\big((1+r_f)(E_0^{re} + \Pi^{re}(R^{re}, K^{re}) - \tilde{c}(E_0^{re}, Y^{re})) - Y^{re}(R^{re}, K^{re}), 0\big)$$

with E_0^{re} as the initial reinsurer's equity and $\tilde{c}(E_0^{re}, Y^{re})$ as a cost function similar to $c(E_0^{ins}, Y)$.

Following section 2.2.3, we define the premium for the reinsurance contract as a tradeoff between insolvency risk and cost of capital. In the case of reinsurer‘s default, the insurer receives the amount $Y^{re} - D^{re}$, where D^{re} is the difference between the contractual and realized payoff, defined as in equation 5.

Analogous to optimization problem 7, we define the premium Π^{re} through

$$\begin{aligned}(\Pi^{re}, E_0^{re}) = \operatorname{argmin}_{\Pi, E_0^{re}} |\Pi - B_0\big(\mathbb{E}[Y^{re}] - \mathbb{E}[D^{re}]\big) - \tilde{c}(E_0^{re}, Y^{re})| \\ \text{subject to } E_{1-}^{re}(R^{re}, K^{re}) \geq RCR(Y^{re}) \\ \Pi, E_0^{re} \geq 0,\end{aligned}$$

where $M_q(Y^{re}(R^{re}, K^{re}))$ is a risk measurement connected with regulatory restrictions.

Following Froot and O'Connell (2008), the reinsurer is in a competitive market. The reinsurer creates an area of supply where the insurer chooses the utility-maximizing product. This space is defined by

$$\begin{aligned}G^{re} := \Big\{ E_0^{re}, \Pi^{re} : (E_0^{re}, \Pi^{re}) \in \big(E_{1-}^{re}(R^{re}, K^{re}) - RCR(Y^{re}) \geq 0 \wedge \\ \mathbb{E}[E_1^{re}(R^{re}, K^{re})] \geq E_{bound}^{re}\big) \Big\}\end{aligned}$$

with

$$\begin{aligned}E_{bound}^{re} &= (1 + r_f) E_0^{re}, \\ E_{1-}^{re} &= (1 + r_f)(E_0^{re} + \Pi^{re} - \tilde{c}).\end{aligned}$$

We expand the model by including the capital market since it can be considered more risk-bearing than a (re)insurer due to its volume and the risk affinity of investors. We look at an indemnity Catastrophe Bond (Cat Bond) following the standard one-period model. The insurer can enter a contract with the investors in the capital market at $t = 0$ by paying a premium Π^{cm} and receiving Y^{cm} in $t = 1$. Y^{cm} has an excess of loss design, see equation 2, depending on a retention R^{cm} and a cap K^{cm}. Additionally, Y^{cm} is fully covered, so there is no positive default probability, and the capital market is competitive. The insurer finds a supplier for all possible R^{cm} and K^{cm} combinations and can choose the best match for itself.

In time $t = 0$, the investors pay a Cat Bond notional N to a trust account which generates interest at the risk-free rate. Since the payout of the capital market is fully covered, the notional is $N = K^{cm} - R^{cm}$. In time $t = 1$, the insurer receives Y^{cm} from the trust account, and the investors get the

1. the rest of the trust account $N - Y^{cm}$,
2. the annual earned interest of the trust account $r_f N$ and

3. a risk premium sN, where s is the Cat Bond spread.

Point 1 is thereby the Cat Bond principal and points 2 and 3 are the Cat Bond coupon, so $P = (N - Y^{cm})$ and $C = \big(r_f + s\big)N$. The investor receives the payments from points 1 and 2 from the trust account, and only point 3 is a payment from the insurer. Therefore, point 3 (risk-adjusted discounted) is the premium paid by the insurer. The Cat Bond price CB is defined as the present value of the future cash-flows, so

$$\begin{aligned} CB &= \frac{1}{1+\gamma}\big(\mathbb{E}[P] + C\big) \\ &= \frac{1}{1+\gamma}\big(\mathbb{E}[N - Y^{cm}] + C\big) \\ &= \frac{1}{1+\gamma}\big(C + N - \mathbb{E}[Y^{cm}]\big) \\ &= \frac{1}{1+\gamma}\big((r_f + s)N + N - \mathbb{E}[Y^{cm}]\big) \\ &= \frac{1}{1+\gamma}\big((1 + r_f + s)N - \mathbb{E}[Y^{cm}]\big), \end{aligned}$$

where γ is a risk-adjusted discount rate and $\gamma \geq r_f$ holds. γ is an exogenous variable and can be derived from observable Cat Bond yields (e.g., secondary market) or an asset pricing model. As mentioned, the premium Π^{cm} the insurer pays is the risk-adjusted discounted spread $\frac{1}{1+\gamma}sN$. So, we transform the equation to

$$\Pi^{cm} := \frac{1}{1+\gamma}sN = CB - \frac{1+r_f}{1+\gamma}N + \frac{1}{1+\gamma}\mathbb{E}[Y^{cm}], \tag{11}$$

where $CB - \frac{1+r_f}{1+\gamma}N$ defines the risk premium and $\frac{1}{1+\gamma}\mathbb{E}[Y^{cm}]$ is the risk adjusted expected payoff.

The platform Artemis (2022) shows an average multiple[8] of around 2.5 for the year 2021. This means that the premium equals $\Pi^{cm} = \frac{2.5}{1+\gamma}\mathbb{E}[Y^{cm}]$ (and thus $CB - \frac{1+r_f}{1+\gamma}N = \frac{1.5}{1+\gamma}\mathbb{E}[Y^{cm}]$). Unfortunately, the assumption that the risk premium is only a multiple of the expected loss is too simple. In general, the risk premium has a frictional term and a risk term analogous to the cost of capital in section 2.2.4. To our knowledge, there is yet no model that explains these frictions in economic terms. We address this aspect in more detail in section 3.

2.3.2 Interaction Company, (Re) Insurer and capital market

We define

$$\tilde{Y}(R, K, R^{re}, K^{re}, R^{cm}, K^{cm}) = Y(R, K) - Y^{re}(R^{re}, K^{re}) - Y^{cm}(R^{cm}, K^{cm})$$

[8]The multiple is the ratio of how many times the expected loss the investors are receiving in $t = 1$.

and

$$\Pi = \Pi^{re} + \Pi^{cm}.$$

$\tilde{Y}$ together with $D^{re}(R^{re}, K^{re})$ is the loss which remain by the insurer and $\Pi = \Pi^{re} + \Pi^{cm}$ is premium paid by the insurer for hedging. We rewrite the terminal equity of the insurer to

$$\tilde{E}_1^{ins} = max\Big((1+r_f)\big(E_0^{ins} + \tilde{\pi} - \Pi - c(R, K, E_0^{ins})\big) - \tilde{Y} - D^{re}, 0\Big), \tag{12}$$

where $\tilde{\pi}$ is the premium paid by the company to the insurer. In general, π and $\tilde{\pi}$ can differ. Note, Y in equation 5 changes to $\tilde{Y}$. For a given R and K, we find the insurer's optimal hedging strategy $W(R, K)$ by solving

$$\begin{aligned} W(R,K) := \operatorname{argmax}_{R^{re},K^{re},R^{cm},K^{cm}} \quad & U^{ins}(\tilde{E}^{ins}(R^{re}, K^{re}, R^{cm}, K^{cm})) \\ \text{subject to} \quad & (E_0^{re}, \Pi^{re}) \in G^{re} \\ & R^{re} \geq 0 \\ & K^{re} > R^{re} \\ & R^{cm} \geq K^{re} \\ & K^{cm} > R^{cm}, \end{aligned}$$

where (E_0^{re}, Π^{re}) solves the optimization problem for the premium.

Assuming a fully competitive primary insurance market, the insurer cannot pass costs resulting from its hedging to the company.[9] With knowing the optimal hedging strategy of the insurer, we update the supply space to

$$\tilde{G}^{ins} := \Big\{\tilde{E}_0^{ins}, \tilde{\pi} : (\tilde{E}_0^{ins}, \tilde{\pi}) \in \big(\tilde{E}_{1-}^{ins}(R, K, W) - M_q\big(\tilde{Y}\big)\big) \geq 0 \wedge \\ \mathbb{E}[\tilde{E}_1^{ins}(R, K, W)] \geq \tilde{E}_{bound}^{ins}\big)\Big\}$$

and adjust optimization problem 10 to

$$\begin{aligned} (R^*, K^*) := \operatorname{argmax}_{R,K} \quad & U(R, K) \\ \text{subject to} \quad & (\tilde{\pi}, \tilde{E}_0^{ins}) \in \tilde{G}^{ins} \\ & R \geq 0 \\ & K > R \\ & \pi \geq \tilde{\pi}, \end{aligned} \tag{13}$$

where $(\tilde{\pi}, \tilde{E}_0^{ins})$ solves the optimization problem for the premium.

[9] For the interaction between insurer and company, Π is replaced by the fair premium $E[\tilde{Y} - D^{re}]$.

2.3.3 How to involve the government?

The literature offers various theories on government involvement in the market. Cummins (2006) describes three public policy theories about government's role in addressing market failures in the insurance industry: laissez-faire, public interest, and market enhancement.

