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ON-DEMAND INSURANCE

Alexander Braun
University of St. Gallen

Niklas Haeusle
University of St. Gallen

Paul Thistle[1]
University of Nevada Las Vegas

ABSTRACT:
On-demand insurance contracts are crucial for the sharing economy. These contracts make use of a simple mechanism in a practical way: People differ in their frequency of exposure as well as the probability of loss; the extra dimension of heterogeneity can be used to screen the insured which leads to welfare improvements (the utility possibility frontier shifts outward). The conditioning on time periods relaxes the incentive compatibility constraint, since high frequency types are risking being uncovered for some time by choosing a different contract. We provide sufficient conditions when a full coverage equilibrium is reached. We also analyze various types of on-demand contracts which are used in practice, both in a Rothschild-Stiglitz and in a Wilson-Miyasaki-Spence framework.

[1] Braun: University of St. Gallen, I.VW-HSGBüro 53-006, Tannenstrasse 19, 9000 St. Gallen, Switzerland, Phone: 41 71 224 3653, Email: alexander.braun@unisg.ch
Haeusle: University of St. Gallen , I.VW-HSG, Büro 53-106, Tannenstrasse 19, 9000 St. Gallen. Switzerland. Phone: +41 71 224 3658, Email: niklas.haeusle@unisg.ch
Thistle: Department of Finance, University of Nevada Las Vegas 4505bS. Maryland Parkway, Box 456008, LasVegas, NV 89154. Phone: 702-895-3856, Fax:702-895-4603, Email: paul.thistle@unlv.edu

On-Demand Insurance

On-demand insurance is one of several innovations that the InsurTech sector recently brought to the insurance industry (see Braun and Schreiber, 2017). It allows individuals to flexibly secure coverage for episodic risks whenever needed. Individuals may therefore seek exclusive protection once their items or they themselves are at risk, e.g., when they drive their car part-time for Uber or Lyft. In contrast to consumers of classical insurance products with annual premiums, those of on-demand policies are only charged for the actual time of usage. Claims are valid as long as coverage was active when the damage happened. The number of firms offering on-demand insurance has rapidly grown in recent years and now spans various product lines such as pay-per-mile car insurance (*Metromile*, *Cuvva*, *Friday*), pay-per-use policies for bicycles and cameras (*Lings*, *Getsafe*), homeshare and rideshare coverage (*Slice*, *Sure*) and even commercial insurance products (*Tapoly*). Some observers estimate that the on-demand insurance market could grow to $190 billion by 2026 (see, e.g., Garth, 2019).

Beyond its standalone appeal for technology-affine insurance customers, on-demand insurance exhibits an even greater potential: it is the natural form of insurance for the sharing economy. The sharing, collaborative or access economy is designed to allow individuals the consumption of a wide variety of products and services (from cars and houses to tools etc.) without ownership.[2] A key segment is car sharing, which can take place in two forms. On the one hand, peer-to-peer platforms such as Turo are matchmakers between car owners and car seekers. On the

[2] In a narrow sense, sharing means collective consumption that does not involve profit-seeking lenders and intermediaries. Hence, Bardhi, and Eckhardt (2012) introduced the term access economy, which underlines that consumers are in fact participating in a heavily intermediated market and pay to utilize the property or service of someone else for a specific time span.

other hand, centralized mobility service providers such as Share Now run car fleets that they distribute across urban centers so that customers can access them on the spot through a smart phone app. In both cases, insurance coverage must be delivered in the form of new on-demand contracts, either for a fixed period (in the case of peer-to-peer car sharing) or in a pay-as-you-go format (for mobility services). Standard one-year insurance policies are not suitable for this purpose.

We analyze on-demand insurance contracts in markets with adverse selection. Individuals may differ in their riskiness and in their amount of exposure to the risk; both are private information. We adopt the contract-theoretic approaches of Rothschild-Stiglitz (1976) as well as Wilson (1977), Miyazaki (1977) and Spence (1978). Much of the existing literature in this framework analyses "standard" insurance contracts, which cover a single risk and are fully characterized by the price of the policy and the quantity of coverage. It is implicitly assumed that the coverage is in force at all times during the policy period. In contrast, on-demand policies are in force only during selected times during the policy period. That is, on-demand policies are characterized by the price, the quantity of coverage and a determination of the subperiods during which the coverage is in force. The objective of this paper is to examine the efficiency effects of the introduction of on-demand policies and to characterize the equilibrium policies.

We contribute to the large literature on risk classification and categorization in insurance markets (e.g., Hoy, 1982, 1989, Crocker and Snow 1985, 1986, Rothschild, 2011, 2015, among others).[3] This literature shows that, under certain conditions, risk categorization improves welfare.[4] Our analysis is most closely related to the work by Bond and Crocker (1991), Crocker and Snow (2011) and Crocker and Zhu (2021). Bond and Crocker (1991) allow insurance companies to

[3] See Crocker and Snow (2013) and Dionne and Rothschild (2014) for reviews of this literature.

[4] Costless categorization always increases welfare, while costly categorization may or may not increase welfare. Rothschild (2011) shows that, if the government provides the appropriate partial social insurance policy, then private insurers have the incentive to engage in costly risk categorization if, and only if, it is welfare improving.

classify risks based on the observable consumption of goods, which is correlated with the insurable risk. They term this idea “endogenous categorization”.[5] Consumption may provide information about the individual’s privately known loss propensity and allow more efficient sorting of individuals into risk categories. Bond and Crocker (1991) show that this may lead to first best allocations, if the adverse selection problem is “small.” Crocker and Snow (2011) examine the bundling of different losses and perils into one insurance policy. The different risks and perils have different deductibles, allowing for screening in multiple dimensions. They show that multidimensional screening relaxes the self-selection constraint, improving the efficiency of allocations. This also implies that the Rothschild-Stiglitz equilibrium exists for a wider range of populations. Crocker and Zhu (2021) analyze the efficiency effects of voluntary risk classification. Consumers have the option of taking a costless, but imperfectly informative underwriting test, the results of which are used to classify individuals. They show that voluntary classification is Pareto superior to no classification and to mandatory classification. Voluntary classification allows the insurer to use both the choice to take the test and the test results to underwrite policies.

The insurance policies examined in these papers can be characterized as “price-quantity-plus” contracts, adding a dimension beyond just price and quantity.[6] The contracts examined in Bond and Crocker (1991) add the correlated consumption goods. The contracts examined in Crocker and Snow (2011) are multi-dimensional, covering multiple risks. The contracts examined

[5] Bond and Crocker (1991) allow for both moral hazard and adverse selection, whereas we only consider an adverse selection problem.

[6] These papers, as is ours, are also related to the literature on contract form as a screening mechanism. Smith and Stutzer (1990) as well as Ligon and Thistle (2005) show that participating policies sold by mutual insurance companies offer individuals an additional possibility to signal their type. In the resulting equilibrium, low-risk individuals purchase policies from mutual insurers and high-risk individuals those from stock insurers. Posey and Thistle (2017) analyze automobile insurance markets with asymmetric information and show that the policyholders’ choice between full and limited tort coverage effectively represents a screening mechanism. Finally, Gemmo et al. (2017) draw on a Wilson-Miyazaki-Spence framework to examine transparency contracts, which allow for risk screening with wearables or telematics devices. They find that consumers who are prepared to share private information receive full insurance coverage, implying a Pareto improvement in terms of utilitarian welfare.

in Crocker and Zhu (2021) add the choice to take the underwriting test as well as its result. In our analysis, on-demand contracts are characterized by price and quantity plus the time dimension. There are two important distinctions compared to multidimensional risk bundling. First, on-demand contracts have fewer degrees of freedom with regard to contract design. This limits the possibilities of effective screening with on-demand contracts. Second, the time dimension makes this problem inherently dynamic. An important condition in our analysis is whether the high-risk individuals' uncovered loss under the low-risk policy is greater than the difference in the total loss probabilities. If this condition holds, then the time dimension provides enough information to separate the low risks and high risks. If so, both high and low risk types may receive full coverage at actuarially fair prices. The fairly priced, full coverage policies can be achieved in a Wilson-Miyazaki-Spence (WMS) equilibrium and in a Rothschild-Stiglitz (RS) equilibrium. If the condition does not hold, the time dimension alone does not provide enough information to separate the low risks from the high risks and some rationing of coverage for the low risks is needed. Nonetheless, adding the time dimension relaxes the self-selection constraint so that the utility possibility frontier for on-demand contracts lies outside the utility possibility frontier for standard insurance policies. As a result, low risks receive more coverage in both the WMS and the RS equilibrium.

