Imperfect Competition in Markets with Adverse or Advantageous Selection

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July 19, 2022

Abstract

This paper proposes a spatial model of imperfect competition in markets with adverse or advantageous selection. The model shows that a reduction in competition exacerbates the inefficiency created by adverse selection, but can ameliorate the inefficiency created by advantageous selection. However, reduced competition never corrects the inefficiency perfectly. In contrast, the inefficiency can be corrected perfectly through a corrective tax when there is perfect competition. Our results have implications for competition policy in credit and insurance markets as they caution against viewing imperfect competition as a solution to the inefficiencies created by selection.

Keywords: *Imperfect Competition, Selection Markets, Price Discrimination, Credit Markets, Insurance Markets*

JEL Classification: D82, D43, C7, L1

*I thank Mark Armstrong, Franklin Allen, Ioana Chioveanu, Margaret Meyer, Andrew Rhodes, John Vickers, Ben Waltmann, Lucy White, Josef Zechner and various seminar and conference audiences for helpful comments and suggestions. Remaining errors are mine. I gratefully acknowledge financial support from the Economic and Social Research Council and Merton College, Oxford. Vienna University of Economics and Business, Vienna Graduate School of Finance, email: markus.parlasca@wu.ac.at.

1 Introduction

Traditionally, market failures have been attributed either to imperfect competition or to imperfect information.[1] Recently, there has been growing interest in the interaction of these two imperfections as several markets exhibit both simultaneously, such as the market for health insurance (Dafny, 2010; Cabral et al., 2018; Einav et al., 2021) or credit markets (Crawford et al., 2018). Competition authorities study these markets closely (European Commision, 2007; Competition and Markets Authority, 2016).

While we know that imperfect information can lead to inefficiency in markets with perfect competition and that imperfect competition can lead to inefficiency in markets with full information, less is known about markets with both imperfections. Specifically, we do not know whether (and if so when) imperfect competition exacerbates or ameliorates the inefficiency created by imperfect information. Thus, it is not clear whether competition policy should tolerate intermediate degrees of competition which has been suggested on grounds that this can correct inefficiencies arising from imperfect information.[2]

This paper studies imperfect competition in markets with adverse or advantageous selection, i.e. in a specific setting with imperfect information. In these selection markets, consumers have private information on a fixed characteristic which affects their willingness-to-pay (WTP) and the cost a firm incurs when selling to that consumer. Selection can take two forms. A market exhibits adverse (advantageous) selection if the firm incurs a higher (lower) cost when selling to agents with a high WTP than when selling to agents with a low WTP. For example, adverse selection can arise in the market for health insurance as consumers with private information on their poor health have a high WTP for insurance and high expected medical costs, or in credit markets when borrowers have private information on the riskiness of the project they are trying to finance (Stiglitz and Weiss, 1981). Advantageous selection can arise in insurance markets when agents who are more risk averse take actions to mitigate their risk (Hemenway, 1990; De Meza and Webb, 2001)[3] or in credit markets when borrowers have privat information on the quality of the project they are trying to finance (De Meza and Webb, 1987).[4] Empirically, advantageous selection has been detected in markets for health insurance (Fang et al., 2008) and credit markets (Mahoney and Weyl, 2017).

Under perfect competition, the equilibrium in a market with selection can be inefficient. In a market with adverse selection, underprovision can arise and the market can even break down

[1]Evidence of a recent decline in competition is provided by De Loecker et al. (2020), Gutierrez and Philippon (2017), and Barkai (2020). Imperfect information exists, for example, in the market for used cars (Akerlof, 1970), various insurance markets (Einav and Finkelstein, 2011), and consumer credit markets (Adams et al., 2009).

[2]For example, Mahoney and Weyl (2017) state: "Policies to correct market power and selection can be misguided when these forces coexist." (p. 637)

[3]For example, highly risk averse agents may drive more carefully or choose a healthier lifestyle (Einav and Finkelstein, 2011). We provide micro-foundations for adverse and advantageous selection in insurance markets in Appendix B.2 and in credit markets in Appendix B.1.

[4]Formally, whether adverse or advantageous selection arises in credit markets depends on how the distribution of project returns differs across projects. Stiglitz and Weiss (1981) assume that all projects have the same expected return and that the distribution of returns of a riskier project is a mean preserving spread of the distribution of project returns of a safer project. De Meza and Webb (1987) assume that the return distributions of better projects first order stochastic dominates the return distribution of worse projects. We discuss this in more detail in Appendix B.1.

completely (Akerlof, 1970). In a market with advantageous selection, overprovision can arise (De Meza and Webb, 2001). Firms lower their price to steal the rival firm's existing consumers, which are profitable, even when this lower price also attracts consumers which previously did not purchase from either firm and are not profitable. In equilibrium, there is inefficient overprovision, i.e. some consumers purchase the good even though the cost of selling to these consumers exceeds their WTP.[5]

This paper proposes a spatial model of imperfect competition in markets with adverse or advantageous selection to investigate whether imperfect competition exacerbates or ameliorates the inefficiency created by selection. The model shows that while in markets with adverse selection a reduction in competition can exacerbate the inefficiency, in markets with advantageous selection it can ameliorate it, but it never cancels it out perfectly. Thus, our results caution against viewing imperfect competition as a solution to inefficiencies introduced by selection.

Model preview: Our model extends models of spatial competition to capture selection. As in canonical models of spatial competition (Hotelling, 1929; Thisse and Vives, 1988), two firms, which are located at the edge of the unit interval, sell differentiated products to consumers who differ in their WTP and their location, which captures brand preferences or physical distance from a shop. As in the literature on selection markets, consumers have private information on a characteristic which determines their WTP and the cost a firm incurs when selling to that consumer. Thus, the model merges components which are common in the literature on the respective imperfection.

Our model gives rise to a distinction between the efficient allocation and the efficient quantity. The efficient allocation is reached when a consumer purchases from her preferred firm if and only if her WTP exceeds the cost a firm incurs when selling to that consumer and otherwise she does not purchase. The efficient quantity is the quantity traded in the efficient allocation. Thus, in the efficient allocation the quantity must be efficient, but the efficient quantity can be reached by an inefficient allocation. For example, an allocation with underprovision for consumers with some brand preferences and overprovision for consumers with other brand preferences, is an inefficient allocation but can correspond to the efficient quantity. In markets without selection, this distinction is not important as an equilibrium satisfies either both notions of efficiency or neither.[6] In models with selection, this distinction is important.

Results preview: In our model, imperfect competition can never perfectly correct the inefficiency introduced by selection. In markets with adverse selection, imperfect competition exacerbates underprovision. In markets with advantageous selection, imperfect competition can ameliorate overprovision, but surprisingly cannot correct this inefficiency perfectly. While reductions in competition reduce the equilibrium quantity and therefore can reach the efficient quantity, the efficient allocation is never reached. Instead, overprovision for consumers with

[5] An excellent survey on markets with selection under perfect competition and monopoly is provided by Einav and Finkelstein (2011).

[6] While in markets without selection there exists inefficient allocations with the efficient quantity, this never arises in equilibrium when firms price discriminate and also not when they use uniform prices. Intuitively, this distinction becomes important only when firms may find it optimal to set prices such that there is overprovision. This is not the case in markets without selection or with adverse selection but is possible in markets with advantageous selection.

some brand preferences coexists with underprovision for consumers with other brand preferences. The mechanism and the result which consumers experience over- or underprovision depend on whether firms can price discriminate or not.

The model shows that the efficiency properties of the equilibrium under imperfect competition are influenced by whether firms price discriminate or not. While regardless of whether firms price discriminate or not, the result holds that in markets with advantageous selection imperfect competition causes overprovision for some consumers to coexist with underprovision for others, the result which consumers experience over- or underprovision is reversed depending on whether firms can price discriminate. When firms can price discriminate, they compete fiercely for consumers with weak brand preferences while charging high prices to those with strong brand preferences. The result is that overprovision for consumers with weak brand preferences coexists with underprovision for consumers with strong brand preferences. When firms cannot price discriminate, they can only compete for consumers with weak brand preferences by offering a low price to all consumers. The result is overprovision for consumers with strong brand preferences and underprovision for those with weak brand preferences.

While in our model no degree of imperfect competition can reach the efficient allocation, the efficient allocation can be reached through a corrective tax when there is perfect competition. Moreover, in markets with advantageous selection, the corrective tax has the additional benefit of raising government revenue without causing a deadweight loss. This creates room to reduce distortionary taxes in other markets - an additional efficiency gain. Thus, combining the corrective tax with tough competition policy to achieve perfect competition results in higher welfare than tolerating intermediate degrees of competition. Moreover, it has lower informational requirements. Hence, our paper cautions against viewing imperfect competition as a solution to inefficiencies introduced by selection.

More broadly, our model highlights how imperfect information affects a firm's pricing decisions under imperfect competition. While in markets without selection firms contemplate how many consumers a price cut would attract, in markets with selection firms additionally consider which type of consumer they attract. Our model predicts that, when firms use uniform prices, those consumers who switch to a firm in response to its price cut have on average a higher WTP than the consumers in the rival's demand. Thus, switching consumers are more strongly selected than the rival's demand. For adverse (advantageous) selection this mutes (strengthens) a firm's incentives to cut price.

Related literature: Imperfect competition in selection markets has previously been studied by Mahoney and Weyl (2017), Crawford et al. (2018), and Lester et al. (2019). Our paper is distinct by (i) its spatial approach, (ii) considering price discrimination, (iii) considering alternative policy solutions such as a corrective tax.

The paper closest to us is Mahoney and Weyl (2017) (henceforth MW). Like our paper, they develop a theoretical model to study imperfect competition in markets with adverse or advantageous selection. Unlike our paper, they take a reduced form approach rather than a spatial approach. While the differences in the model are subtle, they lead to diverging results for advantageous selection. Results for adverse selection are similar.

MW take a reduced form approach and model equilibrium prices p^* as a weighted average of the price under perfect competition p^{PC} and the price a monopolist would set p^M, i.e. $p^* = \theta p^M + (1-\theta)p^{PC}$, where $\theta \in [0,1]$ indexes the degree of competition and higher values of θ correspond to less intense competition.[7] They argue that this reduced form approach nests Cournot competition and differentiated Bertrand competition and thus yields "results that are robust to the details of the industrial organization."[8]

However, MW's approach rests on the assumption that the firm's average cost from selling to its customers is identical to the average cost the firm would incur when selling to all consumers who purchase from any firm (Cournot) or similarly that the average cost of selling to those consumers who switch to the firm when it cuts its price is identical to the average cost the rival firm incurs when selling to its customers (differentiated Bertrand). This means that switching consumers are assumed to be representative of the consumers purchasing from the rival firm. We term this the "representative switching assumption."

We construct a spatial model of imperfect competition in selection markets from first principles. The model differs from MW in two aspects which lead to diverging results for the case of advantageous selection. First, our model implies that switching consumers are not representative of the consumers purchasing from the rival and second, our model distinguishes between the efficient allocation and the efficient quantity.

First, in contrast to MW's representative switching assumption, our model implies that switching consumers have on average a higher WTP than the consumers who purchase from the rival firm. In a selection market, this means that the cost of selling to switching consumers differs from the cost of selling to all consumers who purchase from the rival firm. Thus, switching consumers are not representative. Intuitively, this arises because the consumers who switch are those consumers with the weakest brand preferences for the rival firm. Thus, they purchased only if their WTP exceeds the sum of the rival's price and the large disutility from purchasing from a distant firm. This contrasts to consumers with a strong preference for the rival firm who purchased already at a lower WTP since they purchase if their WTP exceeds the sum of the rival's price and the low disutility from purchasing from a firm that is located close. In a selection market, consumers who are not representative in terms of WTP are also not representative in terms of cost. Thus, in our spatial model, switching consumers are more strongly selected than the consumers who purchase from the rival firm in the sense that in markets with advantageous selection switching consumers have a lower average cost than the consumers who purchase from the rival, while in markets with adverse selection they have a higher average cost.

Second, our model gives rise to a distinction between the efficient allocation and the efficient quantity. This distinction is not present in MW's model where the efficiency properties of an equilibrium are fully summarized by the equilibrium quantity. In contrast, in our model efficiency depends not just on the total quantity, but also on which type of consumer purchases. Thus, in our model it is possible that an equilibrium reaches the efficient quantity, but does so via an inefficient allocation where overprovision for some consumers coexists with underprovision for

[7]This corresponds to equation (1) on p.640 in Mahoney and Weyl (2017). Their approach is a variant of the conduct parameter approach developed by Bresnahan (1989) and Weyl and Fabinger (2013).

[8]Quoted from p.638 in Mahoney and Weyl (2017).

other consumers. This distinction is of first order importance in selection markets where the cost a firm incurs depends on the type of consumer it sells to, but is less important in markets without selection where costs are derived from the firm's production function.

For markets with advantageous selection, these subtle differences lead to diverging results. While in MW there exists an intermediate degree of competition at which the equilibrium results in the efficient allocation, this is not the case in our model. The reason is that our model distinguishes between the efficient quantity and the efficient allocation. As in MW, in our model reductions in competition (starting from perfect competition) can ameliorate overprovision. As in MW, these reductions can even result in the efficient quantity. New in our papers is the result that the efficient allocation is never reached. That means overprovision for some consumers coexists with underprovision for others. Thus, while the theory of the second best result applies that in the presence of one imperfection (imperfection information) a second imperfection can increase welfare (Lipsey and Lancaster, 1956), our paper shows that there is a limit to this reasoning as in the context of selection markets a second imperfection cannot restore the efficient allocation. This observation leads us to consider alternative policy tools such as a corrective tax, which is not considered in MW.

For markets with adverse selection, our results are similar to MW's. As in MW, in our model reductions in competition (starting from perfect competition) exacerbate underprovision in the sense of reducing the equilibrium quantity. Therefore, as in MW, reductions in competition cannot result in the efficient quantity. This in turn means that reductions in competition cannot result in the efficient allocation either. Our paper adds the result that there exists an additional source of inefficiency - allocative inefficiency. This adds a new channel which supports MW's result that reductions in competition exacerbate the inefficiency created by adverse selection.

Empirical results support the theoretical prediction that a reduction in competition results in higher prices in markets with adverse selection. Crawford et al. (2018) develop a structural model of credit markets and estimate it with data on bank loans to small and medium firms in Italy. Their estimates highlight the presence of adverse selection and imperfect competition in this market. In line with theoretical predictions, they find that a reduction in competition, modelled as a merger of two banks, leads to higher prices in their estimated model.[9]

While MW and our model both find that reductions in competition exacerbate the inefficiency created by adverse selection, the literature shows that this prediction can be reversed when firms can make offers which differ not just in price (as in Akerlof (1970), MW, and our model) but also in quantity. Rothschild and Stiglitz (1976) show that this second dimension allows firms to screen consumers, i.e. offer each consumer a menu of price quantity combinations such that consumers self select. Lester et al. (2019) show that, in this setting, increases in competition can reduce welfare as competition makes it harder to sustain pooling contracts. Intuitively, there exist cases where under perfect competition no pooling equilibrium exists as firms can profitably deviate and attract only a subset of consumers (cream skimming), while a monopolist would offer a pooling contract. Since in a pooling contract all gains from trade are realized, monopoly

[9] While Crawford et al.'s (2018) model permits them to study how a merger, higher funding costs for banks, or stronger selection affect equilibrium prices, their model does not permit them to study how efficiency or social welfare are affected by these changes. Our theoretical model generates insights on efficiency and social welfare.

results in higher total surplus than perfect competition. Building a search theoretical model of imperfect competition, Lester et al. (2019) find that in markets with severe adverse selection, intermediate degrees of competition achieve the highest total surplus, while in markets with mild adverse selection, monopoly achieves the highest total surplus.[10] Overall, Lester et al. (2019)'s results are complementary to MW's and ours as taken together they highlight that the welfare effects of increases in competition depend on the type of contracts firms can offer. In markets with indivisible goods (e.g. used cars) or where the quantity is regulated (as in some health insurance markets), the results from MW and our model apply, while in markets where firms make offers which differ in a second dimension other than price (e.g. insurance with different levels of coverage) Lester et al. (2019)'s sorting results apply.[11]

We extend the literature on imperfect competition in selection markets by considering price discrimination. This is increasingly relevant because recent advances in information technology are viewed as making price discrimination feasible in more settings (Vives and Ye, 2021). Our key impossibility result that no degree of competition can achieve the efficient allocation in a selection market holds also when firms can price discriminate. However, the prediction which consumers experience over- or underprovision is reversed depending on whether firms price discriminate or not.