Laissez-faire means that any market-based equilibrium, irrespective of how imperfect, provides a more efficient allocation of resources within the economy than an equilibrium involving government intervention. Stigler (1971) argues in this context that a government intervention in markets results primarily from the rent-seeking behaviour of special interest groups. The public interest theory suggests that market failures can lead to suboptimal allocation of resources and that government intervention targeted at addressing the market failures can improve welfare. Although laissez-faire policy suggests that private sector coordination is optimal, public interest theory suggests that, in specific instances, the government can improve upon the market equilibrium by substituting for private sector coordination. The market enhancement theory takes a middle position. A conclusion of Ibragimov et al. (2009) is that government support in helping to reach a coordinated diversification equilibrium may play an important role in maintaining functioning markets for catastrophe insurance. Market failures can create suboptimal allocations of wealth and private sector coordination is not always effective. According to their reasoning, public policy helps to reach a coordinated diversification equilibrium. In the field of natural catastrophes or terrorism, we can already see that it is challenging to build up a market without the government (see, e.g., Lakdawalla and Zanjani (2005),Cummins (2006)). If a market does not exist, the government might need to cover the loss due to political pressure or to prevent the economy from collapsing. One example is the Corona pandemic, where (especially in economically strong countries) the taxpayer took over a significant portion of the loss. So, the government might provide early incentives to encourage market development to reduce costs for the taxpayer at the time of the event and protect the economy. We follow here the thoughts of Ibragimov et al. (2009).

Government's policy may depend on political, economic, social, or media factors. It may be difficult to predict (and could also be country-specific) what kind of help is supported by the broad public.[10] This could lead to investments that promise short-term survival but cost the taxpayer long-term. A prediction of actions could be derived using the model of Bryson et al. (2006), which describes that multiple parties in society - i.e., nonprofits, the media, the community, and the government - must work together to address challenges effectively and humanely. Besides, following Arrow and Lind (1970), at least economically strong countries are acting risk-neutral.

Generally, there are direct and indirect ways of action. Direct actions give money into the market, whereas indirect actions try to influence the market through a third party, e.g., general reduction of (minimum capital) requirements may support the development of a market. Also, the government could support the creation of alternative risk transfer instruments, e.g,

[10] For example, whether it is more likely that support for small and medium-sized companies or large companies is accepted.

captives.[11] Another issue is the perception of risk within the population. The government can impact this, e.g., through special information programs.[12] Viscusi (1995) shows that in some cases, the structure of how government actions feed into the risk-belief function is essential. So, determining the magnitude of the impact is also an important issue and not just the direction.[13] However, we do not discuss indirect measures here and focus on the direct measures.

Most of the criticism of government compensation focuses on ex-post compensation on an ad hoc basis, see, e.g., Epstein (1996) or Langendonck (2007). Following Harrington (2000), ex-post government relief reduces incentives to purchase insurance. Bruggeman et al. (2012) proposed a variety of forms for ex-ante government intervention, in contrast to the ex-post and ad-hoc approach. (1) the government can rely on the private insurance market (which means no government action is taken). (2) the government can offer compulsory insurance, which truncates the damage L (or in the case of fully comprehensive insurance, it eliminates the private insurance market). (3) the government can also provide catastrophe insurance itself or underwrite catastrophe losses through an additional layer of insurance, such as a reinsurer. Table 1 gives an overview of the government's actions we consider in this paper. Additionally, Bruggeman et al. (2012) thinks about new forms of government intervention, such as acting as a lender of last resort or a combination of the possibilities.

Table 1: Overview of government's actions considered in this paper

Government's action	time
Relying on private insurance market	-
Direct compensation of disaster victims (disaster relief)	ex-post
(Partly) compulsory insurance (disaster relief)	ex-ante
Providing coverage as a (re)insurer (risk pooling)	ex-ante

We denote with ω the government decision variable. Optimization problem 13 is adjusted to

$$\begin{aligned} (R^*, K^*) := \operatorname{argmax}_{R,K} \quad & U(R, K, \omega) \\ \text{subject to} \qquad & (\tilde{\pi}, E_0^{ins}, \omega) \in \tilde{G}^{ins} \\ & R \geq 0 \\ & K \geq R, \end{aligned} \tag{14}$$

[11]Captives allow (large) companies to access the reinsurance market and thus skip expensive primary insurers. Captives are especially interesting when there is a hardened (or no) market.

[12]Viscusi (1995) shows that biases in risk perception potentially have a major impact on insurance and risk behaviour. Slovic et al. (2016) indicates that disasters are systematically misjudged among the population.

[13]Misperceptions dramatically affect the trade-off between compensating differences and the magnitude of the loss, but only slightly affect the trade-off between compensating differences and the magnitude of the probability.

where $(\tilde{\pi}, E_0^{ins})$ solves the optimization problem for the premium. Summarized, we look for the strategy that maximizes the company's utility knowing the government's action ω.

Example 1. *(Government's decision)*
We denote with $A(I(\omega))$ the government's return on its action, where $I(\omega)$ is the government investment (e.g., a stimulus package in $t = 0$). We assume that the government makes an optimal decision concerning its action function, so $A_\omega = 0$ is fulfilled as a necessary condition and $A_{\omega,\omega} < 0$ as a sufficient condition. Accordingly, the structure of A is essential, but especially socioeconomic processes or political self-interests are difficult to represent mathematically. Looking only economic factors, as proposed in Arrow and Lind (1970), the action function can be defined as $A(I) = \psi(b, \omega) - I(\omega)$. b is the input of the company to the economy of the country and ψ weight this input (like a utility function). At this point, the importance of the government's economic situation becomes obvious. Economically constrained countries are limited in their investment opportunities. This can lead to the fact that the optimal strategy ω^* cannot be implemented since the related investment $I(\omega^*)$ is above the possibilities. Economically strong countries might not have these constraints. Furthermore, economically strong countries could tend to act in a risk-neutral way, see, e.g., Arrow and Lind (1970). A risk-neutral evaluation leads to $\psi(\cdot) = \mathbb{E}[\cdot]$. In contrast, economically weaker countries might evaluate $\psi(\cdot)$ differently. [14]

Example 2. *(disaster relief)*
Charpentier and Le Maux (2014) presents a model in which one insurer fully covers all claims. In the case of insolvency of the insurer, the government distributes the default loss ex-post over all policyholders, including those who have no losses, in the form of taxes. Each loss increases the tax, thus creating an expense for the victims and other policyholders. Orientated on the idea of Charpentier and Le Maux (2014), we introduce an ex-ante and ex-post disaster relief for the companies. If the loss exceed a limit of R^{rel}, the government pays all losses above directly to the companies. Only the loss up to R^{rel} remains in the free market, so R^{rel} implies an upper bound for K.[15] *Unlike risk pooling, companies do not pay a premium here; instead, they are automatically involved as part of the public economy (e.g., through taxes). R^{rel} could be set up ex-ante or ex-post. By ex-ante, the underlying heavy-tailed distribution is "truncated". The company remains responsible for its survival below this limit, e.g., with an insurance contract. Ex-post has the goal of absorbing the imbalance of some companies. The difference between the ex-ante and the ex-post strategy is the flow of information to the companies. In the case of ex-post relief, the company behaves as if there is no government (optimization problem 13) or relies on the government to cover the loss ex-post, which reduces the willingness to purchase coverage (see, e.g., Harrington (2000)). In the ex-ante relief, the company knows the limit R^{rel} at time $t = 0$ and can adjust the hedging accordingly. In $t = 1$, the payment from the government to the company is $Y(R^{rel}, \inf)$ with Y from equation 2. Generally, a too high R^{rel} leads to a company's insolvency since it cannot absorb the damage below; a too low R^{rel} prevents the formation of an insurance market.*

[14] For example, the survival of the company has high priority and thus $\psi(\cdot)$ weights these events more.
[15] R^{rel} can be seen as retention that remains in the free market.

Example 3. *(risk pooling)*
Lewis and Murdock (1996) propose an approach where the government expands private insurance capacity through a form of reinsurance with an excess of loss format. The program allows the private sector to "crowd out" the government if it can ensure that layer of catastrophe risk well. Here, a risk pool for the insurer would be, next to reinsurance and Cat Bond, a third possible layer. The insurer pays a premium Π^{pool} *to participate in the pool. The pool is activated when the insurer's loss extends* R^{pool}*. It must hold that* $R^{pool} > K^{cm}_{pre}$*, where* K^{cm}_{pre} *should be estimated as if no government aid is available (e.g., solving optimization problem 13). This ensures that the government does not undercut the free market. The pool pays all losses up to* K^{pool} *and has no default risk. This is similar to an excess of loss reinsurance contract with retention* $\{R^{pool}$*, cap* $K^{pool}\}$ *and the government's payout to the insurer is* $Y(R^{pool}, K^{pool})$ *with* Y *from equation 2. Section 2.3.1 need to be extend by this possibility. The activation of the pool and the strength of the effect are strongly dependent on the design of the previous layers (reinsurance and capital market) as well as on the premium (*Π^{pool}*).*

3 Empirical Analysis

We calibrate the model with real market data. Next to the loss distribution, we propose an adequate way to estimate the frictional and the risk costs for (re)insurance companies and the investors in the capital market.

3.1 Distributions

Natural Catastrophe

In progress

Pandemic

Chetty et al. (2020) provide weekly data for U.S. small businesses' revenue and consumer spending between January 2020 and January 2022. Based on the revenue, we build a loss index and estimate a generalized Pareto distribution, see Figure 1. We obtain a scale parameter of 22.1922 and a shape parameter of -0.00937.[16]

[16] To derive a loss distribution, we convert the data to a revenue index RI_t, with starting by $RI_0 = 100$ index points on 19 January 2020. Using the revenue index, we define a loss index LI_t through $LI_t = max(RI_0 - RI_t, 0)$ and normalize the loss index by $LI_t^{norm} = \frac{LI_t}{max(LI_t)}$.

