A Model of Anchoring and Adjustment for Decision-Making under Risk *

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August 8, 2022

Abstract

We introduce a general model of anchoring and adjustment for decision-making under risk. To evaluate a lottery, agents anchor on the unweighted average of the utils associated with all possible outcomes. They adjust from the anchor (insufficiently) using information on the outcomes' probabilities. The resulting model implies behavior which is observationally similar to established prospect theory models. However, it is not rank-dependent and naturally applies to lotteries with any given number of outcomes, including continuous lotteries. We consider applications to decisions in simple lotteries, show how the model can explain several well-known choice anomalies, and apply it to the equity premium puzzle. We also use historical data on asset returns to show that equity premiums can be explained by reasonable combinations of preference parameters. The anchoring model is a flexible tool for modeling a simplified choice process for decisions under risk.

Keywords: Decision Theory · Probability Weighting · Anchoring · Heuristics

JEL Classifications: D11 · D81 · D91 · G41

*This paper benefited from constructive comments from Ben Collier, Jürgen Eichberger, Benjamin Enke, Charles Sprenger, Justin Sydnor, and seminar participants at the University of Munich, the 2021 American Risk and Insurance Association Annual Meeting, and the 2022 Annual Meeting of the Verband der Hochschullehrer für Betriebswirtschaft. All remaining errors are ours.

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1 Introduction

People have difficulty making decisions under risk. The mathematical operations required to properly evaluate a risky lottery are complex, and decision-makers often deviate from the predictions made by expected utility theory (EUT). One way economists rationalize these suboptimal choices is to incorporate distortions such as probability weighting or reference dependence. These models are descriptively helpful, yet they often add more complexity to the decision process. But do people really mentally conduct sophisticated mathematical operations when making decisions under risk? A simpler – and more psychologically digestible – explanation is that agents engage in a less cognitively demanding decision process by applying heuristics to the problem. Empirical evidence of heuristic thinking appears in many different contexts (Lacetera et al., 2012; Barberis et al., 1998; Gathergood et al., 2019; Benartzi and Thaler, 2001). These empirical tests often develop new models specific to the task being studied; few general models of heuristic decision-making exist (Shleifer, 2012).

In this paper, we introduce a general model of decision-making under risk based on Tversky and Kahneman's (1974) anchoring and adjustment heuristic. When using this heuristic to evaluate a lottery, agents endogenously form an anchor and adjust from this anchor in the direction of the true expected utility of the lottery. The anchoring model predicts behavior similar to established prospect theory models with probability weighting, but offers several advantages. Most importantly, the probability weights implied by the model do not depend on the ranking of the outcomes (that is, the model is not rank-dependent), which is in line with recent experimental research (Bernheim and Sprenger, 2020).[1] This also makes it easier to implement than rank-dependent theories like cumulative prospect theory (CPT, Tversky and Kahneman, 1992). In contrast to many other theories without rank-dependence, the anchoring model naturally incorporates situations with more than two potential outcomes, including continuous lotteries. These advantages make the anchoring model an attractive tool for modeling decisions under risk.

Our model begins with the idea that people have a clear sense of how they would feel about particular outcomes, but they have trouble accurately weighting those feelings by the outcomes' respective likelihoods. To simplify the decision-making process, agents use the information on outcomes and the information on probabilities separately when evaluating a lottery. They use the outcomes to form an anchor and treat each outcome as equally likely. They use the probabilities to consider whether the lottery looks more or less attractive than this anchor and adjust their evaluation accordingly. If the adjustment is incomplete, behavior is incongruent with expected utility maximization. Rather, agents make choices which look like utility maximization with weighted probabilities. In contrast to substantive processing models, such as EUT and CPT, we assume agents make those choices by using heuristics to simplify the decision process.[2] This helps to address a fundamental criticism of economic decision models, that people are unlikely to conduct

[1]Note, however, that other tests of rank dependence, such as Weber and Kirsner (1997) and Diecidue et al. (2007) come to a different conclusion.

[2]We use the terminology of Forgas (1995) to differentiate two broad categories of decision models in our context. "Substantive" models assume that people conduct complex arithmetic operations with a high level of cognitive effort

complex mathematical operations with substantial cognitive processing every time they make a decision under risk (Brandstätter et al., 2006). In addition, we refrain from imposing restrictive structure or unnecessary assumptions to allow for a flexible model with many potential applications and extensions. Ultimately, this paper introduces a mathematically tractable model of people making decisions under risk without a high cognitive burden.

A simple example helps to illustrate the decision process. Imagine a risk-neutral agent who considers a lottery with a 70% chance of a \$50 gain and a 30% chance of a \$10 gain.[3] The correct evaluation of this lottery is $(0.7 \times 50) + (0.3 \times 10) = 38$. The agent, however, does not make this complex calculation in her head. This is in line with several studies documenting that people often have significant problems doing even simple calculations involving probabilities (e.g., Galesic and Garcia-Retamero, 2010). Rather, the agent simplifies the problem by forming an anchor at $(50 + 10)/2 = 30$, the mean of the outcomes. She recognizes, though, that the \$50 outcome is more likely than the \$10 outcome. To reflect the higher chance of the good outcome, she adjusts her evaluation of the lottery upwards. Research tells us that her adjustment is likely to be insufficient (e.g., Epley and Gilovich, 2006). Rather than adjusting her anchor upwards by 8 to reach the correct evaluation of 38, she adds something smaller. Perhaps she only adds 5 to her anchor, ultimately evaluating the lottery at 35. This is the essence of our model: agents establish an anchor with equally-likely outcomes and insufficiently adjust from it in the direction of the correct evaluation. The rest of the paper will formalize this intuition.

To demonstrate the versatility of the anchoring model, we offer two applications. In the first, we consider decisions in simple discrete lotteries and show how the model can explain several well-known choice anomalies, including the effect of probability in the fourfold pattern of risk attitudes, both Allais paradoxes, the St. Petersburg paradox, and event-splitting effects. In the second application, we consider the equity premium puzzle (Mehra and Prescott, 1985; Siegel and Thaler, 1997), so-named because classic models of economic decision-making (i.e., EUT) can only explain small differences in expected returns between stocks and less risky assets such as bonds. The application demonstrates the ease with which the anchoring model can explain complex decision processes. We resolve the equity premium puzzle in a way that is familiar from the solution using probability weighting in CPT (see Barberis and Huang, 2008), but does not require the cumulative form of probability weighting.

Our model connects to the economics literature on modeling heuristic decision processes.[4] We consider a stylized psychological pattern in judgment and decision-making, the anchoring and adjustment heuristic, and formalize it such that it renders a general model of decision-making

when evaluating a lottery. "Heuristic" models assume that people use simplifying heuristics to reduce the cognitive burden of the decision process.

[3]We use a risk-neutral agent for simplicity in this example. Our model incorporates a utility function but does not impose a functional form. The applications described in this paper maintain the common assumption of diminishing marginal utility. Risk aversion in the utility function combines with the risk attitudes generated by the anchoring and adjustment process.

[4]Heuristic models have also been applied in other disciplines. In game theory, models of adaptive learning have great descriptive properties (e.g., Camerer and Ho, 1999). The psychology literature also includes several quantified models of heuristic decision-making under risk (e.g., Denrell, 2007; Brandstätter et al., 2006).

under risk. This is comparable to work formalizing the representativeness heuristic (Barberis et al., 1998; Rabin, 2002; Gennaioli and Shleifer, 2010; Rabin and Vayanos, 2010) or the role of salience in decisions under risk (Bordalo et al., 2012).[5] The latter also describes a process of how probability weights form endogenously in a choice situation. It focuses on the salience of a state of nature, measured as the variability of achievable outcomes in it, and derives probability weights from it which are then used in a sophisticated evaluation of each choice alternative. This is in contrast to our decision model in which probability weights are independent of the set of potential choice alternatives and result from the heuristic decision process without any sophisticated calculations on the part of the decision-maker.

Tversky and Kahneman (1974) were the first to consider the anchoring and adjustment heuristic in the context of probability judgments. While they propose that people use the heuristic to estimate probabilities, they do not apply it to the process of making decisions under risk. Building on their work, Einhorn and Hogarth (1986) and Hogarth and Einhorn (1990) use the anchoring and adjustment heuristic to model how agents weigh probabilities in decisions under risk or ambiguity. They concentrate on the individual evaluation of each outcome's probability as part of a larger, more complex decision process. Our model, in contrast, stipulates that agents will use the anchoring and adjustment heuristic to holistically evaluate an entire lottery, without individually evaluating each element. This offers a simple model of heuristic decision-making under risk.

The anchoring model also reconciles evidence of heuristic behavior with evidence of substantive processing models featuring probability weighting. Because it renders behavior which is observationally equivalent to probability weighting, the anchoring model contributes to the literature on the potential origins of probability weighting, a phenomenon that has been linked to various mechanisms.[6] In this literature, our model is most closely related to that of behavioral inattention and Bayesian updating (Gabaix, 2014, 2019; Enke and Graeber, 2021; Khaw et al., 2021). The reduced form of Bayesian shrinkage is formally equivalent to anchoring and adjustment. This lets Gabaix (2019) formulate a general model in which outcomes are weighted by a convex combination of their actual probability with some prior that also allows for an endogenous updating parameter. If only binary outcomes are considered and the agent uses the ignorance prior, as in the model of Enke and Graeber (2021), then the two approaches are formally equivalent. For multiple outcomes, however, the models tend to differ, because Bayesian approaches typically apply different updating parameters to different outcomes which would be inconsistent with our model's motivation.[7]

[5]Other work on heuristic processing involves topics such as rehearsal and associativeness in models of limited memory (Mullainathan, 2002), considering selective attention to model learning and belief formation (Schwartzstein, 2014), and modeling associative memory and anchoring and adjustment in economic decisions (Bordalo et al., 2020).

[6]These mechanisms include insensitivity to changes in probability (Tversky and Wakker, 1995; Gonzalez and Wu, 1999; Abdellaoui et al., 2011), the optimal reaction to errors in cognitive processing (Steiner and Stewart, 2016), evolutionary mechanics (Herold and Netzer, 2011), implicit Bayesian updating on objectively-provided quantities (e.g., Fennell and Baddeley, 2012; Khaw et al., 2021), the neurological recoding of probability information with bounded neurological capacity (Zhang et al., 2020; Juechems et al., 2021), and the idea that the perception of a probability is influenced by the uncertainty reduction it provides (Akrenius, 2020).

[7]This also makes such models more closely related to salience-based models of decisions under risk.

Our model is distinct from Bayesian updating models in how agents approach the decision process. Bayesian updating models are fundamentally based on the idea that agents use a substantive process to make decisions. Those models consider anchored and adjusted probabilities as an optimal response to cognitive costs or imprecision. In contrast, agents in our model approach the decision heuristically, easing the cognitive burden by not using complex arithmetic to combine outcomes and probabilities. They use information on outcomes to form an anchor, and use information on probability to determine the stength and direction of adjustment. We are agnostic about what determines the degree of adjustment; it is likely some combination of endogenous factors (such as cognition) and exogenous factors (such as the decision context). Because substantive processing and heuristic behavior likely coexist and are applied depending on the decision situation (Payne et al., 1993; Stanovich and West, 2000), we see both approaches as complementary. In this paper, we demonstrate that the heuristic process of anchoring and adjustment can not only be applied to decisions under risk, but also leads to choices that look like agents engage in probability weighting.

2 Theory

2.1 Basic Model

Agents face a lottery L which they seek to evaluate. For the formal expositions of this paper, we consider lottery outcomes in terms of final wealth, though the model can be extended to other preference motives such as reference dependence (see Section 6). We assume that agents have a utility function over outcomes $U : \mathbb{R}^+ \to \mathbb{R}$. We further assume that $U(0)$ is finite, that $U'(x) > 0$, and that $U''(x) \leq 0 \;\forall\; x$ such that the function is increasing and is either linear or concave in shape. However, we do not assume that agents use this utility function for a direct evaluation of the lottery. Agents instead evaluate the lottery with value $V(L)$ which is formed heuristically. The function V represents the actions of agents such that for two lotteries L_i and L_j it holds that $L_i \succeq L_j$ if and only if $V(L_i) \geq V(L_j)$. The expected utility of the lottery, denoted $EU(L)$, merely determines the correct evaluation of the lottery. If $EU(L)$ were to guide the choices of the agents, they would maximize the expected experienced utility.

Our core argument is that agents do not engage in the complex math necessary to correctly evaluate the expected utility of a lottery. Rather, agents use information on outcomes and probabilities separately in a heuristic process of anchoring and adjustment to come to an evaluation. The key psychological intuition behind this assumption is that probabilities and outcomes are separable dimensions of a lottery (see, e.g., Garner, 2014, for a definition). Agents thus do not need to integrate probabilities and outcomes immediately to form an evaluation. Rather, they can combine information on outcomes and information on probabilities separately from one another and then integrate the results of these combinations – similar to how an agent can judge an object's shape and color separately before coming to an overall impression (Handel and Imai, 1972). Previous evidence on decisions under risk supports this claim. Fiedler and Glöckner (2012), for example, report in an eye tracking analysis that both outcome size and probability increase the time focused on an outcome, but that the interaction of the two does not.

Agents use the outcome information to form an anchor for the lottery, which we denote as $A(L)$. It is worth noting that this anchor is not exogenously given, but rather is formed endogenously from parameters of the lottery. In the sections below, we introduce a process for how this anchor is formed when evaluating discrete or continuous lotteries. The main assumption we propose is that when forming the anchor, agents only concentrate on the outcomes of the lottery and treat each state as equally likely. Agents adjust the anchor to form the lottery evaluation $V(L)$. They use the probability information to determine the direction and the strength of the adjustment. However, as is evident from the literature on the anchoring and adjustment heuristic, the adjustment is not sufficient (e.g., Epley and Gilovich, 2006; Tversky and Kahneman, 1974). Rather than fully adjusting their evaluation, agents only do it to degree $\gamma \in [0, 1]$. Mathematically, we can express this process as:

$$V(L) = A(L) + \gamma[EU(L) - A(L)]. \tag{1}$$

Importantly, the comparison made in the second term of Equation (1) does not assume agents actually calculate $EU(L)$. Note that $EU(L)$ and $A(L)$ differ only in their probabilities – $EU(L)$ is based on the objective outcome probabilities, while $A(L)$ treats all outcomes as equally likely. So agents simply compare whether the objective probability of each outcome is higher or lower than the simplified probability used in setting the anchor. If the objective probability of a favorable outcome is higher, then agents adjust their evaluation of the lottery upwards; if it is lower, they adjust it downwards. The degree of adjustment depends on γ. The proposed process without a complex mathematical treatment of probabilities reflects that many individuals have difficulties comprehending and utilizing probabilistic information (Peters, 2012) and use it infrequently in their daily lives (Jonas, 2018). Our model posits that people compare probabilities to each other rather than engage in the more complex operation of using them as weights on the outcomes. This is in line with evidence showing that people perform better when comparing probabilities than when performing other calculations with them (compare, e.g., questions 5 and 6 in Galesic and Garcia-Retamero, 2010, with the remaining questions).