The remainder of this paper is structured as follows. The next section provides an illustrative example. In the third section, we develop the model with two risk types. We analyze fixed-period on-demand policies, where the subperiods covered are offered by the insurer and the policyholder knows with uncertainty that those are the periods in which he will be at risk. We also analyze fixed-duration on-demand policies, where the insurer offers to cover a certain number of subperiods and the policyholder can select the ones for which he or she would like to be insured.

In both cases, individuals are assumed to commit to their coverage at the beginning of the policy period. Finally, we discuss pay-as-you-go insurance, where a commitment from the policyholder is not required.[7]

We want to highlight that all three types of on-demand insurance policies we examine can be found in practice. Fixed-period contracts are popular for example with embedded travel insurance. If you book a trip online, you often have the option to directly add travel insurance. Here, the insured cannot choose the contract period himself, but there is also no uncertainty when he or she will actually require the coverage. Moreover, fixed-duration contracts can be applied when the usage (exposure) pattern throughout the year is known in advance. An example is insurance for small businesses, which have part-time (but repeating) opening times. Finally, pay-as-you go contracts are becoming very popular in the car insurance space, where drivers usually do not know their future usage.

An Illustrative Example

Assume that there are two individuals who have wealth W and face a random insurable loss of fixed size D, which can occur only once during the policy period.[8] Linda has a 20% risk of an accident during the only month in the year she needs to use a vehicle. Helen uses her vehicle 10 months of the year and has a total accident risk of 30% for the whole year. At the beginning of the year, both commit to the time periods during which they will have coverage, with an associated

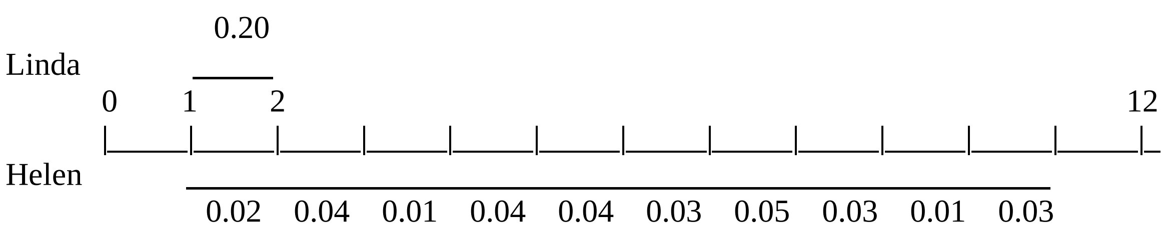
[7] Industry practitioners also use the term usage-based insurance (UBI) in this context. UBI is commonly defined as the combination of flexible coverage duration *and* observable risk behavior. In telematics car insurance, e.g., these elements are termed pay-as-you-drive (PAYD) and pay-how-you-drive (PHYD) (Śliwiński and Kuryłowicz, 2021). Our analysis focuses on the time dimension rather than the transparency dimension. The latter is treated in Gemmo et al. (2017).

[8] This is the same as assuming that the insurance policy is terminated without reinstatement after the first loss has been reported.

total premium ρ. Consumption then happens at the end of the year, after observing whether a loss has been realized and any indemnity paid.

In the traditional Rothschild-Stiglitz setting with standard contracts, Helen has a higher annual risk, reflected by the total loss probability $p_H = 0.3$, so she gets full, fair insurance for a premium of $\rho = 0.3D$. Linda only receives partial, fair insurance, since she has a lower annual risk: her total loss probability is $p_L = 0.2$. A separating equilibrium arises. However, on a monthly basis, Linda is at a higher risk. Linda's joint probability of a loss in any subperiod[9] is $p_L = 0.2 > 0.05 = \max p_H$.

Figure 1
Linda's and Helen's Risk Profiles



This figure illustrates the risk profiles of the two individuals. Consider now the following on-demand contract: Instead of a standard yearly contract, the on-demand contract provides full insurance for one month at the fair premium for Linda. Note that Linda has no coverage for the other months. Helen still keeps her original standard insurance contract, which covers the whole year.

To show that this is incentive compatible, there are various cases to consider. Linda could switch from the one-month on-demand policy to the full-year standard policy. However, the standard policy is not her preferred choice, because it has a higher premium and provides coverage

[9] The joint probability of a loss in a subperiod P(A *and B*) comprises two coinciding events: A) a loss will occur in that subperiod *and* B) no loss occurred in the previous subperiods.

in months when she is not at risk. Helen, on the other hand, could switch from the full-year standard policy to the one-month on-demand policy with the lower premium. This, however, would leave her with no coverage for 9 of the 10 months in which she is at risk. As we will prove in the next section, the disutility from uncovered months is under certain conditions larger than the disutility from the premium differential. Hence, Helen prefers the standard contract. Finally, Helen could buy the one-month policy ten times, but this would cost $(n_H/n_L)\rho_L = 10 \cdot 0.2D = 2D$ instead of the annual premium of $0.3D$ for the standard policy. In this example, the difference in the time periods for which Linda and Helen demand coverage is enough to distinguish between them and offer both fairly priced full coverage.

Now suppose instead of being at risk for only one month, Linda is at risk from January to September, with monthly loss probabilities which sum equals in total to 0.2. Linda can buy an on-demand policy covering January to September. If Helen buys this policy, she is only at risk for an uncovered loss for three months with a probability of 0.07. If the on-demand policy offers full coverage, then Helen will imitate Linda and buy the policy – the premium savings are enough to offset the risk of uncovered losses. In this case, the time dimension *alone* is not enough to screen Linda and Helen. Some rationing of coverage in the on-demand policy is also needed. Yet, as Helen still would face the disincentive of uncovered losses, the indemnity can be larger than if Linda's policy offered full-year coverage.

Fixed-Period Policies

In our basic model, all decisions are made at the beginning of the policy period for all subperiods. The analysis of on-demand contracts is inherently dynamic: the subperiods to insure are dispersed along the time dimension . For fixed-period policies, however, individuals commit at the beginning of the year to all subperiods they would like to cover. This allows us to capture the economics of on-demand insurance without getting bogged down in the complexities of a dynamic model.

We now generalize our example from the previous section and provide a sufficient condition when full coverage is optimal and can be supported by a competitive equilibrium. There is a continuum of individuals, who are assumed to be observationally identical. We assume all individuals are expected utility maximizers with non-satiated, risk averse preferences described by a utility function *U*, with $U' > 0$ and $U'' < 0$. Individuals have initial wealth *W* and face a possible loss of *D*. Individuals are assumed to be one of two types, a type with lower total risk (subscript *L*) and a type with higher total risk (subscript *H*); risk type is private information. The proportion of type *i* individuals is δ_i, where $0 < \delta_H, \delta_L < 1$ and $\delta_H + \delta_L = 1$.

An insurance policy, *C*, is described by a premium, ρ, an indemnity, *I*, and a time dimension, $\boldsymbol{\tau}$. The overall policy period is divided into *T* subperiods, so that $\boldsymbol{\tau}$ is a *T*-vector of zeros and ones describing when the policy is in force ($\tau^t = 1$) or not in force ($\tau^t = 0$). For a standard insurance policy, $\boldsymbol{\tau}$ is a *T*-vector of 1s, indicating the policy is always in force. For an on-demand policy, the insurer offers a contract which specifies the subperiods during which the policy is in force.[10] The premium is paid at the beginning of the policy period. A loss may occur during one of the subperiods, in which case it is indemnified if the policy is in force during that subperiod. All consumption takes place at the end of the policy period. For ease of exposition, we assume that the discount rate is zero.

[10] Formally, a standard policy is a special case of an on-demand policy which is always in force ($\tau_i^t = 1$ for all t). We take "on-demand" to mean that the policy is in force for at least one subperiod and not in force for at least one subperiod. We also distinguish on-demand policies from the null policy (no insurance), which has a premium and indemnity of zero, and $\boldsymbol{\tau}$ equal to a *T*-vector of zeros.