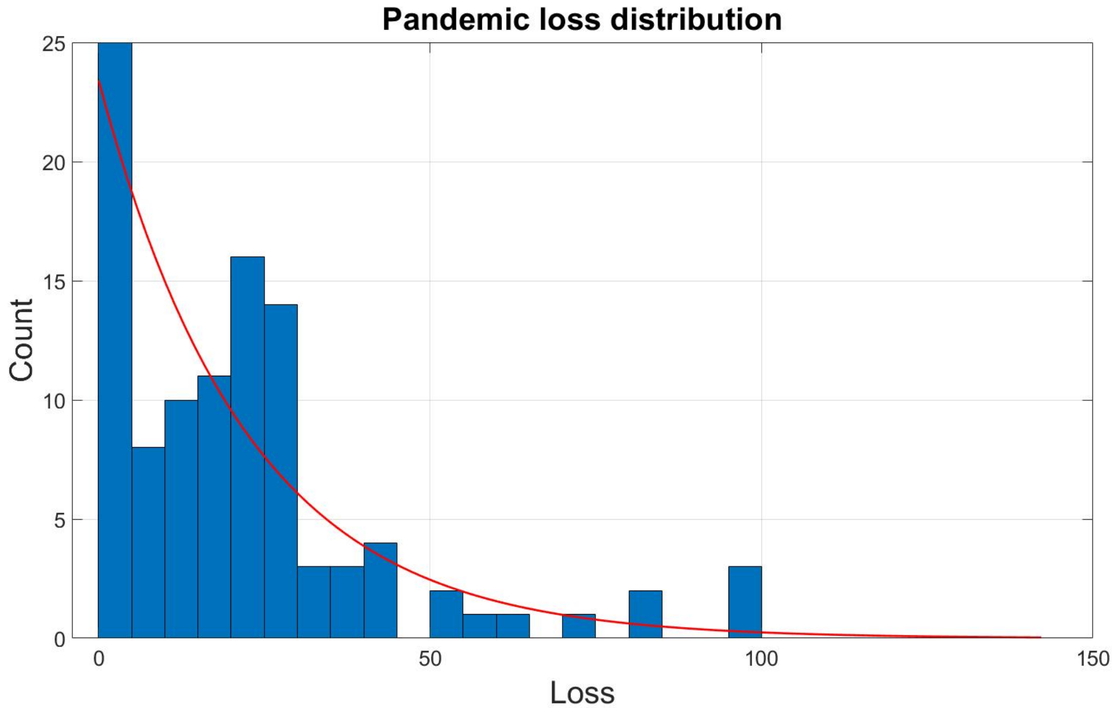

Figure 1: Loss distribution for pandemic risk

Cyber risk

In progress

3.2 Frictional costs of (re)insurer

We defined the (re)insurers cost as $c(E_0^i, Y) = \mathit{capital\ cost}(E_0^i, Y) + k^i$, where i is either the insurer or the reinsurer, and the capital costs as

$$\begin{aligned} \mathit{capital\ cost} &= frictional\ costs + risk\ costs \\ &:= r_f \cdot E_0^i + \alpha^i \cdot E_0^i + \delta \cdot \mu^{risk}(R^i) \cdot E_0^i \end{aligned}$$

with k^i as the operational costs.

We assume that everything that cannot be identified as a risk premium must be frictional costs, see, e.g. Bauer et al. (2013) or Braun et al. (2019b). We estimate the total cost of capital using the FF-3 model of Fama and French (1993) (see, e.g., Cummins and Phillips

(2005))[17] and define the intercept as the frictional costs through

$$r_{j,t} - r_{f,t} = \alpha_{j,fric} + \beta_{j,MKT}MKT_{j,t} + \beta_{j,SMB}SMB_{j,t} + \beta_{j,HML}HML_{j,t} + \epsilon_{j,t},$$

where $r_{j,t}$ is the stock return of (re)insurer j in time t, MKT is the excess market return, SMB is the difference between the average of the returns on small-stock portfolios and the average of the returns on big-stock portfolios and HML is the difference between the average of the returns on the high-BE/ME portfolios and the average of the returns on the low- BE/ME portfolios.[18] We verify the results with the FF-5 five factor model from Fama and French (2015) by adding the factors $RMW_{j,t}$ and $CMA_{j,t}$ where RMW is the difference between the returns on diversified portfolios of stocks with robust and weak profitability and RMW is the difference between the returns on diversified portfolios of the stocks of low and high investment firms (conservative and aggressive) and with a FF-5+ model by adding the $TERM$ and DEF factor from Fama and French (1993) to the FF-5 model.[19] Thereby $TERM$ is a premium computed as the difference between the monthly long-term government bond return and the one month Treasury bill rate measured at the end of the previous and DEF is the default premium computed difference between the return on a market portfolio of long-term corporate bonds and the long-term government bond return. Following Braun et al. (2019a), we compute $TERM$ through the monthly return on the Barclays U.S. Long-Term Government Bond Index in excess of the one-month T-Bill rate. DEF we compute as the difference between Bloomberg Barclays U.S. Corporate High Yield Bond Index and the Barclays U.S. Long-Term Government Bond Index. The other factors are downloaded from Kenneth French's website (see French (2022)) and the stock return are drawn from Thomson Reuters Eikon. Cochrane (2005) advice 240 months or 20 years as the shortest period to test a factor model. So, we retrieve monthly observation between July 1994 and December 2015 for the 20 largest listed P&C insurers in the U.S. and data of the same period for the 5 largest listed reinsurance companies worldwide. [20] Only companies for which at least 36 consecutive months of data are available are considered, which is standard practice, see e.g., Cummins and Phillips (2005). The estimation results are shown in Table 2 and Table 3. An overview of all estimated betas are in the Appendix B, Table 8 and Table 9 We estimate the frictional costs by calculating the mean of all significant intercepts and obtain for the insurer an annual α^{ins} of 7.553% and for the reinsurer an annual α^{re} of 6.1704%. Thus, our estimate for the insurer is in line with the results of Dal Moro (2008) and Swiss Re (2005), which measure costs of 6.57 % and about 7%, respectively.

[17]There are also models like Ben Ammar et al. (2018) that deal with the insurance-specific risk premiums (liability side). In our model, only this risk is considered since the assets of the (re)insurers are invested at the risk-free interest rate. Therefore, we filter out the market risk (asset side) in the calibration process by using the Fama French models.

[18]BE/ME is the ratio of book value of equity (BE) to market value of equity (ME).

[19]Thus, we use all known risk factors from Fama and French (1993) and Fama and French (2015)

[20]According to AM.Best (2021), the world's five largest reinsurers write almost half the gross premiums of the 50 largest reinsurers. Biener et al. (2017) mention that large reinsurers are characterized by high cost efficiency, while small reinsurers exhibit superior efficiency only when specialized. Therefore, to model an average of the large reinsurers, we focus on the largest five in terms of premium volume.

Our objective within the calibration is to show the model with an average (re)insurer. This is problematic since the markets are very heterogeneous. For example, Kielholz (2000) shows that the cost of capital varies strongly within European countries. Also in our analysis in Table 8 and Table 9, we see high variances in the estimated betas as well as the adjusted R^2. Another problem is shown by McKinsey (2022) in their study. Since 2015, insurers have struggled to earn their cost of capital. We can confirm this through our calibrations. If data until 2022 are used, we get negative or non frictions. To prevent this, we took data only until the end of 2015. Also, it has to be questioned whether a factor model can be applied. Roll (1977) already shows that the CAPM and all extensions are misleading and untestable. Therefore, this estimate is not satisfying, but there are no other methods to estimate the cost of capital yet, see Exley and Smith (2006).[21] The sensitivity tests in Tables 6 and 7 show that the frictional costs are stable and slight variations do not significantly impact the results of our model.

Table 2: Time series OLS results for insurers

		FF-3		FF-5		FF-5+	
	Insurer	α^{ins}	p-value	α^{ins}	p-value	α^{ins}	p-value
1	AMERICAN.INTL.GP.	-0.494	0.5054	0.2468	0.8648	0.0197	0.9887
2	ALLEGHANY	0.3321	0.1695	0.3271	0.2051	0.3181	0.2325
3	ALLSTATE.ORD.SHS	0.1211	0.6641	-0.1773	0.5086	-0.3435	0.3183
4	ARCH.CAP.GP.	0.6235	0.0768	0.3691	0.3293	0.1509	0.679
5	BERKSHIRE.HATHAWAY.A.	0.5142	0.0611	0.5209	0.0903	0.4965	0.0883
6	CHUBB	0.5921	0.1444	0.4342	0.3345	0.4362	0.3374
7	CINCINNATI.FINL.	0.3166	0.341	0.008	0.9797	0.0073	0.9815
8	CNA.FINANCIAL	-0.5513	0.1336	-0.5111	0.2248	-0.6213	0.1578
9	ERIE.INS.GROUP	0.3391	0.4383	0.0197	0.9592	-0.1228	0.7794
10	EVEREST.RE.GP.	0.6113	0.1005	0.3329	0.3913	0.2063	0.5891
11	FAIRFAX.FINL.HDG.	0.7434	0.3257	0.5282	0.5437	0.2931	0.752
12	HARTFORD.FINL.SVS.GP.	-0.1665	0.7618	-0.2016	0.7451	-0.6187	0.3029
13	KEMPER	-0.1353	0.676	-0.3072	0.3632	-0.2955	0.3984
14	MARKEL	0.7504	0.0035	0.4754	0.0694	0.4818	0.0959
15	MERCURY.GENERAL	0.2148	0.6065	-0.1089	0.7955	-0.2273	0.5561
16	OLD.REPUBLIC.INTL.	0.2076	0.6205	0.0529	0.8965	-0.2069	0.5933
17	PROGRESSIVE.OHIO	0.5695	0.1505	0.3553	0.3986	0.3175	0.4882
18	SELECTIVE.IN.GP.	0.2219	0.5446	-0.1279	0.7312	-0.1598	0.6479
19	TRAVELERS.COS.	0.3843	0.2746	0.0558	0.8806	0.04	0.916
20	W.R.BERKLEY	0.6856	0.1213	0.2182	0.634	0.1119	0.8034
	Mean	0.6294		0.4981		0.4892	

[21]The goal of the paper is to find a general model for the interactions between the global players. Due to the lack of alternatives, this approach is therefore acceptable.