Our main exposition here assumes a constant relative adjustment rate of γ. This implies that the absolute adjustment will be larger when the difference between the anchor and the actual expected utility of a lottery increases. The parameter γ describes how well the agents adjust the heuristic calculation. If $\gamma = 0$, agents decide only based on the heuristic calculation with equal likelihoods. When $\gamma = 1$ and adjustment is fully sufficient, the agents act according to EUT preferences. Canonical models of EUT preferences and even some formulations of prospect theory assume that preference parameters are constant for an agent and apply to all decision situations.[8] Preference parameters may not be stable and equal in all decision situations, however, and heuristic decision-making is no exception. While there is likely an underlying stable component of the adjustment parameter γ which is idiosyncratic to the agent, the parameter also may depend on the specific decision situation. For example, it may be the case that adjustment is stronger

[8] Note that Kahneman and Tversky (1979) make no such assumption and at times even hint at the opposite.

when the probability distribution of a lottery diverges strongly from the uniform distribution. We discuss such an extension to the anchoring model and its implications in Section 6.1.

The model outlined in Equation (1) is broad and in itself not tractable for applied settings. This is because we have not yet specified how the anchor $A(L)$ is formed. Below, we offer two simple and behaviorally intuitive rules for calculating the anchor in discrete and continuous settings.

2.2 Discrete Lotteries

In the discrete setting, lotteries are of the form $L = (x_i, p_i)_{i=1}^n := (x_1, p_1; ...; x_n, p_n)$. Without loss of generality but with gained convenience for later comparisons to CPT, we assume outcomes are ordered such that $x_1 > x_2 > ... > x_n$. We assume that agents form a lottery's anchor by only focusing on the lottery outcomes, treating every outcome of the lottery as equally likely. Thus, this anchor is the simple average over the utiles of the outcomes.[9]

The agents adjust the anchor in the direction of the actual probability distribution. If the actual probability distribution allocates more probability mass to higher outcomes than the simple average does, agents adjust the anchor upwards. If the opposite is true and the actual probability distribution allocates less probability mass to higher outcomes than the simple average, then the agents adjust the anchor downwards. However, this adjustment is insufficient and only appears to degree $\gamma \in [0, 1]$. Formally, we substitute $A(L) = \frac{1}{n}U(x_i)$ into Equation (1), rearrange, and obtain

$$V(L) = \sum_{i=1}^{n} \left(\gamma p_i + (1 - \gamma)\frac{1}{n} \right) U(x_i). \tag{2}$$

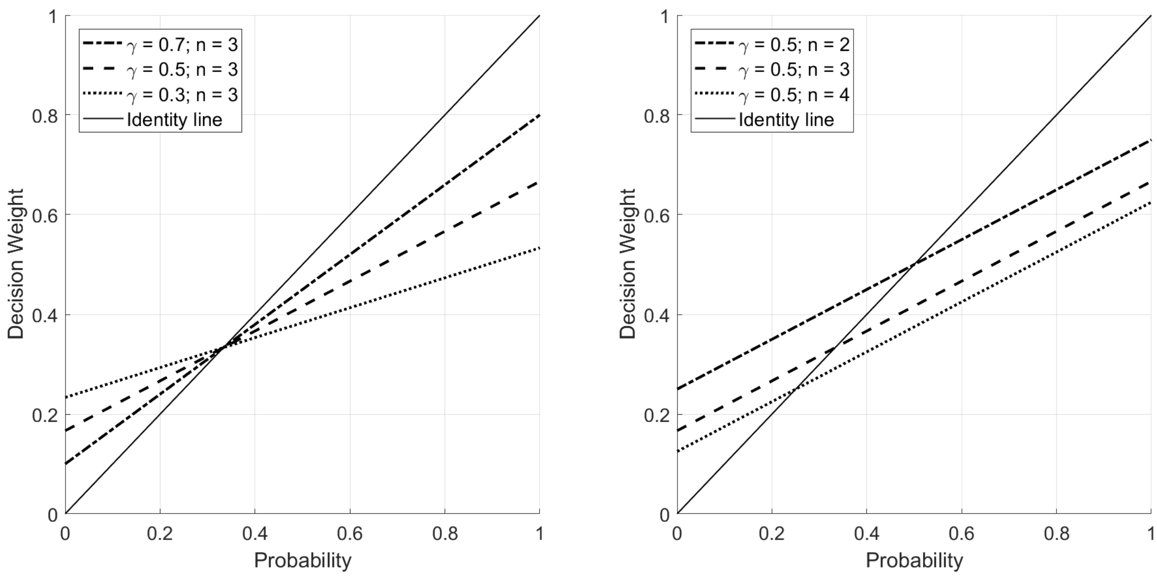
This form clarifies why the anchoring model can lead to behavior which is observationally equivalent to probability weighting, particularly in the sense of separable prospect theory (SPT, Camerer and Ho, 1994). If all possible lotteries have an equal number of outcomes, then the model presented here is formally equivalent to an SPT model in the gain domain with the implied linear probability weighting function $w(p) = \gamma p + (1 - \gamma)\frac{1}{n}$. This function is a special case of the neo-additive probability weighting function of Wakker (2010) with only one open parameter (γ) instead of two.[10] Here, γ regulates both the slope of the implied probability weighting function and the intersection of the function with the y-axis.[11] For $n \geq 2$, increasing γ makes the implied

[9] Recent neuroscientific evidence points out that outcomes might also not be treated linearly when encoded in the brain (Luyckx et al., 2019). Some of these non-linearities can be captured in the utility function assumed for the agents. However, the issue should be analyzed further and is a promising direction for future research.

[10] Using the term "neo-additive" is not entirely correct from a formal perspective. It was originally introduced in the context of decisions under ambiguity by Chateauneuf et al. (2007) who used the abbreviation "neo" to describe a function which is additive on **n**on-**e**xtreme **o**utcomes. This does not describe the behavior of our proposed model correctly, because the model does not imply behavior according to Choquet expected utility, i.e. it does not imply a cumulative form of probability weighting. However, since the mathematical expression of the function implied by our model is equivalent to that of the neo-additive function, we do not coin our own terminology but rather use the one established in the literature.

[11] In the terminology of Abdellaoui et al. (2011), γ influences both the likelihood insensitivity index (which is $1-\gamma$) and the pessimism index (which is $(1 - 0.5n)(1 - \gamma)$). However, Abdellaoui et al. consider a cumulative model in which the probability of the highest gain is always weighted first. This is not the case in this model. Thus, pessimism is an inaccurate term for the elevation of the implied probability weighting function in this model.

probability weighting function have both a steeper slope and a lower intercept. This behavior is shown for $n = 3$ outcomes in panel (a) of Figure 1. The change in slope leads to more sensitivity to changes in the probability, which is intuitive as higher values of γ correspond to behavior closer to EUT. When the number of possible outcomes is changed (and the adjustment parameter γ is kept constant), the intersection with the y-axis decreases while the slope of the implied probability weighting function and thus the sensitivity to changes in the probability remains constant. In addition, a given probability might be over- or underweighted depending on how many possible outcomes there are. This is apparent in panel (b) of Figure 1; a loss probability of 0.40, for example, is overweighted when there are two outcomes, but underweighted when there are three or more outcomes.



(a) Changing γ for constant n

(b) Changing n for constant γ

Figure 1: Probability weighting functions implied by the anchoring model. γ denotes the adjustment factor, while n denotes the number of outcomes in the evaluated lottery.

At first glance, the influence of n in Figure 1(b) implies that more probabilities tend to be underweighted as n increases. However, with an increasing number of possible outcomes, the probability mass also must be distributed across more points. So while with $n = 2$ at least one event must have a probability weakly larger than 50%, this does not have to be the case for 3 events. Thus, it is hard to construct cases in which the probabilities of more than a few outcomes are underweighted strongly. With this in mind, Figure 1(b) tells us that for an outcome with a given underweighted probability, the probability becomes more and more underweighted as the number of other possible outcomes in the lottery increases. The psychological intuition of this is that while the outcome's objective probability does not change, the outcome *appears* less likely to the agents because more alternative outcomes are possible.

While there are similarities between the anchoring model and SPT, there are also three differences between Equation (2) and the common formulations of the SPT preference functional in the gain domain. First, the decision weights do not usually add up to one in SPT. In the model presented here, $\sum_{i=1}^{n} w(p_i) = \sum_{i=1}^{n} \gamma p_i + (1-\gamma)\frac{1}{n} = \gamma \sum_{i=1}^{n} p_i + (1-\gamma)\sum_{i=1}^{n} \frac{1}{n} = 1$. Second, and more importantly, the implied probability weighting function in our model changes with the number of outcomes of a lottery, which is not a common feature of prospect theory models. Third, the anchoring model can easily be applied to continuous lotteries (see Section 2.3); there are no commonly-accepted extensions of SPT to the continuous case.

With the decision weights in the preference functional adding up to one, it could be speculated that the model is observationally equivalent to CPT or Rank Dependent Expected Utility Theory (RDEU, Quiggin, 1982). In the anchoring model, however, the weighting of outcomes is not cumulative. In CPT and RDEU, cumulative weighting results in muted weights of intermediate outcomes. In the anchoring model, the rank of an outcome does not influence the weight of the outcome's probability. Consider the lottery $L = (12, 0.2; 10; 0.2; 5, 0.6)$. In the anchoring model, the weighting function derived above would lead to decision weights $\gamma\frac{1}{5} + (1-\gamma)\frac{1}{3} = \frac{1}{3} - \gamma\frac{2}{15}$ for the first two outcomes, both of which are larger than the original probability of 0.2 for $\gamma < 1$. In a cumulative theory with the same weighting function, the decision weight of the first outcome is the same at $\frac{1}{3} - \gamma\frac{2}{15}$. However, the second-highest outcome, 10, would have a weight of $\gamma\frac{1}{5}$, which is *smaller* than the original probability for $\gamma < 1$.

Because probability weighting is not cumulative in the anchoring model and probabilities are not treated in a rank-dependent manner, the model can thus predict violations of stochastic dominance (Fishburn, 1978).[12] One example for such a prediction could appear for the pair of lotteries $L_1 = (12, 0.4; 0, 0.6)$ and $L_2 = (10, 0.2; 11, 0.2; 0, 0.6)$. Evaluated in isolation (and normalizing $U(0) = 0$), they will render $V(L_1) = \left(\frac{1}{2} - \gamma\frac{3}{30}\right) U(12)$ and $V(L_2) = \left(\frac{1}{3} - \gamma\frac{4}{30}\right)(U(10) + U(11))$. Thus, if the utility of 10, 11 and 12 are not drastically different and γ is not close to 1, the agent will evaluate L_2 higher than L_1 even though L_1 first-order stochastically dominates L_2. This prediction from the model is unrealistic if the agent can choose between both lotteries directly. Decision-makers are likely to recognize the dominance relationship and will thus choose the dominating lottery L_1.

To circumvent issues like this, Kahneman and Tversky (1979) incorporate an editing phase in their original prospect theory formulation. In this phase, certain operations are applied to lotteries to rule out some forms of behavior (like direct violations of dominance). The presence of an editing phase is especially important for our model because we assume agents use a heuristic when evaluating a lottery. While we propose that the use of the anchoring heuristic is the default operation of decision-making, it does not have to be the only one that agents apply. Specifically, if there is a simpler heuristic which the agents can apply, they are likely to use it rather than anchoring. If a lottery is clearly worse than others in a cursory comparison (for example due to first-order stochastic dominance), then this lottery will be eliminated from consideration. Evaluating

[12]However, the specific form of the implied probability weighting function prevents some of the more extreme violations, such as that described by Abdellaoui et al. (2020). This is because the decision weights sum to one.

that lottery through the anchoring heuristic in the form of Equation (2) will never take place. As a consequence, dominance violations in the evaluation of lotteries are more likely to appear when there is no obviously optimal choice. This also reflects empirical findings on the issue. Even though some instances of fairly obvious dominance violations have been documented (see, e.g., Birnbaum, 2005; Rabin and Weizsäcker, 2009), they tend to disappear if they are clearly apparent to the agent. Nevertheless, if the agent must evaluate each lottery in isolation and there is no direct comparison of the lotteries, mistakes in the evaluation, such as dominance violations, could still appear.[13]

2.3 Continuous Lotteries

We can also apply our model to continuous lotteries. A continuous lottery is defined as a probability density function $f_L(x)$ with finite support $[\underline{x}, \overline{x}]$. The analogue of a simple average over a discrete lottery is to assume a uniform distribution over all outcomes in the support of a continuous lottery. As such, the anchor is defined as the average utility of outcomes in the support with each outcome having the same chance of occurring. The uniform distribution is the simplest possible continuous probability distribution in the sense that it contains the lowest amount of information (Jaynes, 1957). It thus implies a low cognitive burden and becomes a likely candidate for the anchor in a heuristic evaluation of the lottery. As in the discrete case, the anchor can also be interpreted as a focus on the possible outcomes of a lottery without any regard of the information regarding probability mass.

The anchor is adjusted in the direction of the true expected utility, taking into account the probability density $f_L(x)$. The strength of the adjustment is again denoted by the parameter γ. The agents evaluate lotteries according to

$$V(L) = \int_{\underline{x}}^{\overline{x}} \left(\gamma f_L(x) + (1-\gamma) \frac{1}{\overline{x} - \underline{x}} \right) U(x) dx. \tag{3}$$

The psychological intuition behind Equation (3) is similar to that of Equation (2). Agents do not explicitly calculate an expected utility according to probability density $f_L(x)$, but simply adjust their heuristic evaluation upwards if $f_L(x)$ is more attractive than the uniform distribution and downwards if $f_L(x)$ is less attractive.