For an individual with an on-demand policy, three states can materialize. In state 1, there is no loss, implying that the premium is paid and the individual's terminal wealth amounts to $W_i^1 = W - \rho_i$. In state 2, there is an insured loss. Accordingly, the premium is paid, the loss is indemnified, and the individual's terminal wealth amounts to $W_i^2 = W - \rho_i - D + I_i$. Finally, in state 3, there is an uninsured loss: the policy was not in force during the subperiod in which the loss occured. In this case, the individual's terminal wealth is $W_i^3 = W - \rho_i - D$. Note that the third state cannot occur with a standard policy.[11]

We assume that at most one loss can occur during the policy period. Let p_i^t ($0 \leq p_i^t < 1$) be the type *i* individuals' probability of the joint events that a loss has not occurred up until $t - 1$ *and* then occurs in subperiod t (between $t - 1$ and t). The p_i^t are assumed to be common knowledge. We assume that $0 \leq p_i^t < 1$ and $0 < \sum_{t=1}^{T} p_i^t < 1$, and we allow $p_i^t = 0$ during some subperiods, since the individual may not be engaged in the risky activity to be insured. For example, if someone drives for a ride-share service only on the weekend, then the probability of a loss is zero on weekdays. We do not assume that $p_H^t > p_L^t$, this may or may not be true for any subperiod. The probabilities of the different states for the overall policy period can be determined as follows.

- The probability of state 1 (no loss) is $P_i^1 = 1 - \sum_{t=1}^{T} p_i^t$.
- The probability of state 2 (covered loss) is $P_i^2 = \sum_{t=1}^{T} p_i^t \tau_i^t$.
- The probability of state 3 (uncovered loss) is $P_i^3 = \sum_{t=1}^{T} p_i^t (1 - \tau_i^t)$.

The high risks have a greater risk of loss and therefore a higher probability of no loss during the overall policy period than the low risks: $P_L^1 > P_H^1$. The probabilities of states 2 and 3 depend on the contract through $\boldsymbol{\tau}$. We write $P_{ij}^2 = \sum_{t=1}^{T} p_i^t \tau_j^t$ and $P_{ij}^3 = \sum_{t=1}^{T} p_i^t (1 - \tau_j^t)$ when a type *i*

[11] In the Crocker and Snow (2011) model with *n* bundled risks, there are *n* + 1 states of the world with *n* degrees of freedom. Our model has three states and only one degree of freedom since $W^3 = W^1 - D$.

individual has the contract C_j ($i \neq j$). We exclusively use the subscript i when a type i individual has the contract C_i.

For a standard insurance policy, full insurance means the indemnity is equal to the loss. For an on-demand policy, full insurance means the indemnity is equal to the loss and the policy is in force during each subperiod with a non-zero risk of loss.

The high and low-risk types can be at risk of a loss during different time periods. Let the "at risk" set, $R_i = \{t \mid p_i^t > 0\}$, be the set of periods in which the probability of loss is positive. While there are exceptions, for most applications we expect that R_H is *not* a subset of R_L: $R_H \nsubseteq R_L$. Hence, in our main analysis, we look at the case in which R_H cannot overlap with R_L, unless it contains more elements.[12] Moreover, we assume that individuals do not choose contracts with superfluous coverage, that is, contracts for which $\tau_i^t = 1$ in subperiods where $p_i^t = 0$.[13] The fairly priced, full insurance contract is denoted C_i^*; for this contract, the premium is $\rho_I = (1 - P_i^1)D$, the indemnity is $I_i = D$ and subperiods are insured if, and only if, they are at risk: $\tau_i^t = 1$ with $t \in R_i$. Finally, we assume that the difference in total risk between the two types is less than the potential uncovered risk for the type H individuals:

Assumption 1: $\sum_{t=1}^{T} p_H^t - \sum_{t=1}^{T} p_L^t \leq \sum_{t=1}^{T} p_H^t (1 - \tau_L^t)$.

This assumption plays an important role in the analysis, which we discuss in detail below.

In the aforementioned setting, the expected utility for a type i individual with the contract C_j is given by

[12] Suppose Linda and Helen both go on vacation to the beach for the month of July. Linda sits on the beach reading, while Helen goes jet skiing and snorkeling. Alternatively, suppose that Linda vacations during July and August, while Helen vacations only during July. In either case, the time periods they are at risk cannot be used screen Linda and Helen. Price-quantity contracts must be used, and we are essentially back to standard insurance. We wish to rule this type of situation out.

[13] This is a tie-breaker rule. A contract that offers superfluous coverage yields the same expected utility as a contract that does not. If some contract C is optimal, then so is every contract that offers the same coverage plus superfluous coverage. This leads to multiple equilibria which are not different in any meaningful economic sense.

$$V_i(C_j) = P_i^1 U(W - \rho_j) + P_{ij}^2 U(W - \rho_j - D + I_j) + P_{ij}^3 U(W - \rho_j - D). \quad (1)$$

The resource constraint of the insurers is

$$RC \qquad \delta_H[\rho_H - P_H^2 I_H] + \delta_L[\rho_L - P_L^2 I_L] \geq 0 \quad (2).$$

The incentive compatibility constraints for the high risks and low risks are

$$IC_H \qquad V_H(C_H) \geq V_H(C_L), \quad (3)$$

$$IC_L \qquad V_L(C_L) \geq V_L(C_H), \quad (4)$$

and the high-risk utility constraint is

$$UH \qquad V_H(C_H) \geq \bar{V}_H. \quad (5)$$

where $\bar{V}_H$ is the lower bound on the high risks' utility.[14] Finally, the welfare maximization problem is

$$\text{OD} \qquad \arg\max_{\{C_H, C_L\}} V_L(C_L) \quad (6)$$

subject to the constraints (2), (3), (4) and (5).

Our first result is described by the following proposition:

Proposition 1: If Assumption 1 holds, then, at the optimum (a), the resource constraint is binding, (b) the incentive compatibility constraints are both slack, and (c) both types receive full insurance contracts (C_H^*, C_L^*).

Proof: All proofs are given in the Appendix.

Part (c) of Proposition 1 is different from the well-known result for standard contracts in that the low risks now receive full instead of partial coverage. The economic reason is straightforward – the types can be screened by the subperiods for which the contract provides coverage. As a result,

[14] Without this lower bound, the solution would be to allocate all resources to the low risks. We consider solutions where $\bar{V}_H \leq V_H(C^P)$, where C^P is the fairly priced pooled full insurance contract. This constraint implies that the high risks do not subsidize the low risks. Solutions to the welfare maximization problem where this inequality is reversed are symmetric, interchanging the roles of the H and L types. These solutions imply overinsurance for the high risks. Overinsurance violates the principal of indemnity in property and liability insurance.

premium and coverage do not need to be part of the screening mechanism and both types can be offered full coverage.

Proposition 1 yields some important additional results.

> **Corollary 1**: If Assumption 1 holds, then (a) the WMS equilibrium contracts are (C_H^*, C_L^*), and (b) there is no cross-subsidization.

In the classical WMS model, if the proportion of high risks is small enough, then low risks may be willing to subsidize the high risks to relax their incentive compatibility constraint and obtain a contract with higher coverage for themselves. This is not the case with on-demand contracts, since the low risks obtain full insurance.

> **Corollary 2**: If Assumption 1 holds, then (a) the RS equilibrium contracts are (C_H^*, C_L^*) and (b) this is true for all values of $\delta_H > 0$.

Since there is no cross-subsidization in the WMS equilibrium, both policies break even individually. Thus, in the presence of on-demand contracts, the WMS and RS equilibria coincide. An important problem with the RS equilibrium for standard contracts is that it does not exist, if the proportion of high risks is too small. On-demand contracts add the time dimension, which improves screening and solves the existence problem.

Note that **Proposition 1** and its corollaries crucially depend on Assumption 1, which does not always hold. We let C_L^{**} denote the low-risk partial insurance contract that satisfies the resource constraint and the incentive compatibility constraint IC_H. Therefore, we now relax Assumption 1 and reconsider the results.

> **Proposition 2**: If Assumption 1 does not hold, then, at the optimum (a), the resource constraint is binding, (b) the incentive compatibility constraint IC_L is slack and the

incentive compatibility constraint IC_H is binding, (c) the high risks receive full insurance and the low risks receive partial insurance (C_H^*, C_L^{**}), and (d) the low risks receive higher coverage under on-demand contracts than under standard contracts.

Assumption 1 determines whether or not the high-risk incentive compatibility constraint is binding, and, thus, whether the low risks receive full or partial coverage. However, even if Assumption 1 does not hold, on-demand contracts relax the incentive compatibility constraint IC_H compared to standard contracts. As a result, the low risks receive higher coverage under on-demand contracts.