Table 3: Time series OLS results for reinsurers

		FF-3		FF-5		FF-5+	
	Reinsurer	α^{re}	p-value	α^{re}	p-value	α^{re}	p-value
1	MUENCHENER.RUCK.	-0.003	0.9949	-0.1368	0.7945	-0.057	0.9071
2	SWISS.RE	-0.2163	0.6358	-0.3892	0.4029	-0.4256	0.3914
3	HANNOVER.RUECK	0.4609	0.3505	0.2118	0.6605	0.3082	0.5386
4	SCOR.SE	-0.4574	0.4704	-0.8329	0.2015	-0.8329	0.1783
5	BERKSHIRE.HATHAWAY.A.	0.5142	0.0611	0.5209	0.0903	0.4965	0.0883
	Mean	0.5142		0.5209		0.4965	

3.3 Frictional costs of the investors in the capital market

We need to find the spread to compute the premium the insurer pays to the investors. Usually the Cat Bond is sold at par, thus $CB = N$ holds and we convert equation 11 to

$$\begin{aligned} & \frac{1}{1+\gamma} sN = N - \frac{1+r_f}{1+\gamma} N + \frac{1}{1+\gamma} \mathbb{E}[Y^{cm}]. \\ \Leftrightarrow \quad & sN = (1+\gamma)N - (1+r_f)N + \mathbb{E}[Y^{cm}] \\ \Leftrightarrow \quad & s = 1 + \gamma - (1+r_f) + \frac{\mathbb{E}[Y^{cm}]}{N} \\ \Leftrightarrow \quad & s = (\gamma - r_f) + \frac{\mathbb{E}[Y^{cm}]}{N}, \end{aligned}$$

where $\gamma - r_f$ is the risk premium and $\frac{\mathbb{E}[Y^{cm}]}{N}$ the expected loss ratio. Once γ is determined, the premium Π^{cm} can be computed. The risk-adjusted discount rate is normally a composition of a frictional term and a risk term which cannot be explained by the approach from Zanjani (2002). Because Cat Bonds have virtually no default risk (100% collateral from U.S. T-bills, taxes at the investor level), the market observed γ contain a large proportion of frictions. Thus, we need a solid model to estimate the frictional term. Since Cat Bond rates are historically observable, a pragmatic approach is to calculate

$$\begin{aligned} & \gamma - r_f = frictions + risk \\ \Leftrightarrow \quad & frictions = (\gamma - r_f) - risk \end{aligned}$$

with the assumption that frictions (further noted as α^{cm}) must cause everything that cannot be explained as a risk premium. Unfortunately, for extreme events such as pandemics and cyber risk exist not enough sufficient market data for a risk premium estimation yet. We use Cat Bond returns as a basis and estimate a lower limit for the frictions[22]. In general, there are two possible estimation approaches:

1. A classical CAPM or APT-style factor model.
2. A consumption-based model.

[22]For the extreme events like a pandemic or cyber, this term might be higher

From an economic point of view, the second one might be preferable since it can consider a downside risk or rare disaster risk extension as done in Dionne et al. (2018) or Wachter (2013). However, empirical implementation becomes problematic. Even if the correlations between Log Consumption Growth with quarterly data and ILS Returns can be estimated, we get values close to zero (and the resulting Euler equation will be zero).[23] A downside risk measure or a rare event risk process will be practically impossible to estimate empirically because of the lack of high-frequency data. It might therefore be preferable to use an APT factor model.

We choose the factor model from Fama and French (1993) with the additional factors $TERM$ and DEF and compute[24]

$$\begin{aligned} r_{ILS,t} - r_{f,t} =& \alpha^{cm} + \beta_{MKT} MKT_t + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{TERM} TERM_t + \\ & \beta_{DEF} DEF_t + \epsilon_t. \end{aligned}$$

For the ILS Returns, we use the AON Total Return Index from January 2001 to December 2020 and verify the results with the Swiss RE Total Return Index from February 2002 to December 2020. Both indices were downloaded from Thomson Reuters Eikon. The results are shown in Table 4. We estimate for the investors at the capital market annual frictions of 4.5%

[23]For the calculation we used the log growth rate of the Personal Consumption Expenditures, provided by fred.stlouisfed.org and the AON Cat Bond return index, quarterly from the 2002 to the end 2020. In total, we used 76 data points. We get a correlation coefficient of 0.023957.

[24]Empirical studies such as Braun et al. (2019a) have shown that the TERM and DEF factors are significant for ILS fund returns.

Table 4: Results for the ILS Index

	Dependent variable:	
	Cat Bond Indices	
	AON Index	SWISS RE Index
MKT	0.017	−0.002
	(0.018)	(0.021)
SMB	−0.009	−0.034
	(0.023)	(0.027)
HML	0.022	0.010
	(0.019)	(0.024)
TERM	0.105^{**}	0.172^{***}
	(0.052)	(0.058)
DEF	0.059^{**}	0.103^{***}
	(0.028)	(0.032)
Constant	0.375^{***}	0.406^{***}
	(0.057)	(0.064)
Observations	240	226
R^2	0.068	0.084
Adjusted R^2	0.048	0.063
Residual Std. Error	0.842 (df = 234)	0.901 (df = 220)
F Statistic	3.395^{***} (df = 5; 234)	4.036^{***} (df = 5; 220)
Note:		$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

3.4 Risk costs

Natural Catastrophe

For catastrophe losses, we assume that losses are uncorrelated with the capital market (see, e.g.,Cummins and Harrington (1985), Froot et al. (1995) , Zanjani (2002)). Therefore, for the (re)insurer $\mu^{risk} = 0$ holds and we define the cost of capital as

$$capital\ cost = \alpha^i \cdot E_0^i \qquad i \in \{insurer, reinsurer\}.$$

For the Cat Bond index, we found risk premiums given by the $TERM$ and DEF factors. Thus, $\mu^{risk} = \beta_{TERM} TERM_T + \beta_{DEF} DEF_T$ holds and the risk-adjusted discount rate is

$$\mathbb{E}[\gamma - r_f] = \alpha^{cm} + \beta_{TERM}\mathbb{E}[TERM] + \beta_{DEF}\mathbb{E}[DEF].$$

Pandemic

To estimate the pandemic risk premium we choose a consumption-based approach. Following Braun et al. (2019b), the consumption-based model calculates the expected excess return via the Euler equation which we modify lightly to[25]

$$\mu^{risk} = \lambda(R) \cdot \rho(r_c, r_{loss}) \cdot \sigma(r_c) \cdot \sigma(r_{loss}) \cdot \eta,$$

where $\lambda(R)$ is a weighting function with $\lambda(0) = 1$, ρ the correlation coefficient, r_c the consumption rate, r_{loss} the loss rate, σ the standard deviations and η the relative risk aversion coefficient.[26]

We use again the data provided by Chetty et al. (2020). For the consumption rate, we use the weekly consumer spending between January 2020 and January 2022 and for the loss rate, we use the weekly revenue data for U.S. small businesses between January 2020 and January 2022. For both rates we compute the log growth, see Figure 2.

[25] If returns are too small, a consumption-based approach with a downside risk estimator like in Dionne et al. (2018) may be a better choice.

[26] η is the market risk coefficient derived from the power utility function. We will use a different utility function in the Empirical Analysis. This is still consistent, since each function matches other players

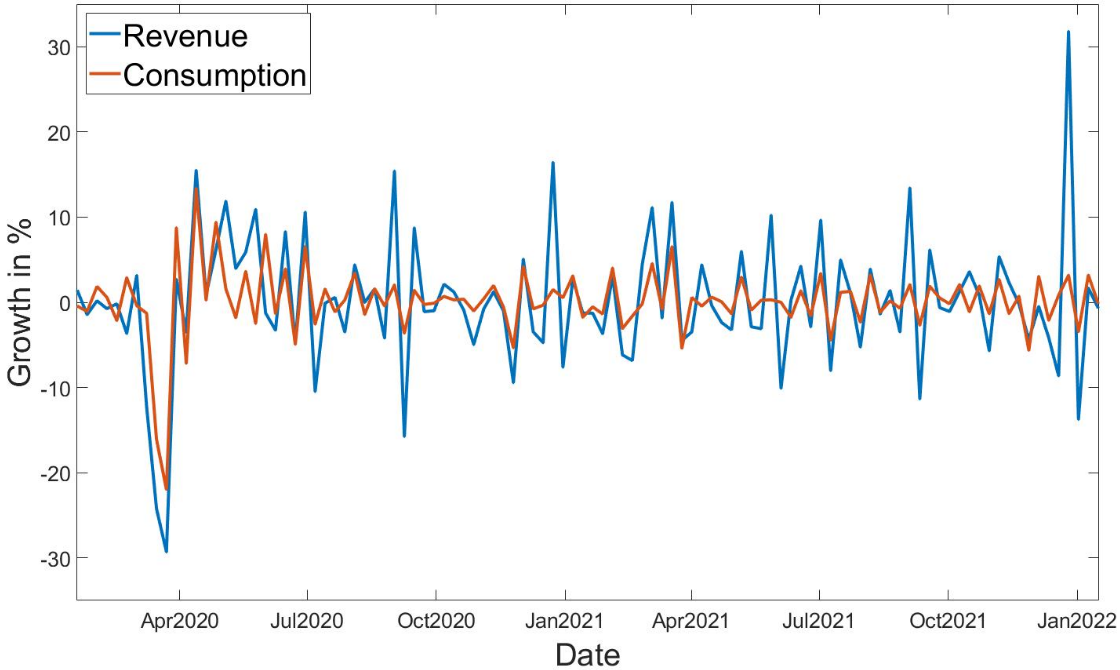

Figure 2: The growth in consumption and revenue between January 2020 and January 2022

While η is a constant factor for the general risk aversion, λ can be viewed as a higher order risk preference, see, e.g., Deck and Schlesinger (2010), Ebert (2013), Deck and Schlesinger (2014). Thereby events which occur rarely should be weighted higher, the correlation does not decrease linearly in retention R. For illustration, a linear λ and a concave λ are given in Figure 3.