In studying price discrimination, our paper is related to the industrial organization literature on firm pricing strategies. Relative to this literature, our paper is distinct through its focus on selection markets. An excellent survey of the literature on price discrimination is Armstrong (2006). Particularly close to our paper is the literature on price discrimination under imperfect competition. While in monopoly and under perfect competition firms are at least weakly better off when they can price discriminate relative to when they can charge only a uniform price, the effects of price discrimination under imperfect competition are more nuanced. In imperfect competition, the constraint of being able to use only uniform prices can act as a valuable commitment device for firms. Thus, there exist cases of oligopoly where equilibrium prices at all locations are lower under price discrimination than under uniform pricing (Thisse and Vives, 1988; Corts, 1998). The empirical relevance of this possibility result is shown by Grennan (2013) in the context of the market for medical devices.

Our spatial model is closely related to Thisse and Vives (1988), but differs in important ways as we study different questions. We investigate how imperfect competition affects efficiency in markets with selection and thus (i) capture selection by making the cost of selling to an agent a function of his WPT, (ii) create the possibility of inefficient over- and underprovision by focusing on an efficient allocation where not all consumers are allocated the good. Thisse and Vives (1988)

[10]Formally, Lester et al. (2019) model imperfect competition via search frictions as in Burdett and Judd (1983), i.e. they assume that only a fraction of consumers receives offers from both firms while the remaining consumers receive offers from only one firm. More intense competition corresponds to more consumers receiving offers from both firms. We model imperfect competition via product differentiation. In our model, (i) all consumers receive offers from all firms, (ii) reducing competition corresponds to stronger brand preferences or to a merger of the firms, (iii) each consumer receives only one offer from each firm.

[11]Veiga and Weyl (2016) show that the mechanism that increases in competition make it harder to sustain pooling equilibria also applies when each firm can offer only one contract with two dimensions, rather than offering a menu of contracts as in Lester et al. (2019). Veiga and Weyl (2016) conclude that in markets with adverse selection welfare is maximised when competition is less than perfect.

study firms' strategic choice of pricing policy and (i) assume constant marginal costs normalized to zero, (ii) study a market where all consumers buy. While they study the strategic choice of pricing policy, we vary it exogenously.

The empirical literature demonstrates that advantageous selection is a real world phenomenon and not merely a theoretical artefact. Evidence on advantageous selection in insurance markets includes Fang et al. (2008) who detect advantageous selection in markets for health insurance, specifically in the Medigap market.[12] It also includes Cawley and Philipson (1999) who show that men with life insurance have a lower mortality rate than those without, which is consistent with advantageous selection. Evidence on advantageous selection in credit markets is provided by Mahoney and Weyl (2017) who use data from Einav et al. (2012) on the US market for subprime auto loans and find evidence of strong advantageous selection.

Layout: Section 2 outlines our model. Section 3 characterises the efficient allocation. Section 4 shows that our model nests the familiar results for perfect competition and monopoly. Section 5 characterises the equilibrium under imperfect competition. Section 6 outlines policy remedies other than tolerating intermediate degrees of competition. While, for clarity of exposition, section 2 - 6 focus on advantageous selection, section 7 presents the corresponding results for adverse selection. Section 8 discusses extension of our model and section 9 concludes.

2 The Model

As in the standard Hotelling model, there are two firms which each produce one good. Firms sell to consumers who differ in their taste for the products. This gives firms a degree of market power over consumers who strongly prefer their product to the alternative. We depart from the standard Hotelling model to capture selection. Instead of deriving a firm's cost from its production function, we let the cost a firm incurs when selling to a consumer depend on the consumer's willingness-to-pay (WTP).

2.1 Specification of the Model

Players

There are two firms, which are located at the ends of the unit interval. Thus, we name the firms L (left) and R (right) respectively. Consumers are heterogeneous along two dimensions and are located on the unit square $(x, y) \in [0, 1]^2$. They differ in their taste $x \in [0, 1]$ and in their willingness-to-pay $y \in [0, 1]$. They have unit demand and thus purchase one good or no good. We assume that the distributions of consumers regarding taste x and WTP y are independent and uniform, i.e. $x \sim U[0, 1]$ and $y \sim U[0, 1]$. We denote the respective CDFs as $F(y)$ and $G(x)$.

Consumers differ in how strongly they prefer purchasing from one firm over the other. This can capture geographical distance (e.g. to a bank branch to apply for a loan), taste differences,

[12] Medigap is a type of private health insurance in the US which Medicare recipients can purchase. Medigap covers health related financial risks which are not covered by Medicare such as deductibles or co-insurance payments.

or switching costs, e.g. created by prior transactions.[13] The common feature in all these cases is that while some consumers switch already for small price differentials, others switch only for large price differentials. Formally, consumers face travel costs $T(d)$ when purchasing from a firm which is distance d away, i.e. $d = x$ or $d = 1 - x$ respectively, and where $\frac{dT}{dd} > 0 \; \forall \; d$. Thus, consumers with $x > \frac{1}{2}$ prefer to purchase from R rather than L. $T(d)$ is continuous and differentiable. As a normalisation, let $T(0) = 0$. We present all results assuming that $T(d) = td$ where $t > 0$ and discuss generalizations in the Appendix.

The defining feature of markets with selection is that consumers have private information on one aspect which affects both their willingness-to-pay (WTP) and the cost a firm incurs when selling to that consumer. For example, consumers with private information that their health is poor have a high WTP for health insurance and high expected medical costs.

In our model, consumers' WTP (y) is private information and the expected cost a firm incurs when selling to a consumer $c(y)$ can depend on y. We assume that $c(y) \geq 0 \; \forall \; y$ and that $c(y)$ is continuous and differentiable. In the benchmark case of no selection, $c(y) = \alpha \; \forall \; y$. If $\frac{dc}{dy} < 0 \; \forall \; y$, the market exhibits advantageous selection and if $\frac{dc}{dy} > 0 \; \forall \; y$ there is adverse selection.[14] We assume that the entire market is characterised by the same form of selection. Thus, we study markets with either adverse selection ($\frac{dc}{dy} > 0 \; \forall \; y$), or advantageous selection ($\frac{dc}{dy} < 0 \; \forall \; y$), but not markets where $\frac{dc}{dy}$ changes sign. .

We assume that $c(y)$ is linear in y, i.e. $c(y) = \alpha + \beta(1 - y)$. In the case of advantageous selection, $\alpha \geq 0$ is the cost of the lowest cost agent and $\beta > 0$ captures the strength of advantageous selection. Adverse selection is captured by $\beta < 0$. While our model is flexible enough to capture advantageous or adverse selection, for clarity of exposition, we first present the model and our results focusing on markets with advantageous selection. We discuss adverse selection thereafter (in section 7).

Actions

Firms compete in prices. They simultaneously choose prices $p_i(x)$ where $i = \{L, R\}$. We distinguish between a case where firms can price-discriminate based on location x, i.e. $p_i(x)$ can vary across x, and a case of no price-discrimination, i.e. $p_i(x) = p_i \; \forall \; x$. We refer to the latter case as uniform pricing. This case arises when firms do not observe x or when firms are not allowed to condition prices on it. Throughout, we assume that willingness-to-pay (y) is the consumer's private information.

Each consumer (x, y) faces a unique price pair $(p_L(x), p_R(x))$ and chooses whether to buy from L, from R, or not at all.

[13]E.g. banks may be able to charge existing consumers a mark-up as they have access to past payment histories which help the bank judge the customer's credit worthiness but which rival banks cannot access. This consideration was central to the Competition and Markets Authority launching the Open Banking initiative under which consumer data needs to be made accessible to rival banks. This aspect is also prominent in the literature on relationship banking (Sharpe, 1990; Boot and Thakor, 2000). In an insurance context, consumers may have acquired a chronic medical condition which the existing insurer must cover but based on which new insurers could refuse to offer insurance.

[14]Different micro-foundations for advantageous selection are possible. For the case of insurance markets, see Einav and Finkelstein (2011) or our exposition in Appendix B. In our setting, $c(y)$ would include administrative costs which is one way to rationalise why some consumers may have WTP below their expected cost.

Pay-offs

Firms maximise profit. Firm L's profit per consumer (x, y) is $\pi_L(x, y) = p_L(x) - c(y)$. Denoting the set of consumers buying from firm L as $\mathcal{D}_L$, firm L's total profit is

$$\Pi_L = \int\int_{(x,y)\in\mathcal{D}_L} \pi_L(x, y)\, f(y)\, dy\; g(x)\, dx = \int\int_{(x,y)\in\mathcal{D}_L} [p_L(x) - c(y)]\, f(y)\, dy\; g(x)\, dx$$

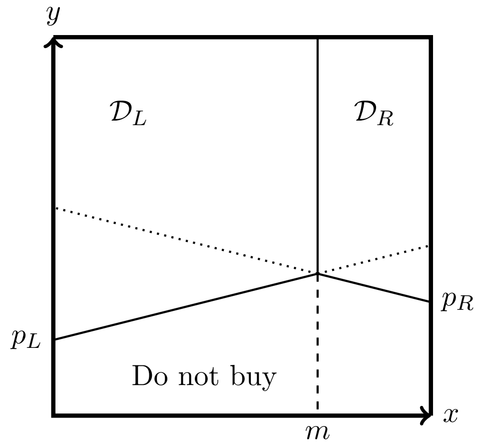
Each consumer maximises her monetary benefit. Normalising the outside option of not purchasing to zero $u_o = 0$, purchasing from L yields $u_L = y - p_L(x) - T(x)$ while purchasing from R results in $u_R = y - p_R(x) - T(1 - x)$.

For any pair of uniform prices (p_L, p_R), there exists a location m at which consumers are indifferent between purchasing from L and R. At all locations $x < m$ consumers prefer purchasing from L to purchasing from R. Whether consumers prefer purchasing from L to the outside option depends on their WTP. Thus, consumers purchase from L if $u_L > u_R$ and $u_L > 0$ both hold. Thus,

$$\Pi_L(p_L, p_R) = \int_0^{m(p_L,p_R)} \int_{p_L+T(x)}^{1} [p_L - c(y)]\, f(y)\, dy\; g(x)\, dx$$

The allocation resulting for an arbitrary pair of uniform prices (p_L, p_R) is depicted below.

Figure 1: Purchasing behaviour for uniform prices (p_L, p_R)



At location m consumers are indifferent between purchasing from L or R.

Since our goal is to study competition, we assume that for every location x there exist benefits from trade for some y. Formally, this means that we restrict our attention to pairs of $c(y)$ and $T(d)$ such that $c(1) + T(\frac{1}{2}) \leq 1$ holds. This ensures that the two firms are not monopolists pricing over disjoint sets of demand, but that there exist some consumers who have gains from trade with both firms. Thus, firms will compete for these consumers.

In contrast to many models of spatial competition, our assumptions do not imply that it is efficient to allocate the good to every consumer when $T(d) = 0\ \forall\ d$.[15] Instead, our assumptions imply that it is efficient to allocate the good only to a fraction of consumers. This departure is

[15]When models of spatial competition assume that a firm's marginal costs are constant and normalized to zero,

necessary to allow us to study whether inefficient overprovision arises in equilibrium which is a first-order issue in markets with advantageous selection. Formally, this departure is generated by our assumptions on costs. When $T(d) = 0 \; \forall \; d$, then for markets with advantageous selection, there exist consumers with low WTP who are not allocated the good in the efficient allocation since the cost incurred when selling to these consumers $c(0) = \alpha + \beta > 0$ exceeds their WTP ($y = 0$). However, there exist consumers with high WTP who are allocated the good in the efficient allocation since $c(1) \leq 1$. Hence, in the efficient allocation a fraction of consumers are allocated the good.

Equilibrium concept

We focus on symmetric Nash equilibria. That means we focus on equilibria where firms set $p_L(x) = p_R(1-x)$ and consumers respond optimally to these prices.

2.2 Switching Consumers

We do not assume switching patterns, i.e. which types of consumers switch in response to a price cut, but derive them from our model. While the literature has assumed that switching consumers are representative of demand (Mahoney and Weyl, 2017), our model predicts that switching consumers are an advantageous selection of demand. This shapes the firms' strategic interaction.

The switching pattern affects whether overprovision arises. If the firm offering the lowest price attracts all consumers (perfect switching, Bertrand competition), inefficient overprovision results.[16] If, however, a price cut only attracts newly entering consumers and no switching consumers, firms never lower prices below the efficient level.[17] While Mahoney and Weyl (2017) assume "representative switching", i.e. that switching consumers are on average equal to the average consumer in demand, our model has the feature that switching consumers differ in a predictable way from the average consumer in demand.[18]

Our model predicts that, in a market with advantageous selection, those consumers who switch to a firm in response to it cutting its price are an advantageous selection of the rival's demand. This arises because a price cut only attracts those consumers from the rival who had the largest distance to travel, thus needed to pay large transport costs, and therefore only chose the rival over the outside option if their WTP is high. Thus, the switching consumers are higher

and that all consumers have a strictly positive WTP (as in Hotelling (1929) and Thisse and Vives (1988)), then for $T(d) = 0 \; \forall \; d$ the efficient allocation is to allocated the good to every consumer. We model costs differently to capture selection.

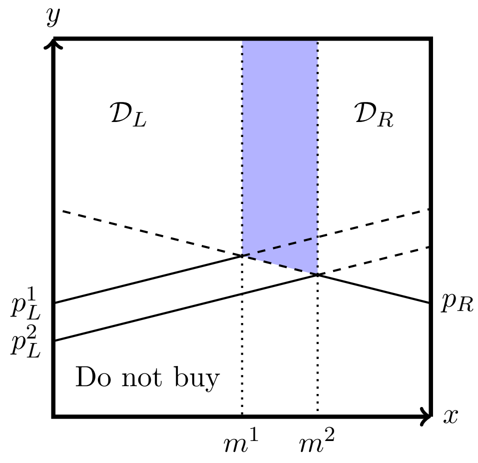
[16] This arises because in markets with advantageous selection (MC increasing, AC below MC), at the efficient allocation ($p = MC$) there still are positive industry profits ($p > AC$). Thus, when firms undercut each other until profits are zero, this results in overprovision. While the newly entering consumers are loss making, firms are compensated by stealing profitable consumers from the rival. At $p = AC$ undercutting stops.

[17] For prices below the efficient level, newly entering consumers are loss making for the firm. As there is no switching, there is no benefit of cutting prices as the firm steals no profitable consumers from the rival. Thus, it is not optimal for the firms to lower prices below the efficient level.

[18] The switching patterns in our model differ not just from the main case, but also from generalisations discussed by Mahoney and Weyl (2017). On p.640, footnote 7, they state that "Even if this assumption [representative switching] fails, so long as average switching consumers have a cost that is strictly between that of average exiting consumers and average purchasing consumers, most of our results are left unchanged." This generalises representative switching to allow switching consumers to have a *lower* average WTP than demand. Our model predicts a *higher* average WTP than demand, thus differs also from this generalisation.

WTP than the average of the rival's demand. Given advantageous selection, this makes switching consumers lower cost than average. Hence, the switching consumers are an advantageous selection of demand. This is depicted below:

Figure 2: Switching Consumers are an Advantageous Selection of Demand



Firm L's price cut from p_L^1 to p_L^2 results in consumers in the blue area switching.
These consumers have on average a higher WTP than the consumers in $\mathcal{D}_R$.

The *extent* to which the margin is selected depends on the degree of competition. The softer competition (in the sense of higher transport costs), the stronger is the selection on the switching margin, i.e. the less representative it is.[19] Thus, the composition of the switching and entering margins is itself determined by the level of competition. It is not a fundamental and we cannot assume it to be fixed across different degrees of competition.

Related, but not shaping the firms' strategic interaction as strongly, is the composition of entering consumers. These consumers, who in response to a price cut start purchasing, initially did not buy the product and thus must have a lower WTP than the average consumer in demand. Thus, in markets with advantageous selection, entering consumers are high cost.