In the continuous case, the anchoring model is only defined for lotteries with finite support. This is a necessary restriction, because the uniform distribution is not defined on an infinite support. However, several interesting economic applications involve cases in which the support of the considered lottery is, at least in theory, unbounded. For example, many models of financial markets assume normally distributed (and thus unbounded) asset returns. So how do agents using heuristic decision processes evaluate lotteries with infinite support? While there is no definite answer at this point, it seems plausible that agents will mentally bound the distribution of such lotteries. Most likely, the agents construct some kind of intra-quantile range of the distribution. Such ranges are commonly used when describing or communicating uncertainty. Examples include the "cone of

[13] This is in line with Mu et al. (2020), who show that in the presence of small-stakes risk aversion, people must either evaluate lotteries in isolation or violate stochastic dominance.

uncertainty" in hurricane risk communication (Broad et al., 2007), the use of empirical quantiles for election forecasts (FiveThirtyEight, 2020), or the "value at risk" used when regulating financial institutions (European Commission, 2015). In these cases, it is likely that agents bound the distribution within an appropriate interval such as a 95% or 99% confidence interval.

The model for continuous lotteries also allows for violations of stochastic dominance. However, this is a more realistic prediction for continuous lotteries than for discrete lotteries. Bhargava et al. (2017), for example, document widespread violations of first- and second-order stochastic dominance in health insurance decisions – situations which naturally feature a continuous distribution of losses. Such mistakes become less prevalent when the decision situations are simplified with graphical aids (Samek and Sydnor, 2020). This is in line with the predictions of our model. As the substantive processing of the decision situation becomes less burdensome, people are less likely to use heuristic decision processes and thus the prevalence of dominance violations should decrease.

Like in the discrete case, the main difference between the model in Equation (3) and continuous versions of CPT (as, e.g., given in Davies and Satchell, 2004) is that Equation (3) is not cumulative in the treatment of the probability distribution. This leads to differences in implications between the continuous models. For SPT, Rieger and Wang (2008) derive a continuous version of the model by letting the number of outcomes converge to infinity.[14] However, their derived model either evaluates every lottery with an infinite value or reduces to EUT. Our model differs from this in the sense that it evaluates continuous lotteries with a transformed probability function and thus does not equate to EUT. Other continuous versions of original prospect theory exist, but use substantially different formulations and are often sensitive to the properties of the lottery's probability distribution (e.g., Rieger and Wang, 2008). In consequence, most non-cumulative versions of prospect theory do not have an easily applicable version for continuous lotteries or lose their distinctive properties. The anchoring model is not cumulative and still allows for a simple application to continuous lotteries while keeping the basic properties of the model.

3 Relationship to Experimental Evidence

Experimental studies of probability weighting have almost exclusively considered decision situations with a discrete number of outcomes. In the discrete case, our model makes two important predictions. First, the model predicts that as the number of distinct outcomes in a lottery increases, the elevation of the probability weighting function decreases. Second, the model predicts that as the adjustment parameter γ increases, the slope of the probability weighting function will likewise increase. In this section, we relate both of these predictions to experimental findings from previous literature.

Experimental studies of probability weighting rarely consider more than three possible outcomes. In fact, the majority of these studies have focused on binary lotteries. Varying the number of events within a study is also rare. In one such study, Enke and Graeber (2021) consider a lottery with two distinct outcomes, but manipulate the representation of one of these outcomes. While the control

[14] See their Theorem 2.

group sees an urn with two different colors of balls, the treatment group sees an urn with balls in ten different colors, nine of which lead to the same outcome. As is predicted by our model (and by that of Enke and Graeber), the probability weighting function is less elevated in the treatment group than in the control group.

The idea that the number of considered events can influence probability judgments is known under the term *partition dependence*. Starting with papers by Attneave (1953) and Fischhoff et al. (1978), researchers have documented that probabilistic judgments for n mutually exclusive and exhaustive events regress towards $1/n$ (see also, amongst others, Fox and Rottenstreich, 2003; Fox and Clemen, 2005). This phenomenon has also been linked to the anchoring heuristic (e.g. Tversky and Kahneman, 1974). However, as is pointed out by Akrenius (2020), evidence from the judgment literature cannot simply be applied to decisions under risk without additional assumptions. Specifically, one must assume that agents still make probability judgments for events even if the probabilities of these events are provided explicitly in a decision situation.

The second prediction of our model considers changes in the parameter γ. In our model, γ determines how much the agents adjust away from the anchor. This makes the parameter an inverse indicator for the prevalence of heuristic processing and, due to the mechanics of the model, it is also an indicator for the slope of the implied probability weighting function. As we describe in Section 2.1, we see γ as a partially-constructed parameter in the sense that the decision situation will influence the magnitude of γ to a certain degree. Taking this perspective, we can consider previous experimental evidence in light of our model's predictions. Fehr-Duda et al. (2010) show that increasing the stake size in lotteries can lead to a decrease in probability weighting by agents. A relatively large body of the literature has established that monetary incentives lead to a smaller anchoring bias in judgment tasks (Wright and Anderson, 1989; Epley et al., 2004; Epley and Gilovich, 2005), specifically if the direction of adjustment is clear (Simmons et al., 2010).[15] Thus, the influence of increased incentives on probability weighting can, at least in part, be caused by the decreased use of heuristics in the agents' decision process. A similar interpretation arises for the result that probability weighting becomes more pronounced in affect-rich environments (as documented in Sunstein, 2003; Hsee and Rottenstreich, 2004; Pachur et al., 2014) because heuristic processing seems more pronounced in such settings (Forgas, 1995; Jaspersen and Aseervatham, 2017). In terms of personal characteristics, probability weighting has been shown to decrease with cognitive ability of the agents (Choi et al., 2018). Since cognitive ability is consistently linked to biases in the decision process (Enke et al., 2020), it seems reasonable that its influence on probability weighting could occur through the anchoring heuristic.

4 Application: Common Lotteries and Choice Anomalies

[15] Some evidence to the contrary exists. In a rigorous study of the phenomenon, Enke et al. (2020) find no effects of regular lab incentives or strongly increased incentives on the prevalence of anchoring in a university student sample. They do, however, find a significant effect on the time used for the decision, which could indicate more substantive processing by the subjects.

4.1 Binary Lotteries and the Fourfold Pattern of Risk Attitudes

We begin by considering how the model treats simple binary lotteries. Because the anchor is the unweighted average of the utils associated with all possible payoffs, risk attitudes are determined by two separate motives: the utility function and the probability weighting implied by the anchoring and adjustment process. Here, we maintain the common assumption of concave utility, which always makes the agent risk averse. The anchoring-implied probability weighting can either increase or decrease the agent's risk aversion. Consider, for example, the decision between the simple binary lottery $(x, p; 0, 1-p)$ and its expected outcome $(px, 1)$. For $p \geq \frac{1}{2}$, the agent will always choose the certain payment, because both the utility function and the implied probability weighting make the agent act risk averse. For $p < \frac{1}{2}$, the utility function implies risk averse behavior, while the implied probability weighting implies risk seeking behavior. The agent will only choose the risky option if the implied overweighting of the risky payment is strong enough to dominate the risk aversion generated by his utility function. Ultimately, which motive dominates will depend on the agent's preferences and the lottery being considered.

We can also compare the predictions for binary lotteries made by the anchoring model to those made by EUT. Agents choose between a lottery $L_y = (y_1, p, y_0, 1-p)$ and $L_x = (x_1, q, x_0, 1-q)$ assuming $y_1 > y_0$, $x_1 > x_0$ and $q \geq p$ without loss of generality. Let $\succeq_{EUT}$ denote the preference relations of EUT agents and $\succeq_A$ denote that of agents who follow the anchoring model with $\gamma < 1$ and the same utility function as the EUT agents. We can then analyze whether $L_y \sim_{EUT} L_x$ has any implications for $\succeq_A$ over the two lotteries. From Equation (1), $L_y \sim_{EUT} L_x$ implies that $L_y \succ_A (\sim_A, \prec_A) L_x$ if and only if $A(L_y) > (=, <) A(L_x)$.[16] That is, the order of the anchors determines the preference order. This can lead to clear-cut predictions. Consider, for example, the case $y_1 > x_1$, $y_0 = x_0$, and $p < q$. In this case, we can normalize the utility function and see that $A(L_y) = 1/2$ and $A(L_x) = p/(2q)$ and thus $L_y \succ_A L_x$. Intuitively, the anchoring agents always consider the smaller probability p more strongly than the larger probability q. If the EUT agents are indifferent between both lotteries, then this tendency will make the anchoring agents choose L_y. Other choices regarding the parameters in the lotteries can lead to different predictions, but the comparative magnitude of the lotteries' anchors will always determine the preference tendency of the anchoring model relative to that under EUT.

One of the preference patterns for binary lotteries attributed to prospect theory is the commonly-observed fourfold pattern of risk attitudes (see, e.g., Harbaugh et al., 2010, for an experimental analysis). According to this pattern, agents appear risk averse for gains and risk seeking for losses when the probability of the more extreme outcome in a binary lottery is high. When this probability is low, people tend to display the opposite risk attitudes in each domain. We analyze this question with the following approach: given an indifference relationship between a binary lottery and a safe payment under EUT, how does the structure of the lottery influence the preference relationship between the two options in the anchoring model? The model predicts the same influence of

[16] Note, however, that the anchoring model will always predict an evaluation of a lottery L between $EU(L)$ and $A(L)$. Whether this prediction holds up to an empirical test should be the subject of further research.

probabilities on risk attitudes as the fourfold pattern: agents are less risk averse for low-probability gains than for high-probability gains and more risk averse for low-probability losses than for high-probability losses. As noted above, the concave utility function in the anchoring model always implies risk aversion. We thus do not stipulate that the anchoring model necessarily leads to risk averse or risk seeking behavior. Rather, we show that the model adjusts the EUT preferences in certain directions given a change in probabilities.

Formally, consider a binary lottery $L_b = (x_1, p; x_2, 1-p)$. Under EUT, there exists a safe payment which makes the agent indifferent between the payment and the lottery. Consistent with convention, we call this payment the EUT certainty equivalent, denoted x^{CE}_{EUT} and implicitly defined as $pU(x_1) + (1-p)U(x_2) = U(x^{CE}_{EUT})$. This lottery is sufficient to analyze the role of probability in the fourfold pattern of risk attitudes. We can use it to analyze gains by setting $x_1 = w + g$ and $x_2 = w$ with $w > 0$ being current endowment of the agent and $g > 0$ being a gain. We can also use the lottery to analyze losses by setting $x_1 = w$ and $x_2 = w - \ell$ with $\ell > 0$ being a loss amount. In both cases, the fourfold pattern stipulates $V(L_b) > V(x^{CE}_{EUT})$ for $p < \frac{1}{2}$ and $V(L_b) < V(x^{CE}_{EUT})$ for $p > \frac{1}{2}$. To show that this prediction is indeed obtained, we observe that $V(x^{CE}_{EUT}) = U(x^{CE}_{EUT}) = pU(x_1)+(1-p)U(x_2)$. The pattern thus predicts $[\gamma p+(1-\gamma)\frac{1}{2}]U(x_1)+[\gamma(1-p)+(1-\gamma)\frac{1}{2}]U(x_2) > pU(x_1) + (1-p)U(x_2)$ for $p < \frac{1}{2}$. Rearranging renders $(\frac{1}{2} - p)[U(x_1) - U(x_2)] > 0$. This is true for $p < \frac{1}{2}$ while the opposite prediction is obtained for $p > \frac{1}{2}$. The anchoring model thus predicts that for low-probability gains and high probability losses ($p < \frac{1}{2}$) agents behave less risk averse than under EUT and that for high-probability gains and low-probability losses ($p > \frac{1}{2}$) agents behave more risk averse than under EUT.

4.2 Allais (1953) Paradoxes

We consider both paradoxes of Allais (1953). The lotteries used for these paradoxes are given in Figure 2. In both of these paradoxes, preferences over lotteries with different numbers of outcomes are compared, so we cannot simply point to SPT for this evaluation. However, the anchoring model also explains both paradoxes.

The first Allais paradox can be summarized by the preference relations $L_{A1} \succ L_{A2}$ and $L_{A3} \prec L_{A4}$ with $x_1 > x_2 > 0$ and $p < 1-q$. According to Equation (2) and normalizing $U(x_1) = 1$ and $U(0) = 0$, these preference relations imply $U(x_2) > \gamma p + (1-\gamma)\frac{1}{3} + (\gamma q + (1-\gamma)\frac{1}{3})U(x_2)$ and $\gamma p + (1-\gamma)\frac{1}{2} > (\gamma(1-q)+(1-\gamma)\frac{1}{2})U(x_2)$. Rearranging and combining renders $\frac{\gamma p+(1-\gamma)\frac{1}{2}}{\gamma(1-q)+(1-\gamma)\frac{1}{2}} > U(x_2) > \frac{\gamma p+(1-\gamma)\frac{1}{3}}{1-\gamma q-(1-\gamma)\frac{1}{3}}$ such that the model allows for the Allais paradox if $\frac{\gamma p+(1-\gamma)\frac{1}{2}}{\gamma(1-q)+(1-\gamma)\frac{1}{2}} > \frac{\gamma p+(1-\gamma)\frac{1}{3}}{1-\gamma q-(1-\gamma)\frac{1}{3}}$. Since the numerator on the left is larger than the numerator on the right, it is sufficient to show that $\gamma(1-q) + (1-\gamma)\frac{1}{2} < 1 - \gamma q - (1-\gamma)\frac{1}{3}$. This inequality reduces to $\frac{1}{6}(1-\gamma) > 0$ which is true for $\gamma \in [0, 1)$. Thus, the model can explain the first Allais paradox for any adjustment parameter γ that implies at least some use of the anchoring heuristic by the agents.