We define the (re)insurers capital costs as

$$\begin{aligned} capital\ cost &= \alpha^i \cdot E_0^i + \lambda(R^i) \cdot \rho(r_c, r_{loss}) \cdot \sigma(r_c) \cdot \sigma(r_{loss}) \cdot \eta \cdot Y^i(R^i, K^i) \qquad i \in \{insurer, reinsurer\} \\ &= \alpha^i \cdot E_0^i + \delta \cdot \mu^{risk} \cdot Y^i(R^i, K^i) \end{aligned}$$

and the risk-adjusted discount rate as

$$\begin{aligned} \mathbb{E}[\gamma - r_f] &= \alpha^{cm} + \lambda(R^{cm}) \cdot \rho(r_c, r_{loss}) \cdot \sigma(r_c) \cdot \sigma(r_{loss}) \cdot \eta \\ &= \alpha^{cm} + \mu^{risk}. \end{aligned}$$

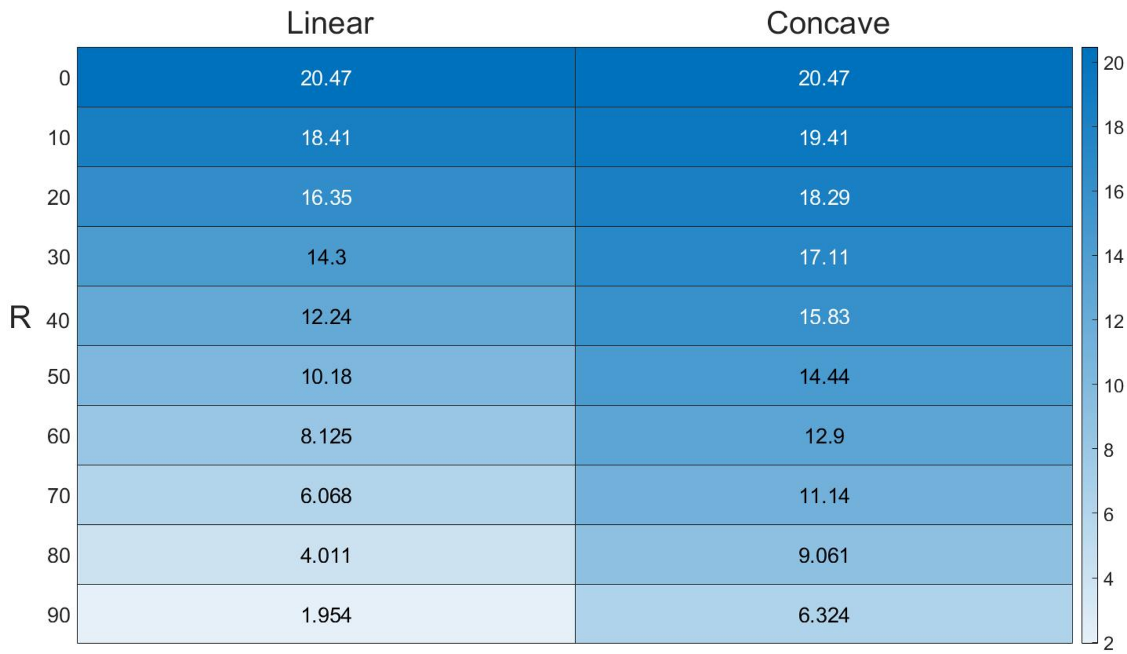

Figure 3: Consumption matrix μ^{risk} for $\eta = 1$. R corresponds to the % quantile of the loss distribution

Cyber risk

In progress

3.5 Results

We consider the exponential utility function

$$U(E) = 1 - exp(-\iota \cdot E)$$

which is a special form of the Expo-power utility from Saha (1993). According to Cerreia-Vioglio et al. (2015), this function has been applied in a variety of fields, such as finance, intertemporal choices, and agriculture economics (including crop insurance). Also in the general insurance frame it is used, see, e.g., Freifelder (1979). Holt and Laury (2002) find that this function is a good fit for data that includes both low and high stakes. An overview over all variables is in Table 5.

Table 5: Initial values

Variable	Symbol	Value
Company's initial equity	E_0^{com}	50
(Re) insurers operation cost	k	0
Risk free rate	r_f	1%
Company's growth rate	r_{com}	2.5%
Frictions insurer	α^{ins}	7.553%
Frictions reinsurer	α^{re}	6.1704%
Frictions capital market	α^{cm}	4.5%
Risk coefficient	ι	0.15
Risk coefficient market	η	1
Diversification factor	δ	1
Simulations	-	$10'000$

Natural Catastrophe

In progress

Pandemic

Figure 4, left side up, show the premium paid by the company and right side up, respectively, the company's utility without reinsurance, capital market and government intervention. The middle shows the absolute difference when the reinsurer and the capital market enter the market and down is the fully hedged market. It can be seen that when the insurer can pass on part of the risk, the premium decreases, and thus the utility of the company increases. In our example, if the insurer does not hedge, the company chooses the coverage with $R^* = 10$ and $K^* = 80$ and pays $\pi = 20.1641$. If the insurer hedges the company chooses the contract with $R^* = 10$ and $K^* = 85$ and the premium decreases, although the protection increases, to $\pi = 19.2209$. In general, the utility for the company first increases with K, and then decreases slightly. The company has a high utility in covering part of the tail, but at the same time wants to to have protection for small losses. Accordingly, the very rare losses above K^* are no factor in the coverage decision. By including hedging, especially the premiums with a high K decrease, and accordingly the utility rises in this area. Nevertheless, it is still not enough to choose a higher cap, but the utilities move closer together. In our example, the company would be insolvent in 88 out of $10'000$ simulations, which corresponds approximately to an event which occurs one time in 100 years. Assuming that the government bears all losses where $L > K^*$, the government has average expenses of 17.5346. Now we suppose that the government announced ex-ante that it pays all damages which are 2.5 times the size of the company's equity, meaning $L > 125$. In that case, the company chooses a K^* of 110, which is an increase of almost 30%, and the governments expenses decreased by around 7.5% to an average of 16.2468. This example shows that if the government acts early, more risk retains in the market and government spending can be reduced.