3 Efficient Allocation

We characterise the efficient allocation, which is the benchmark against which we will judge equilibria, and show that in markets with advantageous selection, the efficient allocation can be implemented with price discrimination, but not with uniform pricing. Therefore, we also characterise the socially optimal uniform price, that is the constrained efficient allocation where the constraint is to not be able to condition prices on location.

Comparisons of the efficient allocation and the equilibrium under imperfect competition are confounded by transport costs, which capture the degree of competition, having multiple effects. Larger transport costs have a strategic effect, which is what we aim to capture, but also

[19] Only for $T(d) = 0$, i.e. perfect competition, does our model have perfect switching which is a case of representative switching. For any $T(d) = td$ with $t > 0$, switching becomes non-representative.

have a direct cost effect, i.e. transport costs reduce total surplus achievable in a market. To avoid biasing our results in favour of competition, we isolate the strategic effect by comparing equilibrium outcomes to the outcome a welfare maximising social planner can achieve when he faces the same transport costs. This section derives this benchmark.

3.1 Welfare

We focus on total surplus as our welfare measure.[20] Thus, welfare is the sum of consumers' monetary benefit and the firms' profits. Allocating the good to a consumer (x, y) generates surplus $s = y - T(d) - c(y)$. The outside option results in zero surplus.

Total surplus is aggregates s across all transactions which take place. Again denoting the set of consumers allocated the good from firm L as $\mathcal{D}_L$ and from R as $\mathcal{D}_R$, total surplus is defined as:

$$\mathcal{S} = \int\int_{(x,y)\in\mathcal{D}_L} [y - T(x) - c(y)] f(y)\, dy\; g(x)\, dx + \int\int_{(x,y)\in\mathcal{D}_R} [y - T(1-x) - c(y)] f(y)\, dy\; g(x)\, dx \tag{1}$$

3.2 Socially Optimal Allocation

The socially optimal allocation is to allocate the good to every agent whose WTP exceeds the sum of his costs and his transport costs, and allocate it from the firm where transport costs are lowest. Mathematically, the consumer should be allocated the good from firm L if and only if the following two conditions both hold:

$$y \geq c(y) + T(x) \tag{2}$$

$$x \leq \frac{1}{2} \tag{3}$$

Thus, there exists a unique socially optimal allocation. This allocation has a familiar threshold structure, where $y^S(x) = c(y) + T(x)$ is a threshold. Then, all (x, y) with $y > y^S(x)$ and $x \leq \frac{1}{2}$ are allocated the good from L, all (x, y) with $y > y^S(x)$ and $x > \frac{1}{2}$ are allocated the good from R, and all others are not allocated the good.

The threshold $y^S(x)$ depends on the extent of selection. While in markets with no selection ($c(y) = \alpha\ \forall\ y$), the threshold increases in line with transport costs ($\frac{dy^S(x)}{dx} = T'(x)$), in markets with advantageous selection, the threshold is flatter than transport costs, but still increasing.

3.3 Socially Optimal Price

We show that while in markets with selection, the socially optimal allocation can be implemented using price discrimination, it cannot be implemented using uniform pricing. Thus, when the planner faces the additional constrained of using only uniform prices, the socially optimal uniform

[20] We discuss alternative welfare measures in section 8.

price can only approximate the efficient allocation. It results in overprovision for consumers with strong brand preferences and underprovision for those who are close to indifferent among the brands. Hence, while in markets without selection, there exists a uniform price which implements the efficient allocation, in markets with advantageous selection, the following proposition holds.

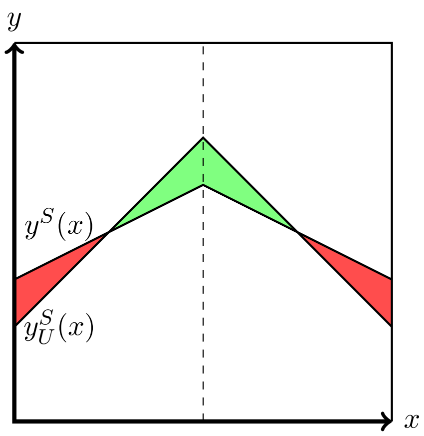
Proposition 1 *In markets with selection, no uniform price can implement the efficient allocation.*

Proof: See Appendix A.1.

Intuition: Suppose the planner sets the uniform price to achieve the efficient allocation among consumers with the strongest brand preferences ($x = 0$). That means that the consumer $(0, p_L)$ is indifferent between purchasing from L and not purchasing. Thus, he generates zero surplus. At any $0 < x < \frac{1}{2}$, for this uniform price the indifferent consumer will be $(x, p_L + T(x))$. That means that the WTP of the agent is by $T(x)$ higher than the WTP of the marginal agent at $x = 0$. If the market exhibits advantageous selection, the higher WTP means that fundamental costs ($c(y)$) are lower. In total, the marginal consumer $(x, p_L + T(x))$ must generate strictly positive surplus as his WTP covers his higher travel costs but the lower fundamental costs are not offset, thus creating positive surplus. Thus, agents with a slightly lower WTP than the marginal consumer at $x > 0$ would also still generate positive surplus. As they are not allocated the good, there is inefficient underprovision.

Thus, when the planner is limited to setting only uniform prices, he is trying to find the constrained efficient allocation. Since uniform prices result in steeper threshold than the efficient allocation, the constrained efficient allocation includes overprovision at $x = 0$ and underprovision at $x = \frac{1}{2}$. This is depicted below.[21]

Figure 3: Social Planner: Advantageous Selection



$y^S(x)$ is the efficient allocation, $y^S_U(x)$ is the allocation resulting from the socially optimal uniform price.

Green areas indicate underprovision, red areas indicate overprovision.

[21] The inefficiency created by the planner not being able to price discriminate is increasing in transport costs and in the degree of selection. Moreover, selection and transport costs magnify each others effect on inefficiency.

Example: When $c(y) = \alpha + \beta(1 - y)$, $T(d) = t\,d$, $F(y) = y$, $G(x) = x$, then:

$$p^S = \frac{\alpha + \beta}{1 + \beta} - \frac{\beta}{4(1 + \beta)}\, t \tag{4}$$

4 Perfect Competition and Monopoly

This section shows that our model of advantageous selection, which generates insights on imperfect competition, also nests the familiar results of perfect competition resulting in overprovision and of monopoly resulting in underprovision. Based on these results, it is tempting to believe that there exists a level of imperfect competition which corrects the overprovision caused by selection and achieves the first best efficient allocation. However, as the next section shows, this is not the case in our model - neither when firms use uniform pricing, nor when firms use price discrimination.

Perfect Competition

In models of spatial competition, the intensity of competition is captured by transport costs $T(d)$. Perfect competition corresponds to $T(d) = 0 \;\forall\; d$. Then the firm with the lower price attracts all consumers, i.e. there is perfect switching, and captures all industry profits. Thus, a firm wants to undercut the rival's price as long as industry profits are positive, i.e. $p > AC$.[22] In equilibrium, both firms price such that $p = AC$. That means that there is inefficient overprovision, since in markets with advantageous selection MC is increasing[23] and $AC < MC$. Overprovision arises because firms continue to undercut each other even when the newly entering consumers are loss making as firms try to steal the rival's profitable existing consumers.

Monopoly

Our model offers two ways to study monopoly, either via joint ownership or via high transport costs. In both cases the equilibrium exhibits inefficient underprovision.

In the joined ownership perspective, one firms sells both products L and R. As this firm internalises the effect that changes in the price of one product have on the demand for the other product, it will never overprovide and as it has market power it even raises prices above the efficient level and thus underprovides.

Proposition 2 *In monopoly, there is inefficient underprovision at all locations x, regardless of the type and strength of selection and for all available pricing strategies.*

Proof: See Appendix A.2.

[22] In our model, average costs are $AC = \frac{\int_p^1 c(y)f(y)dy}{1-F(p)}$.
[23] In our model, marginal costs are $MC = c(p)$.

When the monopolist can price discriminate and there is no selection ($c(y) = \alpha \; \forall \; y$), for $x \leq \frac{1}{2}$ it sets:

$$p_L^M(x) = \frac{1+\alpha}{2} - \frac{T(x)}{2} \tag{5}$$

which is the price that equates marginal revenue and marginal cost at a given location x.[24] This price is decreasing in distance travelled since the transport costs needed to travel larger distances reduce demand. The monopolist partly absorbs this effect of transport costs. As the effect is not absorbed fully, the effective price consumers face (and thus the WTP threshold above which consumers buy) is increasing in distance travelled, i.e. is $y^M(x) = \frac{1+\alpha}{2} + \frac{T(x)}{2}$. Note that $y^M(x)$ increases more slowly than $T(x)$ and is thus flatter than the threshold resulting from a uniform price.

If there is advantageous selection, the monopolist sets:

$$p_L^M(x) = \frac{1 + c\big(p^M(x) + T(x)\big)}{2} - \frac{1}{2}T(x) \tag{6}$$

which for the linear costs example $c(y) = \alpha + \beta(1-y)$ is:[25]

$$p_L^M(x) \;=\; \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{2+\beta}\,T(x) \tag{7}$$

A stronger degree of selection (larger β and equally a steeper marginal cost curve) leads to higher prices, i.e. $\frac{dp_L^M(x)}{d\beta} > 0$.

When the monopolist can only use uniform pricing, he cannot achieve his optimal outcome, i.e. there does not exist a uniform price which implements the monopolist's optimal allocation.[26] Like in the social planner's case, this arises because the monopolist's optimal allocation has a threshold with a different slope than any uniform price achieves.[27] The monopolist chooses a uniform price (p_L^M) to approximate his preferred allocation. For $t \leq \frac{4\,(1-\alpha)}{3+\beta}$

$$p_L^M = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{4\,(2+\beta)}\;t \tag{8}$$

For $t > \frac{4\,(1-\alpha)}{3+\beta}$, the monopolist's optimal price is $p_L^M = \frac{1+2\alpha+\beta}{3+\beta}$ which results in consumers with weak brand preferences not purchasing for any WTP. A stylised example of the allocation under monopoly (for $t \leq \frac{4\,(1-\alpha)}{3+\beta}$) is depicted below. The monopolist's uniform price is always above the planner's uniform price and it results in inefficient underprovision at all locations x.

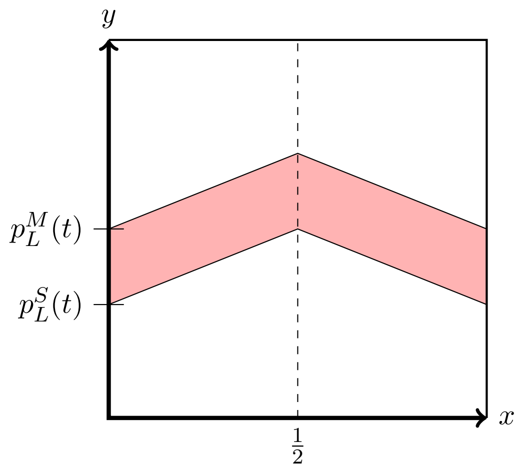
[24] This is the unique optimal price.

[25] This is the uniquely optimal price. The resulting threshold describing the allocation is $y^M(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}\,T(x)$.

[26] This means that the monopolist makes strictly larger profits with price discrimination than with uniform pricing.

[27] While the social planner could implement his optimal allocation with a uniform price in the absence of selection, but not under selection, the monopolist can implement his preferred allocation in neither case. The monopolist's implementation problem arises because absorbing transport costs is impossible with uniform prices. The planner's implementation problem arises because it is impossible to absorb the indirect effect of higher transport costs in markets with selection - i.e. lower fundamental costs.

Figure 4: Monopoly allocation for uniform prices



An alternative way of studying monopoly is to focus on transport costs which are so high that no surplus generating trades at $x = \frac{1}{2}$ exist. Then, firms operate on disjoint sets of demand. The intuition for underprovision also applies in this case.

As this section showed that, in markets with advantageous selection, perfect competition results in inefficient overprovision and monopoly in underprovision, a natural question is whether there exists a level of imperfect competition which corrects the overprovision caused by selection and achieves the first best efficient allocation. While in Mahoney and Weyl (2017) such a level of competition exists, the next section shows that this is not the case in our model - neither when firms use uniform pricing, nor when firms use price discrimination.

5 Imperfect Competition

We find that no degree of imperfect competition can restore the first best efficient allocation in the presence of advantageous selection. While softer competition results in fewer trades occurring in equilibrium, this never exactly offsets the inefficient overprovision which results under perfect competition. Instead, imperfect competition results in inefficient underprovision and overprovision co-existing for different consumer types. This result is robust to different assumptions on the firm's pricing strategy, i.e. it applies both when firms use uniform prices and when they use price discrimination. However, the result regarding which consumers experience underprovision and which experience overprovision does depend on the firms' strategy. While if firms use uniform prices there is overprovision for consumers with strong brand preferences and underprovision for consumers who are indifferent among brands, the reverse is true when firms can price discriminate.

5.1 Uniform Pricing

When firms compete using uniform prices, there exists an intermediate degree of competition at which the oligopoly equilibrium price coincides with the planner's optimal price. However, the planner's optimal price does not implement the efficient allocation. Due to the presence of advantageous selection, the planner's uniform price is itself only constrained efficient, i.e. it approximates the first best efficient allocation under the constrained of using a uniform price. Thus, at this price, consumers with strong brand preferences experience overprovision, while there is underprovision for consumers who are indifferent among the brands. That means the two types of inefficiency co-exist.

In the duopoly game with uniform prices, firm L's profit is given by:

$$\Pi_L(p_L, p_R) = \int_0^{m(p_L,p_R)} \int_{p_L+T(x)}^{1} [p_L - c(y)]\ f(y)\ dy\ g(x)\ dx \tag{9}$$

where $m(p_L, p_R)$ denotes the location x at which consumers are indifferent between purchasing from either firm. Since transport costs are linear ($T(d) = td$ where $t \geq 0$), we get the familiar Hotelling result $m(p_L, p_R) = \frac{1}{2} + \frac{p_R - p_L}{2t}$. Profit maximisation gives rise to the following proposition, where p^* denotes the equilibrium uniform price under imperfect competition, i.e. duopoly.

Proposition 3 *There exists a unique equilibrium.*
If $t \leq \frac{4\,(1-\alpha)}{3+\beta}$, *it is characterized by the equilibrium price*

$$p^* = \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{(2+\beta)}\,\frac{t}{2} - \frac{1}{2+\beta}\sqrt{(1-\alpha-\frac{t}{2})^2 + \frac{t^2}{2}(\beta+2)(\beta+3)} \tag{10}$$

If $t > \frac{4\,(1-\alpha)}{3+\beta}$, *it is characterized by the equilibrium price*

$$p^* = \frac{1+2\alpha+\beta}{3+\beta} \tag{11}$$

Proof: See Appendix A.3.

If $t > \frac{4\,(1-\alpha)}{3+\beta}$, then firms price such that their demand sets are disjoint, i.e. at $x = \frac{1}{2}$ nobody purchases. That corresponds to behaving like a monopolist.

This proposition gives rise to a result on cost pass-through. While in itself only a minor result, our suggestion of a corrective tax as a policy tool will build on this result later on.

Corollary 1 *In oligopoly, there is cost pass-through, i.e.* $0 < \frac{dp^*}{d\alpha}$, *regardless of the presence and strength of selection.*

Proof: See Appendix A.4.

While in the absence of selection, the standard ordering of prices under different ownership structures holds for all levels of transport costs ($p_L^M(t) > p^*(t) > p_L^S(t)\ \forall\ t > 0$) there is no such universal ordering under advantageous selection. When there is advantageous selection, then for very low transport costs the oligopoly equilibrium price is below the planner's optimal price. For large transport costs, the ordering is reversed.[28] This gives rise to the following proposition.

[28]For all levels of transport costs $p_L^M(t)$ exceeds $p^*(t)$ and $p_L^S(t)$.

Proposition 4 *(Advantageous selection)*

(i) There exists a unique level of transport costs, denoted $\tilde{t}$, at which the oligopoly equilibrium price $p^*(t)$ and the socially optimal price $p_L^S(t)$ coincide, i.e. $p_L^S(\tilde{t}) = p^*(\tilde{t})$ where $\tilde{t} > 0$.