The second Allais paradox, often also called the *common ratio violation*, can be summarized by the preference relations $L_{A1} \succ L_{A5}$ and $L_{A6} \prec L_{A7}$. We again normalize $U(x_2) = 1$ and $U(0) = 0$ to see that the preference relations imply $U(x_2) > \gamma h + (1-\gamma)\frac{1}{2}$ and $\gamma hj + (1-\gamma)\frac{1}{2} >$

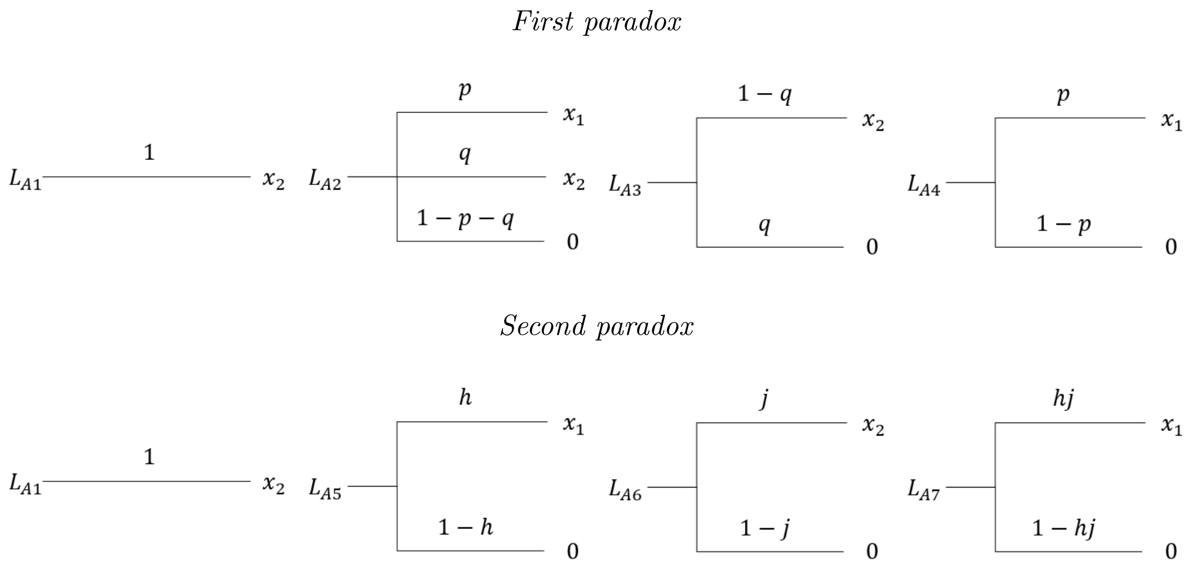


Figure 2: Lotteries used in the two Allais paradoxes

$(\gamma j + (1-\gamma)\frac{1}{2})U(x_2)$. Combining and rearranging renders $\frac{\gamma hj+(1-\gamma)\frac{1}{2}}{\gamma j+(1-\gamma)\frac{1}{2}} > U(x_2) > \gamma h + (1-\gamma)\frac{1}{2}$. This reduces to $\gamma j(h-\frac{1}{2}) + \frac{1}{2}(1-(\gamma h + (1-\gamma)\frac{1}{2})) > 0$ for which $h > \frac{1}{2}$ is a sufficient condition. Since common ratio violations are typically reported for $h = 0.8$ or comparable values (Kahneman and Tversky, 1979), this sufficient condition does not seem restrictive.

4.3 The St. Petersburg Paradox

We also consider the St. Petersburg paradox, which originally led Bernoulli (1738) to introduce the expected utility hypothesis. As reported by Blavatskyy (2005), several theories that feature probability weighting have problems explaining this paradox. The paradox states that an agent's evaluation of the lottery $L_{SP} = (2^i, \frac{1}{2^i})_{i=1}^{\infty}$ is generally not infinite even though $\mathbb{E}[L_{SP}]$ is. Bernoulli (1738) proposes concave utility as a solution to this conflict, but fails to see that if the utility function is unbounded, the lottery can be modified such that the EUT evaluation of the lottery remains infinite, even if U is concave (Menger, 1934).

Nevertheless, there are two common ways of resolving the St. Petersburg paradox under EUT. The first is the observation that the lottery L_{SP} cannot have infinite value, because nobody can offer a truly infinite gamble. Under this circumstance, the lottery is described by $L_{SP}^{finite} = (2^i, \frac{1}{2^i})_{i=1}^{n}$ and EUT will always explain the paradox (Arrow, 1974). However, CPT can still lead to an infinite evaluation of L_{SP}^{finite} because the slope of the probability weighting function is potentially infinite (Blavatskyy, 2005; Rieger and Wang, 2006). This criticism does not apply to the anchoring model introduced here. Agents evaluate the finite lottery as $V(L_{SP}^{finite}) = \gamma \sum_{i=1}^{n} \frac{1}{2^i}U(2^i) + (1-\gamma)\sum_{i=1}^{n}\frac{1}{n}U(2^i)$. This can be seen as the convex combination of the EUT evaluation of the outcomes of lottery L_{SP}^{finite} with two different associated probability distributions. The first distribution is the original St. Petersburg lottery, while the second is a uniform distribution over all possible outcomes of the lottery. Both probability distributions lead to finite expected values. Applying the result of Arrow (1974) to both expected utility evaluations separately and observing that the convex combination of two finite values is in itself finite shows that $V(L_{SP}^{finite})$ is finite.

The second possible resolution under EUT is to bound the utility function U. This implies existence of a finite value W such that $U(x) < W \,\forall\, x$ (Menger, 1934). However, due to the potentially infinite slope of the probability weighting function, this might again be insufficient for finite evaluation in CPT. Nevertheless, the solution is sufficient under the anchoring model. According to Equation (2), the agent evaluates the St. Petersburg lottery as $V(L_{SP}) = \lim_{n\to\infty} \gamma \sum_{i=1}^{n} \frac{1}{2^i} U(2^i) + (1-\gamma)\frac{1}{n}\sum_{i=1}^{n} U(2^i)$. Due to the bounded utility function, we know $V(L_{SP}) < \lim_{n\to\infty} \gamma \sum_{i=1}^{n} \frac{W}{2^i} + (1-\gamma)\frac{1}{n}\sum_{i=1}^{n} W = \gamma W + (1-\gamma)W = W$ which is clearly finite.

4.4 Different Numbers of Outcomes and Event-Splitting Effects

To understand how the anchoring model reacts to lotteries with different numbers of outcomes, we analyze a mean-preserving spread. Consider a binary lottery of the form $L_x = (x, p, y, 1-p)$. We can apply a mean-preserving spread to one of the outcomes, say x, to obtain $L'_x = (x+\varepsilon, p/2, x-\varepsilon, p/2, y, 1-p)$. An EUT decision-maker dislikes this spread, because $\Delta_{EUT} = U(x) - \frac{1}{2}(U(x+\varepsilon) + U(x-\varepsilon))$ is always positive. For the anchoring model, the agent will prefer L'_x to L_x if it holds that $(\gamma p + (1-\gamma)\frac{2}{3})\Delta_{EUT} < \frac{1}{6}(1-\gamma)[U(x) - U(y)]$. This expression provides intuition as to how the anchoring model reacts to an increase in outcomes. First, there can only be a preference for L'_x if $x > y$. That is, the new lottery needs to have more favorable outcomes than the original one. Second, the effect of the adjustment parameter is not trivial. If Δ_{EUT} is large enough, such that $\frac{1}{6}[U(x) - U(y)] - \frac{2}{3}\Delta_{EUT}$ is negative, then the increased risk in outcomes will decrease the anchor of the lottery. In this case, the agent will always prefer L_x. This shows that even with additional favorable outcomes, the anchoring model does not universally prefer an increase in outcomes. Because the anchor is formed over the utiles of the outcomes, it is also important how much additional risk is introduced.

A related phenomenon is the prevalence of event-splitting effects (ESEs). In any discrete lottery, one event can be split into two subevents, each with the same outcome as the original event and whose probabilities sum to the probability of the original event, as shown in Figure 3. ESEs occur when agents appear to evaluate the original lottery L_{xC} differently than the split lottery L_{xS}. In Figure 3, $0 < \lambda < 1$ and the other parameters are chosen such that $p > q$ and $x > y$ or $p < q$ and $x < y$. ESEs obtain if $L_y \succ L_{xC}$ and $L_y \prec L_{xS}$. Starmer and Sugden (1993) conduct an experiment using $y = 11$ and $x = 7$. In two tests with different probabilities and different values of λ, they find a significant share of their subjects make choices consistent with ESEs.

Under CPT, such ESEs are impossible due to the cumulative nature of probability weighting. They are possible in SPT, in the sense that the evaluation of L_{xC} can be lower than that of L_{xS}. However, this will only be the case if the probability weighting function is subproportional over the relevant range of probabilities. Particularly for the higher levels of probabilities in Starmer and Sugden (1993), this will not always be given. In the anchoring model, however, a lower evaluation of L_{xC} than that of L_{xS} will always be predicted. Normalizing $U(0) = 0$ and $U(x) = 1$, we can see that $V(L_{xC}) = \gamma q + (1-\gamma)\frac{1}{2}$. Similarly, $V(L_{xS}) = \gamma(\lambda q + (1-\lambda)q) + (1-\gamma)\frac{2}{3} = \gamma q + (1-\gamma)\frac{2}{3} > V(L_{xC})$.

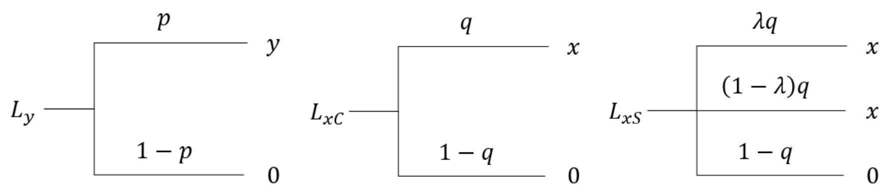


Figure 3: Lotteries used for demonstrating event-splitting effects.

5 Application: Equity Premiums

5.1 Motivation

A common decision under risk which many people face is the choice of investment in capital markets. One of the most discussed irregularities in this market is the equity premium puzzle (Mehra and Prescott, 1985; Siegel and Thaler, 1997). Equities seem to be priced at a lower level than common levels of risk aversion in EUT would suggest, rendering a high premium for the ownership of risky equities over that of safe bonds. The level of risk aversion necessary to explain the observed equity premium cannot be reconciled with other empirical observations when the EUT model is assumed (Rabin, 2000; Chetty, 2006). The most commonly-cited behavioral solution to the puzzle is a combination of CPT preferences (including an emphasis on loss aversion) with myopic evaluation periods of about one year (Benartzi and Thaler, 1995; Barberis et al., 2001).

We embrace myopia as a necessary part of the behavioral explanation of the equity premium puzzle and build on the results of Benartzi and Thaler (1995). We consider a one-period model of the stock market and calibrate it using one-year returns. We investigate the influence of anchoring and adjustment on the equity premium using a two-step approach. In the first step, we analyze a stylized stock market under EUT and the anchoring model to understand the differences in asset evaluation between both theoretical models. In the second step, we use stock market data to analyze whether anchoring and adjustment can resolve the equity premium puzzle similar to the role of loss aversion in Benartzi and Thaler (1995).

5.2 Theoretical Model

For the theoretical analysis, we consider a simple structure which has been used to study equity premiums in non-canonical models (see, e.g., Chateauneuf et al., 2007). We assume a market with two assets: a bond of price 1 and an equity of price q. The bond generates a certain payoff $r > 1$, while the equity generates risky payoffs $\tilde{x}$ distributed according to probability density function $f(x)$ with support $[\underline{x}, \overline{x}]$ with $\underline{x} < r < \overline{x}$. The goal of the model is to consider the equity premium $\rho(q^*) = \frac{\int_{\underline{x}}^{\overline{x}} x f(x) dx}{q^*} - r$ in which q^* is the equilibrium price of the equity and smaller values of q^* lead to larger values for $\rho(q^*)$.

Agents have initial wealth w and purchase a quantity of a equities and b bonds. Because bonds generate positive returns, agents exhaust their budget such that $w = aq + b$. Agents then decide on their investment allocations according to

$$max_{a,b}\left[V(L)=\int_{\underline{x}}^{\overline{x}}\left(\gamma f(x)+(1-\gamma)\frac{1}{\overline{x}-\underline{x}}\right)U(br+ax)dx\right]. \tag{4}$$

We substitute the budget constraint to simplify the maximization problem to a single variable such that agents now determine only the equity portion according to

$$max_{a}\left[V(L)=\int_{\underline{x}}^{\overline{x}}\left(\gamma f(x)+(1-\gamma)\frac{1}{\overline{x}-\underline{x}}\right)U(wr+a[x-qr])dx\right]. \tag{5}$$

Since we do not attempt a full analysis of the financial market, but rather aim to illustrate how the anchoring model could affect the equity premium, we make a series of simplifying assumptions. Specifically, we assume that the exogenous aggregate endowment of equity A is positive and the aggregate endowment of bonds B is zero. We also assume that there is a single agent acting as the investor.[17] To find the anchoring model's equilibrium price q_A^* with the agent maximizing $V(L)$, the condition

$$V'(A)=\int_{\underline{x}}^{\overline{x}}\left(\gamma f(x)+(1-\gamma)\frac{1}{\overline{x}-\underline{x}}\right)U'(wr+A[x-q_A^*r])[x-q_A^*r]dx=0 \tag{6}$$

must hold. In this equilibrium, $w=q_A^*A$ holds and can be substituted such that an explicit solution for the equity price can be derived:

$$q_A^*=\frac{\int_{\underline{x}}^{\overline{x}}\left(\gamma f(x)+(1-\gamma)\frac{1}{\overline{x}-\underline{x}}\right)U'(Ax)xdx}{r\int_{\underline{x}}^{\overline{x}}\left(\gamma f(x)+(1-\gamma)\frac{1}{\overline{x}-\underline{x}}\right)U'(Ax)dx}. \tag{7}$$

The benchmark value of the equity price is given by the case of EUT, which can be derived from Equation (7) by setting $\gamma=1$. The resulting EUT equilibrium price q_{EUT}^* is

$$q_{EUT}^*=\frac{\int_{\underline{x}}^{\overline{x}}f(x)U'(Ax)xdx}{r\int_{\underline{x}}^{\overline{x}}f(x)U'(Ax)dx}. \tag{8}$$

First, we consider the case of agents with a risk neutral utility function such that the marginal utility is constant in all wealth levels. The resulting equity price is

$$q_A^*=\frac{\gamma\int_{\underline{x}}^{\overline{x}}f(x)xdx+(1-\gamma)\int_{\underline{x}}^{\overline{x}}\frac{1}{\overline{x}-\underline{x}}xdx}{r}=q_{EUT}^*+(1-\gamma)\frac{\int_{\underline{x}}^{\overline{x}}\left(\frac{1}{\overline{x}-\underline{x}}-f(x)\right)xdx}{r} \tag{9}$$

Whether or not the equity price is smaller or larger than the benchmark value derived from EUT thus depends on the sign of the expression $\int_{\underline{x}}^{\overline{x}}\left(\frac{1}{\overline{x}-\underline{x}}-f(x)\right)xdx$. Since this expression is equivalent

[17] This latter assumption can be relaxed to allow for multiple investors. If k agents each have equal preferences and starting wealth, they will purchase A/k stocks. Similarly, we could assume $B>0$ to allow for bond purchases. The simplifying assumptions help to concentrate on the equity premium implied by the anchoring model.

to $\frac{1}{2}(\underline{x}+\overline{x})-\mu(\tilde{x})$, we can see that the the equity price is lower than under EUT (which implies an equity premium higher than EUT) if the mean of the equity returns is above the midpoint of the support interval $[\underline{x},\overline{x}]$. Correspondingly, if the mean of the equity returns is below the midpoint of the support, the anchoring model predicts a lower equity premium than EUT. Intuitively, this makes sense – if the actual probability distribution has more probability mass above the midpoint of its support, then the uniform distribution will overweight bad outcomes and thus the equity will look worse to the agents than it actually is at the anchor. Because the adjustment in the direction of the correct return distribution is insufficient, the agents are willing to pay a smaller price than would be adequate under the EUT model.