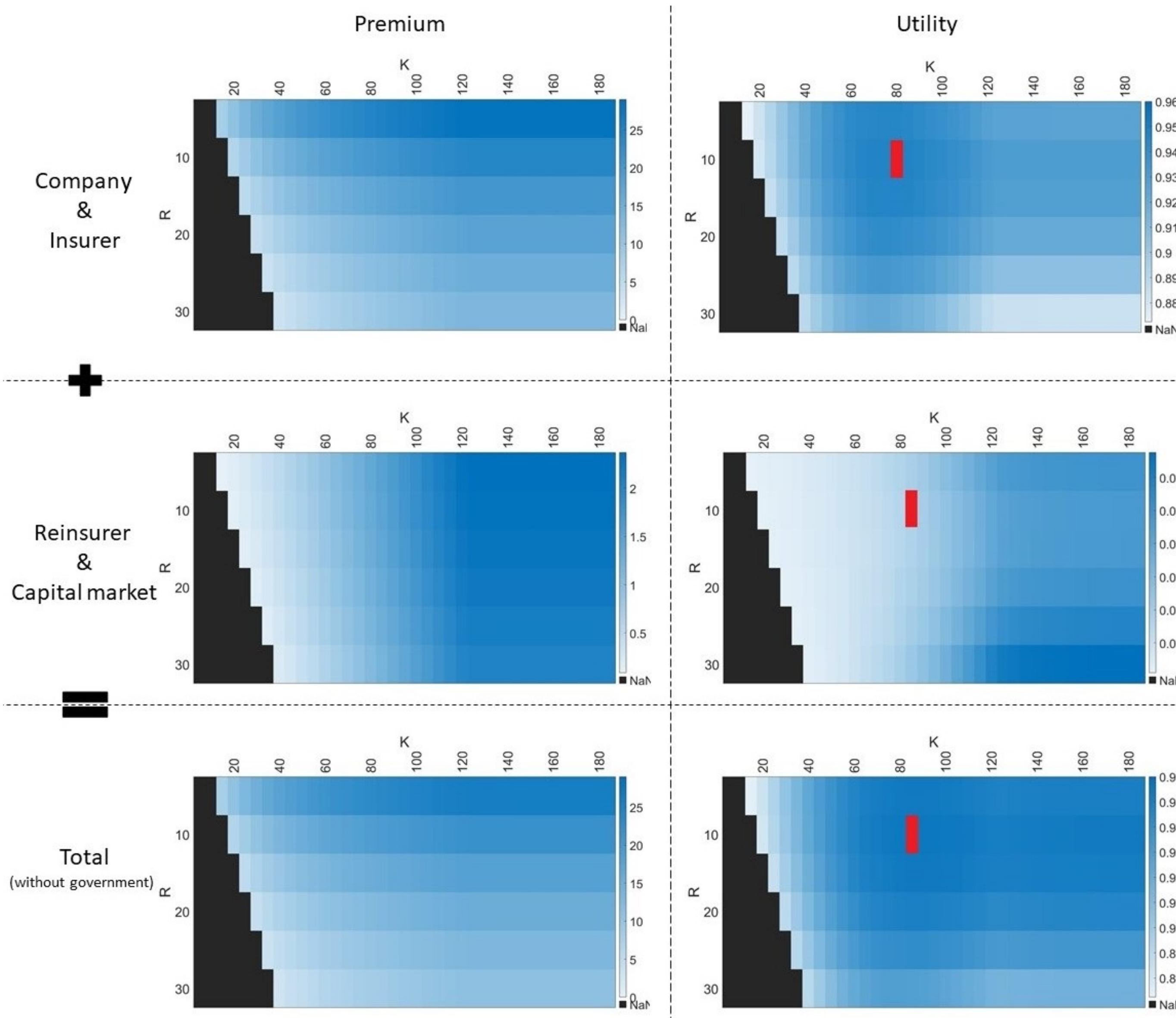

Figure 4: Premium and utility for different R and K. In red, the best choice.
Upper part; The market without insurer's hedging; Middle part: Market change if the insurer change; Down: Total market without government

Cyber risk

In progress

4 Conclusion

Against the background of the worldwide coronavirus, we analyze in this paper how (global) extreme events can be insured. These events can be identified mainly by a heavy tailed distribution and a correlation to the capital market. Based on the fundamental literature as for example Zanjani (2002), we derive a theoretical framework that includes a company, the insurer and furthermore interactions with a reinsurer, the investors in the capital market and the government. Finally, we calibrate this model to actual market data and show that the offer of the (re)insurer is strongly affected by the cost of capital. In particular, a correlation of the risk to the capital market drastically amplifies this effect. At the same time, investors in the capital market expect a high return. These aspects make the purchase of extreme event coverage prohibitively expensive or unattractive for a company. Finally, it is shown that, particularly for the most extreme events, government backstops in the highest loss layers are necessary for a private insurance market to share heavy tail risk in the first place.

A Proofs

A.1 Reforming premium

We first need to rewrite

$$
\begin{aligned}
\mathbb{E}[D^{ins}] &= \mathbb{E}[max\big(0, Y - \mathbb{E}[Y] - \mathbb{E}[E_1^{ins}]\big)] \\
&= \mathbb{E}[max\big(0, Y - \mathbb{E}[Y] - (1+r_f)(E_0^{ins} + \pi - c) + \mathbb{E}[Y]] \\
&= \mathbb{E}[max\big(0, Y - (1+r_f)(E_0^{ins} + \pi - c)] \\
&= \mathbb{E}[\mathbb{1}_{Y+(1+r_f)c>(1+r_f)(E_0^{ins}+\pi)}(Y - (1+r_f)(E_0^{ins} + \pi - c)] \\
&= \mathbb{E}[\mathbb{1}_{Y+(1+r_f)c>(1+r_f)(E_0^{ins}+\pi)}Y] - \mathbb{P}\Big(Y + (1+r_f)c > (1+r_f)(E_0^{ins} + \pi)\Big)(1+r_f)(E_0^{ins} + \pi - c)
\end{aligned}
$$

So, according to the definition of the premium we get

$$
\begin{aligned}
\pi &= B_0\big(\mathbb{E}[Y] - \mathbb{E}[D^{ins}]\big) + c \\
&= B_0\Big(\mathbb{E}[Y] - \mathbb{E}[\mathbb{1}_{Y+(1+r_f)c>(1+r_f)(E_0^{ins}+\pi)}Y] + \\
&\quad \mathbb{P}\big(Y + (1+r_f)c > (1+r_f)(E_0^{ins} + \pi)\big)(1+r_f)(E_0^{ins} + \pi - c)\Big) + c \\
&= B_0\big(\mathbb{E}[Y] - \mathbb{E}[\mathbb{1}_{Y+(1+r_f)c>(1+r_f)(E_0^{ins}+\pi)}Y]\big) + \mathbb{P}\big(Y + (1+r_f)c > (1+r_f)(E_0^{ins} + \pi)\big)(E_0^{ins} + \pi - c) + c.
\end{aligned}
$$

We rerange after the premium and obtain

$$
\begin{aligned}
\pi - \mathbb{P}\Big(Y + (1+r_f)c > (1+r_f)(E_0^{ins} + \pi)\Big)\pi &= B_0\big(\mathbb{E}[Y] - \mathbb{E}[\mathbb{1}_{Y+(1+r_f)c>(1+r_f)(E_0^{ins}+\pi)}Y]\big) \\
&\quad + \mathbb{P}\Big(Y + (1+r_f)c > (1+r_f)(E_0^{ins} + \pi)\Big)E_0^{ins} \\
&\quad - \mathbb{P}\Big(Y + (1+r_f)c > (1+r_f)(E_0^{ins} + \pi)\Big)c + c
\end{aligned}
$$

We remember that $1 - \mathbb{P}\Big(Y + (1+r_f)c > (1+r_f)(E_0^{ins} + \pi)\Big) = \mathbb{P}\Big(Y + (1+r_f)c \leq (1+r_f)(E_0^{ins} + \pi)\Big)$, so

$$
\begin{aligned}
\mathbb{P}\Big(Y + (1+r_f)c \leq (1+r_f)(E_0^{ins} + \pi)\Big)\pi &= B_0\big(\mathbb{E}[Y] - \mathbb{E}[\mathbb{1}_{Y+(1+r_f)c>(1+r_f)(E_0^{ins}+\pi)}Y]\big) \\
&\quad + \mathbb{P}\Big(Y + (1+r_f)c > (1+r_f)(E_0^{ins} + \pi)\Big)E_0^{ins} \\
&\quad + \mathbb{P}\Big(Y + (1+r_f)c \leq (1+r_f)(E_0^{ins} + \pi)\Big)c
\end{aligned}
$$

and we get

$$\pi = \frac{B_0\Big(\mathbb{E}[Y] - \mathbb{E}[\mathbb{1}_{Y+(1+r_f)c>(1+r_f)(E_0^{ins}+\pi)}Y]\Big)}{\mathbb{P}\Big(Y + (1+r_f)c \le (1+r_f)(E_0^{ins}+\pi)\Big)} + \frac{\mathbb{P}\Big(Y + (1+r_f)c > (1+r_f)(E_0^{ins}+\pi)\Big)E_0^{ins}}{\mathbb{P}\Big(Y + (1+r_f)c \le (1+r_f)(E_0^{ins}+\pi)\Big)}$$
$$+ \frac{\mathbb{P}\Big(Y + (1+r_f)c \le (1+r_f)(E_0^{ins}+\pi)\Big)c}{\mathbb{P}\Big(Y + (1+r_f)c \le (1+r_f)(E_0^{ins}+\pi)\Big)}.$$

We shorten this term to

$$\pi = \frac{B_0\Big(\mathbb{E}[Y] - \mathbb{E}[\mathbb{1}_{Y+(1+r_f)c>(1+r_f)(E_0^{ins}+\pi)}Y]\Big) + E_0^{ins}}{\mathbb{P}\Big(Y + (1+r_f)c \le (1+r_f)(E_0^{ins}+\pi)\Big)} - E_0^{ins} + c$$

We see, there is no closed form for π since it occurs always on both sides.

A.2 Company optimal solution

First order conditions leads to $\nabla\mathbb{E}[U(E_1^{com}(R,K))] = \mathbf{0}$, so $\frac{\mathbb{E}[U(E_1^{com}(R,K))]}{dR} = 0$ and $\frac{\mathbb{E}[U(E_1^{com}(R,K))]}{dK} = 0$.