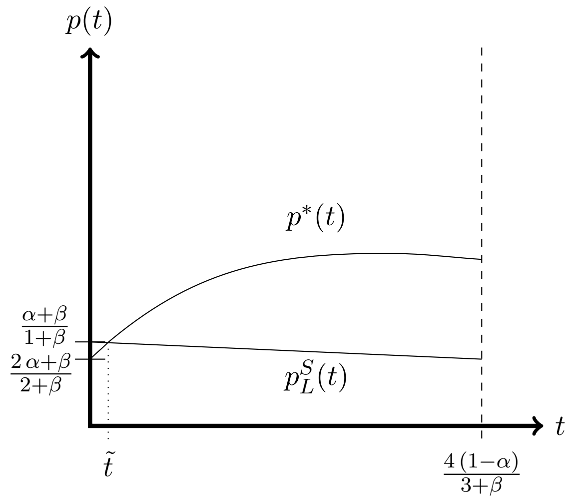
(ii) However, the oligopoly equilibrium allocation and the socially optimal allocation do not coincide for any level of transport costs t.

Proof: See Appendix A.5.

The first part of proposition 4 follows from a fixed point argument which builds on the limit results of perfect competition and monopoly as well as on the continuity of $p^*(t)$. Since we know that perfect competition ($t = 0$) results in $p^*(0) < p_L^S(0)$ and that $t = \frac{4\,(1-\alpha)}{3+\beta}$ results in monopoly pricing even though surplus generating trades exist, i.e. $p^*(\frac{4\,(1-\alpha)}{3+\beta}) > p_L^S(\frac{4\,(1-\alpha)}{3+\beta})$, there must be a level of t where $p^*(t) = p_L^S(t)$.

The intuition above also holds in our spatial model, but the formal exposition is complicated by larger transport costs having two effects. Larger transport costs reduce competition, which tends to raise prices, but they also alter demand which in this setting leads firms to lower their price. As a result, $p^*(t)$ is continuous but not monotone in t.[29] However, the intuition of $p^*(t)$ and $p_L^S(t)$ only crossing once is preserved in our model.

Figure 5: Uniform Price Comparison



The second part of proposition 4 shows that no degree of imperfect competition can restore the efficient allocation when there is advantageous selection. This builds on proposition 1 that no uniform price can implement the efficient allocation. The planner's optimal price is thus only constrained efficient. It is only an approximation to the efficient allocation. Hence, while for $\tilde{t}$ we have $p_L^S(\tilde{t}) = p^*(\tilde{t})$, the fact that $p_L^S(\tilde{t})$ is itself only an approximation to the efficient allocation means that there is overprovision for low x and underprovision for medium x (consumers who are indifferent between the brands) as in Figure 3.

[29] The effective price at $x = \frac{1}{2}$, $p^*(t) + \frac{1}{2}t$, is monotonically increasing in t.

Thus, proposition 4 cautions against viewing intermediate degrees of competition as desirable in markets with selection. Since $\tilde{t}$ achieves only the constrained efficient allocation, this leaves room for other policy interventions to achieve higher total surplus by reaching the first best efficient allocation. Section 6 explores this and finds that a corrective tax combined with perfect competition can achieve the first-best and is thus a socially more desirable policy.

Our arguments above rely on firms using uniform prices. It is natural to wonder whether the impossibility result that no degree of imperfect competition achieves the first best also holds when firms can use price discrimination, i.e. are not restricted to charging all consumers the same price. We now turn to the case of price discrimination and show that while the firms' strategies are altered and over-and underprovision occur for different types of consumers than under uniform pricing, the impossibility result continues to apply.

5.2 Price Discrimination

When firms use price discrimination, they compete very fiercely for consumers who are indifferent among brands and use their market power to set high prices for consumers with strong brand preferences. Since with price discrimination firms can target prices at consumers depending on brand preferences, these two motives are not conflicting, which they are under uniform prices. As a result, overprovision and underprovision co-exist.

There is overprovision for consumers who have small brand preferences. While intermediate degrees of competition reduce the range of brand preferences in which overprovision arises, for any degree of imperfect competition overprovision continues to exist for those consumers who are exactly indifferent between brands.

There is underprovision for consumers with strong brand preferences. For softer degrees of competition, underprovision arises for a larger range of brand preferences. The extent of underprovision can be large. Already for intermediate degrees of competition, consumers with strong brand preferences experience the same degree of unverprovision that arises in monopoly.

Characterisation of the Equilibrium

When firms use price discrimination, they effectively compete for separate markets at every location x. The transport cost differential $T(x) - T(1-x)$ affects how much market power a firm has at that location. Consumers with strong brand preferences for L (low x) will only buy from R if R sets a price which compensates for the extra travel, i.e. $p_R(x) \leq p_L(x) - [T(1-x) - T(x)]$, i.e. if R undercuts L by $T(1-x) - T(x)$. As firm R also needs to break even, R cannot profitably attract consumers when firm L prices no higher than break even price plus transport cost differential. Let $p_i^B(x)$ denote the lowest possible price firm i can charge at x and still break even, i.e. make a non-negative profit, in the hypothetical case where firm j does not sell at that location x. For example, in the absence of selection ($c(y) = \alpha \forall y$) the break even price is $p_i^B(x) = \alpha$ and R cannot profitably undercut any $p_L(x) \leq \alpha + T(1-x) - T(x)$. This allows firm L to charge a mark-up at all $x < \frac{1}{2}$. If transport costs are sufficiently high, i.e. $T(d)$ is sufficiently steep, then at some locations x firm L can even charge the same price a monopolist would charge. Towards the middle of the unit interval, competition erodes the firm's mark-ups.

The idea of transport cost differentials protecting firms' pricing power also applies when markets exhibit selection and gives rise to the following proposition:

Proposition 5 *In duopoly, there exists an equilibrium with the following prices:*

$$\begin{aligned}
&\textit{For } x \in [0, x_L) && p_L(x) = p_L^M(x) = \tfrac{1+\alpha+\beta}{2+\beta} - \tfrac{1+\beta}{2+\beta} tx \\
& && p_R(x) \geq p_L^M(x) + T(x) - T(1-x) \\
&\textit{For } x \in [x_L, \tfrac{1}{2}] && p_L(x) = \tfrac{2\alpha+\beta}{2+\beta} + \tfrac{2}{2+\beta} t - \tfrac{4+\beta}{2+\beta} tx \\
& && p_R(x) = p_R^B(x) = \tfrac{2\alpha+\beta}{2+\beta} - \tfrac{\beta}{2+\beta} t + \tfrac{\beta t}{2+\beta} tx \\
&\textit{where } x_L = \begin{cases} \frac{2}{3} - \frac{1-\alpha}{3t} & \textit{if} \quad t > \frac{1}{2}(1-\alpha) \\ 0 & \textit{if} \quad t \leq \frac{1}{2}(1-\alpha) \end{cases}
\end{aligned}$$

This means that at $x < \frac{1}{2}$ consumers purchase either from firm L or do not purchase at all. The equilibrium prices on $x > \frac{1}{2}$ and the resulting allocation follow from the symmetric set-up of the model.[30]

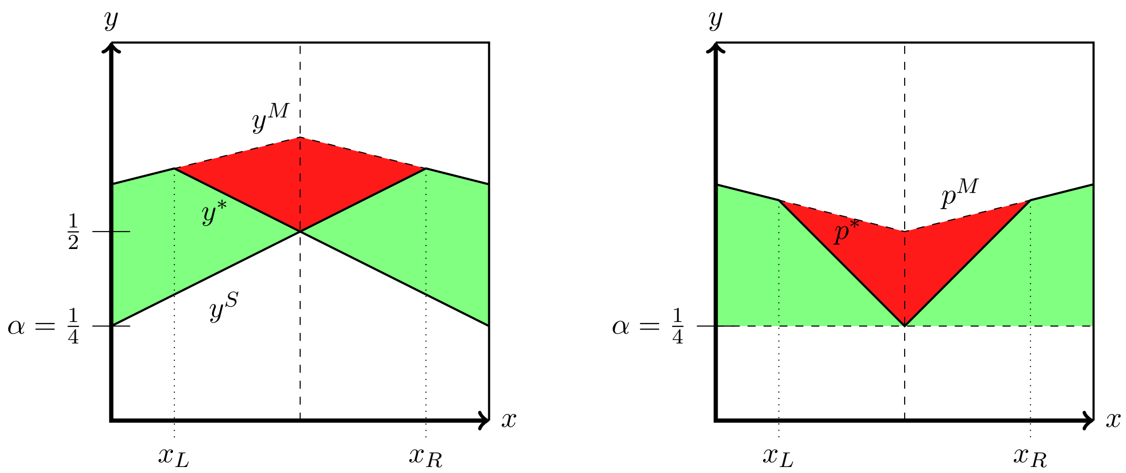
Proof: See Appendix A.6.

Proposition 5 means that competition lowers price only for consumers in the region $x \in [x_L, \frac{1}{2}]$. For $x \in [0, x_L)$ firm L prices like a monopolist as firm R cannot offer a price which both breaks even and attracts consumers.[31] For $x \in [x_L, \frac{1}{2}]$ firm L does not price like a monopolist as firm R would undercut L, steal consumers and make a profit. In equilibrium, firm L sets a price $p_L(x) < p_L^M(x)$ which is just low enough such that firm R would make a loss from any price that attracts consumers. This means that firm L covers its cost and makes an additional mark-up which equals the size of the transport cost differential.

Diagrammatically:

Figure 6: Equilibrium under Duopoly: No Selection

(a) Allocation　　　　(b) Prices



[30] This means that at $x > \frac{1}{2}$ all consumers either purchase from firm R or do not purchase at all and that firm R sets the monopolist's price unless transport costs are so low that firm L would undercut firm R. To prevent this, firm R sets a lower price.

[31] Formally, this means that any $p_R(x)$ which does not attract consumers given $p_L^M(x)$ is an equilibrium price. Hence the inequality in Proposition 5 in $p_R(x) \geq p_L^M(x) + T(x) - T(1-x)$.

The social optimum is y^S or p^S, Monopoly results in y^M and p^M. The equilibrium outcome in imperfect competition or duopoly is y^* and p^*.

Light green shaded area is the inefficiency in duopoly relative to the socially optimal allocation.

Solid red shaded area is the efficiency gain in duopoly relative to monopoly.

The equilibrium has interesting monotonicity features:

Corollary 2

(i) The equilibrium price is monotonically decreasing in distance from the firm.

(ii) The number of consumers purchasing the good can be non-monotone in distance from the firm.

Proof: See Appendix A.7.

The difference between price and quantity effects described in Corollary 2 arises because transport costs drive a wedge between the price consumers pay to the firm $p(x)$ and the total cost consumers incur when purchasing $p(x) + T(d)$. The price consumers pay to the firm, $p(x)$, is monotonically decreasing in distance travelled but the total cost incurred is not. At $x \in [0, x_L]$ the firm prices like a monopolist and thus partially absorbs transport costs. That means $p(x)$ is decreasing in x but at a slower rate than transport costs. Thus $y(x)$ is increasing. When there is effective competition ($x \in [x_L, 1 - x_L]$), prices decrease more strongly than transport costs rise. Thus, prices are decreasing in distance and quantity is decreasing, too. Hence, the non-monotonicity arises because for $x < x_L$, distance only affects a monopolist's pricing while for $x \in [x_L, \frac{1}{2}]$ it acts to intensify competition.

Comparative Static: When competition is less fierce (transport costs are higher) firms engage in monopoly pricing at more locations, i.e. $\frac{dx_L}{dt} > 0$. This arises because the price a monopolist would set at a given x increases by less than transport costs, while the price the competitor needs to break even increases exactly by transport costs. Thus, at a given x there is less room for profitably undercutting which means that already at x closer to $\frac{1}{2}$ no room for undercutting remains and monopoly pricing occurs. Even if duopoly pricing occurs, the larger transport costs result in higher prices, $\frac{dp_L(x)}{dt} > 0$.

Efficiency Properties of the Equilibrium

While the structure of the equilibrium does not depend on the presence of selection, the efficiency properties are affected by the presence of selection.

Proposition 6 *In the absence of selection, the duopoly equilibrium exhibits inefficient underprovision at all x other than $x = \frac{1}{2}$ and efficient provision at $x = \frac{1}{2}$.*

Proof: See Appendix A.8.

This proposition holds for all transport costs.[32] This result is visualised in Figure 6 where the inefficiency of duopoly in terms of unexploited surplus generating trades is captured by the

[32]We only require that $T(d)$ is strictly increasing in distance travelled, i.e. that $\frac{dT}{dd} > 0\ \forall d$.

area between $y^*(x)$ and $y_S(x)$, or correspondingly in terms of mark-ups is captured by the area between $p^*(x)$ and α (both are visualised as the light green shaded areas). The efficiency gain of duopoly relative to monopoly is captured by the gap between $y^*(x)$ and $y_M(x)$ or respectively $p^*(x)$ and $p_M(x)$ (solid red shaded area).

When a market is characterised by advantageous selection, there exists inefficient *over*provision for agents with no or only small brand preferences while simultaneously there is *under*provision for consumers with strong brand preferences. Formally:

Proposition 7 *In markets with advantageous selection, the duopoly equilibrium allocation has the following efficiency properties:*

(i) If $T(1) \geq \frac{\beta}{2+2\beta}(1-\alpha)$*, there exists a unique* x_e *which solves* $\beta(1-\alpha) = (2+2\beta)\,T(1-x_e) - (2+\beta)\,T(x_e)$*. Then*

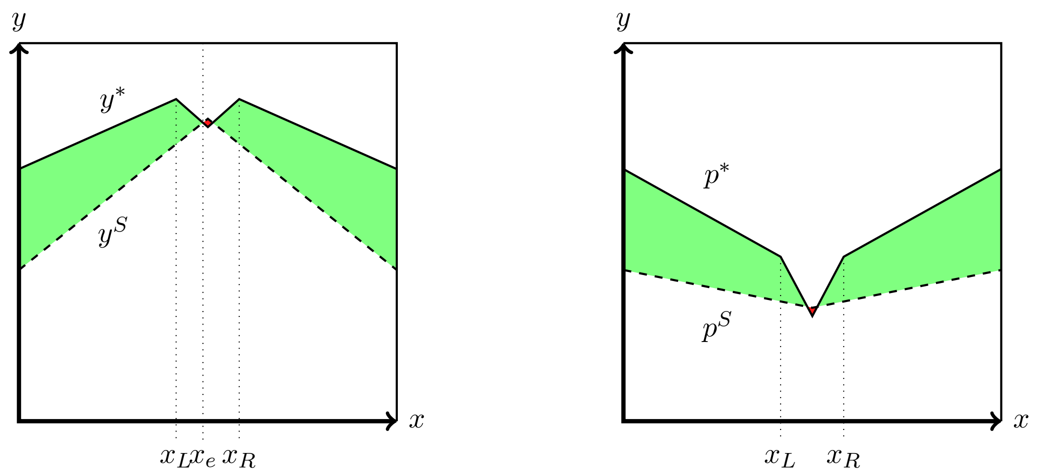
$$\text{For all} \quad x \in \ [0, x_e) \quad \text{there is inefficient underprovision.}$$
$$\text{For all} \quad x \in \ (x_e, \frac{1}{2}] \quad \text{there is inefficient overprovision.}$$

(ii) If $T(1) < \frac{\beta}{2+2\beta}(1-\alpha)$*, then* $x_e = 0$ *and there is inefficient overprovision at all* x*.*

Proof: See Appendix A.9.

Figure 7: Equilibrium under Duopoly: Advantageous Selection

(a) Allocation (b) Prices



where y^* is the threshold characterising the duopoly equilibrium allocation, i.e. imperfect competition, y^S is the threshold which describes the socially optimal allocation.

The green areas highlight inefficient *under*provision and red areas (small, around $x = \frac{1}{2}$) highlight inefficient *over*provision.[33]

Since $T(d) = t\,d$, we have $x_e = \frac{2}{4+3\beta} + \frac{\beta}{t}\,\frac{(2t+\alpha-1)}{(4+3\beta)}$.

[33]Under- and overprovision refer to comparisons the equilibrium under imperfect competition to the socially optimal allocation.