In the WMS equilibrium for standard contracts, the low risks receive partial coverage. There is no cross-subsidization, if the population proportion of high risks is large enough. Otherwise, the low risks subsidize the high risks. That is, there is a δ_H^0 such that the low risks subsidize the high risks for $\delta_H < \delta_H^0$. If Assumption 1 does not hold, this is also true for on-demand contracts:

Corollary 3: If Assumption 1 does not hold, then (a) the WMS equilibrium contracts are (C_H^*, C_L^{**}), meaning the high risks receive full insurance and the low risks receive partial insurance, (b) there is a δ_H^* such that the low risks subsidize the high risks for $\delta_H < \delta_H^*$, (c) the low risks receive higher coverage under on-demand contracts than under standard contracts, and (d) the presence of on-demand contracts leads to a lower critical fraction of high risks in the market, below which cross-subsidization occurs: $\delta_H^* < \delta_H^0$.

In the WMS equilibrium for standard contracts and on-demand contracts, the low risks receive partial coverage. However, the level of coverage for the low risks is higher under on-demand contracts than under standard contracts.

In the RS equilibrium for standard contracts, both policies break even individually. The high risks receive full coverage, while the low risks receive partial coverage. Moreover, there is a

δ_H^{00} such the RS equilibrium does not exist for $\delta_H < \delta_H^{00}$. We have similar results for the RS equilibrium under on-demand contracts, if Assumption 1 does not hold. Let C_L' denote the fixed-period policy for the low risks that offers partial coverage ($I_L' < D$), earns zero expected profit ($\rho_L' - P_L^2 I_L' = 0$), and satisfies the incentive compatibility constraint IC_H as an equality ($V_H(C_H^*) = V_H(C_L')$). This policy provides less than full coverage for the low risks.

Corollary 4: If Assumption 1 does not hold, then (a) the RS equilibrium contracts are (C_H^*, C_L'), meaning the high risks receive full insurance and the low risks receive partial insurance, (b) there is a δ_H' such that the equilibrium does not exist for $\delta_H < \delta_H'$, (c) the low risks receive higher coverage under on-demand contracts than under standard contracts, and (d) the presence of on-demand contracts leads to a lower critical fraction of high risks in the market, below which the RS equilibrium does not exist: $\delta_H' < \delta_H^{00}$.

As with the WMS equilibrium, the low risks receive higher coverage under the on-demand contracts than under standard contracts. Whether or not Assumption 1 holds, on-demand contracts relax the incentive compatibility constraint relative to standard contracts.

We now turn to the welfare comparison of standard and on-demand contracts. As on-demand contracts relax the incentive compatibility constraints, we obtain the following result:

Proposition 3: The utility possibility frontier for fixed-period on-demand insurance contracts lies outside the utility possibility frontier for standard insurance contracts.

Under standard contracts, the high-risk individuals receive full insurance and the low-risk individuals receive partial insurance (Crocker and Snow, 1986).[15] When on-demand contracts are available, the low risks receive full insurance or partial insurance, depending on whether or not Assumption 1 holds. In either case, however, the low risks receive greater coverage than in the case where insurers only offer standard policies. In particular, for any WMS equilibrium in standard contracts, there is a pair of on-demand contracts (with the same tax and subsidy) that

[15] Except at the pooled policy C_P^0, where both types receive full insurance.

offers more coverage to the low risks. The equilibrium in on-demand policies is thus Pareto superior to the classical equilibrium in standard contracts.[16]

Fixed-Duration Policies

In this section, we extend our analysis to fixed-duration contracts. In contrast to fixed-period contracts, the insurance company now does no longer determine the covered subperiods. Instead, it fixes the number of available subperiods as part of the contract, and the policyholders decide for which specific subperiods they would like to be covered. Through the course of the following analysis, we will compare fixed-duration polices to fixed-period and to standard policies.

Let $\tilde{C}_i$ denote the fixed-duration policy. A fixed-duration policy consists of a premium, ρ_i, an indemnity, I_i, and the number of subperiods for which the policy is in force, n_i ($n_i \leq T$). Given n_i, policyholders report the vector $\tilde{\boldsymbol{\tau}}_i$ to the insurer, i.e., they tell the insurer for which exact subperiods they want the policy to be in force

Given the number of subperiods offered in the contract, individuals choose the $\tilde{\tau}_i^t$ to minimize their potential uncovered risk: $\tilde{\boldsymbol{\tau}}_{\boldsymbol{i}} = \arg\min \sum_{t=1}^{T} p_i^t (1 - \tau_i^{t\prime})(D - I_i)$ subject to $\sum_{t=1}^{T} \tilde{\tau}_i^t \leq n_i$. The inequality is weak, because n_i may be greater than the number of at-risk subperiods R_i. In this case, subperiods will be insured ($\tilde{\tau}_i^t$ = 1) if, and only if, they are at risk ($t \in R_i$). We retain Assumption 1, with τ_L^t replaced by $\tilde{\tau}_L^t$. The fairly priced, full insurance contracts are now denoted $\tilde{C}_i^*$. The premium is $\rho_i = (1 - P_i^1)D$, the indemnity is $I_i = D$, and the number of subperiods covered equals the number of at-risk subperiods, $n_i = \#R_i$.

[16] The exception is the pooled equilibrium.

As with fixed-period on-demand contracts and standard contracts, full coverage is optimal for the high-risk types. For the low-risk types, in contrast, full or partial coverage is optimal, depending on whether or not Assumption 1 holds.

Proposition 4:

(a) If Assumption 1 holds, then at the optimum (*i*) the resource constraint is binding, (*ii*) the incentive compatibility constraints are slack, and (*iii*) both types receive the full insurance contracts ($\tilde{C}_H^*$, $\tilde{C}_L^*$).

(b) If Assumption 1 does not hold, then at the optimum (*i*) the resource constraint is binding, (*ii*) the incentive constraint IC_H is binding, (*iii*) under the optimal contracts ($\tilde{C}_H^*$, $\tilde{C}_L^{**}$), the high risks receive full insurance and the low risks receive partial insurance, and (*iv*) the low risks receive higher coverage through on-demand contracts than through standard contracts.

If Assumption 1 holds, the at-risk time periods covered can be used to screen types. If Assumption 1 does not hold, some price-quantity rationing is needed in addition to the at-risk time periods. Proposition 4 leads to the following Corollaries on WMS and RS equilibrium outcomes:

Corollary 5: If Assumption 1 holds, then (a) (*i*) the WMS equilibrium contracts are ($\tilde{C}_H^*$, $\tilde{C}_L^*$), and (*ii*) there is no cross-subsidization. (b)(*i*) The RS equilibrium contracts are ($\tilde{C}_H^*$, $\tilde{C}_L^*$), and (*ii*) this is true for all values of $\delta_H > 0$.

Corollary 6: If Assumption 1 does not hold, then

(a) (*i*) the WMS equilibrium contracts are ($\tilde{C}_H^*$, $\tilde{C}_L^{**}$) such that the high risks receive full insurance and the low risks receive partial insurance, (*ii*) there is a $\tilde{\delta}_H^*$ such that the low risks subsidize the high risks for $\delta_H < \tilde{\delta}_H^*$, (*iii*) the low risks receive higher coverage under on-demand contracts than under standard contracts, and (*iv*) the presence of on-demand contracts leads to a lower critical fraction of high risks in the market, below which cross-subsidization occurs: $\tilde{\delta}_H^* < \delta_H'$.

(b) (*i*) The RS equilibrium contracts are ($\tilde{C}_H^*$, $\tilde{C}_L^{'}$) such that the high risks receive full insurance and the low risks receive partial insurance, (*ii*) there is a δ_H' such the RS equilibrium does not exist for $\delta_H < \delta_H'$, (*iii*) the low risks receive higher coverage under on-demand contracts than under standard contracts, and (*iv*) the presence of on-demand contracts leads to a lower critical fraction of high risks in the market, below which the RS equilibrium does not exist: $\delta_H' < \delta_H^{00}$.

If Assumption 1 does not hold, then the low risks receive partial coverage. The partial coverage may take the form of an indemnity which is smaller than the size of the potential loss or some uninsured at-risk periods or some combination of the two. In particular, low risks may be willing to trade off a higher indemnity in covered subperiods for leaving some subperiods uninsured. For example, suppose the conditional probabilities of loss are one in ten for subperiod 1, one in ten million for subperiod 2 and zero for the remaining subperiods. The most important issue in determining the tradeoff is which alternative does the most to relax the incentive compatibility constraint. If, for example, the types Hs have a high conditional probability of loss in period 2, foregoing coverage in that subperiod would likely have a substantial effect in deterring the type Hs from wanting to choose the low-risk policy.

We now turn to the welfare analysis of the fixed-duration policies compared to both fixed-period contracts and standard contracts.