A.2.1 Case R

$$\begin{aligned}\frac{d\mathbb{E}[U(E_1^{com})]}{dR} &= \mathbb{E}[U_{E_1^{com}}\frac{dE_1^{com}}{dR}] \\ &= \mathbb{E}[U_{E_1^{com}}(-(1+r_{com})\frac{d\pi}{dR} + \frac{dY}{dR} - \frac{dD^{ins}}{dR}))\big).\end{aligned}$$

We can compute

$$\frac{dY}{dR} = \begin{cases} 0 & \text{if } L < R \\ -1 & \text{if } R \le L \le K \\ -1 & \text{if } L > K \end{cases},$$

so $\frac{dY}{dR} = -\mathbb{1}_{(L\ge R)}$. This leads to

$$\begin{aligned}\frac{\mathbb{E}[dU]}{dR} &= \mathbb{E}[U_{E_1^{com}}(-(1+r_{com})\frac{d\pi}{dR} - \frac{dD^{ins}}{dR} - \mathbb{1}_{(L\ge R)})] \\ &= \mathbb{E}[-(1+r_{com})U_{E_1^{com}}\frac{d\pi}{dR}] - \mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dR}] - \mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L\ge R)}]\end{aligned}$$

$(1+r_{com})\frac{d\pi}{dR}$ is a deterministic term, so

$$\begin{aligned}\frac{\mathbb{E}[dU]}{dR} &= -(1+r_{com})\frac{d\pi}{dR}\mathbb{E}[U_{E_1^{com}}] - \mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dR}] - \mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L\geq R)}]\\ &= 0.\end{aligned}$$

This expression we transform to

$$\mathbb{E}[U_{E_1^{com}}] = \frac{-\mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L\geq R)}] - \mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dR}]}{(1+r_{com})\frac{d\pi}{dR}}.$$

A.2.2 Case K

$$\begin{aligned}\frac{d\mathbb{E}[U(E_1^{com})]}{dK} &= \mathbb{E}[U_{E_1^{com}}\frac{dE_1^{com}}{dK}]\\ &= \mathbb{E}[U_{E_1^{com}}(-(1+r_{com})\frac{d\pi}{dK} + \frac{dY}{dK} - \frac{dD^{ins}}{dK})].\end{aligned}$$

We compute

$$\frac{dY}{dK} = \begin{cases} 0 & \text{if } L < R \\ 0 & \text{if } R \leq L \leq K \\ 1 & \text{if } L > K \end{cases}$$

so $\frac{dY}{dK} = \mathbb{1}_{(L>K)}$. This leads to

$$\begin{aligned}\frac{\mathbb{E}[dU]}{dK} &= \mathbb{E}[U_{E_1^{com}}(-(1+r_{com})\frac{d\pi}{dK} - \frac{dD^{ins}}{dK} + \mathbb{1}_{(L>K)})]\\ &= \mathbb{E}[-(1+r_{com})U_{E_1^{com}}\frac{d\pi}{dK}] - \mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dK}] + \mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L>K)}]\end{aligned}$$

$\frac{d\pi}{dK}$ is a deterministic term, so

$$\begin{aligned}\frac{\mathbb{E}[dU]}{dK} &= -(1+r_{com})\frac{d\pi}{dK}\mathbb{E}[U_{E_1^{com}}] - \mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dK}] + \mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L>K)}]\\ &= 0.\end{aligned}$$

This expression we transform to

$$\mathbb{E}[U_{E_1^{com}}] = \frac{\mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L>K)}] - \mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dK}]}{(1+r_{com})\frac{d\pi}{dK}}.$$

A.3 Insurer optimal solution

First order conditions leads to $\nabla\mathbb{E}[U^{ins}(E_1^{ins})] = \mathbf{0}$, so $\mathbb{E}[\frac{dU^{ins}(E_1^{ins})}{dR}] = 0$ and $\mathbb{E}[\frac{dU^{ins}(E_1^{ins})}{dK}] = 0$.

A.3.1 Case R

$$\begin{aligned}\mathbb{E}[\frac{dU^{ins}(E_1^{ins})}{dR}] &= \mathbb{E}[U^{ins}_{E_1^{ins}}\frac{dE_1^{ins}}{dR}] \\ &= \mathbb{E}[U^{ins}_{E_1^{ins}}\big((1+r_f)(\frac{d\pi}{dR} - \frac{dc}{dR}) - \frac{dY}{dR}\big)]\end{aligned}$$

We compute

$$\frac{dY}{dR} = \begin{cases} 0 & \text{if } L < R \\ -1 & \text{if } R \leq L \leq K \\ -1 & \text{if } L > K \end{cases},$$

so $\frac{dY}{dR} = -\mathbb{1}_{(L\geq R)}$. This leads to

$$\begin{aligned}\mathbb{E}[\frac{dU^{ins}(E_1^{ins})}{dR}] &= \mathbb{E}[U^{ins}_{E_1^{ins}}\big((1+r_f)(\frac{d\pi}{dR} - \frac{dc}{dR}) + \mathbb{1}_{(L\geq R)}\big)] \\ &= \mathbb{E}[U^{ins}_{E_1^{ins}}(1+r_f)(\frac{d\pi}{dR} - \frac{dc}{dR})] + \mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L\geq R)}] \\ &= 0,\end{aligned}$$

what we reform to

$$\mathbb{E}[U^{ins}_{E_1^{ins}}] = \frac{-\mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L\geq R)}]}{(1+r_f)(\frac{d\pi}{dR} - \frac{dc}{dR})}$$

A.3.2 Case K

$$\begin{aligned}\mathbb{E}[\frac{dU^{ins}(E_1^{ins})}{dK}] &= \mathbb{E}[U^{ins}_{E_1^{ins}}\frac{dE_1^{ins}}{dK}] \\ &= \mathbb{E}[U^{ins}_{E_1^{ins}}\big((1+r_f)(\frac{d\pi}{dK} - \frac{dc}{dK}) - \frac{dY}{dK}\big)]\end{aligned}$$

We compute

$$\frac{dY}{dK} = \begin{cases} 0 & \text{if } L < R \\ 0 & \text{if } R \leq L \leq K \\ 1 & \text{if } L > K \end{cases},$$

so $\frac{dY}{dK} = \mathbb{1}_{(L>K)}$. This leads to

$$
\begin{aligned}
\mathbb{E}[\frac{dU^{ins}(E_1^{ins})}{dK}] &= \mathbb{E}[U^{ins}_{E_1^{ins}}\big((1+r_f)(\frac{d\pi}{dK} - \frac{dc}{dK}) - \mathbb{1}_{(L>K)}] \\
&= \mathbb{E}[U^{ins}_{E_1^{ins}}(1+r_f)(\frac{d\pi}{dK} - \frac{dc}{dK})] - \mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L>K)}] \\
&= 0,
\end{aligned}
$$

what we reform to

$$
\frac{d\pi}{dK}\mathbb{E}[U^{ins}_{E_1^{ins}}] = \frac{\mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L>K)}]}{(1+r_f)(\frac{d\pi}{dK} - \frac{dc}{dK})}
$$

A.4 Proof Lemma 1

Proof. We set $r_f = 0$ and $r_{com} = 0$. We assume, we found the optimal R and K for the insurer. We can reform the insurance part to

$$
\begin{aligned}
\frac{d\pi}{dR} &= -\frac{\mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L\geq R)}]}{\mathbb{E}[U^{ins}_{E_1^{ins}}]} + \frac{dc}{dR} \\
\frac{d\pi}{dK} &= \frac{\mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L>K)}]}{\mathbb{E}[U^{ins}_{E_1^{ins}}]} + \frac{dc}{dK}
\end{aligned}
$$

We put this in into the equation for the company and reform it to

$$
\begin{aligned}
\frac{\mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L\geq R)}]}{\mathbb{E}[U^{ins}_{E_1^{ins}}]} - \frac{dc}{dR} &= \frac{\mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L\geq R)}] + \mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dR}]}{\mathbb{E}[U_{E_1^{com}}]}, \\
\frac{\mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L>K)}]}{\mathbb{E}[U^{ins}_{E_1^{ins}}]} + \frac{dc}{dK} &= \frac{\mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L>K)}] - \mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dK}]}{\mathbb{E}[U_{E_1^{com}}]},
\end{aligned}
$$

We reform again to

$$
\begin{aligned}
\frac{\mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L\geq R)}]}{\mathbb{E}[U^{ins}_{E_1^{ins}}]} - \frac{\mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L\geq R)}]}{\mathbb{E}[U_{E_1^{com}}]} &= \frac{\mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dR}]}{\mathbb{E}[U_{E_1^{com}}]} + \frac{dc}{dR}, \\
\frac{\mathbb{E}[U^{ins}_{E_1^{ins}}\mathbb{1}_{(L>K)}]}{\mathbb{E}[U^{ins}_{E_1^{ins}}]} - \frac{\mathbb{E}[U_{E_1^{com}}\mathbb{1}_{(L>K)}]}{\mathbb{E}[U_{E_1^{com}}]} &= -\frac{\mathbb{E}[U_{E_1^{com}}\frac{dD^{ins}}{dK}]}{\mathbb{E}[U_{E_1^{com}}]} - \frac{dc}{dK},
\end{aligned}
$$

Since on the left side the terms are normalized in each case, and the indicator terms are identical, the result equals always 0. If we look at the right side, it equals zero only if both terms are identical with different signs or equal to zero. According to the definition, $U_{E_1^{com}}$

is always positive. With increasing R the insurer takes less risk, therefore his default risk decreases, so $\frac{dD^{ins}}{dR} \leq 0$. With the same logic follows $\frac{dD^{ins}}{dK} \geq 0$. Therefore, in case R the first term is always negative or equal to zero, and in case K it is greater than or equal to zero. If less risk is transferred, the costs decrease, with more risk they increase. Accordingly $\frac{dc}{dR} < 0$ and $\frac{dc}{dK} > 0$. It follows that on the right side both terms have the same sign, so they cannot add up to zero, and at least the change of the costs is always greater than 0. Therefore the equation is violated. □

B Additional Figures and Tables

Table 6: Sensitivity test for insurer

α^{ins}	5%	7.553%	10 %
R^*	10	10	10
K^*	90	80	70
π^*	19.6849	20.3709	20.3923

Table 7: Sensitivity test for (re)insurer; return variables are R^*, K^*,π^*

		α^{ins}		
		5%	7.553%	10 %
α^{re}	5%	10/90/19.6849	10/90/19.6849	10/90/19.6849
	6.1704 %	10/95/19.9415	10/85/20.0392	10/85/20.0392
	10 %	15/95/16.0639	10/85/20.4651	10/70/20.3923