Thus, inefficient overprovision and underprovision can coexist and do not cancel each other out perfectly. The inefficiency from underprovision at $x \in [0, x_e)$ arises because firms have market power over consumers with strong brand preferences (akin to "captured consumers") and exploit this to raise prices. This argument is not specific to selection markets, but our results show that it also applies in the presence of selection. The inefficiency from overprovision at $x \in (x_e, \frac{1}{2}]$ is driven by forces specific to selection markets which are familiar from our discussion of perfect competition in markets with advantageous selection: Consumers at x close to $\frac{1}{2}$ are close to indifferent between the firms' products. Thus, firms compete fiercely for these consumers until profits are completely eroded. Since we can view each x as a separate market with advantageous selection, competition eroding profits completely at an x means that at that x there is inefficient overprovision.[34]

This gives rise to two impossibility results which caution against viewing imperfect competition as an effective policy solution for the problem of inefficient overprovision in markets with advantageous selection.

Proposition 8 *Impossibility Results: Advantageous Selection*

(i) In markets with advantageous selection, the duopoly equilibrium does not *coincide with the socially efficient allocation for any degree of competition.*

(ii) In markets with advantageous selection, no degree of imperfect competition removes overprovision at all x.

Proof: See Appendix A.10.

While our model is in line with the familiar results that perfect competition leads to overprovision at all x and that monopoly leads to underprovision at all x, it yields new predictions for the intermediate case of imperfect competition. No level of competition achieves the efficient allocation at all x simultaneously. Instead, for intermediate degrees of competition there exists a cut-off location x_e and at all locations x with consumers with strong brand preferences ($x < x_e$ or $x > 1 - x_e$) there is *under*provision while at all locations x with consumers with less strong brand preferences ($x_e < x < 1 - x_e$) there is inefficient *over*provision.

Less intense competition (higher transport costs) increase the set of x in which underprovision arises (x_e increases), but there always remains some x at which overprovision exists. E.g. at $x = \frac{1}{2}$, there is overprovision for all degrees of imperfect competition. Only monopoly would remove this overprovision. Thus, imperfect competition introduces a new type of inefficiency at some x (underprovision) without removing overprovision at other x. Thus, these inefficiencies do not cancel each other out perfectly at all x, but co-exist.

This section showed that neither under uniform prices, nor under price discrimination does there exist a degree of imperfect competition which achieves the first best efficient allocation. Given that it is impossible to reach the first best allocation by tolerating lower degrees of

[34] Since in markets with advantageous selection $MC > AC$, when profits are eroded at $p = AC$ the price is below MC and thus there is inefficient overprovision.

competition, a natural question is whether alternative policy tools exist which can achieve the first best. The next section focuses on such alternative policy tools.

6 Policy Remedies

Since no degree of imperfect competition can perfectly correct the inefficiency created by advantageous selection, this section discusses alternative policy solutions. First, we study a corrective tax, i.e. a tax or subsidy which applies equally to all trades. Then, we turn to "risk adjustment", which is a tax or subsidy conditioned on an individual's characteristics.

6.1 Corrective Tax

We find that the combination of a corrective tax and fierce competition achieves the first best efficient allocation and thus results in higher total surplus than intermediate degrees of competition, as these only achieve the constrained efficient allocation. Thus, competition policy should not be lenient towards firms because they operate in a market with advantageous selection. Instead, competition policy should continue to focus on achieving perfect competition. The distortions arising from selection can be better addressed through a tax than through altered competition policy. The corrective tax has the additional benefit of generating government revenue, holds regardless of whether firms use uniform prices or price discrimination, and has lower informational requirements.

Proposition 9 *There exists a corrective tax which achieves the first best efficient allocation.*

Proof: See Appendix A.11.

A tax per unit sold (τ) is treated by firms like an upward shift in costs. If $c(y) = \alpha + \beta(1-y)$, it corresponds to an increase in α. Since corollary 1 established that there is cost pass-through, i.e. that $0 < \frac{dp^*(t)}{d\alpha}$, the corrective tax leads firms to raise prices and, when correctly calculated and combined with perfect competition, the tax can restore the first-best efficient allocation.

Since $c(y) = \alpha + \beta(1-y)$ where $\beta > 0$, there is inefficient overprovision under perfect competition, i.e.

$$p_L^S(t=0) = \frac{\alpha + \beta}{1+\beta} > p^*(t=0) = \frac{2\,\alpha + \beta}{2+\beta} \tag{12}$$

Since a per unit tax (τ) results in equilibrium prices $p^*(t=0) = \frac{2\,(\alpha+\tau)+\beta}{2+\beta}$, we can solve for the tax rate which achieves the first-best efficient allocation:[35]

$$\tau^* = \frac{\beta\,(1-\alpha)}{2\,(1+\beta)} \tag{13}$$

Regardless of whether firms use uniform pricing or price discriminate, a per unit tax can always achieve the first-best efficient allocation. The optimal tax rate is the same in both cases. Thus, the policy of using a corrective tax is robust regardless of which pricing strategy firms use.

The corrective tax not only results in larger total surplus than imperfect competition, it also has lower informational requirements. In order to calculate the level of competition which achieves the constraint efficient allocation ($\tilde{t}$), the policy maker needs to know costs $c(y)$ and both type distributions $F(y)$ and $G(x)$. In order to calculate the optimal corrective tax, which achieves the first-best efficient allocation (τ^*), the policy maker only needs to know costs $c(y)$ and the WTP distribution $F(y)$, but not the preference distribution $G(x)$. This arises because perfect competition has the additional advantage that it makes all consumers face the same total cost of purchasing $p + T(x) = p\ \forall\ x$, and thus results in consumers at all x behaving equally. This makes it unnecessary to know $G(x)$.

There are further benefits of a corrective tax not captured in our model. A corrective tax generates government revenue which can be used to reduce distortionary taxes in other markets and thus enhances efficiency. Attempts to correct overprovision through less fierce competition have no comparable benefit as instead of government revenue they increase firm profits and create a deadweight loss (transport costs).

6.2 Risk Adjustment

An alternative policy to the corrective tax or to tolerating intermediate degrees of competition is risk adjustment. Under risk adjustment, firms receive a subsidy conditional on the type of consumers they insure such that the cost a firm incurs from insuring a consumer is independent from the consumer's type. Thus, risk adjustment effectively removes selection.

While risk adjustment is used in practice, e.g. in health insurance markets, a corrective tax has lower informational requirements and is likely more robust to gaming of risk scores by insurance providers. The corrective tax shifts every agent's cost equally and thus preserves heterogeneity in costs. Hence, to implement the tax, neither the policy maker nor the firm needs to know the agent's type y. Under risk adjustment, firms receive a subsidy conditional on y. Thus, both the firm and the policy maker need to know y. In practice, a consumer's riskiness is not observed and subsidy payments are conditioned on risk scores which insurers assign. This generates an incentive problem as the insurer prefers to make the consumer appear to have large

[35] More generally, for any transport cost t, there exists a tax rate or subsidy rate which achieves the constrained efficient allocation, i.e. for which $p_L^S(t) = p^*(t)$. This tax rate is implicitly defined by $p_L^S(\alpha,\beta,t) = p^*(\alpha+\tau,\beta,t)$ or equally by $\frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4\,(1+\beta)}t = \frac{1+\alpha+\tau+\beta}{2+\beta} + \frac{3+\beta}{2(2+\beta)}t - \frac{1}{2(2+\beta)}\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha-\tau)t + 4(1-\alpha-\tau)^2}$. However, the first best efficient allocation is only achieved under perfect competition ($t = 0$) combined with tax rate (13). Moreover, all cases with $t > 0$ are further complicated by considerations whether firms use uniform pricing or price discrimination and by whether the tax rate has to apply equally to all x or can discriminate based on x.

expected claims in order to get a large subsidy. The tax does not require risk scores and therefore does not generate this incentive problem.

Uniform pricing: Our model predicts that non-representative switching patterns matter when calculating the effect of risk adjustment. One way of implementing risk adjustment is to adjust every agent's cost to equal the average cost of all agents purchasing. When costs are linear, this is equal to the cost of the average infra-marginal consumer. However, when using uniform prices, firms' pricing decisions are driven by the average marginal consumer, which is calculated across switching and entering consumers. When the average marginal consumer is a lower type than the average infra-marginal consumer, as is the case when distributions are uniform, then in a market with advantageous selection, firms respond to this form of risk adjustment by lowering their prices.

Price discrimination: Calculations of the effect of risk adjustment, or equally comparative statics on the strength of selection, face the obstacle that a change in β corresponds to both altering the slope of marginal costs and also to altering the level of costs for a given demand. That means $\frac{dx_e}{d\beta}$ captures both a pure cost effect, i.e. an increase in total cost for a given level of demand, and an increase in the strength of selection. We need to isolate the effect of increased selection. We achieve this through an indirect approach. $\frac{dx_e}{d\alpha}$ captures a pure cost effect with no change in selection. We find that $\frac{dx_e}{d\alpha} > 0$. We also find that $\frac{dx_e}{d\beta} < 0$. Taken together, this means that the selection effect has the opposite sign of a pure cost effect and more than outweighs it. Thus, purely increasing the strength of selection while not altering average cost in the market likely leads to inefficient overprovision occurring at more locations x. Risk adjustment, i.e. a reduction in the strength of selection, has the opposing effect. Formally:

Corollary 3 *If, in markets with advantageous selection, selection is more pronounced, overprovision occurs at more locations x for given transport costs $T(d)$.*

Proof: See Appendix A.12.

7 Adverse Selection

Using our spatial model to study markets with adverse selection, we find that our result of cautioning against viewing imperfect competition as a policy remedy for inefficiencies introduced by selection is even stronger in markets with adverse selection than in markets with advantageous selection. While under advantageous selection the oligopoly equilibrium when firms use uniform prices can achieve the second-best (constraint efficient) - but never the first-best - under adverse selection the oligopoly equilibrium can achieve neither the first-best nor the second-best. As the mechanisms are similar to the case of advantageous selection, we highlight results briefly and focus on the key differences.

The model captures adverse selection when $c(y)$ is increasing in y. We focus on linear costs $c(y) = \alpha + \beta(1 - y)$ where $\beta < 0$ ensures adverse selection.[36]

[36] Our assumption that costs are positive implies that $\beta > (-1)$.

Switching Consumers: In markets with adverse (advantageous) selection, the consumers who switch to a firm when it cuts its price are an adverse (advantageous) selection of the rival's demand. The mechanism is the same in both cases. Switching consumers have, on average, higher WTP than the rival's demand as they needed to cover larger transport costs than the average consumer in demand. In markets with adverse selection, serving switching consumers is thus more costly than serving the average consumer in demand or in the population. This mutes firms' incentives to cut their price.

Uniform Pricing and Efficiency: The result that no uniform price can implement the efficient allocation in markets with selection (Proposition 1) applies also under adverse selection. While with advantageous selection, the threshold describing the efficient allocation is flatter than transport costs (and thus than any allocation implemented by uniform prices), under adverse selection the threshold describing the efficient allocation is steeper than transport costs. While any uniform price results in an allocation where consumers purchase if $p + T(x) < y$, i.e. the threshold WTP above which consumers buy is increasing in line with transport costs, the efficient allocation additionally takes into account that higher WTP consumers are more costly to serve. Thus, in the efficient allocation consumers are allocated the good if $c(y) + T(x) < y$, which results in a steeper threshold.

In the constraint efficient allocation, i.e. the allocation resulting from the socially optimal uniform price, over- and underprovision co-exist, but their pattern is the reverse of the pattern under advantageous selection. While under advantageous selection, the constraint efficient allocation exhibited underprovision for consumers with weak brand preferences and overprovision for those with strong brand preferences, under adverse selection there is overprovision for consumers with weak brand preferences and underprovision for those with strong brand preferences.

Monopoly: In monopoly, there is inefficient underprovision at all locations x. This result, formalised in Proposition 2, holds for both adverse and advantageous selection, for any strength of selection and for both uniform pricing and price discrimination.

Imperfect Competition: Uniform Pricing: While, under adverse selection, the equilibrium uniform price continues to be characterised by Proposition 3 and continues to satisfy Corollary 1 on cost pass-through, the efficiency properties differ from the case of advantageous selection (Proposition 4 differs from Proposition 10). As in the case with advantageous selection, with adverse selection the oligopoly equilibrium price never achieves the efficient allocation (Proposition 4 (ii) and Proposition 10 (ii)). However, in contrast to the case with advantageous selection, with adverse selection the equilibrium price never achieves the second best (Proposition 4 (i) and Proposition 10 (i)). Thus, for adverse selection we have:

Proposition 10 *(Adverse selection)*

(i) The oligopoly equilibrium price $p^*(t)$ and the socially optimal price $p_L^S(t)$ never coincide.

(ii) The oligopoly equilibrium allocation and the socially optimal allocation never coincide.

Proof: See Appendix A.13.

This result cautions against viewing imperfect competition as a solution to the inefficiencies introduced by selection even more strongly than in the case of advantageous selection.

Imperfect Competition: Price Discrimination: While under adverse selection, the equilibrium prices continue to be characterized by Proposition 5 and Corollary 2 continues to apply, the efficiency properties differ from the case of advantageous selection (Proposition 7). Whereas under advantageous selection over- and underprovision coexisted for different consumers, under adverse selection there is underprovision for all consumers.

Proposition 11 *In equilibrium, there is inefficient underprovision at all x.*

Proof: See Appendix A.14.

Thus, our results on price discrimination caution against viewing imperfect competition as a solution to the inefficiencies introduced by selection. The results for adverse selection are even stronger than for advantageous selection. While for advantageous selection the inefficiency cancelled out at one location (though not at all locations), for adverse selection the inefficiency is not removed at any location x.

Proposition 12 *Impossibility Result: Adverse Selection*

(i) In markets with adverse selection, the duopoly equilibrium does not coincide with the socially efficient allocation for any degree of competition.

(ii) In markets with adverse selection, no degree of imperfect competition removes underprovision at any x.

The combination of fierce competition and a corrective tax can achieve the first-best also in markets with adverse selection (i.e. Proposition 9 applies). However, whereas in markets with advantageous selection the tax is positive and raises revenue, thus creating room to reduce distortionary taxes in other markets, under adverse selection the tax is negative, i.e. is a subsidy.

8 Discussion

Our results, which are in favour of competition, hold despite the fact that we make several assumptions which tend to understate the social benefit of competition.

We assume that the social planner maximises total surplus. Under any alternative welfare measure which places more weight on consumer surplus relative to producer surplus, competition is even more beneficial. This arises because while overprovision, triggered by fierce competition, and underprovision, triggered by soft competition, both constitute a reduction in total surplus, their distributional implications differ. When there is overprovision, consumer surplus is larger than in the efficient allocation but producer surplus is reduced more strongly, hence total surplus is lower than in the efficient allocation. Hence, fierce levels of competition are socially desirable from a consumer surplus standpoint even when they lead to overprovision.[37]

[37]Industry profits are weakly positive, ensuring that it is optimal for firms to provide goods in this market.

We focus purely on the short term strategic effect of different degrees of competition. Competition can have the additional benefit of spurring innovation and thus lowering costs. This seems particularly relevant in markets with advantageous selection as inefficient overprovision can only occur if, for some agents, costs exceed their willingness-to-pay. This is rationalised, e.g. in an insurance setting, by administrative cost increasing costs above expected claims.[38] When competition has the additional benefit of eroding administrative costs, competition results in the efficient allocation (everyone purchasing insurance) even in the absence of a corrective tax.

In our model, competition has a direct cost effect and a strategic effect. Our approach was to isolate the strategic effect by judging the duopoly equilibrium relative to allocations a planner can achieve when he faces the same transport costs. An alternative approach to focus on total surplus would include the direct cost and would thus view competition as even more beneficial.

Note that our exposition treated costs as identical for all agents with the same WTP. However, our model is more general. We can allow for cost heterogeneity at a given WTP. Then, $c(y)$ can be interpreted as the average cost at WTP y. Since all agents at a given (x, y) will make the same choice, both approaches are valid. While the approach with cost heterogeneity at a given y seems more realistic, we phrased this paper in terms of no cost heterogeneity at a given y. This makes the exposition more concise and avoids confusion between heterogeneity in costs *at* y (which is possible but not central to our model) and heterogeneity in costs *across* y (which is the defining feature of markets with selection and thus central to our model).