A key insight from the risk-neutral case is that the anchoring model predicts that the equity premium should decrease with the expected skewness of the equity's returns. Informally put, a positively-skewed asset has a longer right tail than left tail. Increasing the skewness of a return while keeping the mean and variance constant increases $\overline{x}$ more strongly than it decreases $\underline{x}$ and thus more skewed returns have a higher midpoint of the support interval $[\underline{x},\overline{x}]$. This increases equity prices under the anchoring model and thus decreases the equity premium. The effect is similar to that observed under CPT (Barberis and Huang, 2008) and empirical evidence in support of this skewness effect abounds (e.g., Bali et al., 2011; Boyer et al., 2010; Conrad et al., 2013).

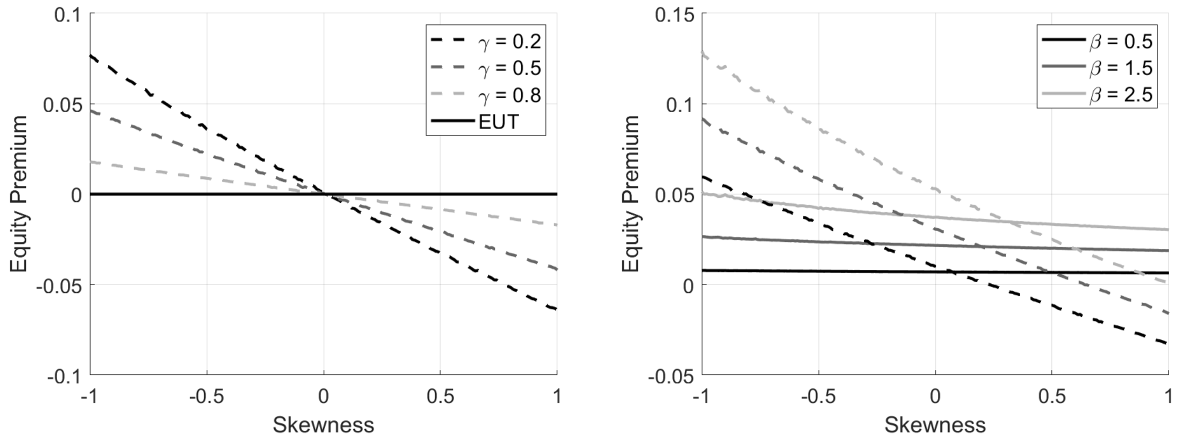
A short numerical example helps to illustrate the magnitude of the effect of skewness on the equity premium. Based on the simple model above, we set $r=1.04$, $\mu(\tilde{x})=1.08$, and $\sigma(\tilde{x})=0.125$. In the unskewed case, we assume $\tilde{x}$ is distributed normally. To vary the skewness, we apply the procedure of Jaspersen and Peter (2017) which utilizes polynomial transformations of the normal distribution to generate distributions with equal mean and variance, but different skewness. To simulate agents who consider previous asset returns in making their investment decision, we draw 50 realizations from the distribution of $\tilde{x}$ and use those as a numerical approximation of the continuous case to calculate q_A^* and q_{EUT}^*.[18] We repeat this process 1,000 times to minimize random fluctuations.

The resulting equity premiums, defined as $\mu(\tilde{x})/q-r$, for different values of γ can be seen in panel (a) of Figure 4. The anchoring model shows a clear negative impact of skewness on the equity premium – as the distribution becomes more positively skewed, the equity premium decreases. Skewness has a larger impact on the equity premium as the adjustment parameter γ decreases (i.e., as the agent tends more toward heuristic decision-making). The figure also shows that the impact is economically significant. At $\gamma=0.5$, for example, very small levels of negative skewness can already increase the equity premium by a percentage point or more.

Under risk aversion, the problem becomes more complicated. From Equations (7) and (8), we can see that

$$q_A^*=\frac{q_{EUT}^*\gamma r\int_{\underline{x}}^{\overline{x}}f(x)U'(Ax)dx+(1-\gamma)\int_{\underline{x}}^{\overline{x}}\frac{1}{\overline{x}-\underline{x}}U'(Ax)xdx}{r\int_{\underline{x}}^{\overline{x}}\left(\gamma f(x)+(1-\gamma)\frac{1}{\overline{x}-\underline{x}}\right)U'(Ax)dx}. \tag{10}$$

[18] How many realizations are sampled does not change the qualitative results of our analysis but does influence the magnitude of the influence of skewness on the equity premium. This issue is explored further in Appendix A.



(a) Risk neutral

(b) Constant relative risk aversion (assuming $\gamma = 0.5$)

Figure 4: Equity premiums according to EUT (solid lines) and the anchoring model (dashed lines). The model assumes $r = 1.04$, $\mu(\tilde{x}) = 1.08$, $\sigma(\tilde{x}) = 0.125$, and varies the skewness of the returns according to the procedure by Jaspersen and Peter (2017). $\tilde{x}$ is evaluated based on 50 draws from the distribution and the analysis uses 1,000 replications.

The equity price under the anchoring model is thus smaller than under EUT if and only if $\int_{\underline{x}}^{\overline{x}} \frac{1}{\overline{x}-\underline{x}} U'(Ax)x dx < q^*_{EUT} r \int_{\underline{x}}^{\overline{x}} \frac{1}{\overline{x}-\underline{x}} U'(Ax)dx$. Substituting Equation (8) and rearranging renders

$$q^*_A < q^*_{EUT} \;\Leftrightarrow\; \frac{\int_{\underline{x}}^{\overline{x}} \frac{1}{\overline{x}-\underline{x}} U'(Ax)x dx}{r \int_{\underline{x}}^{\overline{x}} \frac{1}{\overline{x}-\underline{x}} U'(Ax)dx} < \frac{\int_{\underline{x}}^{\overline{x}} f(x)U'(Ax)x dx}{r \int_{\underline{x}}^{\overline{x}} f(x)U'(Ax)dx}. \tag{11}$$

This condition is equivalent to the condition that $q^*_{uniform} < q^*_{EUT}$. That is, the anchoring model will lead to a lower equity price (and thus a higher equity premium) if and only if an EUT agent has a lower willingness to pay for the equity when the equity's returns are distributed uniformly over the support than when the returns are distributed according to the actual probability distribution function $f(x)$.

Using the same procedure as before, we analyze the risk-averse case in panel (b) of Figure 4. We assume a constant relative risk aversion utility function with coefficient β and consider the risk premium of the EUT agent (solid line) and the anchoring agent with $\gamma = 0.5$ (dashed line). Two effects are visible. First, skewness decreases the equity premium, consistent with empirical evidence. This effect is stronger than the risk-neutral case in panel (a) because the risk-averse agent weighs the lower bound more strongly than the upper bound. Second, anchoring and adjustment results in higher equity premiums even for unskewed returns. This is because for a normal distribution, the anchoring model overweights the outcomes in both tails. However, the risk-averse utility function emphasizes low outcomes more strongly than high outcomes such that the two overweighted tails do not cancel each other out. As can be seen in the figure, the equity premium difference between EUT and the anchoring model increases as utility curvature increases.

5.3 Empirical Analysis

Given that two effects allow the anchoring model to imply higher equity premiums than EUT, we now ask which parameters in the model are necessary to resolve the equity premium puzzle. We take a similar approach to Benartzi and Thaler (1995). We consider monthly value-weighted returns in the S&P 500 (including dividends), 1-year treasury bonds, and 90-day treasury bills (T-bills) from 1941 until 2019.[19] Throughout the analysis here, we only consider nominal returns.[20] Using random starting points, we draw 10,000 12-month returns (with replacement) from each of these time series. Summary statistics for both monthly and annual returns are given in Table 1.

Table 1: Summary statistics for monthly and annual asset returns

	Mean	Variance	Skewness	Min	Max
Monthly Returns					
Stocks	0.0099	0.0017	-0.4174	-0.2158	0.1681
1-year Treasury Bonds	0.0037	2.41E-5	2.6769	-0.0172	0.0561
90-day Treasury Bills	0.0033	8.63E-6	1.2802	-0.0093	0.0213
Annual Returns (random starting month)					
Stocks	0.1290	0.0276	-0.1102	-0.4254	0.6146
1-year Treasury Bonds	0.0470	0.0015	1.1394	-0.0078	0.2221
90-day Treasury Bills	0.0420	0.0012	0.9481	-0.0068	0.1725

Note: The table reports descriptive statistics for monthly and annual returns of the different asset classes considered in the analysis for the time period between 1941 and 2019. Stock returns refer to the S&P 500 including dividend payments. All returns are reported in nominal terms. Annual returns are reported based on 10,000 draws (with replacement) of random starting months between January 1941 and January 2019.

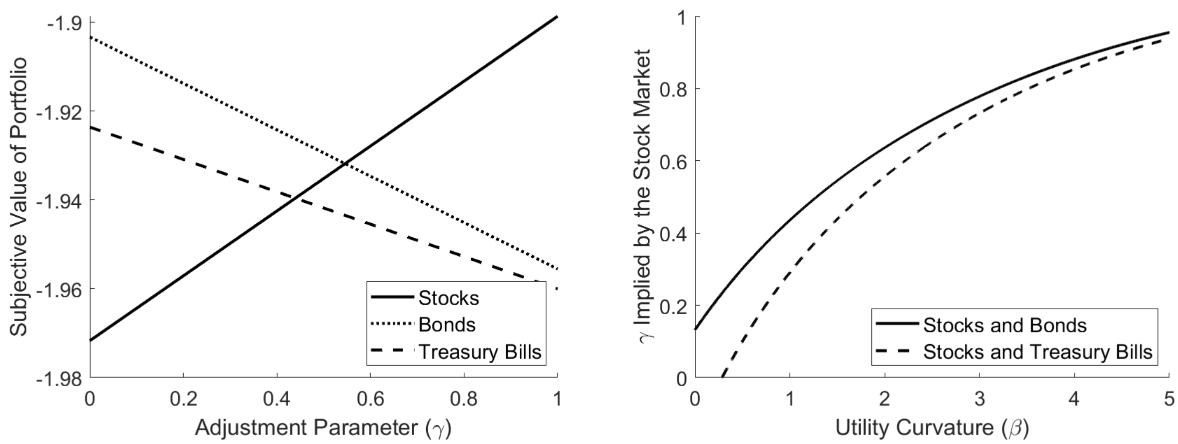
The table shows that stock investments lead to higher monthly and annual returns than bonds or T-bills, but also pose a higher risk to the portfolio. Bonds have a slightly higher return and higher risk than T-bills. Stock returns (both monthly and annual) are negatively skewed, while the returns to bonds and T-bills are both more strongly skewed in the positive direction. This positive skew comes from the limited downside risk of the return distributions. For both asset classes, the minimum annual return is higher than -1%.

To understand how the anchoring model evaluates the different asset classes, we first consider their subjective evaluations for different values of the anchoring parameter γ. We use the 10,000 randomly drawn annual returns for a discrete numerical approximation of the continuous evaluation according to Equation (3). We assume a risk averse decision-maker with iso-elastic utility and relative risk aversion of $\beta = 1.5$. The resulting subjective values of the different single-asset

[19]We access equity and bond index data through the Center for Research in Security Prices (CRSP). CRSP provides historical U.S. treasury data and stock index data starting in 1926, while bond index data begins in 1941. The analysis reported here is for the full range of the last 79 years; long-range analyses are the standard in the literature on the equity premium puzzle. The results are qualitatively robust to using different time time periods of the last 60, 40 or 20 years, see Appendix B.

[20]The focus on nominal returns follows the argument by Benartzi and Thaler (1995) that virtually all performance reports of capital market assets are given in nominal terms. This implies that investors are used to focusing on nominal returns and that they will thus evaluate assets based on them. We nevertheless provide an analysis using real returns in Appendix C.

portfolios for $\gamma \in [0,1]$ are illustrated in panel (a) of Figure 5. As γ increases, the valuation of the stock portfolio increases and is thus less attractive under the anchoring model than under EUT. Both the effect of negative skewness and that of overweighted tails of the return distribution contribute to this change in evaluation. For bonds and T-bills, the value decreases in γ. The effect of the positive skewness in the returns outweighs the effect of overweighted tails so the anchoring model leads to a higher subjective value of those portfolios than EUT. The difference in the influence of γ on the subjective value for the different asset classes leads to a crossing point of stocks and bonds at $\gamma = 0.54$ and to a crossing point of stocks and T-bills at $\gamma = 0.44$. This indicates that for $\beta = 1.5$ the equity premium puzzle can be resolved by the anchoring model with an adjustment parameter γ around 0.5.



(a) Subjective value of portfolios consisting of one exclusive asset class ($\beta = 1.5$)

(b) β / γ combinations which leads to indifference between two single asset class portfolios

Figure 5: Panel (a) shows the subjective value according to the anchoring model of three single-asset portfolios for different values of γ. We assume agents have an iso-elastic utility function with a relative risk aversion of $\beta = 1.5$. At $\gamma = 1$, the anchoring model is equal to EUT. Panel (b) reports the value of γ for which the agents are indifferent between a portfolio with only stocks and a portfolio with only 1-year bonds or only 90-day T-bills, respectively. Agents again have an iso-elastic utility function with relative risk aversion as indicated on the x-axis. Analyses in both panels are based on 10,000 draws of 12-month nominal returns (with replacement) from a random starting month between January 1941 and January 2019.