Proposition 5:

(a) If Assumption 1 holds, then (*i*) fixed-period on-demand contracts and fixed-duration on-demand contracts have the same utility possibility frontier, and (*ii*) the utility possibility frontier for both types of on-demand contracts lies outside the utility possibility frontier for standard contracts.

(b) If Assumption 1 does not hold, then (*i*) the utility possibility frontier for fixed-period on-demand contracts lies weakly outside the utility possibility frontier for fixed-duration on-demand contracts, and (*ii*) the utility possibility frontier for fixed-duration on-demand contracts lies outside the utility possibility frontier for standard contracts.

The first part of Proposition 5 follows from the fact that the optimal fixed-period and optimal fixed-duration contacts are the fairly priced full insurance contracts for both types. The second part is obtained, because fixed-period contracts do not tighten the incentive compatibility constraints compared to fixed-duration contracts, and fixed-duration contracts relax the incentive compatibility constraint compared to standard contracts. As a result, the utility possibility frontiers for both types of on-demand contracts lie outside the utility possibility frontier for standard contracts.

Assumption 1 plays a critical role in our main results for the optimal contracts. In particular, Assumption 1 determines whether the incentive compatibility constraint IC_H is slack or binding. Observe that Assumption 1 can be rewritten as $\sum_{t=1}^{T} p_L^t \geq \sum_{t=1}^{T} p_H^t \tau_L^t$. The left-hand side of the inequality is the total probability of a loss for type L individuals. The right-hand side is the probability of a covered loss for the high risks under the low-risk contract. One way to interpret this condition is that, if it holds, then the high risks' probability of a covered loss under the low-risk contract is "too small" for the high risks to want to imitate the low risks at full coverage. We should also point out that Assumption 1 can be verified *ex ante*. This follows from the fact that, if it holds, full coverage is optimal. If so, then low risk coverage is in force during all at risk periods, $p_L^t > 0$ if, and only if, τ_L^t =1. It is then a simple matter to verify whether or not Assumption 1 holds.

More broadly, on-demand contracts rely on the time dimension to screen high risks from low risks. There are situations where this works well, for example, if the time periods when they are at risk are different for the high risks and low risks (e.g., $R_H \cap R_L = \emptyset$). In this case, the time dimension separates the high risk and low risks. There are also situations where this does not work at all, for example, if the low risks and high risk are both at risk during the same subperiods ($R_H = R_L$). In this case, the time dimension provides no information that can be used to separate the high risks and low risks. These are the extreme cases. When there is some overlap between R_H and R_L ($R_H \cap R_L \neq \emptyset$ but $R_H \not\subseteq R_L$), the ability of the time dimension to screen types is less clear. Assumption 1 provides the dividing line between two situations. When Assumption 1 holds, the time dimension provides enough information to completely separate the high and low risks, and the low risks obtain full insurance. When Assumption 1 does not hold the time dimension alone does not provide enough information to separate the high and low risks, and rationing of the low risks' coverage is also required for screening. Nevertheless, as long as the at-risk time periods for the two types are not fully congruent, the time dimension does provide some information that can be exploited for screening. This relaxes the incentive compatibility constraint and leads to greater coverage for the low risks.

Pay-As-You-Go-Insurance

The on-demand insurance products that we examined until now require an ex-ante commitment to the covered subperiods by both insurers and policyholders. This captures the economics of on-demand insurance when the at-risk periods are known in advance. In the context of uncertainty about the actual exposure intervals, however, this is not a desirable product feature. A ride-share driver, for example, might want to remain flexible with regard to the days when he or she offers the service. This is at odds with an insurance contract that requires him or her to fix the covered subperiods in advance. Similarly, the demand for bike insurance is likely to be weather dependent and hence stochastic from an ex-ante perspective. In the following, we therefore extend our analysis to pay-as-you go insurance. Pay-as-you-go insurance implies a fully dynamic contract, which requires a commitment by the insurance company but no commitment by the policyholder. Thus, pay-as-you-go insurance has similarities with multiperiod insurance contracting, e.g., Cooper and Hayes (1987), D'Arcy and Doherty (1990), Dionne and Doherty (1994), Nilssen (2000), DiGaridel-Thornton (2005), Jia and Wu (2019).[17] These models examine multiperiod

[17] See Jia and Wu (2019), Table 1 and Appendix 1 for a summary of the theoretical and empirical literatures.

extensions of the standard insurance policy. Dynamic models with a one-sided commitment by the insurer predict “highballing” or front-loading of premiums. We show that if premiums are fully front loaded, pay-as-you-go insurance reduces to fixed-duration on-demand insurance.

To illustrate the general mechanics of pay-as-you-go insurance, we return to the introductory example of Helen and Linda in Figure 1. Linda is at risk for one one subperiod, with a probabilty of loss of 0.20. Helen has a total probaility of loss of 0.30, spread out over 10 subperiods. Assume the price of full insurance for a single month under contract C_L is $0.2D$. Suppose that under contract C_H, the pricing is level for all 10 months; it is naively set to $0.03D$ per month. In the presence of a one-sided commitment, Linda will buy contract C_H for one month and then stop paying. This is profitable for her, since $0.03D < 0.2D$. However, the insurance company would make negative profits if this contract menue is offered. As both IC_L and *RC* are violated, this cannot be an equilibrium. The solution is to frontload the payments of contract C_H to satisfy the incentive compatibility constraint of the low-risk types. This means Helen and Linda both pay 0.2D upfront and Helen can add more coverage at a premium of 0.1 D later on.

In the formal model, we retain most of the features from the previous analyses. Nonsatiated, risk-averse consumers have wealth *W* and are subject to the risk of a loss of size *D*, and at most one loss occurs during the policy period. The probability that the policyholder suffers no loss until subperiod $t-1$ *and* then a loss occurs between $t-1$ and t is p_i^t, with $\sum_{t=1}^{T} p_H^t > \sum_{t=1}^{T} p_L^t$. The proportion of high (low) risk indiduals is δ_H (δ_L). The set of at-risk subperiods is R_i and we again focus on the case where $R_H \nsubseteq R_L$. All consumption takes place at the end of the policy period. We assume that the discount rate is zero.

An insurance contract, C_i therefore consists of the maximum number of covered subperiods (n_i), a premium vector $\boldsymbol{\rho_i} = (\rho_i^1, \dots\ \rho_i^T)$, containing the premium for each subperiod, and an indemnity I_i. The subperiod premiums may be zero, even for periods during which the policy is in force. The resource constraint needs to hold over the full policy period, but not necessarily in every single subperiod. The insurer fully commits to the terms of the contract for the policy period, whereas the insured decides at the beginning of each subperiod, whether or not to pay the premium and obtain coverage. Formally, the policyholder chooses $\tau_i^t \in \{0, 1\}$ at the beginning of each subperiod, subject to the overall limit of n_j subperiods granted in the contract.[18] If the policy is in force, then the policyholder pays the premium for that subperiod. To account for this freedom of choice, , let Π_i be the power set of R_i and let π_i^k be a subset of Π_i ($\pi_i^k \in \Pi_i$). In addition, let $\#\pi_i^k$ denote the number of elements in the subset π_i^k. We assume $\tau_i^t = 1$ for $t \in \pi_i^k$ and = 0 otherwise. If a type i individual purchases a contract C_j and chooses to cover the subperiods included in π_i^k ($\#\pi_i^k \leq n_j$), then their expected utility is

$$V_i(C_j; \pi_i^k) = \sum_{t=1}^{T}(1 - p_i^t)U(W - \sum_{t\in\pi_i^k} \rho_j^t) + \sum_{t\in\pi_i^k} p_i^t U(W - \sum_{t\in\pi_i^k} \rho_j^t - D + I_j) + \sum_{t\notin\pi_i^k} p_i^t U(W - \sum_{t\in\pi_i^k} \rho_j^t - D). \quad (7)$$

The incentive compatibility constraints need to hold for all π_i^kconsistent with the contract. The individual will choose to cover the periods that yield the highest expected utility, that is, the periods with the highest loss probabilities. Define

$$V_i(C_j) = \max_{\pi_i^k} V_i(C_j;\, \pi_i^k),$$

$$s.t. \quad \pi_i^k \in \Pi_i,$$

$$\#\pi_i^k \leq n_j.$$

[18] This differs from the fixed-duration model, where the policyholder needs to make the coverage decision for all subperiods at the outset of the insurance contract.