9 Conclusion

This paper developed a spatial model of imperfect competition in markets with adverse or advantageous selection and used it to investigate whether imperfect competition exacerbates or ameliorates the inefficiency created by selection. The model shows that a reduction in competition exacerbates the inefficiency created by adverse selection, but can ameliorate the inefficiency created by advantageous selection. However, reduced competition never corrects the inefficiency perfectly. In contrast, the inefficiency can be corrected perfectly through a corrective tax when there is perfect competition.

While our model is flexible enough to capture advantageous or adverse selection, our exposition focused mainly (though not exclusively) on advantageous selection. We find that, even when perfect competition results in inefficient overprovision in markets with advantageous selection, no degree of imperfect competition can achieve the efficient allocation. Even if the efficient quantity is reached, imperfect competition leads to over- and underprovision coexisting for consumers with different preferences.

Our results, which are in favour of competition, hold despite the fact that we make several assumptions which tend to understate the social benefit of competition. We assume that the social planner maximises total surplus. Under any alternative welfare measure which places more weight on consumer surplus relative to producer surplus, competition is even more beneficial. We

[38] In the insurance literature, this is referred to as firms charging a loading factor.

focus purely on the short term strategic effect of different degrees of competition. Competition can have the additional benefit of spurring innovation and thus lowering costs. We assume that firms incur the same cost for serving a given consumer. When there are cost differences, competition can have the additional benefit of allowing the more efficient firm to win market share from the less efficient firm.

Overall, our results caution against viewing imperfect competition as a solution to inefficiencies introduced by selection. Combining tough competition policy to achieve perfect competition with a corrective tax can reach the efficient allocation which cannot be reached by tolerating intermediate degrees of competition. Moreover, the corrective tax has lower informational requirements. This suggests that if we view the build up of credit prior to the Global Financial Crisis as inefficient overprovision in a market with advantageous selection, then we should not conclude that competition in credit markets was excessive pre financial crisis, but rather should conclude that we lacked the appropriate corrective taxation.

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A Proofs

A.1 Proof of Proposition 1

Sketch proof: Any uniform price results in an allocation where consumers buy iff $y > p_L + T(x)$. Thus, the WTP threshold above which consumers buy has slope $T'(x)$. Without selection, the efficient allocation is characterised by a threshold with slope $\frac{dy^S(x)}{dx} = T'(x)$ and since the slopes are equal there does exist a uniform price which implements the efficient allocation ($p_L = \alpha$). With advantageous (adverse) selection, the threshold describing the efficient allocation is flatter (steeper) than transport costs. Thus, the threshold resulting from a uniform price and the threshold describing the efficient allocation never coincide. *Q.E.D.*

Formal proof: First, we establish three Lemmata. Then, we combine them to prove Proposition 1. Throughout the proof, we focus on $x \leq \frac{1}{2}$. $x > \frac{1}{2}$ follows by symmetry.

Lemma A.1 *For any uniform price p_L, consumers purchase the good if and only if their WTP exceeds a threshold $y_{L,U}(x)$ where $\frac{dy_{L,U}(x)}{dx} = T'(x)$.*

Proof: A consumer purchases the good if and only if

$$y > p_L + T(x) \tag{14}$$

By definition

$$y_{L,U}(x) = p_L + T(x) \tag{15}$$

Thus

$$\frac{dy_{L,U}(x)}{dx} = T'(x) \tag{16}$$

Q.E.D.

Lemma A.2 *In markets without selection, there exists a unique efficient allocation. In it, consumers are allocated the good if and only if their WTP exceeds a threshold $y^S(x)$ where $\frac{dy^S(x)}{dx} = T'(x)$.*

Proof: It is efficient to allocate the good to a consumer if and only if

$$y > c(y) + T(x) \tag{17}$$

By definition

$$y^S(x) = c\big(y^S(x)\big) + T(x) \tag{18}$$

In the absence of selection, $c(y) = \alpha \; \forall \; y$. Thus,

$$y^S(x) = \alpha + T(x) \tag{19}$$

Therefore,

$$\frac{dy^S(x)}{dx} = T'(x) \tag{20}$$

Q.E.D.

Lemma A.3 *In markets with advantageous selection, there exists a unique efficient allocation. In it, consumers are allocated the good if and only if their WTP exceeds a threshold $y^S(x)$ where $\frac{dy^S(x)}{dx} < T'(x)$.*

Proof: It is efficient to allocate the good to a consumer if and only if

$$y > c(y) + T(x) \tag{21}$$

By definition

$$y^S(x) = c\big(y^S(x)\big) + T(x) \tag{22}$$

In the presence of advantageous selection $\frac{dc}{dy} < 0$. Thus, totally differentiating and rearranging yields

$$\frac{dy^S(x)}{dx} = \frac{1}{1 - \frac{dc}{dy}} T'(x) \tag{23}$$

where $\frac{1}{1-\frac{dc}{dy}} < 1$ due to the presence of advantageous selection. *Q.E.D.*

Lemma A.4 *In markets with adverse selection, there exists a unique efficient allocation. In it, consumers are allocated the good if and only if their WTP exceeds a threshold $y^S(x)$ where $\frac{dy^S(x)}{dx} > T'(x)$.*

Proof: It is efficient to allocate the good to a consumer if and only if

$$y > c(y) + T(x) \tag{24}$$

By definition

$$y^S(x) = c\big(y^S(x)\big) + T(x) \tag{25}$$

In the presence of adverse selection $\frac{dc}{dy} > 0$. Thus, totally differentiating and rearranging yields

$$\frac{dy^S(x)}{dx} = \frac{1}{1 - \frac{dc}{dy}} T'(x) \tag{26}$$

where $\frac{1}{1-\frac{dc}{dy}} > 1$ due to the presence of adverse selection. *Q.E.D.*

By Lemma A.1 and A.2, in markets without selection, the equilibrium allocation and the socially optimal allocation are characterised by thresholds with the same gradient. Thus, there exists a uniform price at which also the levels coincide at all x.

By Lemma A.1, A.3, and A.4, in markets with selection, the equilibrium allocation and the socially optimal allocation are characterised by thresholds with different gradients. Thus, if the levels of the allocation coincide at one location x, they diverge at all other locations. Hence, there does not exist a uniform price which can implement the socially optimal allocation.[39] *Q.E.D.*

A.2 Proof of Proposition 2

First, we prove this proposition for the case where firms use uniform prices. Then, we prove it for the case where firms can price discriminate.

Lemma A.5 *When firms use uniform prices, the monopolist's optimal price is*

$$p_L^M = \tfrac{1+\alpha+\beta}{2+\beta} - \tfrac{1+\beta}{4(2+\beta)}t \quad if \quad t \leq \tfrac{4(1-\alpha)}{3+\beta}$$
$$p_L^M = \tfrac{1+2\alpha+\beta}{3+\beta} \quad if \quad t > \tfrac{4(1-\alpha)}{3+\beta}$$

Proof: The monopolist solves $\max_{p_L} \pi$ where

$$\pi = \int_0^{n(p_L)} \int_{p_L+T(x)}^1 [p_L - c(y)]\, f(y)\, dy\; g(x)\, dx \tag{27}$$

where $n(p_L)$ is defined as solving $p_L + T(n) = 1$ and must satisfy $0 \leq n \leq \frac{1}{2}$. We consider the case of $n = \frac{1}{2}$ and $n < \frac{1}{2}$ separately.

For $n = \frac{1}{2}$, (27) becomes:

$$\pi = -\Big(\frac{2+\beta}{4}\Big)p_L^2 + \frac{1}{2}(1+\alpha+\beta)p_L - \frac{1+\beta}{8}tp_L - \frac{1}{2}\alpha - \frac{1}{4}\beta + \frac{\alpha+\beta}{8}t - \frac{\beta}{48}t^2 \tag{28}$$

Solving the FOC $\frac{d\pi}{dp_L} = 0$ yields the monopolist's optimal uniform price

$$p_L^M = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{4(2+\beta)}\, t \tag{29}$$

This is a profit maximum since it satisfies the SOC. I.e. $\frac{d^2\pi}{dp_L^2} = -(1+\frac{1}{2}\beta)$ and thus $\frac{d^2\pi}{dp_L^2} < 0$ is equivalent to $(-2) < \beta$ which holds for any type and strength of selection. $n = \frac{1}{2}$ is satisfied iff $t \leq \frac{4(1-\alpha)}{3+\beta}$.

For $n < \frac{1}{2}$, i.e. $n = \frac{1-p_L}{t}$, (27) becomes

$$\pi = \frac{1}{t}\Big[-\frac{1}{2}a - \frac{1}{6}\beta + \big(\frac{1}{2}+\frac{1}{6}\beta\big)p_L^3 - \big(1+\frac{1}{2}\alpha+\frac{1}{2}\beta\big)p_L^2 + \big(\frac{1}{2}+\alpha+\frac{1}{2}\beta\big)p_L\Big] \tag{30}$$

Solving the FOC $\frac{d\pi_L}{dp_L} = 0$ yields

$$p_L^M = \frac{2+\alpha+\beta}{3+\beta} \pm \frac{1-\alpha}{3+\beta} \tag{31}$$

[39] The socially optimal uniform price as depicted for the case of advantageous selection in Figure 3 achieves the efficient allocation at $x = \frac{1}{4}$, results in inefficient overprovision at all $x \in [0, \frac{1}{4})$ and in inefficient underprovision at all $x \in (\frac{1}{4}, \frac{1}{2}]$.

where only $p_L^M = \frac{2+\alpha+\beta}{3+\beta} - \frac{1-\alpha}{3+\beta} = \frac{1+2\alpha+\beta}{3+\beta}$ satisfies the SOC that $\frac{d^2\pi}{dp_L^2} < 0$. $n < \frac{1}{2}$ is satisfied if $t > \frac{4(1-\alpha)}{3+\beta}$. *Q.E.D.*

Lemma A.6 *In monopoly, when firms use uniform prices, there is inefficient underprovision at all locations x regardless of the type and strength of selection.*

Proof: To show:

$$p_L^M + tx > y^S(x) \; \forall \; x \tag{32}$$

The socially optimal allocation is characterised by a WTP threshold $y^S(x)$ above which consumers are allocated the good. $y^S(x)$ is defined by

$$y^S(x) = t\,x + c\big(y^S(x)\big) \tag{33}$$

which solves to

$$y^S(x) = \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}\,t\,x \tag{34}$$

Part I) $t \leq \frac{4(1-\alpha)}{3+\beta}$, adverse selection: Inequality (32) becomes:

$$\frac{1+\alpha+\beta}{2+\beta} - \frac{\alpha+\beta}{1+\beta} + \frac{\beta}{1+\beta}tx - \frac{1+\beta}{4(2+\beta)}t > 0 \tag{35}$$

Since $0 > \beta > (-1)$, $\frac{\beta}{1+\beta} < 0$ and thus showing that (35) holds at $x = \frac{1}{2}$ is sufficient to show that (35) holds at all x.

$$\frac{1+\alpha+\beta}{2+\beta} - \frac{\alpha+\beta}{1+\beta} + \frac{\beta}{1+\beta}\frac{t}{2} - \frac{1+\beta}{4(2+\beta)}t > 0 \tag{36}$$

which simplifies to

$$1-\alpha-\frac{t}{2} > -\frac{t}{4}(\beta+1)^2 \tag{37}$$

where the left side must be positive by assumption that $c(1) + T(\frac{1}{2}) \leq 1$ and the right side must be negative. Hence, the inequality holds. *Q.E.D.*

Part II) $t > \frac{4(1-\alpha)}{3+\beta}$, adverse selection: Inequality (32) becomes:

$$\frac{1+2\alpha+\beta}{3+\beta} - \frac{\alpha+\beta}{1+\beta} + \frac{\beta}{1+\beta}tx > 0 \tag{38}$$

Since $0 > \beta > (-1)$, $\frac{\beta}{1+\beta} < 0$ and thus showing that (38) holds at $x = \frac{1}{2}$ is sufficient to show that (38) holds at all x. For $x = \frac{1}{2}$, equation (38) simplifies to

$$(1-\alpha-\frac{t}{2})\,(1-\beta) > -\frac{1}{2}\,t\,(\beta+1)^2 \tag{39}$$

which holds for all parameters. *Q.E.D.*

Part III) Advantageous selection. The inequality (32) is equivalent to

$$t < \frac{4(1-\alpha)}{(1+\beta)^2 - (8\beta + 4\beta^2)x} \tag{40}$$

For the monopolist, there is meaningful demand only if $t \leq \frac{4(1-\alpha)}{3+\beta}$. Thus if $\frac{4(1-\alpha)}{3+\beta} < \frac{4(1-\alpha)}{(1+\beta)^2-(8\beta+4\beta^2)x}$, then also equation (40) holds. The condition

$$\frac{4(1-\alpha)}{3+\beta} < \frac{4(1-\alpha)}{(1+\beta)^2 - (8\beta + 4\beta^2)x} \tag{41}$$

simplifies to

$$\beta^2 - 8\beta x - 4\beta^2 x < 2 - \beta \tag{42}$$

which holds at all x for all $\beta < 1$. *Q.E.D.*

Lemma A.7 *When firms can price discriminate, the monopolist's optimal price is*

$$p_L^M(x) = \tfrac{1+\alpha+\beta}{2+\beta} - \tfrac{1+\beta}{2+\beta}tx$$

and it sets $p_R(x)$ such that nobody buys from R.

The resulting allocation is that all consumers with $y > \hat{y}(x)$ buy from L where

$$\hat{y}(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}tx \tag{43}$$

Proof: At every location x, the monopolist prices such that $MR = MC$. This must always be a lower quantity than the efficient allocation which is characterised by $p = MC$. This argument replicates the familiar logic behind underprovision of monopoly, but while the argument is usually applied across the entire market, here it applies at every individual location x.

Formally, the monopolist's problem at a location x is to choose a p to maximise

$$\pi(x) = \int_{p+tx}^{1} [p - \alpha - \beta(1-y)]\, dy \tag{44}$$

which simplifies to:

$$\pi = (-\alpha) - \frac{1}{2}\beta - \big(1 + \frac{1}{2}\beta\big)p_L^2 + (1+\alpha+\beta)p_L - (1+\beta)txp_L + (\alpha+\beta)tx - \frac{1}{2}\beta t^2 x^2 \tag{45}$$

The FOC, $\frac{d\pi}{dp} = 0$ yields

$$p_L^M(x) = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{2+\beta}tx \tag{46}$$

which is the unique profit maximum since the SOC holds, i.e. $\frac{d^2\pi}{dp^2} = -(2+\beta) < 0$. *Q.E.D.*

Lemma A.8 *In monopoly, when firms can price discriminate, there is inefficient underprovision at all x regardless of the type and strength of selection.*

Proof: To show:

$$\hat{y}(x) > y^S(x) \; \forall \, x \in \left[0, \frac{1}{2}\right] \tag{47}$$

Which equals

$$\frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta} tx > \frac{\alpha+\beta}{1+\beta} + \frac{tx}{1+\beta} \tag{48}$$

which simplifies to

$$tx < 1 - \alpha \tag{49}$$

It is sufficient to establish that (49) holds at $x = \frac{1}{2}$:

$$t < 2(1-\alpha) \tag{50}$$

which is identical to the assumption that $c(1) + T(\frac{1}{2}) \leq 1$ and thus always holds. *Q.E.D.*

A.3 Proof of Proposition 3

Given our assumptions, we can simplify L's profit function:

$$\pi_L(p_L, p_R) = \int_0^{m(p_L,p_R)} \int_{p_L+T(x)}^1 [p_L - c(y)] \, f(y) \, dy \, g(x) \, dx \tag{51}$$

to:

$$\pi_L(p_L, p_R) = \int_0^{m(p_L,p_R)} \int_{p_L+tx}^1 p_L - \alpha - \beta(1-y) \; dy \; dx \tag{52}$$

and to:

$$\begin{aligned} \pi_L(p_L, p_R) =& \frac{1}{2} \, t \, m(p_L, p_R)^2 \left(\alpha + \beta - (1+\beta)p_L\right) \; - \; \frac{1}{6} \, \beta \, t^2 \, m(p_L, p_R)^3 \\ &+ m(p_L, p_R) \left((1+\alpha+\beta)p_L - (1+\frac{\beta}{2})p_L^2 - \alpha - \frac{1}{2}\beta\right) \end{aligned} \tag{53}$$

where $m(p_L, p_R)$ is the consumer who is indifferent between consuming from firm L and firm R. Thus:

$$m(p_L, p_R) = \frac{1}{2} + \frac{p_R - p_L}{2t} \tag{54}$$

and $\frac{dm}{dp_L} = -\frac{1}{2t}$.