While $\beta = 1.5$ seems to be a reasonable choice for the risk aversion parameter (see, e.g., Chetty, 2006), the value was chosen arbitrarily. In a second analysis, we vary β and determine the level of γ for which the agent is indifferent between the stock portfolio and the bonds or the Treasury bills portfolio. Panel (b) of Figure 5 shows that the anchoring model can resolve the equity premium puzzle for a broad range of utility curvatures. The required value of γ is increasing in β. This is intuitively appealing, because the equity premium puzzle disappears for high levels of risk aversion and thus there is less need for heuristic processing to resolve the equity premium puzzle when β

is high. If bonds are taken as the risk-free asset class, the anchoring model can resolve the equity premium even if there is no curvature in the utility function.

The results of this section highlight two convenient features of the anchoring model when aiming to explain equity premiums. First, the model helps explain the empirically-documented negative effect of skewness on the equity premium (Bali et al., 2011; Boyer et al., 2010; Conrad et al., 2013). Second, the model can explain the equity premium puzzle even for sensibly low values of utility curvature. This highlights the appeal of the anchoring model for financial applications. Additionally, the anchoring model can be applied easily to financial markets even when incorporating continuous outcome distributions.

6 Discussion and Possible Extensions

We propose a model which leads to behavior that looks like probability weighting while making no direct assumptions about the underlying probability weighting function. Instead, we formalize a heuristic which results in observationally equivalent behavior. By relying on a simple heuristic, the anchoring model assumes less cognitive processing than more complex decision-making models. In addition to its empirical applications, one advantage of the anchoring model at this stage are the few necessary assumptions and the model's resulting flexibility. Because the model outlined in Section 2 is so simple, however, it is not always able to fully capture certain empirically-documented behaviors. Here, we describe some of these situations and suggest extensions or ways in which future research might adapt the anchoring model to explain them.

6.1 Inverse S-Shaped Probability Weighting Functions

When γ is constant for an agent, the anchoring model implies linear probability weighting functions as those displayed in Figure 1. This differentiates the model from the traditional perspective of probability weighting functions with an inverted S-shape. While much of the research on decisions under risk has focused on such inverse S-shaped probability weighting functions (Baillon et al., 2020; Wakker, 2010), there is some empirical support for linear probability weighting functions (e.g., Barseghyan et al., 2013; Bleichrodt and Pinto, 2000). Nevertheless, the linear shape of the implied probability weighting function can be seen as a limitation of the anchoring model. Here, we introduce an extension to the anchoring model that makes γ dependent on the probability distribution of the considered lottery. This extension implies inverse S-shaped probability weighting functions, while keeping the characteristic properties of the original model.

The extension of the model has a simple psychological intuition. When the probabilities of all possible outcomes are far enough from the edges of the unit interval, the the agents put most weight on the anchor. However, if some outcomes have extreme probabilities, the agent will see the anchor as not very representative of the probability distribution and thus have a stronger adjustment process, represented by a higher value of γ. To measure how extreme the probabilities are, we consider the lowest probability in the lottery. For any value of n, if the lowest probability is $\underline{p}$, then the highest probability is at maximum $1 - (n-1)\underline{p}$. Thus, $\underline{p}$ is always the probability (weakly) closest to the edge of the unit interval and constitutes a good inverse measure of how extreme a probability distribution is for both low and high probabilities in a lottery.

We propose a simple linear functional form for $\gamma(\underline{p})$, namely

$$\gamma(\underline{p}) = 1 - gn\underline{p}. \tag{12}$$

In this equation, $g \in (0, 1]$ is a measure of how strong the anchoring process is. For $g = 1$, the adjustment parameter γ can be between 0 and 1, allowing for very strong anchoring when the probability distribution of a lottery is close to the uniform distribution. If g is decreased, γ is restricted to the interval $(0, g]$, such that the anchor has a smaller influence on the agents' decisions. We display the function $\gamma(\underline{p})$ for $n = 2$ and $n = 3$ and various values of g in Figure 6. Note that for any discrete lottery, the smallest probability can never be larger than $1/n$ such that $\gamma(\underline{p})$ can never be negative.

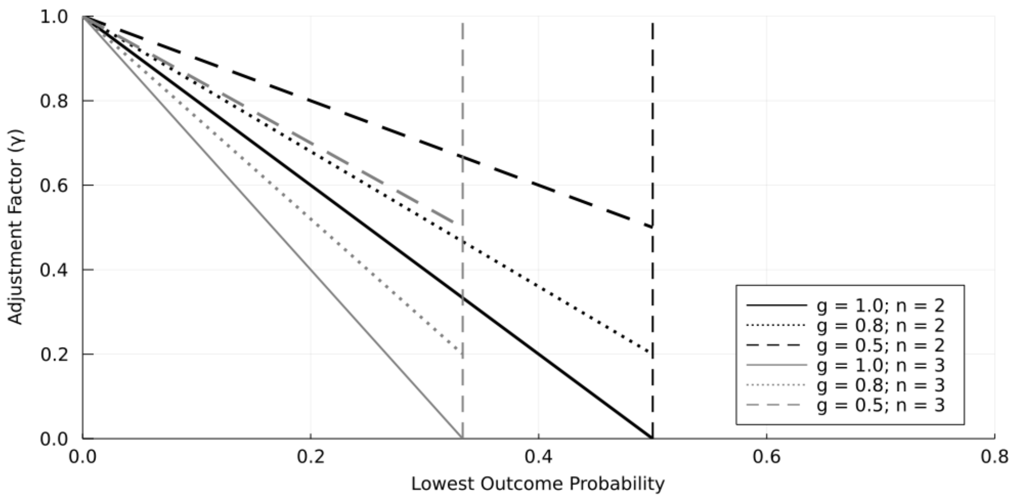
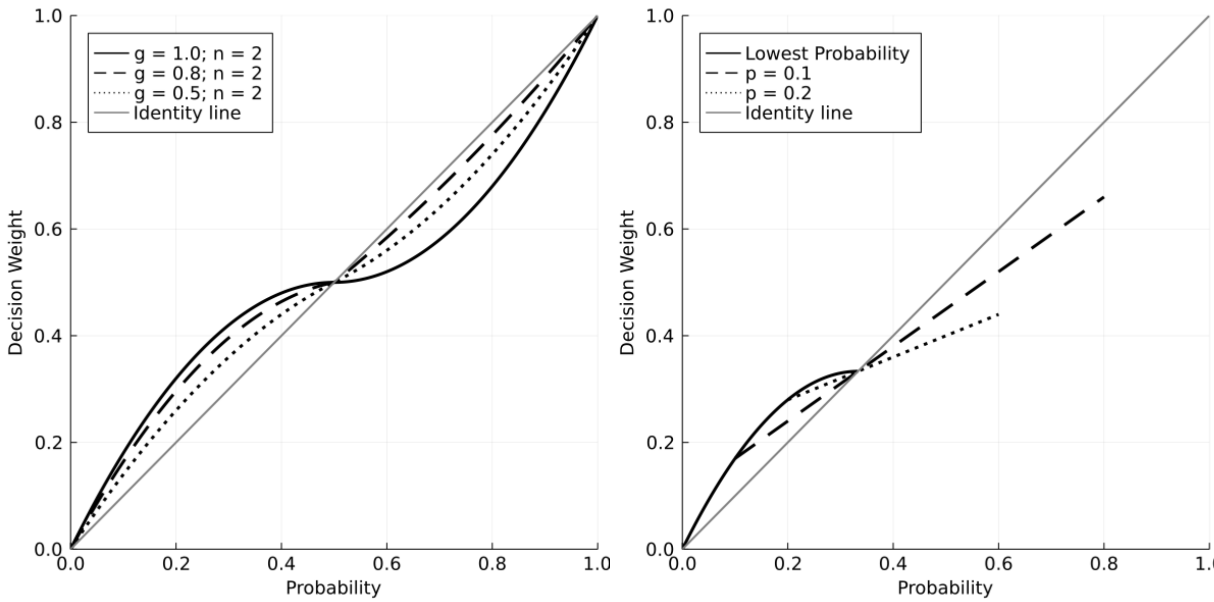


Figure 6: The figure shows possible functions for $\gamma(\underline{p})$ for the cases $n = 2$ (shown in black) and $n = 3$ (shown in gray). Vertical lines denote the limiting cases of $\underline{p} = 1/2$ and $\underline{p} = 1/3$, respectively.

To see what the endogenous value of γ implies, we consider the cases of $n = 2$ and $n = 3$ in Figure 7. For binary lotteries, the two probabilities are always symmetric around 0.5. This makes us able to derive implied probability weighting functions that can be used as in separable prospect theory and do not need to be differentiated for possible values of $\underline{p}$. This, however, is unique to the $n = 2$ case. For lotteries with a larger number of outcomes, the same probability weighting function does not apply for lotteries with different values of $\underline{p}$. Nevertheless, we first focus on this simple case due to the ease of exposition. As we can see in panel (a) of Figure 7, modeling the adjustment factor as in Equation (12) implies a probability weighting function with the characteristic inverse S-shape. Decreasing g increases γ and thus leads to less influence of the anchor. In the implied probability weighting functions, this results in a less pronounced curvature. g thus acts as an open parameter which describes how much the agent is influenced by the anchoring heuristic. Its function is similar to the role of γ in the main model. It is thus likely that the same factors which we discuss as possible influences for γ in Section 3.



(a) $n = 2$ for various values of g

(b) $n = 3$ and $g = 1$ for two sample values of $\underline{p}$

Figure 7: Implied probability weighting functions when γ is endogenous to the lowest probability of the lottery as in Equation (12). Panel (a) shows the implied probability weights for all probabilities of the lottery. The solid black line in Panel (b) shows the implied probability weight of the lowest probability. The dashed and dotted lines each show the implied probability weights when the lowest probability is 10% and 20%, respectively.

We now focus on the slightly more complicated case with $n = 3$. Panel (b) of Figure 7 shows the implied probability weighting function for the lowest probability as the solid line and potential implied probability weighting functions for the other outcomes as dashed and dotted lines. For the lowest probability, the decision weights again follow (the beginning of) an inverse S-shaped function. The decision weights of the other outcomes' probability weights are then given by a straight line with slope $\gamma(\underline{p})$. This is exemplified for $\underline{p} = 10\%$ and $\underline{p} = 20\%$ in the figure. While panel (b) only shows the decision weights for $g = 1$, the described process is also possible for smaller values of g. In such cases, the curvature of the solid black line will be less pronounced and for any given value of $\underline{p}$, the slope of the implied probability weighting function will be closer to that of the identity line. It is worth noting that while the decision weights for an individual lottery lie on a straight line when $n > 2$, the agents' treatment of probabilities across lotteries will still be non-linear, because different values of $\underline{p}$ lead to different straight lines on which the decision weights lie. Technically, even the decision weights for binary lotteries lie on a straight line which connects $w(\underline{p})$ with $w(1 - \underline{p})$. However, observing decisions across different lotteries with different values of $\underline{p}$ renders the curves shown in panel (a) of Figure 7.

The probability weighting function implied by the anchoring model and Equation (12) is closer to the commonly considered functional forms. Nevertheless, it still retains the properties of the anchoring model. The elevation and intersection with the identity line of the implied probability weighting function still decreases in n. Further, the decision weights still add up to 1, because for any given lottery, the value of γ is the same for all probabilities. Lastly, the slope of the implied probability weighting function for the lowest probability never exceeds 2, irrespective of the number of outcomes in the lottery.[21] This excludes undesirable behavioral predictions when probabilities close to the origin are involved, such as in the St. Petersburg Paradox as described in Section 4.3.

6.2 Crossing Points and Alternative Anchors

The implied probability weighting function of the anchoring model crosses the diagonal at $1/n$. This is a direct consequence of the individual and separable probability weights adding up to 1. A linear probability weighting function with given slope γ is of the form $w(p) = \gamma p + b$. Requiring that $\sum_{i=1}^{n} w(p_i) = 1$ implies $b = (1-\gamma)\frac{1}{n}$. From $w(p) = \gamma p + (1-\gamma)\frac{1}{n}$ it follows that $w(1/n) = 1/n$. However, for binary lotteries, an implied crossing point of $1/2$ is seemingly at odds with the empirical evidence. For example, Bleichrodt and Pinto (2000) find an intersection below 0.4 when correcting for the curvature of the utility function. That study, however, works in an RDEU theoretical framework. In RDEU, a crossing point of 0.4 implies that when the higher ranked of two outcomes has a probability of 40%, it gets a decision weight of 0.4. However, when the lower ranked of two outcomes has a probability of 40%, its decision weight will be higher than 0.4. In general, tests of the crossing point of the probability weighting function cannot be considered outside of the theoretical framework assumed for the test. Within rank-dependent frameworks such as RDEU and CPT, crossing points below $1/2$ for binary lotteries are arguably more tests of rank-dependence than they are tests of the crossing point.

Within our theoretical framework of separable probability weights, the crossing point is only identified if there is at least one comparison between a risky lottery and a certain payment.[22] If only risky lotteries are compared, any crossing point is possible. To see this, note that the same action which maximizes $V(L)$ also maximizes any positive transformation of $V(L)$. We could thus also maximize $\alpha V(L)$ with $\alpha > 0$ and reach the same conclusions as considered in all the examples above. If only risky lotteries are compared, we can write $\alpha V(L) = \sum_{i=1}^{n} \hat{w}(p_i)U(x_i)$ with $\hat{w}(p) = \alpha\gamma p + \alpha(1-\gamma)\frac{1}{n}$. The function $\hat{w}(p)$ has a crossing point smaller (larger) than $1/n$ if $\alpha < 1$ ($\alpha > 1$). Thus, as long as only risky lotteries are compared (as is, for example, the case in Bleichrodt and Pinto, 2000), the crossing point has no impact on the decisions implied by the anchoring model.

The crossing point obtains a meaningful interpretation once comparisons between a safe outcome and a risky lottery are made. In this case, having a lower crossing point *ceteris paribus* implies

[21] To see this, note that the implied probability weighting function for the lowest probability is $w(\underline{p}) = (1+g)\underline{p} - gn\underline{p}^2$. The slope of this function is $w'(\underline{p}) = 1 + g - 2gn\underline{p}$. The slope is thus decreasing in $\underline{p}$ and reaches its maximum at $\lim_{\underline{p}\to 0} w'(\underline{p}) = 1 + g \leq 2$.

[22] The adjective "risky" describes a lottery which is non-degenerate in the sense that at least two outcomes have a non-negative probability of occurring.

a lower evaluation of an uncertain lottery than a certain payment. This is a natural extension of the anchoring model which could provide an interesting avenue for future research. Such an “uncertainty effect” is also the explanation proposed by Gneezy et al. (2006) for their empirical findings and is at the heart of the certainty preference model of Schmidt (1998). One possible way to achieve it is using an alternative anchor for which agents adjust the unweighted average of the utils downward due to the presence of uncertainty in the lottery. Alternatively, the anchor could be calculated pessimistically. This would imply taking an average in which low outcomes have more weight than high outcomes.