Now consider a frontloaded premium. This forces the premium to stay constant for all π^k, that is $\sum_{t\in\pi_i^k}\rho_j^t = const.\ \forall k$. If that is the case, any consumer will pick the maximum amount of allowed risk periods, since the coverage can only improve by doing so, while the costs do not increase. Therefore, we obtain:

$$V_i(C_j) = \max_{\pi_i^k} V_i(C_j;\ \pi_i^k),$$

$$s.t.\quad \pi_i^k \in \Pi_i,$$

$$\#\pi_i^k = n_j$$

This is just a different notation of the fixed-duration contract. The many incentive compatibility constraints arising from the flexibility collapse into one constraint per type and we are back to the case of fixed duration contracts. Consequently, with frontloaded premiums, the optimization problem is to maximize $V_L(C_L)$ subject to the constraints (2), (3), (4) and (5). Intuitively, if a large portion of the premium is paid upfront, subsequent subperiods are cheap enough to discourage picking a subset of the intended contract.

Assumption 1 still plays an important role in the analysis. The analogs of Propostions 1, 2, and 3, and their Corollaries continue to hold in the pay-as-you-go model.

Summary and Conclusions

In this paper, we formally characterize different types of on-demand insurance contracts (fixed period, fixed duration, pay-as-you-go) and analyze their welfare and equilibrium effects in markets with adverse selection. Individuals may differ with respect to their riskiness and the time spent exposed to the risk; both are private information. Since on-demand policies add the time dimension, they can be described as "price-quantity-plus" policies. The existing literature on price-quantity-plus contracts (e.g., Bond and Crocker ,1991, Crocker and Snow, 2011, Croker and Zhu, 2021) suggests that adding an additional screening dimension relaxes the incentive compatibility constraint and leads to improved allocations. We show that, with the addition of the time dimension, this is also true for on-demand insurance.

Under fixed-period on-demand contracts, both the insurer and the policyholder ex ante commit to the subperiods during which the policy will be in force. Fixed-duration polices, in contrast, determine the number of subperiods as part of the contract and the policyholder chooses ex ante which subperiods he or she would like to see covered. This decision, however, cannot be changed during the contract term. This leads to strong results on the optimal insurance policies and the contracts that characterize the WMS and RS equilibria.

The key assumption in our analysis is whether the difference between the total loss probabilities of the high-risk types and the low-risk types is less than the potential uncovered risk for the high-risk types under the low-risk policy. If this condition holds, then the time dimension provides enough information to completely separate the two consumer groups. The welfare maximizing outcome is that both types receive full coverage. This full coverage outcome can be supported by both the WMS and RS equilibria. The RS equilibrium exists regardless of the proportion of high risks.

If the key assumption does not hold, then the time dimension alone does not provide enough information to completely separate the high risks and low risks, and some rationing of coverage for the low risks is required. However, the time dimension still provides some information for screening and relaxes the incentive compatibility constraint. The low risks receive more coverage than under standard policies. This is true for the welfare maximizing policies, as well as the WMS and RS equilibrium contracts. Under on-demand policies, the RS equilibria exist when they would not exist under standard policies. Whether or not the key assumption holds, fixed-period and fixed-duration on-demand insurance leads to more coverage for the low risks and is a Pareto improvement over standard policies.

We also discuss pay-as-you-go insurance. Pay-as-you-go insurance relaxes the full commitment requirement on the side of the policyholder. Hence, whether a subperiod is covered or not can be determined at the beginning of that particular subperiod, rather than at the beginning of the overall policy period. We argue that frontloading of the policy is an optimal equilibrium outcome. Under the same condition on the probabilities, the time dimension completely separates the two risk types and leads to full coverage for both types as an optimal and equilibrium outcome. That is, we obtain essentially the same results for pay-as-you-go insurance as for fixed-duration on-demand insurance.

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Appendix
Proofs of Propositions

Proposition 1: If Assumption 1 holds, then at the optimum (a) the resource constraint is binding, (b) the incentive compatibility constraints are both slack, and (c) both types receive the full insurance contracts (C_H^*, C_L^*).

Proof: (a) If the resource constraint is slack, then, since they are non-satiated, both types can be made better off. (c) Consider feasible contracts (C_H, C_L) for which the resource constraint is binding. Assume the incentive compatibility constraints hold, so that type H's prefer contract C_H and type Ls prefer contract C_L. Assume that contract C_i has premium ρ_i, offers indemnity I_i ($0 < I_i \leq D$) and has some at risk subperiod which is uninsured. That is, there is a $t' \in R_i$ for which $\tau_i^{t'} = 0$. Now consider the contract C_i' for which period t' is covered. The incremental premium is $p_i^{t'} I_i$ so the individual's expected wealth is unchanged. But this shifts the probability from P_i^3 to P_i^2 which increases expected utility. Then for the optimal contracts, we have $t \in R_i$ implies $\tau_i^t = 1$. This implies $P_i^2 = (1 - P_i^1)$ and $P_i^3 = 0$. Repeating this argument *seriatim* proves that prices are actuarially fair. Since coverage is fairly priced, full insurance is optimal for both types. The optimum contracts are (C_H^*, C_L^*).

We now show that, under Assumption 1, the incentive compatibility constraints hold at (C_H^*, C_L^*) and are indeed slack. We first show that the upward incentive compatibility constraint,

$$V_L(C_L) \geq V_L(C_H), \tag{A1}$$

is slack. Both types are fully insured and the total loss probability is lower for the low risk, $P_L^2 = \sum_{t=1}^{n} p_{L,t}\tau_{L,t} < \sum_{t=1}^{n} p_{H,t}\tau_{H,t} = P_H^2$. This implies that

$$U(W - P_L^2 D) > U(W - P_H^2 D). \tag{A2}$$

Now to prove that the downward incentive compatibility constraint is slack, we need to show that:

$$P_H^1 U(W_H^1) + P_H^2 U(W_H^2) > P_H^1 U(W_L^1) + P_{HL}^2 U(W_L^2) + P_{HL}^3 U(W_L^3) \quad \text{(A3)}$$

Observe that $R_H \not\subseteq R_L$ implies $P_{HL}^3 \neq 0$. Since the high risks are fully insured, (A3) can be rewritten as

$$U(W - P_H^2 D) \geq (P_H^1 + P_{HL}^2) U(W - P_L^2 D) + P_{HL}^3 U(W - P_L^2 D - D) \quad \text{(A4)}$$

Using the fact that the low risks are fully insured and the strict concavity of the utility function we have:

$$U\left([P_H^1 + P_{HL}^2](W - P_L^2 D) + P_{HL}^3 (W - P_L^2 D - D)\right) = U(W - P_L^2 D - P_{HL}^3 D) >$$

$$[P_H^1 + P_{HL}^2] U(W - P_L^2 D) + P_{HL}^3 U(W - P_L^2 D - D) \quad \text{(A5)}$$

We can write the high risks expected utility at full insurance as

$$U(W - P_H^2 D) = U(W - P_L^2 D - (P_H^2 - P_L^2) D) \quad \text{(A6)}$$

By Assumption 1, $(P_H^2 - P_L^2) D \leq P_{HL}^3 D$, and the strict inequality in (A5)we have

$$U(W - P_H^2 D) > U(W - P_L^2 D - P_{HL}^3 D) \quad \text{(A7)}$$

Therefore, the downwards compatibility constraint is slack. This completes the proof. ||

Corollary 1: If Assumption 1 holds, then (a) the WMS equilibrium contracts are (C_H^*, C_L^*), and (b) there is no cross-subsidization.

Proof: (a) The WMS equilibrium is the solution to the maximization problem OD, with the high risk individual rationality constraint $\bar{V}_H = V_H(C_H^*)$. (b) At the WMS equilibrium, the high risk individual rationality constraint *UH* is binding. ||

Corollary 2: If Assumption 1 holds, then (a) the RS equilibrium contracts are (C_H^*, C_L^*) and (b) this is true for all values of $\delta_H > 0$.

Proof: This follows from the fact that there are no cross-subsidies at the WMS equilibrium for all values of δ_H. ||

Proposition 2: If Assumption 1 does not hold, then at the optimum (a) the resource constraint is binding, (b) the incentive compatibility constraint IC_H is binding, (c) at the optimal contracts (C_H^*, C_L^{**}) the high risks receive full insurance and the low risks receive partial insurance, and (d) the low risks receive higher coverage under on-demand contracts than under standard contracts.

Proof: (a) This follows from the non-satiation of individuals.