Then, firm L's best response function is given by $\frac{d\pi_L}{dp_L} = 0$, i.e.

$$\begin{aligned} \frac{d\pi_L}{dp_L} =& \, t \; m(p_L, p_R) \; \frac{dm}{dp_L} \; \left(\alpha + \beta - (1+\beta)p_L\right) \; - \; \frac{1}{2} \; t \; m(p_L, p_R)^2 \; (1+\beta) \\ & - \; \frac{1}{2} \; \beta \; t^2 m(p_L, p_R)^2 \; \frac{dm}{dp_L} \\ & + \; \frac{dm}{dp_L} \; \left((1+\alpha+\beta)p_L - (1+\frac{\beta}{2})p_L^2 - \alpha - \frac{1}{2}\beta\right) + m(p_L, p_R) \; \left(1 + \alpha + \beta - (2+\beta)p_L\right) = 0 \end{aligned} \tag{55}$$

This equation simplifies when using that $\frac{dm}{dp_L} = -\frac{1}{2t}$. Moreover, to solve for symmetric Nash equilibria we can use that $p_L = p_R = p^*$ and thus that $m = \frac{1}{2}$. This simplifies (55) to:

$$p_L = \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{2\,(2+\beta)}t \pm \frac{1}{2\,(2+\beta)}\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha)t + 4(1-\alpha)^2} \quad (56)$$

Thus, there are two candidate solutions for a symmetric Nash equilibrium. These candidate solutions were derived assuming that at every location x some consumers buy. To be valid, these candidate solutions need to be such that at all x someone buys the good. To check this, it is sufficient to check that at $x = \frac{1}{2}$ someone buys:

$$p + T\Big(\frac{1}{2}\Big) < 1 \quad (57)$$

Using the negative square-root solution, this for all:

$$t < \frac{4\,(1-\alpha)}{3+\beta} \quad (58)$$

The positive square-root solution never satisfies (57).

Thus, the negative square root solution, i.e. equation (10), is the unique candidate symmetric Nash equilibrium. To establish that it actually is a Nash equilibrium, we check the firm's second order condition. To derive $\frac{d^2\pi_L}{dp_L^2}$, start from a version of $\frac{d\pi_L}{dp_L}$ where we used that $\frac{dm}{dp_L} = -\frac{1}{2t}$, but did not use conditions which only hold in equilibrium i.e. that $p_L = p_R = p^*$ or respectively $m = \frac{1}{2}$:

$$\begin{aligned}\frac{d\pi_L}{dp_L} =& -\frac{1}{2}m(p_L,p_R)\big(\alpha+\beta-(1+\beta)p_L\big) - \frac{1+\beta}{2}tm(p_L,p_R)^2 + \frac{1}{4}\beta tm(p_L,p_R)^2 \\ & - \frac{1}{2t}\big((1+\alpha+\beta)p_L - (1+\frac{\beta}{2})p_L^2 - \alpha - \frac{1}{2}\beta\big) + m(p_L,p_R)\,\big(1+\alpha+\beta-(2+\beta)p_L\big)\end{aligned} \quad (59)$$

From (59) we can calculate the second derivative

$$\begin{aligned}\frac{d^2\pi_L}{dp_L^2} =& -\frac{1}{2}\frac{dm}{dp_L}\big(\alpha+\beta-(1+\beta)p_L\big) + \frac{1}{2}m(p_L,p_R)(1+\beta) - (1+\beta)\,tm(p_L,p_R)\,\frac{dm}{dp_L} \\ & + \frac{1}{2}\,\beta\,t\,m(p_L,p_R)\,\frac{dm}{dp_L} \\ & - \frac{1}{2t}\big((1+\alpha+\beta)-(2+\beta)p_L\big) + \frac{dm}{dp_L}\big(1+\alpha+\beta-(2+\beta)p_L\big) - m(p_L,p_R)(2+\beta)\end{aligned} \quad (60)$$

which using that $\frac{dm}{dp_L} = -\frac{1}{2t}$ and that $p_L = p_R = p$ and thus $m = \frac{1}{2}$ simplifies to:

$$\frac{d^2\pi_L}{dp_L^2} = \frac{1}{4t}(-4 - 3\alpha - 3\beta - 2t - \frac{1}{2}\beta t + 7p + 3\beta p) \quad (61)$$

The second order condition is that $\frac{d^2\pi_L}{dp_L^2} < 0$ which equals

$$p < \frac{4+3\alpha+3\beta}{7+3\beta} + \frac{2+\frac{1}{2}\beta}{7+3\beta}\,t \quad (62)$$

which holds at $t = 0$ and for all other $t < \frac{4\,(1-\alpha)}{3+\beta}$.

Thus, there exists a unique symmetric Nash equilibrium. In this equilibrium, the price is as given in equation (10):

$$p^* = \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{(2+\beta)}\,\frac{t}{2} - \frac{1}{2+\beta}\sqrt{(1-\alpha-\frac{t}{2})^2 + \frac{t^2}{2}(\beta+2)(\beta+3)}$$

Q.E.D.

A.4 Proof of Corollary 1

To show: $0 < \frac{dp^*}{d\alpha}$. We first consider the cases of $t < \frac{4(1-\alpha)}{3+\beta}$ and then turn to $t > \frac{4(1-\alpha)}{3+\beta}$

If $t < \frac{4(1-\alpha)}{3+\beta}$, then p^* is given by equation (10) and thus

$$\frac{dp^*}{d\alpha} = \frac{1}{2+\beta} + \frac{1-\alpha-\frac{t}{2}}{\sqrt{(1-\alpha-\frac{t}{2})^2 + \frac{t^2}{2}(\beta+2)(\beta+3)}} \tag{63}$$

Therefore, $\frac{dp^*}{d\alpha} > 0$ simplifies to

$$(1-\alpha-\frac{t}{2})(2+\beta) > -\sqrt{(1-\alpha-\frac{t}{2})^2 + \frac{t^2}{2}(\beta+2)(\beta+3)} \tag{64}$$

Since $(1-\alpha-\frac{t}{2}) > 0$ by the assumption that $c(1) + T(\frac{1}{2}) \leq 1$, and $(2+\beta) > 0$ by $\beta > (-1)$, the left side must be positive. The right side must be negative. Hence, the inequality always holds.

If $t > \frac{4(1-\alpha)}{3+\beta}$, then $p^* = \frac{1+2\alpha+\beta}{3+\beta}$ and thus

$$\frac{dp^*}{d\alpha} = \frac{2}{3+\beta} \tag{65}$$

Therefore, $\frac{dp^*}{d\alpha} > 0$ simplifies to $2 > 0$ which is true. *Q.E.D.*

A.5 Proof of Proposition 4

Part i: *There exists a unique level of transport costs, denoted $\tilde{t}$, at which the oligopoly equilibrium price $p^*(t)$ and the socially optimal* price *$p_L^S(t)$ coincide, i.e. $p_L^S(\tilde{t}) = p^*(\tilde{t})$ where $\tilde{t} > 0$.*

Lemma A.9 *$p_L^S(0) > p^*(0)$.*

Proof: $p_L^S(0) = \frac{\alpha+\beta}{1+\beta}$; $p^*(0) = \frac{2\alpha+\beta}{2+\beta}$. Thus, $p_L^S(0) > p^*(0)$ is equivalent to

$$(1-\alpha)\,\beta > 0 \tag{66}$$

which is true under advantageous selection ($\beta > 0$) since by the assumption that $c(1) + T(\frac{1}{2}) \leq 1$ we must have $\alpha < 1$. *Q.E.D.*

Lemma A.10 $p_L^S(0) < p^*(\frac{4(1-\alpha)}{3+\beta})$.

Proof:

$$p^*(\frac{4(1-\alpha)}{3+\beta}) = \frac{1+\alpha+\beta}{2+\beta} + \frac{2\,(1-\alpha)}{2+\beta} - \frac{(1-\alpha)}{(2+\beta)\,(3+\beta)}(3\beta+7) \tag{67}$$

Thus, $p_L^S(0) < p^*(\frac{4(1-\alpha)}{3+\beta})$ simplifies to

$$(\beta+2)\,(1-\beta) > 0 \tag{68}$$

which holds. *Q.E.D.*

Lemma A.11 $p_L^S(t)$ *is decreasing in* t.

Proof: This statement follows directly from

$$p_L^S(t) = \frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4\,(1+\beta)}\,t \tag{69}$$

Q.E.D.

Lemma A.11 implies that $p_L^S(\frac{4(1-\alpha)}{3+\beta}) < p_L^S(0)$. Thus, combined with Lemma A.10, we have that $p_L^S(\frac{4(1-\alpha)}{3+\beta}) < p^*(\frac{4(1-\alpha)}{3+\beta})$.

Lemma A.12 $p^*(t)$ *is strictly concave.*

Proof: Since

$$p^* = \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{2\,(2+\beta)}t - \frac{1}{2\,(2+\beta)}\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha)t + 4(1-\alpha)^2}$$

we can calculate that

$$\frac{dp^*}{dt} = \frac{3+\beta}{2\,(2+\beta)} + \frac{2(1-\alpha)\ -\ (2\beta^2+10\beta+13)\,t}{2\,(2+\beta)\,\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha)t + 4(1-\alpha)^2}} \tag{70}$$

and defining $\phi = (2\beta^2+10\beta+13)t^2 - 4(1-\alpha)t + 4(1-\alpha)^2$ we have that

$$\frac{d^2p^*}{dt^2} = \frac{-(2\beta^2+10\beta+13)\,\phi + \left[(2\beta^2+10\beta+13)\,t - 2\,(1-\alpha)\right]^2}{2\,(2+\beta)\,\phi^{\frac{3}{2}}} \tag{71}$$

Then $\frac{d^2p^*}{dt^2} < 0$ is equivalent to

$$\left[(2\beta^2+10\beta+13)t - 2(1-\alpha)\right]^2 < (2\beta^2+10\beta+13)\,\phi \tag{72}$$

or equally

$$0 < 2\beta^2+10\beta+13 \tag{73}$$

which holds for any $\beta \geq 0$. *Q.E.D.*

Proof of Proposition 4, Part i: Lemma A.9 - A.12 prove that there exists a unique crossing of $p_L^S(t)$ and $p^*(t)$. Figure 5 provides the visual intuition. *Q.E.D.*

Part ii: *The oligopoly equilibrium allocation and the socially optimal allocation do not coincide for any level of transport costs t.*

Proof of Proposition 4, Part ii: This follows from proposition 1. In markets with advantageous selection, no uniform price can implement the efficient allocation. Even $p_L^S(t)$ only results in the constraint efficient allocation. Thus, at $\tilde{t}$, we have $p^*(\tilde{t}) = p_L^S(\tilde{t})$, i.e. the duopoly equilibrium results in the constraint efficient allocation but by proposition 1 it does not result in the first best efficient allocation. *Q.E.D.*

A.6 Proof of Proposition 5

The proof proceeds in steps. First, we characterise the price firm L would charge if firm R did not exist and term this monopolist pricing. Second, we characterise the locations x at which firm R can profitably undercut L's monopolist prices. Then, we show how firm L responds and that the resulting equilibrium behaviour is as described in proposition 5.

Step 1: Firm L's pricing if firm R does not exist.

At a location x, firm L's profit is

$$\pi_L(x) = \int_{p_L(x)+T(x)}^{1} [p_L(x) - c(y)]\, f(y)\, dy \tag{74}$$

$$\pi_L(x) = p_L - p_L^2 - p_L T(x) - \left(\alpha + \frac{\beta}{2} - (\alpha+\beta)p_L - (\alpha+\beta)T(x) + \frac{\beta}{2}p_L^2 + \beta p_L T(x) + \frac{\beta}{2}T(x)^2\right) \tag{75}$$

which by solving $\frac{d\pi_L}{dp_L} = 0$ yields

$$p_L^M(x) = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{2+\beta}\, T(x) \tag{76}$$

which is the unique profit maximum since $\frac{d^2\pi_L}{dp_L^2} = -(2+\beta) < 0$.

Step 2: Locations at which firm R can profitably undercut the monopolist price $p_L^M(x)$

We introduce two auxiliary concepts. $\hat{p}_R(x)$ is the maximum undercutting price of firm R, i.e. the highest price firm R can set and still attract consumers given that firm L prices like a monopolist. Formally,

$$\hat{p}_R(x) = p_L^M(x) + T(x) - T(1-x) \tag{77}$$

Using (76) this becomes

$$\hat{p}_R(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}T(x) - T(1-x) \tag{78}$$

As second auxiliary concept, $p_R^B(x)$ is the price firm R needs to charge to make zero profit, i.e.

break even, given that firm L does not sell at that location x. This is the lowest possible price firm R will ever offer. Solving the corresponding zero-profit condition $\pi_R(p_R^B(x), x) = 0$ results in

$$p_R^B(x) = \frac{2\alpha + \beta}{2+\beta} - \frac{\beta}{2+\beta}\, T(1-x) \tag{79}$$

Firm R undercuts $p_L^M(x)$ if and only if[40]

$$\hat{p}_R(x) \geq p_R^B(x) \tag{80}$$

which simplifies to

$$1 - \alpha + T(x) \;\geq\; 2\,T(1-x) \tag{81}$$

Since, under the assumptions made, profit functions are well behaved, there exists a unique threshold x_L defined by

$$1 - \alpha + T(x_L) \;=\; 2\,T(1-x_L) \tag{82}$$

For all $x \leq x_L$ firm R does not undercut while for $x > x_L$ firm R does undercut.

Thus, $x_L = 0$ iff $1 - \alpha \geq 2T(1)$, i.e. transport cost are low, and otherwise x_L is the solution to $1 - \alpha + T(x_L) \;=\; 2\,T(1-x_L)$.

Step 3: Firm L's response to being undercut.

At all $x \in [x_L, \frac{1}{2}]$, firm L will respond to R undercutting its price by in turn undercutting R. Firms continue to undercut each other until a price is reached at which firm R no longer finds it profitable to undercut. Since for $x < \frac{1}{2}$, $T(x) < T(1-x)$, it follows that at all $x \in [x_L, \frac{1}{2}]$ the equilibrium price is:

$$p_L(x) = p_R^B(x) + T(1-x) - T(x) \tag{83}$$

equally

$$p_L(x) = \frac{2\alpha + \beta}{2+\beta} + \frac{2}{2+\beta} T(1-x) - T(x) \tag{84}$$

This proves the existence and uniqueness of the equilibrium described in Proposition 5. *Q.E.D.*

A.7 Proof of Corollary 2

Part i: *The equilibrium price is monotonically decreasing in distance from the firm.*

Proof of Part i:

When $x \in [0, x_L]$: $\frac{dp_L^M}{dx} = -\frac{1+\beta}{2+\beta}\frac{dT(x)}{dd} < 0 \; \forall x$ since $\frac{dT(x)}{dd} > 0$, and for advantageous selection $\beta > 0$, or for adverse selection $0 < \beta < (-1)$.

When $x \in (x_L, \frac{1}{2}]$: $\frac{dp_L}{dx} = -\frac{4+\beta}{2+\beta}\, t < 0 \; \forall\, x$. *Q.E.D.*

Part ii: *The number of consumers purchasing the good can be non-monotone in distance from the firm.*

[40] If R undercuts, R uses the highest possible price which attracts consumers.

Proof of Part ii: We prove this by focusing on $p(x) + T(x)$, i.e. the total cost a consumer faces. The total cost and the number of consumers buying are inversely related.