Given the possible explanation provided above, it is natural to discuss anchors other than the unweighted average as possible components to the anchoring and adjustment model. It is worth noting, though, that the anchor is likely never a single outcome of the lottery, such as the minimum, maximum, or mode. If a single outcome were to act as the anchor, the model would predict that this outcome’s probability is always overweighted, irrespective of its value. Such implied probability weighting, however, has no empirical support. Combinations of multiple values are possible, in theory. We focus on the unweighted average of the utils here, because it is an intuitive and easily calculated anchor. However, other combinations of possible outcomes may be an interesting avenue to be explored in future work.

Lastly, one can consider anchors forming from previous decisions. Most experimental evidence for the anchoring and adjustment heuristic is sequential in nature. This is in part due to convenience – sequential processes let experimenters induce an anchor they can then test for. However, it is likely that anchors can also form sequentially in less controlled settings. The Bayesian updating literature, for example, considers the prior of the decision-maker the anchor, thus allowing for making predictions about sequential decision processes. Sequentially formed anchors can accommodate context effects such as choice set effects, priming, and framing.[23] Generally speaking, our model does not preclude a sequential anchor, because it could simply replace $A(L)$ in Equation (1). However, none of the expositions in this paper are intended to address such a process and it does not always align with the psychological intuition of treating outcome information and probability information separately. While this precludes context effects, the tightly-defined anchor in our model has the advantage that the model delivers predictions without requiring the specification of the anchor, thus making the model more self-contained. This is particularly advantageous in settings where no previous stage in the decision process exists and an anchor can thus not be formed sequentially.

6.3 Decisions under Ambiguity

The anchoring model also could be extended to situations in which agents do not have full information about the outcome probabilities and exhibit ambiguity aversion. Ellsberg (1961) pioneered

[23]The literature on sequential decisions under risk also considers contrast effects (Windschitl et al., 2002; Park et al., 2021). Here, prior choices or similar risks may set anchors which are so obviously unsuitable for the currently considered lottery that agents use them as contrast points rather than anchors in their decision process. This situation could also appear in our applications when the anchor formed by treating each outcome as equally likely is obviously unrepresentative of the lottery. An evaluation would then likely follow a model similar to that introduced in Section 6.1.

research into decision-making under ambiguity and the anchoring heuristic has since played an occasional role (e.g., Tversky and Kahneman, 1974; Einhorn and Hogarth, 1986). The mechanism we describe in Section 2, and the decision model outlined in Equations (2) and (3), can apply when the probability distribution is not fully known. Without any knowledge about a distribution, it seems normatively advisable and descriptively sensible that agents assume a distribution which has the least amount of information: the uniform distribution (Jaynes, 1957). If some information is available, then agents can reasonably form a point estimate (p_i) of the probability of outcome x_i, but will express some uncertainty about this point estimate. Uncertainty about the probability, theoretically, can be described by a mixture between the point estimate and the uniform distribution (represented by γ). The psychological mechanism leading to the adjustment process implies that γ increases with the information that agents have about the probability distribution (i.e., with the precision of the probability point estimate).[24] The anchoring model predicts that, at least in two-outcome lotteries, agents' ambiguity attitudes would display a fourfold pattern similar to the one discussed in Section 4.1 which seems to be evident in the literature (see Trautmann and Van De Kuilen, 2015, for an overview). One difference from existing ambiguity models, however, is that the anchoring model does not allow for a "general" source of ambiguity aversion (e.g., Chateauneuf et al., 2007). Thus, the anchoring heuristic is better viewed as a contributing factor to ambiguity aversion rather than the sole driving force.

6.4 Extensions to the Utility Function

The anchoring model also allows for flexibility in the utility function, rather than in the adjustment process. Our exposition in Section 2 assumes a well-behaved and reference-independent utility function for evaluating outcomes. The model, however, can accommodate any utility function and still lead to behavior which is observationally equivalent to probability weighting. Such an adjustment could be used to address challenges to EUT, such as Rabin's paradox (Rabin, 2000).[25] For gambles other than 50/50, the anchoring model could play the same role as probability weighting to help explain Rabin's paradox even with a standard utility formulation (for 50/50 gambles, the anchoring model reduces to EUT). Extending the anchoring model to incorporate a reference-dependent utility function would allow it to resolve Rabin's paradox even for 50/50 gambles. Such

[24] Abdellaoui et al. (2011) indeed show that there are significant differences in behavior when the amount of information on the underlying probabilities changes. Specifically, they show that agents become less sensitive to changes in probability when they are less knowledgeable about underlying probabilities, which would be in line with a decreasing value of γ.

[25] Rabin's argument states that if agents require a certain risk premium on a 50/50 gamble of winning or losing a small amount of money for all wealth levels up to a certain amount w, then EUT can only accommodate this preference pattern with a very high degree of risk aversion for all wealth levels below w. This makes the utility function so curved that any amount of money above w creates very little utility. This dictates that at wealth level w, the agents reject 50/50 gambles of losing $10,000 or gaining an amount in the billions of dollars. Because no person would refuse this gamble, EUT only allows small risk premiums on gambles with small stakes. Note, however, that while the paradox is often stated using 50/50 gambles, such a restriction is in no way necessary for the paradox. In fact, real-world evidence for Rabin's paradox is commonly reported for gambles which are not 50/50 (e.g., Sydnor, 2010).

a result would be consistent with the literature which shows that while probability weighting does exist, it contributes little to explaining Rabin's paradox for 50/50 gambles (Bleichrodt et al., 2019).

6.5 Stochasticity between Choices

A common observation in decision-making under risk is stochastic behavior by agents. When facing the same or very similar decisions multiple times, they will often make different decisions (Agranov and Ortoleva, 2017). Here, we briefly discuss one way in which such behavior can be modeled in our framework. For this, we adopt the framework of Gul and Pesendorfer (2006) to our context. Given a choice from a set of lotteries $\mathcal{L}$, we are interested in finding a probability distribution $P(\mathcal{L})$ which provides the likelihood that each individual element of $\mathcal{L}$ is chosen. Rather than assuming a stable preference functional, we assume that the agent has a probability distribution ξ over all admissible values of γ, with support $\Gamma \subseteq [0, 1]$. The probability of choosing a lottery $L_i \in \mathcal{L}$ then is

$$P(\mathcal{L})(L_i) = \xi(\gamma \in \Gamma : V(L_i|\gamma) > V(L_j|\gamma) \; \forall \; L_j \in \mathcal{L} \setminus L_i). \tag{13}$$

The advantage of this approach to model stochastic behavior is that it easily integrates with other aspects of our model. For example, the probability distribution ξ can be influenced by aspects of the decision situation, making the tendency to use the anchoring and adjustment heuristic partially constructed as discussed in Section 3. Similarly, the agent can also follow the extended model from Section 6.1 and have a probability distribution over possible values of g.

7 Conclusion

In this paper, we propose a model of anchoring and adjustment for decisions under risk. The model we develop can be applied to both discrete and continuous lotteries. It explains a number of different choice anomalies observed in behavioral economics and also provides a solution for the equity premium puzzle.

The anchoring model offers a connection between a heuristic decision strategy and substantive processing models with probability weighting. This leads to two attractive features in the proposed model. First, the implied objective function is familiar to behavioral economists due to its close relationship to the preference functional under SPT, but also has some idiosyncratic features which are not incorporated in previous formulations of prospect theory. The most striking of these features are the dependency of the probability weighting function on the number of outcomes in the lottery and the ease of application to continuous lotteries. Second, the connection between heuristic decision processing and probability weighting allows for a reinterpretation of results in both fields. This offers potential new avenues for developing empirical hypotheses about heuristic processing and probability weighting.

The anchoring model can make predictions in a variety of applications. We show both a theoretical and an empirical application to equity premiums in capital markets. The model can easily be adapted to make predictions about individual investor behavior, such as giving a heuristic-based explanation for the interest in right-skewed investments like IPOs (Wang et al., 2018) and various cryptocurrencies (Chan et al., 2017). Applications are, however, not limited to investment choices. The model can, for example, explain the positive intercept of the implied probability weighting function estimated by Barseghyan et al. (2013) from insurance choices. Since the anchoring model predicts a high evaluation for actions with positively skewed returns, it is also able to explain certain labor choices. Becoming an artist or an entrepreneur, for example, offers only low expected returns to labor, but strongly positively skewed ones (Moskowitz and Vissing-Jørgensen, 2002; Elberse, 2008). While one would normally not consider career choices to be made heuristically, these two careers are often accompanied by a strong affective response, which does make such a decision strategy more likely (Forgas, 1995; Jaspersen and Aseervatham, 2017). Applications also are not limited to individual decisions. Strategic decisions in organizations are also made under risk; heuristic decision strategies have a long research tradition in this field (Cyert and March, 1963).

The intent of this paper is not to establish the final model of anchoring and adjustment in decision-making under risk. Rather, we intend to introduce a flexible model of this well-known heuristic decision process to the literature and leave it open for possible refinements and extensions. However, even in the very simple form introduced here, the model can explain a broad range of empirically-observed phenomena, making it an interesting model for applied behavioral economists in its own right.

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A Sample Size in the Influence of Skewness on Equity Premium

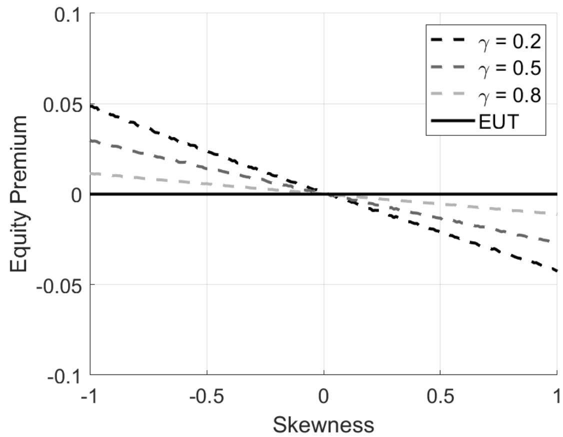
We consider the influence of skewness on the equity premium by using a small number of empirically-drawn values as a numerical approximation to the continuous lottery implied by the investment. This process has two intentions. First, it is supposed to resemble the process in which an investor considers a limited number of empirical observations to evaluate the investment. Second, in a more technical argument, the polynomial transformations of the normal distribution are unbounded such that evaluating the continuous return lottery would require truncating the distribution. This truncation, in turn, would lead to changes in the moments of the distribution, which would affect the analysis. In this appendix, we show that while the choice of sample size in the main analysis influences the magnitude of our reported result, it does not influence the main take-aways.

Figure A.1 shows the results reported in Figure 4 when instead of 50 realizations, the agents consider 20 realizations (panels (a) and (b)) or 100 realizations (panels (c) and (d)) in numerically approximating the continuous lottery. The main message of the figure is that using more realizations for the evaluation does not affect the general shape of the graphs, but it does make the results more extreme. The reason is that the anchor is influenced by the most extreme realizations drawn from the distribution. When more realizations are drawn, the minimum and maximum draw will be more extreme and thus the anchor will lead to stronger effects of skewness on the equity premiums.

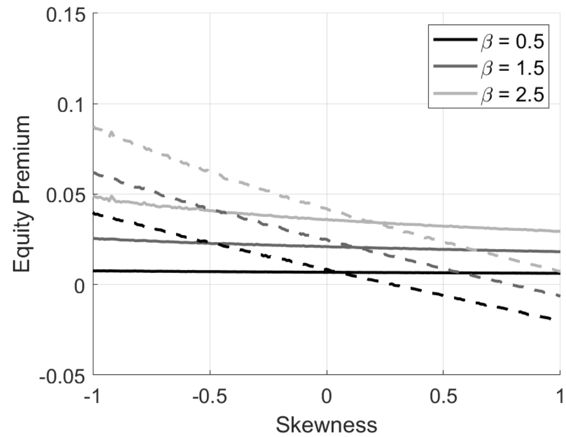
The alternative is to sample a large number of realizations from the return distribution and correctly approximate the return distribution numerically. As is described in Section 2.3, this requires truncation of the lottery. We show the results for such an analysis in panels (a) and (b) of Figure A.2. Truncation is carried out at the 5^{th} and 95^{th} percentile. Truncation, however, affects mean, variance and skewness of the distribution to various degrees. Thus, we also truncate the distribution for the evaluation according to EUT so there is no difference in the underlying asset between both decision models.

Results are again qualitatively comparable, but less extreme in their magnitude than in the main analysis. This is due to the effect of truncation on the moments of the return distribution which is shown in panels (c) through (e) of Figure A.2. Panel (c) shows the effect of truncation on the mean of the distribution. The effect differs depending on the skewness of the untruncated distribution. The mean is increased for negatively skewed distributions and decreased for positively skewed distribution. This has little effect on the equity premium curves, because we calculate the equity premium for both the anchoring model and EUT with the same truncated distributions and both models handle differences in means similarly.