The Lagrangian for the optimization problem is

$$F = \max P_L^1 U(W_L^1) + P_L^2 U(W_L^2) + P_L^3 U(W_L^3) \tag{A8}$$

$$+\gamma_1[\delta(\rho_H - P_H^2 I_H) + (1-\delta)(\rho_L - P_L^2 I_L)]$$

$$+\gamma_2[P_L^1 U(W_L^1) + P_L^2 U(W_L^2) + P_L^3 U(W_L^3) - P_L^1 U(W_H^1) - P_{LH}^2 U(W_H^2) - P_{LH}^3 U(W_H^3)]$$

$$+\gamma_3[P_H^1 U(W_H^1) + P_H^2 U(W_H^2) + P_H^3 U(W_H^3) - P_H^1 U(W_L^1) - P_{HL}^2 U(W_L^2) - P_{HL}^3 U(W_L^3)]$$

$$+\gamma_4[P_H^1 U(W_H^1) + P_H^2 U(W_H^2) + P_H^3 U(W_H^3) - \bar{V}_H]$$

The first order conditions are

$$\frac{\partial F}{\partial \rho_L} = -P_L^1 U'(W_L^1) - P_L^2 U'(W_L^2) - P_L^3 U'(W_L^3) + \gamma_1(1-\delta) - \gamma_2 \Big(P_L^1 U'(W_L^1) +$$

$$P_L^2 U'(W_L^2) + P_L^3 U'(W_L^3)\Big) + \gamma_3(P_H^1 U'(W_L^1) + P_{HL}^2 U'(W_L^2) + P_{HL}^3 U'(W_L^3)) = 0 \tag{A9}$$

$$\frac{\partial F}{\partial \rho_H} = \gamma_1 \delta + \gamma_2(P_L^1 U'(W_H^1) + P_{LH}^2 U'(W_H^2) + P_{LH}^3 U'(W_H^3))$$

$$-(\gamma_3 + \gamma_4)(P_H^1 U'(W_H^1) + P_H^2 U'(W_H^2) + P_H^3 U'(W_H^3)) = 0 \quad \text{(A10}$$

$$\frac{\partial F}{\partial I_L} = P_L^2 U'(W_L^2) - \gamma_1(1-\delta)P_L^2 + \gamma_2 P_L^2 U'(W_L^2) - \gamma_3 P_{HL}^2 U(W_L^2) = 0 \tag{A11}$$

$$\frac{\partial F}{\partial I_H} = -\gamma_1 \delta P_H^2 - \gamma_2 P_{LH}^2 U'(W_H^2) + \gamma_3 P_{LH}^2 U'(W_H^2) + \gamma_4 P_{LH}^2 U'(W_H^2) = 0 \tag{A12}$$

along with the conditions for τ_H and τ_L and the complementary slackness conditions (we do not need these for the proof).

We first show that the incentive constraint IC_H must be binding. If Assumption 1 does not hold, then both incentive compatibility constraints cannot be slack. We can rewrite (A11) as:

$$U'(W_L^2) - \gamma_3 \frac{P_{HL}^2}{P_L^2} U'(W_L^2) = \gamma_1(1-\delta) \quad \text{(A13)}$$

Substituting this into (B9) and rearranging yields

$$P_L^1 U'(W_L^1) + (1-P_L^2)U'(W_L^2) + P_L^3 U'(W_L^3) = \quad \text{(A14)}$$

$$\gamma_3[P_H^1 U'(W_L^1) + P_{HL}^2 U'(W_L^2) - \frac{P_{HL}^2}{P_L^2} U'(W_L^2) + P_{HL}^3 U'(W_L^3)]$$

The LHS of (B14) is strictly positive. If IC_H is slack, then $\gamma_3 = 0$, which creates a contradiction. Therefore, $\gamma_3 > 0$ and IC_H is binding. This proves part (b).

Now let C_P be the pooled fair-odds full insurance policy. Both incentive constraints are binding at the solution (C_P, C_P). We must have $\bar{V}_H = V_H(C_P)$ for this to be a solution. For $\bar{V}_H < V_H(C_P)$, than full coverage. Using the first order conditions (A10) and (A12), straightforward but tedious manipulation yields

$$(1-P_H^2)U'(W_H^1) + P_H^3(U'(W_H^3)) - U'(W_H^1)) = (1-P_H^2)U'(W_H^2) = 0 \quad \text{(A15)}$$

Since the constraint IC_H is binding, we must have $P_H^3 = 0$. Suppose that the contract C_H leaves some at-risk period for the high risk uncovered. Then, as argued in the proof of Proposition1, adding coverage increases the high risks expected utility without violating the resources constraint. But doing so would make the constraint IC_H slack. So we must have $t \in R_H$ implies $\tau_H^t = 1$ and $P_H^3 = 0$. Therefore, the high risks have full coverage. This proves part (c).

Now we need to show that the low risks receive higher coverage under on-demand policies than under standard policies. To do this, we need to show that the constraint IC_H for on-demand contracts, evaluated at the solution to the problem in standard contracts, is slack. Let (C_H^0, C_L^0) be the solution to the welfare maximization problem in standard contracts. In both economic environments, the probability of no loss is $P_i^1 = 1 - \sum_{t=1}^{T} p_i^t$, *i* = *H*, *L*. Under standard contracts, we have $P_i^2 = \sum_{t=1}^{T} p_i^t$ and $P_i^3 = 0$, for *i* = *H*, *L*. For on-demand contacts, we have $P_H^2 = \sum_{t=1}^{T} p_H^t$ and $P_H^3 = 0$ for the high risks and $P_L^2 = \sum_{t=1}^{T} p_L^t \tau_L^t$ and $P_L^3 = \sum_{t=1}^{T} p_L^t (1 - \tau_L^t)$ for the low risks; we can have $P_L^3 \neq 0$.

Since high risks are fully insured in both economic environments, we can write:

$$P_H^1 U(W - (1 - P_L^1) I_L^0) + P_{HL}^2 U(W - (1 - P_L^1) I_L^0 - D) \tag{A16}$$

$$+ P_{HL}^3 U(W - (1 - P_L^1) I_L^0 + I_L^0 - D) + K =$$

$$P_H^1 U(W - (1 - P_L^1) I_L^0) + P_H^2 U(W - (1 - P_L^1) I_L^0 + I_L^0 - D)$$

The LHS of (A14) is for on-demand contracts and the RHS is for standard contracts. We need to show that *K* > 0. Eliminating the first term on each side of (A14) and dividing by P_H^2 yields

$$(P_{HL}^2 / P_H^2) U(W - (1 - P_L^1) I_L^0 - D) + (P_{HL}^3 / P_H^2) U(W - (1 - P_L^1) I_L^0 + I_L^0 - D) + K =$$

$$U(W - (1 - P_L^1) I_L^0 + I_L^0 - D) \tag{A17}$$

Since (P_{HL}^2 / P_H^2) and (P_{HL}^3 / P_H^2) are conditional probabilities, the concavity of the utility function implies that

$$U((P_{HL}^2 / P_H^2)(W - (1 - P_L^1) I_L^0 - D) + (P_{HL}^3 / P_H^2)(W - (1 - P_L^1) I_L^0 + I_L^0 - D)) + K = \tag{A18}$$

$$U(W - (1 - P_L^1) I_L^0 + I_L^0 - D)$$

This implies that

$$U([W - (1 - P_L^1) I_L^0 + (P_{HL}^2 / P_H^2) I_L^0 - D) + K = \tag{A19}$$

$$U(W - (1 - P_L^1) I_L^0 + I_L^0 - D)$$

If $R_H \nsubseteq R_L$ then $(P_{HL}^2/P_H^2) < 1$ and (A17) holds as an equality if, and only if, $K > 0$. This proves part (d) and completes the proof of the Proposition. ‖

Corollary 3: If Assumption 1 does not hold, then (a) the WMS equilibrium contracts are (C_H^*, C_L^{**}), (b) there is a δ_H^* such that the low risks subsidize the high risks for $\delta_H < \delta_H^*$, (c) the low risks receive higher coverage under on-demand contracts than under standard contracts and (d) $\delta_H^* < \delta_H^0$.

Proof: Parts (a) and (b) follow from the fact that the WMS equilibrium is the solution to the maximization problem OD, with the constraint *UH* set to $\bar{V}_H = V_H(C_H^*)$. Parts (a) and (b) follow from standard arguments. Parts (c) and (d) follow from the fact that, compared to standard contracts, on-demand contracts relax IC_H. ‖

Corollary 4: If Assumption 1 does not hold, then (a) at the RS equilibrium contracts are $(C_H^*, C_L^{'})$, the high risks receive full insurance and the low risks receive partial insurance, (b) there is a δ_H' such the equilibrium does not exist for $\delta_H < \delta_H'$, (c) the low risks receive higher coverage under on-demand contracts than under standard contracts and (d) $\delta_H' < \delta_H^{00}$.