Table 8: Time series OLS results for insurers, all betas

	α^{ins}	MKT-RF	SMB	HML	RMW	CMA	TERM	DEF	adj. R^2
1	-0.494	2.0393 ***	-0.4443 .	1.6153 *	-	-	-	-	0.2014
2	0.3321	0.4092 ***	0.1242	0.3346 **	-	-	-	-	0.1801
3	0.1211	1.0479 ***	-0.4114 ***	1.0832 ***	-	-	-	-	0.4671
4	0.6235 .	0.2667 **	0.4658 **	0.3388 *	-	-	-	-	0.1175
5	0.5142 .	0.6372 ***	-0.5357 ***	0.4484 ***	-	-	-	-	0.3184
6	0.5921	0.8448 ***	-0.1373	0.5641 **	-	-	-	-	0.273
7	0.3166	0.608 ***	-0.1943	0.8784 ***	-	-	-	-	0.2896
8	-0.5513	1.2272 ***	-0.1176	1.2359 ***	-	-	-	-	0.5155
9	0.3391	0.4538 ***	0.2723 .	0.1453	-	-	-	-	0.1019
10	0.6113	0.5468 ***	-0.1789	0.7323 ***	-	-	-	-	0.208
11	0.7434	0.9627 ***	0.2398	0.379 *	-	-	-	-	0.1255
12	-0.1665	2.0012 ***	-0.4274	1.6292 ***	-	-	-	-	0.5029
13	-0.1353	1.0123 ***	0.1567	0.9612 ***	-	-	-	-	0.3861
14	0.7504 **	0.5184 ***	-0.1187	0.5131 ***	-	-	-	-	0.2016
15	0.2148	0.7344 ***	-0.1026	0.8461 ***	-	-	-	-	0.2683
16	0.2076	0.8451 ***	-0.0545	0.9514 ***	-	-	-	-	0.3057
17	0.5695	0.9965 ***	-0.6294 ***	0.6298 ***	-	-	-	-	0.3149
18	0.2219	0.741 ***	0.5005 ***	1.1021 ***	-	-	-	-	0.3536
19	0.3843	0.8713 ***	-0.602 .	0.6122 ***	-	-	-	-	0.3341
20	0.6856	0.4072 **	-0.1529	1.0853 ***	-	-	-	-	0.2043
21	0.2468	1.7013 ***	-0.9267 .	2.2785	-1.3116	-0.2872	-	-	0.2116
22	0.3271	0.4093 ***	0.1365	0.3079 *	0.0035	0.0021	-	-	0.1748
23	-0.1773	1.1827 ***	-0.2184	0.9119 ***	0.4795 *	0.2294	-	-	0.4723
24	0.3691	0.388 ***	0.5748 ***	-0.0014	0.358 .	0.272	-	-	0.1262
25	0.5209 .	0.6279 ***	-0.4661 ***	0.6131 ***	0.0973	-0.2026	-	-	0.308
26	0.4342	0.9162 ***	-0.0539	0.4348 *	0.1993	0.2217	-	-	0.2703
27	0.008	0.7654 ***	-0.1958	0.4938 *	0.1777	0.8746 ***	-	-	0.3244
28	-0.5111	1.1967 ***	-0.0119	1.3952 ***	0.1328	-0.4276	-	-	0.5183
29	0.0197	0.6008 ***	0.4796 **	-0.1658	0.5623 **	0.128	-	-	0.1227
30	0.3329	0.6728 ***	-0.0448	0.4882 **	0.346 .	0.3883	-	-	0.2145

31	0.5282	1.0623 ***	0.2839	0.0454	0.1018	0.612	-	-	0.1262
32	-0.2016	2.0109 ***	-0.3406	1.7402 ***	0.1683	-0.192	-	-	0.4989
33	-0.3072	1.0959 ***	0.2	0.7352 **	0.1773	0.3178	-	-	0.3862
34	0.4754 .	0.6462 ***	0.033	0.2939 *	0.4295 **	0.2327	-	-	0.2187
35	-0.1089	0.8823 ***	0.1308	0.6407 **	0.6381 **	0.0147	-	-	0.2929
36	0.0529	0.9148 ***	0.0518	0.8411 ***	0.2699	0.0718	-	-	0.3048
37	0.3553	1.0934 ***	-0.5044 **	0.5488 *	0.2911	0.2796	-	-	0.307
38	-0.1279	0.9092 ***	0.5966 ***	0.6102 **	0.3606 *	0.6351 **	-	-	0.3722
39	0.0558	1.0284 ***	-0.4698 .	0.3928 .	0.4199	0.4846	-	-	0.3422
40	0.2182	0.6258 ***	0.0413	0.6273 *	0.5546 **	0.7297 .	-	-	0.2299
41	0.0197	1.804 ***	-0.8653 .	2.3076	-1.346	-0.2827	0.7471	-0.1604	0.2082
42	0.3181	0.4096 ***	0.1374	0.3077 *	0.0023	0.0029	0.0343	0.0036	0.1683
43	-0.3435	1.209 ***	-0.1932	0.9159 ***	0.456 **	0.2408	0.6071	0.011	0.4765
44	0.1509	0.3655 **	0.575 ***	-0.0034	0.3178 .	0.2908	0.9149 *	0.1771	0.1458
45	0.4965 .	0.7136 ***	-0.4294 **	0.6426 ***	0.0911	-0.2144	-0.0117	-0.2133	0.3087
46	0.4362	1.0724 ***	0.0089	0.4901 *	0.1942	0.1958	-0.2003	-0.4113 .	0.2772
47	0.0073	0.9767 ***	-0.1106	0.5684 **	0.1704	0.8399 ***	-0.2581	-0.555 ***	0.3469
48	-0.6213	1.1334 ***	-0.0277	1.3693 ***	0.12	-0.4068	0.5023	0.2196	0.5189
49	-0.1228	0.6916 ***	0.5217 **	-0.1291	0.5318 *	0.1212	0.4805	-0.1579	0.1288
50	0.2063	0.8436 ***	0.0288	0.5533 **	0.3156 .	0.3659	0.3212	-0.3745 .	0.2331
51	0.2931	1.0382 ***	0.2949	0.0295	0.0706	0.6382	0.9344	0.1764	0.1263
52	-0.6187	1.6249 ***	-0.4804 *	1.6096 ***	0.1002	-0.0853	2.1967	1.2363	0.5363
53	-0.2955	1.3586 ***	0.3049 .	0.8284 ***	0.17	0.2734	-0.3691	-0.696 ***	0.4112
54	0.4818 .	0.6622 ***	0.0389	0.2998 *	0.4298 **	0.2295	-0.0445	-0.0451	0.2127
55	-0.2273	1.1609 ***	0.2536	0.7354 ***	0.6125 **	-0.0199	0.1118	-0.6752 **	0.3327
56	-0.2069	1.0718 ***	0.138	0.8883 ***	0.2292	0.0704	0.8055 .	-0.2876	0.3315
57	0.3175	1.3455 ***	-0.3994 *	0.6367 **	0.2774	0.2417	-0.1655	-0.6443 **	0.3259
58	-0.1598	1.1954 ***	0.7149 ***	0.7103 ***	0.3466 .	0.591 *	-0.2306	-0.7368 **	0.4077
59	0.04	1.1989 ***	-0.3996 .	0.4526 .	0.412	0.458	-0.1497	-0.4405 *	0.3512
60	0.1119	0.915 ***	0.1673	0.7262 **	0.5304 *	0.6921	0.0522	-0.7089 **	0.264

Table 9: Time series OLS results for reinsurers, all betas

	α^{re}	MKT-RF	SMB	HML	RMW	CMA	TERM	DEF	adj. R^2
1	-0.003	0.9796 ***	-0.1956	0.1549	-	-	-	-	0.1915
2	-0.2163	1.3079 ***	-0.151	0.7371 *	-	-	-	-	0.3892
3	0.4609	0.8044 ***	-0.0855	0.1825	-	-	-	-	0.1476
4	-0.4574	0.9194 ***	-0.1194	0.3397	-	-	-	-	0.1567
5	0.5142 .	0.6372 ***	-0.5357 ***	0.4484 ***	-	-	-	-	0.3184
6	-0.1368	1.0596 ***	-0.3444	-0.1227	-0.1932	0.9241 *	-	-	0.2118
7	-0.3892	1.3928 ***	-0.1307	0.5384	0.1076	0.4688	-	-	0.3898
8	0.2118	0.9328 ***	-0.0615	-0.1063	0.2352	0.5336	-	-	0.1515
9	-0.8329	1.1198 ***	-0.2609	-0.2855	-0.0699	1.6232 **	-	-	0.2151
10	0.5209 .	0.6279 ***	-0.4661 ***	0.6131 ***	0.0973	-0.2026	-	-	0.308
11	-0.057	1.197 ***	-0.296	-0.0716	-0.187	0.894 *	-0.4765	-0.3995	0.2125
12	-0.4256	1.4717 ***	-0.0957	0.5651	0.1	0.4592	0.0427	-0.1898	0.3873
13	0.3082	0.6683 **	-0.1721	-0.198	0.2566	0.5704	-0.0139	0.6448 *	0.1716
14	-0.8329	1.1214 ***	-0.2603	-0.2849	-0.0699	1.623 **	-0.0021	-0.0042	0.2088
15	0.4965 .	0.7136 ***	-0.4294 **	0.6426 ***	0.0911	-0.2144	-0.0117	-0.2133	0.3087

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