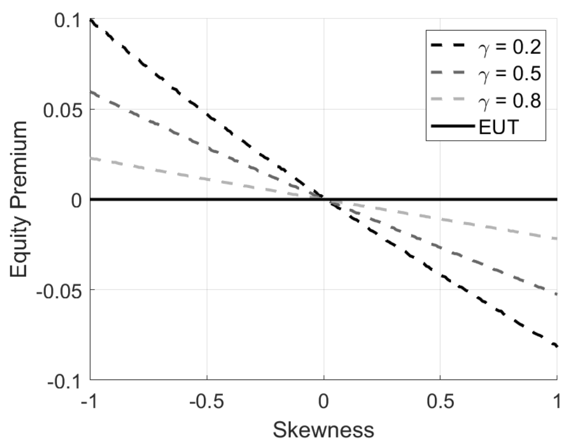
Panel (d) shows that truncation decreases variance. This mutes the effect of variance in the anchoring model which is responsible for the rightward shift of the anchoring lines in panel (b). This effect, however, is nearly constant across the different levels of skewness considered, such that there is little effect on the slopes of the anchoring model's equity premium curves. Panel (e) of Figure A.2 shows how truncation of the distribution affects the skewness. Here we can see a large and systematic effect. The truncated distribution will only have about half the skewness of the



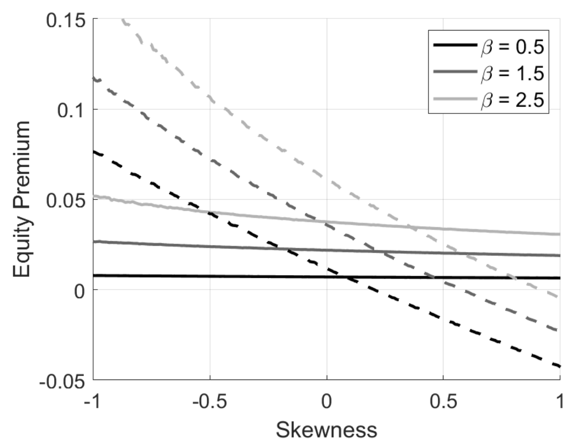
(a) Risk neutral case, sample of 20 draws



(b) Risk averse case (with $\gamma = 0.5$), sample of 20 draws



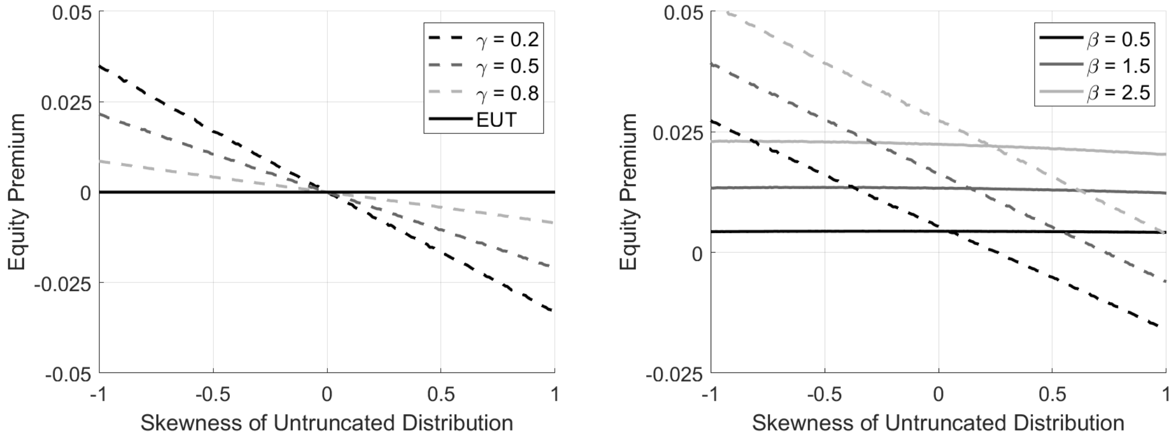
(c) Risk neutral case, sample of 100 draws



(d) Risk averse case (with $\gamma = 0.5$), sample of 100 draws

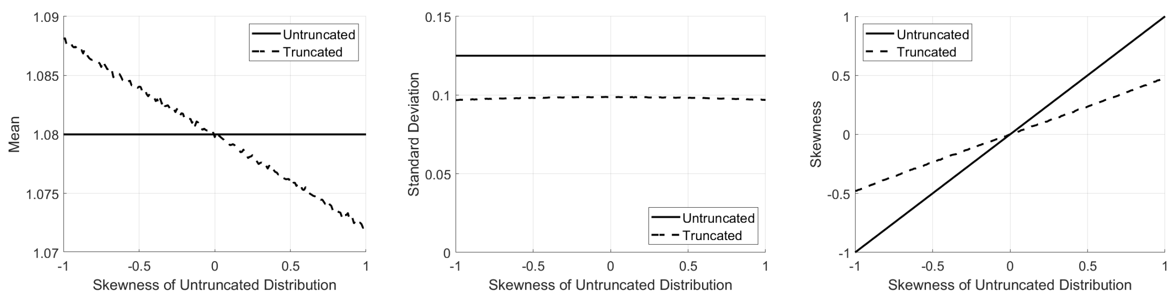
Figure A.1: Equity premiums according to EUT (solid lines) and the anchoring model (dashed lines). The model assumes $r = 1.04$, $\mu(\tilde{x}) = 1.08$, $\sigma(\tilde{x}) = 0.125$, and varies the skewness of the returns according to the procedure by Jaspersen and Peter (2017). $\tilde{x}$ is evaluated based on the indicated number of draws from the distribution and the analysis uses 1,000 replications.

untruncated distribution. This explains the difference in slopes between the results in panels (a) and (b) of Figure A.2 compared to Figure 4 in the main analysis. The anchoring model reacts so much less to skewness, because the distribution that is actually considered by the model is far less skewed than the distribution initially generated for the returns.



(a) Risk neutral case, sample of 20 draws

(b) Risk averse case (with $\gamma = 0.5$), sample of 20 draws



(c) Effect of truncation on mean

(d) Effect of truncation on standard deviation

(e) Effect of truncation on skewness

Figure A.2: Equity premiums according to EUT (solid lines) and the anchoring model (dashed lines). For the untruncated distribution, the model assumes $r = 1.04$, $\mu(\tilde{x}) = 1.08$, $\sigma(\tilde{x}) = 0.125$, and varies the skewness of the returns according to the procedure by Jaspersen and Peter (2017). $\tilde{x}$ is evaluated based on 1,000,000 draws from the distribution and the distributions are truncated at the 5^{th} and 95^{th} percentile.

B Equity Premium Analysis for Different Time Periods

This appendix repeats the analysis reported in Section 5.3 using different time periods for the sampling of the stock returns. The results of the main analysis, using the maximal available range of years, are reported for reference. As can be seen in Table B.1, the returns for both stocks and bonds are relatively stable for the full time period, 60 years of data and 40 years of data, but are markedly smaller and more extremely skewed when only the most recent 20 years of data are considered.

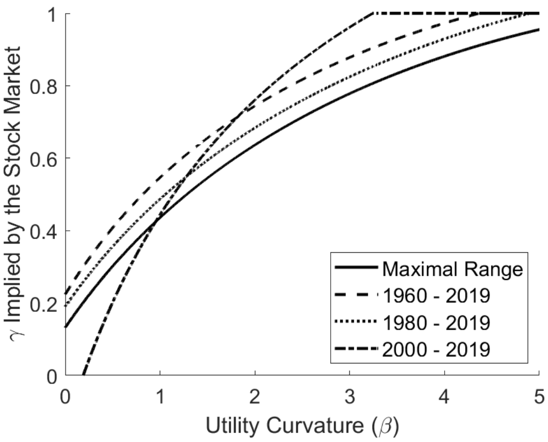
The results in Figure B.1 are qualitatively comparable to Figure 5 in the main analysis. Using shorter time periods of the last 40 or 60 years leads to even larger required values of γ than in the main analysis. This means that the equity premium puzzle is easier to solve using the anchoring

Table B.1: Summary statistics for annual asset returns with different time periods

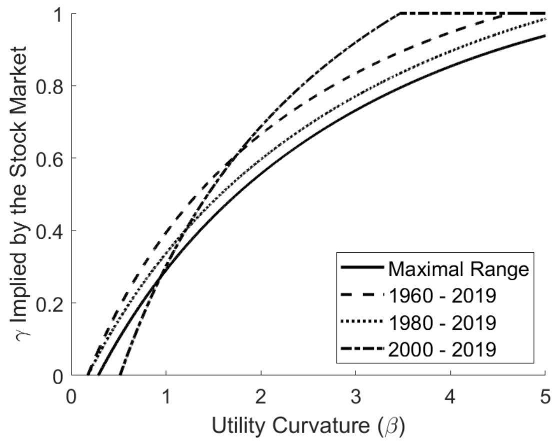
	Mean	Variance	Skewness	Min	Max
Maximal Range (1941 - 2019)					
Stocks	0.1290	0.0276	-0.1102	-0.4254	0.6146
1-year Treasury Bonds	0.0470	0.0015	1.1394	-0.0078	0.2221
90-day Treasury Bills	0.0420	0.0012	0.9481	-0.0068	0.1725
60 Years (1960 - 2019)					
Stocks	0.1166	0.0259	-0.3582	-0.4254	0.6125
1-year Treasury Bonds	0.0560	0.0015	0.9423	0.0000	0.2221
90-day Treasury Bills	0.0506	0.0012	0.6922	0.0003	0.1725
40 Years (1980 - 2019)					
Stocks	0.1284	0.0272	-0.4305	-0.4254	0.6125
1-year Treasury Bonds	0.0541	0.0021	1.0259	0.0000	0.2221
90-day Treasury Bills	0.0467	0.0016	0.8934	0.0003	0.1725
20 Years (2000 - 2019)					
Stocks	0.0743	0.0273	-0.7313	-0.4254	0.5282
1-year Treasury Bonds	0.0217	0.0005	1.2317	0.0000	0.0888
90-day Treasury Bills	0.0169	0.0003	1.0495	0.0003	0.0642

Note: The table reports descriptive statistics for annual returns of the different asset classes considered in the equity premium analysis for different time periods. Stock returns refer to the S&P 500 including dividend payments. All returns are reported in nominal terms adjusted on a monthly basis. Annual returns are reported based on 10,000 draws (with replacement) of random starting months in the given periods.

model – for a given level of utility curvature, less heuristic processing is necessary for a solution. When considering the last 20 years, the curve becomes steeper, but also has a lower intercept.



(a) Stocks and Bonds



(b) Stocks and 90 day T-bills

Figure B.1: Figures report the β / γ combinations for which the agents are indifferent between a portfolio with only stocks and a portfolio with only 1-year bonds (panel (a)) or only 90-day T-bills (panel (b)). Agents have an iso-elastic utility function with relative risk aversion as indicated on the x-axis. Analyses in both panels are based on 10,000 draws of 12-month real returns (with replacement) from a random starting month in the given time periods.

C Equity Premium Analysis with Real Returns

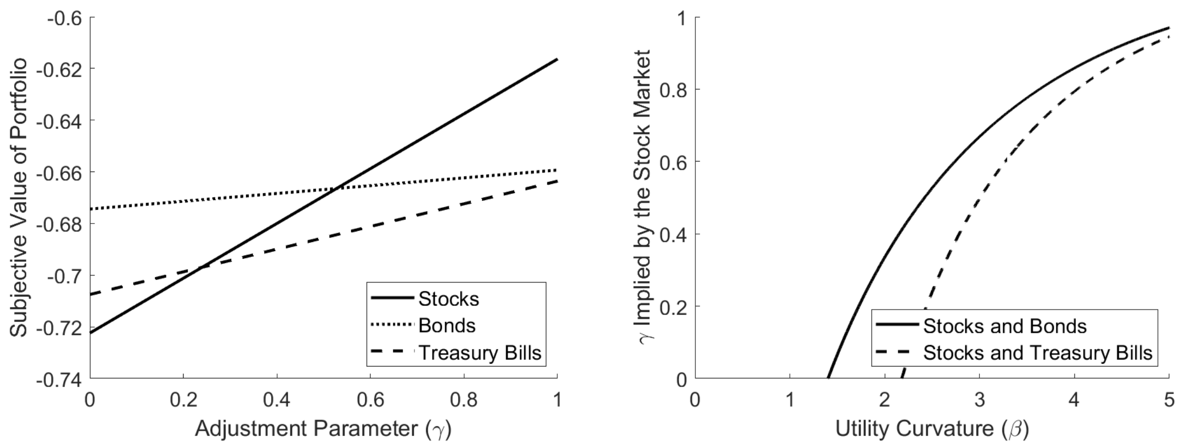
In this appendix, we repeat the empirical analysis of the equity premium puzzle using real instead of nominal returns. Real returns are calculated using monthly changes in the consumer price index. Table C.1 shows the descriptive statistics of the monthly and annual returns for the three considered asset classes in real terms. All asset classes see about equal absolute decreases in their returns and only very little change in their variance. However, while the skewness of the stock returns changes little, bonds and treasury bills are now negatively skewed instead of positively skewed. This change is especially notable for T-bills, turning their return distribution from being strongly right-skewed to strongly left-skewed.

Table C.1: Summary statistics for monthly and annual real asset returns

	Mean	Variance	Skewness	Min	Max
Monthly Returns					
Stocks	0.0069	0.0017	-0.4144	-0.2178	0.1567
1-year Treasury Bonds	0.0007	4.06E-5	-0.3722	-0.0550	0.0443
90-day Treasury Bills	0.0003	2.29E-5	-2.3699	-0.0552	0.0206
Annual Returns (random starting month)					
Stocks	0.0900	0.0288	-0.1409	-0.4542	0.5719
1-year Treasury Bonds	0.0096	0.0017	-0.3440	-0.1572	0.1634
90-day Treasury Bills	0.0047	0.0012	-1.2612	-0.1612	0.0980

Note: The table reports descriptive statistics for monthly and annual returns of the different asset classes considered in the analysis for the time period between 1941 and 2019. Stock returns refer to the S&P 500 including dividend payments. All returns are reported in real terms adjusted on a monthly basis. Annual returns are reported based on 10,000 draws (with replacement) of random starting months between January 1941 and January 2019.

The absolute changes in all asset classes are about equal and all assets now have negatively skewed returns, so stock investments are relatively more attractive. Under the assumption of iso-elastic utility, this leads to an even more pronounced equity premium puzzle. We thus consider a decision-maker with $\beta = 2.5$ (instead of 1.5) in panel (a) of Figure C.1 to see possible indifference between the different portfolios for both bonds and T-bills. Even with this change in risk aversion, the necessary γ values for indifference are still shifted downwards such that a stronger anchoring bias is necessary to explain the equity premium puzzle. All three portfolios now have an increasing subjective value in γ. However, due to the more pronounced tails, the slope is still largest for the stock portfolio and thus the anchoring model still offers an explanation of the equity premium puzzle. In fact, as can be seen in panel (b) of Figure C.1, particularly when bonds are considered as the alternative to stock investments, the anchoring model can still explain the equity premium puzzle with sensible parameter combinations of β and γ.



(a) Subjective value of portfolios consisting of one exclusive asset class ($\beta = 2.5$)

(b) β / γ combinations which leads to indifference between two single asset class portfolios

Figure C.1: Panel (a) shows the subjective value according to the anchoring model of three portfolios consisting of a single asset class for different values of γ. We assume agents have an iso-elastic utility function with a relative risk aversion of $\beta = 2.5$. At $\gamma = 1$, the anchoring model is equal to EUT. Panel (b) reports the value of γ for which the agents are indifferent between a portfolio with only stocks and a portfolio with only 1-year bonds or only 90-day T-bills, respectively. Agents again have an iso-elastic utility function with relative risk aversion as indicated on the x-axis. Analyses in both panels are based on 10,000 draws of 12-month real returns (with replacement) from a random starting month between January 1941 and January 2019.