Proof: The RS equilibrium policies are the solution to the problems of maximizing low risk expected utility subject to the constraints IC_H, IC_L, U_H with $\bar{V}_H = V_H(C_H^*)$, and the resource constraints

$$RC_H \qquad \rho_H - P_H^2 I_H = 0 \qquad \text{(A20)}$$

$$RC_L \qquad \rho_L - P_L^2 I_L = 0 \qquad \text{(A21)}$$

Parts (a) and (b) are proved in Rothschild and Stiglitz (1967). Parts (c) and (d) follow from the fact that, compared to standard contracts, on-demand contracts relax IC_H. ‖

Proposition 3: The utility possibility frontier for the on-demand fixed-period contracts lies outside the utility possibility frontier for the standard contracts.

Proof: If Assumption 1 then the optimal policies are full insurance for both types. If Assumption 1 does not hold then the high risk receive full insurance and the low risk receive partial insurance. The partial insurance for the low risks under on-demand contracts provides more coverage for the low risk than the partial insurance under standard contracts. For any given value of $\bar{V}$, the high risks are no worse off under on-demand contracts and the low risks are strictly better off. ||

Proposition 4: (a) If Assumption 1 holds, then at the optimum (*i*) the resource constraint is binding, (*ii*) the incentive compatibility constraints are slack, and (*iii*) both types receive the full insurance contracts ($\tilde{C}_H^*$, $\tilde{C}_L^*$).
(b) If Assumption 1 does not hold, then at the optimum (*i*) the resource constraint is binding, (*ii*) the incentive constraint IC_H is binding, (*iii*) at the optimal contracts ($\tilde{C}_H^*$, $\tilde{C}_L^{**}$) the high risks receive full insurance and the low risks receive partial insurance, and (*iv*) the low risks receive higher coverage under on-demand contracts than under standard contracts.

Proof: Part (a). (*i*) Nonsatiation implies the resource constraint is bonding. (*ii*), (*iii*) We need to show that $n_i = \#R_i$ for $i = H, L$. Assume the incentive constraints hold. Let $(\tilde{C}_H, \tilde{C}_L)$ be a proposed solution such that $ni < \#Ri$. Individuals choose $\tilde{\boldsymbol{\tau}}_{\boldsymbol{i}} = arg\ min\ \sum_{t=1}^{T} p_i^t (1 - \tau_i^{t\prime})(D - I_i)$ subject to $\sum_{t=1}^{T} \tilde{\tau}_i^t \leq n_i$. Now consider policies $(\tilde{C}_H', \tilde{C}_L')$ such that $n_i' = n_i + 1$. Then individuals add coverage in the uncovered subperiod with the highest conditional probability of loss. This shifts probability from state 3 to state 2, which increases expected utility. Therefore, $(\tilde{C}_H, \tilde{C}_L)$ cannot be a solution. This implies that $n_i = \#R_i$ and that $\tilde{\tau}_i^t = 1$ if and only if, $p_i^t > 0$ at any solution. The remainder of the proof follows from the same argument as Proposition 1. The proof of part (b) follows from the same argument as the proof of Proposition 2. ||

Corollary 5: If Assumption 1 holds, then (a)(*i*) the WMS equilibrium contracts are ($\tilde{C}_H^*$, $\tilde{C}_L^*$), and (*ii*) there is no cross-subsidization. (b)(*i*) The RS equilibrium contracts are ($\tilde{C}_H^*$, $\tilde{C}_L^*$), and (*ii*) this is true for all values of $\delta_H > 0$.

Proof: The proof of part (a) follows from the same argument as the proof of Corollary 1. The proof of part (b) follows from the same argument as the proof of Corollary 3. ‖

Corollary 6: If Assumption 1 does not hold, then (a) (*i*) the WMS equilibrium contracts are $(\tilde{C}_H^*, \tilde{C}_L^{**})$ such that the high risks receive full insurance and the low risks receive partial insurance, and (*ii*) there is a $\tilde{\delta}_H^*$ such that the low risks subsidize the high risks for $\delta_H < \tilde{\delta}_H^*$, (*iii*) the low risks receive higher coverage under on-demand contracts than under standard contracts, and (*iv*) $\tilde{\delta}_H^* < \delta_H'$.
(b) (*i*)_The RS equilibrium contracts are $(\tilde{C}_H^*, \tilde{C}_L')$ such that the high risks receive full insurance and the low risks receive partial insurance, (*ii*) there is a δ_H' such the equilibrium does not exist for $\delta_H < \delta_H'$, (*iii*) the low risks receive higher coverage under on-demand contracts than under standard contracts, and (*iv*) $\delta_H' < \delta_H^{00}$.

Proof: The proof of part (a) follows from the same argument as the proof of Corollary 2. The proof of part (b) follows from the same argument as the proof of Corollary 4. ‖

Proposition 5:
(a) If Assumption 1 holds, then (*i*) fixed-period on-demand contracts and fixed-duration on-demand contracts have the same utility possibility frontier, and (*ii*) utility possibility frontier for the fixed-duration on-demand contracts lies outside the utility possibility fronter for the standard contracts.

(b) If Assumption 1 does not hold, then (*i*) the utility possibility frontier for the fixed-period on-demand contractslies weakly outside the utility possibility frontier for the fixed-duration on-demand contracts, and (*ii*) the utility possibility frontier for the fixed-duration on-demand contracts lies outside the utility possibility frontier for the standard contracts.

Proof: (a) Both (*i*) and (*ii*) follow from the fact that if Assumption 1 holds then the optimal on-demand policies are full insurance for both types both types.

(b) We want to show that fixed-period contracts relax the incentive compatibility constraint, IC_H, compared to fixed-duration contracts. If so, then the low risks will obtain higher partial coverage and will be better off under the fixed-period contracts,

We know from Propositions 2 and 4 that high types always get full insurance. Hence we can focus our attention on the expected utility the high risks receive from the low risk contracts. Then we have the difference

$$V_H\left(C_L^{**}(\tilde{\rho}_L, \tilde{I}_L, \tilde{\tau}_L)\right) - V_H\left(\tilde{C}_L^{**}(\tilde{\rho}_L, \tilde{I}_L, \tilde{n}_L, \tilde{\tau}_L)\right) = K \quad \text{(A22)}$$

where the first term is the high risk expected utility under the low risk fixed-period contract, evaluated at the fixed-duration optimal values, and the second term is the high risk expeced utility under the fixed-duration contract. We need to show that $K \geq 0$.

Let $\widetilde{W}_L^s$ be the wealth in state s under the fixed-duration contract $\tilde{C}_L^{**}$. Then we can write (A22) as

$$P_H^1 U(\widetilde{W}_L^1) + P_{HL}^2 U(\widetilde{W}_L^2) + P_{HL}^3 U(\widetilde{W}_L^3) + K = \quad \text{(A23)}$$

$$\tilde{P}_H^1 U(\widetilde{W}_L^1) + \tilde{P}_{HL}^2 U(\widetilde{W}_L^2) + \tilde{P}_{HL}^3 U(\widetilde{W}_L^3)$$

Eliminating the first term on each side of (A23) and using the definitions of P_{HL}^3 and $\tilde{P}_{HL}^3$, this can be rewritten as

$$(P_{HL}^2 - \tilde{P}_{HL}^2)[U(\widetilde{W}_L^2) - U(\widetilde{W}_L^3)] = -K \quad \text{(A24)}$$

Then K is nonnegative if, and only if, $(P_{HL}^2 - \tilde{P}_{HL}^2) \leq 0$ or $[\sum_{t=1}^{n} p_{H,t}\tau_L^t] \leq [\sum_{t=1}^{n} p_{H,t}\tilde{\tau}_L^t]$. This follows from the definition of $\tilde{\boldsymbol{\tau}}_L$. Then fixed-period contracts weakly relaxes the incentive compatibility constraint, IC_H, relative to fixed-duration contracts, the low risks will obtain at least as much partial coverage and are weakly better off under the fixed-period contracts. This proves part (b)(*i*). Using the same argument as Proposition 2(d), we can show that fixed-duration on-demand contracts relax IC_H compared to standard contracts, leading to higher partial coverage for the low risks and making the low risks better off. This proves part (b)(*ii*) and completes the proof of Proposition 5. ||