When $x \in [0, x_L]$, the total cost consumers face is:

$$p_L^M(x) + T(x) = \frac{1+\alpha+\beta}{2+\beta} - \frac{1+\beta}{2+\beta}T(x) + T(x) \tag{85}$$

$$= \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}T(x) \tag{86}$$

Thus, defining total cost consumers face as $\bar{p} = p_L^M(x) + T(x)$:

$$\frac{d\bar{p}}{dx} = \frac{1}{2+\beta}\frac{dT}{dd} > 0 \,\forall\, x \tag{87}$$

When $x \in (x_L, \frac{1}{2}]$, the total cost consumers face is:

$$\bar{p} = p_L(x) + T(x) = \frac{2\alpha+\beta}{2+\beta} + \frac{2}{2+\beta}\,T(1-x) \tag{88}$$

Thus,

$$\frac{d\bar{p}}{dx} = \frac{2}{2+\beta}\Big(-\frac{dT}{dd}\Big) < 0 \,\forall\, x \tag{89}$$

Since for $x \in [0, x_L]$, $\frac{d\bar{p}}{dx} > 0$ and for $x \in (x_L, \frac{1}{2}]$, $\frac{d\bar{p}}{dx} < 0$, it follows that $\bar{p}$ is non-monotone in x when $x_L > 0$ and thus the number of consumers purchasing is non-monotone. We know from Proposition 5 that for $T(1) > \frac{1-\alpha}{2}$, $x_L > 0$ and then the non-monotonicity arises. If $T(1) \leq \frac{1-\alpha}{2}$, then $x_L = 0$ and the non-monotonicity does not arise. This proves that the non-monotonicity is a possibility, not a certainty, and therefore proves Corollary 2. *Q.E.D.*

A.8 Proof of Proposition 6

Part 1: *In the absence of selection, the duopoly equilibrium exhibits inefficient underprovision at all $x < \frac{1}{2}$.*

Part 2: *In the absence of selection, the duopoly equilibrium achieves the efficient allocation at $x = \frac{1}{2}$.*

In the no selection case, denote costs as $c(y) = \alpha \,\forall\, y$. For $x \in [0, x_L]$, the equilibrium price is $p_L^M(x) = \frac{1+\alpha-T(x)}{2}$ where x_L is the solution to $T(x_L) = 2T(1-x_L) - 1 + \alpha$. This applies provided that $T(1) > \frac{1-\alpha}{2}$. If $T(1) < \frac{1-\alpha}{2}$, then $x_L = 0$. For $x \in [x_L, \frac{1}{2}]$, $p_L(x) = \alpha + T(1-x) - T(x)$.

Proof of Part 2: To show: $p_L(x) = \alpha$ at $x = \frac{1}{2} \,\forall\, T(d)$.

Substituting $x = \frac{1}{2}$ into $p_L(x) = \alpha + T(1-x) + T(x)$ yields:

$$p_L(x) = \alpha + T(\frac{1}{2}) - T(\frac{1}{2}) = \alpha \tag{90}$$

Q.E.D.

Proof of Part 1: To show: $p_L^M(x) > \alpha \; \forall \; x \leq x_L$ and $p_L(x) > \alpha \; \forall \; x \in (x_L, \frac{1}{2})$.

$$p_L^M(x) = \frac{1 + \alpha - T(x)}{2} > \alpha \tag{91}$$

solves to $1 > \alpha + T(x)$, which holds for all x by the the assumption that $c(1) + T(\frac{1}{2}) \leq 1$.

$$p_L(x) = \alpha + T(1-x) - T(x) > \alpha \tag{92}$$

solves to $T(1-x) > T(x)$ which is true for all $x < \frac{1}{2}$. *Q.E.D.*

A.9 Proof of Proposition 7

In the efficient allocation, there is a WTP threshold $y^S(x)$ above which all consumers are allocated the good. The threshold is characterised by

$$y^S(x) = T(x) + c\big(y^S(x)\big) \tag{93}$$

which solves to

$$y^S(x) = \frac{\alpha + \beta}{1 + \beta} + \frac{1}{1 + \beta}\, T(x) \tag{94}$$

There is inefficient overprovision at a location x if and only if

$$p_L(x) + T(x) < y^S(x) \tag{95}$$

where we can focus on prices for $x > x_L$ since $x < x_L$ results in monopoly pricing which always means inefficient underprovision. Thus, (95) becomes

$$\frac{2\alpha + \beta}{2 + \beta} + \frac{2}{2 + \beta}\, T(1-x) < \frac{\alpha + \beta}{1 + \beta} + \frac{1}{1 + \beta}\, T(x) \tag{96}$$

which simplifies to

$$\beta\,(1 - \alpha) > (2 + 2\beta)\, T(1-x) - (2 + \beta)\, T(x) \tag{97}$$

For larger x, the left hand side of this inequality is constant while the right hand side decreases. Thus, there exists at most one crossing. Hence, there are two cases. If $T(1) < \frac{\beta}{(2+2\beta)}(1-\alpha)$, then there is inefficient overprovision at $x = 0$ and thus at all x. If $T(1) > \frac{\beta}{(2+2\beta)}(1-\alpha)$, then there exists a unique x_e defined by solving

$$\beta(1 - \alpha) = (2 + 2\beta)\, T(1 - x_e) - (2 + \beta) T(x_e) \tag{98}$$

such that for all $x < x_e$ there is inefficient underprovision and for all $x > x_e$ there is inefficient overprovision. *Q.E.D.*

A.10 Proof of Proposition 8

The two parts of Proposition 8 are related. We prove both parts simultaneously by showing that, in markets with advantageous selection, there is always inefficient overprovision at $x = \frac{1}{2}$, i.e. for all levels of transport costs.

The efficient allocation is characterised by the WTP threshold $y^S(x)$ which solves

$$y^S(x) = T(x) + c\big(y^S(x)\big) \tag{99}$$

which simplifies to

$$y^S(x) = \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}T(x) \tag{100}$$

In the oligopoly equilibrium, the WTP threshold above which consumers buy is $\bar{p}(x) = p(x)+T(x)$. Consumers at $x = \frac{1}{2}$ face $p_L(x)$, not $p_L^M(x)$. Thus, at $x = \frac{1}{2}$ we have

$$\bar{p}\Big(\frac{1}{2}\Big) = p_L\Big(\frac{1}{2}\Big) + T\Big(\frac{1}{2}\Big) \tag{101}$$

$$\bar{p}\Big(\frac{1}{2}\Big) = \frac{2\alpha+\beta}{2+\beta} + \frac{2}{2+\beta}\,T\Big(\frac{1}{2}\Big) \tag{102}$$

There is inefficient overprovision if and only if

$$\bar{p}\Big(\frac{1}{2}\Big) < y^S\Big(\frac{1}{2}\Big) \tag{103}$$

$$\frac{2\alpha+\beta}{2+\beta} + \frac{2}{2+\beta}\,T\Big(\frac{1}{2}\Big) < \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}\,T\Big(\frac{1}{2}\Big) \tag{104}$$

which simplifies to

$$\beta\left[1-\alpha-T\Big(\frac{1}{2}\Big)\right] > 0 \tag{105}$$

which holds in all markets with advantageous selection ($\beta > 0$) since by the the assumption that $c(1) + T(\frac{1}{2}) \leq 1$ can be restated as $1-\alpha-T\Big(\frac{1}{2}\Big) > 0$. Thus, provided there is any, possibly very small, degree of advantageous selection ($\beta > 0$), there is inefficient overprovision at $x = \frac{1}{2}$. This overprovision is not removed by any level of transport costs. *Q.E.D.*

A.11 Proof of Proposition 9

We know that, in markets with advantageous (adverse) selection, there is inefficient overprovision (underprovision) if $t = 0$, i.e. $p^S(0) > p^*(0)$. This holds when firms use uniform prices (Proposition 4) and when they price discriminate (Proposition 7). For both pricing strategies, the socially optimal allocation and the equilibrium allocation are characterised by a threshold WTP above which consumers are allocated the good and the threshold is independent of x, i.e. is perfectly horizontal. The level of the threshold differs between the socially optimal allocation and the equilibrium.

Since taxes shift the equilibrium threshold without changing its slope (analogous to cost pass through described in Corollary 1), there exists a tax rate for which the equilibrium allocation and the socially optimal allocation coincide. *Q.E.D.*

Example: Consider a market with advantageous selection, i.e. let $c(y) = \alpha + \beta(1-y)$ with $\beta > 0$. Then, there is inefficient overprovision under perfect competition, i.e.

$$p^S(0) = \frac{\alpha+\beta}{1+\beta} > p^*(0) = \frac{2\,\alpha+\beta}{2+\beta}$$

Since a per unit tax (τ) results in equilibrium prices $p^*(0) = \frac{2\,(\alpha+\tau)+\beta}{2+\beta}$, we can solve for the tax rate which achieves the first-best efficient allocation:[41]

$$\frac{2(\alpha+\tau)+\beta}{2+\beta} = \frac{\alpha+\beta}{1+\beta} \tag{106}$$

which solves to

$$\tau^* = \frac{\beta\,(1-\alpha)}{2\,(1+\beta)}$$

A.12 Proof of Corollary 3

We established in Proposition 7 that inefficient overprovision occurs at all $x \in [x_e, \frac{1}{2}]$ where x_e is the solution to $\beta(1-\alpha) = (2+2\beta)\,T(1-x_e) - (2+\beta)\,T(x_e)$ provided that $T(1) \geq \frac{\beta}{2+2\beta}(1-\alpha)$, and otherwise $x_e = 0$.

Lemma A.13 $\frac{dx_e}{d\alpha} > 0$.

Proof:

$$\beta(1-\alpha) = (2+2\beta)\,T(1-x_e) - (2+\beta)\,T(x_e) \tag{107}$$

totally differentiate

$$-\,\beta\,d\alpha = (2+2\beta)\,\frac{dT(1-x_e)}{dd}\,(-dx_e) - (2+\beta)\,\frac{dT(x_e)}{dd}dx_e \tag{108}$$

which solves to

$$\frac{dx_e}{d\alpha} = \frac{\beta}{(2+2\beta)\frac{dT(1-x_e)}{dd} + (2+\beta)\frac{dT(x_e)}{dd}} > 0\;\forall\,x \tag{109}$$

Q.E.D.

Lemma A.14 $\frac{dx_e}{d\beta} < 0$.

Proof:

$$\beta(1-\alpha) = (2+2\beta)\,T(1-x_e) - (2+\beta)\,T(x_e) \tag{110}$$

[41] More generally, for any transport cost t, there exists a tax rate or subsidy rate which achieves the constrained efficient allocation, i.e. for which $p^S(t) = p^*(t)$. This tax rate is implicitly defined by $p^S(\alpha,\beta,t) = p^*(\alpha+\tau,\beta,t)$ or equally by $\frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4\,(1+\beta)}t = \frac{1+\alpha+\tau+\beta}{2+\beta} + \frac{3+\beta}{2(2+\beta)}t - \frac{1}{2(2+\beta)}\sqrt{(2\beta^2+10\beta+13)t^2 - 4(1-\alpha-\tau)t + 4(1-\alpha-\tau)^2}$. However, the first best efficient allocation is only achieved under perfect competition ($t=0$) combined with tax rate (13). Moreover, all cases with $t > 0$ are further complicated by considerations whether firms use uniform prices or price discrimination and by whether the tax rate has to apply equally to all x or can discriminate based on x.

totally differentiate

$$(1-\alpha)\ d\beta = (2+2\beta)\ \frac{dT(1-x_e)}{dd}\ (-dx_e) + 2\ T(1-x_e)\ d\beta - (2+\beta)\ \frac{dT(x_e)}{dd}\ dx_e - T(x_e)\ d\beta \quad (111)$$

which solves to

$$\frac{dx_e}{d\beta} = -\frac{1-\alpha+T(x_e)-2T(1-x_e)}{(2+2\beta)\frac{dT(1-x_e)}{dd}+(2+\beta)\frac{dT(x_e)}{dd}} \quad (112)$$

where $(2+2\beta)\frac{dT(1-x_e)}{dd}+(2+\beta)\frac{dT(x_e)}{dd} > 0\ \forall\ x$ since $\frac{dT}{dd} > 0$.

We now show that $1-\alpha+T(x_e)-2T(1-x_e) > 0$, which then also proves that $\frac{dx_e}{d\beta} < 0\ \forall\ x$. To show that $1-\alpha+T(x_e)-2T(1-x_e) > 0$, rewrite it as:

$$1-\alpha+T(x_e) > 2T(1-x_e) \quad (113)$$

Recall that x_L is the border between monopoly pricing and effective competition and that overprovision can only arise under effective competition, never under monopoly. Thus, we must have $x_e > x_L$. Recall that x_L was defined by

$$1-\alpha+T(x_L) = 2T(1-x_L)$$

Since $1-\alpha+T(x)$ is increasing in x and $2T(1-x)$ is decreasing in x, for all $x > x_L$ we must have that

$$1-\alpha+T(x) > 2\ T(1-x) \quad (114)$$

and since $x_e > x_L$, we must have

$$1-\alpha+T(x_e) > 2\ T(1-x_e) \quad (115)$$

Thus, we have shown that $\frac{dx_e}{d\beta} < 0\ \forall\ x$. *Q.E.D.*

Since $\frac{dx_e}{d\beta}$ captures both a pure cost effect and a selection effect and $\frac{dx_e}{d\alpha}$ captures only a pure cost effect, we interpret the results that $\frac{dx_e}{d\alpha} > 0$ but $\frac{dx_e}{d\beta} < 0$ as indicating that the selection effect has the opposite sign of the pure cost effect and more than outweighs it. *Q.E.D.*

A.13 Proof of Proposition 10

Proof of Part (i): We prove a stronger statement:

Lemma A.15 *The oligopoly equilibrium uniform price is strictly higher than the socially optimal uniform price.*

Proof: We consider the two regimes $t \leq \frac{4(1-\alpha)}{3+\beta}$ and $t > \frac{4(1-\alpha)}{3+\beta}$ in turn. For each, we show that $p^S < p^*$.

For $t \leq \frac{4(1-\alpha)}{3+\beta}$, $p^S < p^*$ becomes:

$$\frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4(1+\beta)}t < \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{2+\beta}\frac{t}{2} - \frac{1}{2+\beta}\sqrt{(1-\alpha-\frac{t}{2})^2 + \frac{t^2}{2}(\beta+2)(\beta+3)} \quad (116)$$

which simplifies to

$$0 < (-\beta)\,(2+\beta)\,(1-\alpha-\frac{t}{2})^2 + \frac{t}{2}(3\beta^2+10\beta+8)(1-\alpha-\frac{t}{2}) + \frac{t^2}{16}(\beta+2)\left[\beta^2(\beta+2)+8(\beta+1)\right] \quad (117)$$

which holds sind all terms on the right are positive.

For $t > \frac{4(1-\alpha)}{3+\beta}$, $p^S < p^*$ becomes:

$$\frac{\alpha+\beta}{1+\beta} - \frac{\beta}{4(1+\beta)}t < \frac{1+2\alpha+\beta}{3+\beta} \quad (118)$$

which simplifies to

$$(1-\alpha) > \frac{\beta(3+\beta)}{\beta-1}\,\frac{t}{4} \quad (119)$$

which holds since $(1-\alpha) > 0$ and the right side is negative since $(\beta - 1) < 0$ while the other terms on the right are positive. *Q.E.D.*

Proof of Part (ii): We prove a stronger statement:

Lemma A.16 *The oligopoly equilibrium uniform price results in inefficient underprovision at all x.*

Proof: Recall that the socially optimal allocation is defined by a threshold $y^S(x) = \frac{\alpha+\beta}{1+\beta} + \frac{tx}{1+\beta}$.

For $t < \frac{4(1-\alpha)}{3+\beta}$, the oligopoly equilibrium uniform price results in an allocation described by threshold $y^*_U(x)$

$$y^*_U(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{3+\beta}{2+\beta}\frac{t}{2} + tx - \frac{1}{2+\beta}\sqrt{(1-\alpha-\frac{t}{2})^2 + \frac{t^2}{2}(\beta+2)(\beta+3)} \quad (120)$$

The goal is to show that $y^*_U(x) - y^S(x) > 0 \,\forall\, x \leq \frac{1}{2}$. This simplifies to

$$(-\beta)(2+\beta)(1-\alpha-\frac{t}{2})^2 + t(\beta+2)(2x\beta+\beta+2)(1-\alpha-\frac{t}{2}) + \frac{t^2}{4}(\beta+2)\left[(\beta+2)(2x\beta+\beta+2)^2 - 2(1+\beta)^2(\beta+3)\right] > 0 \quad (121)$$

All terms are positive. To show that $\left[(\beta+2)(2x\beta+\beta+2)^2 - 2(1+\beta)^2(\beta+3)\right]$ is positive it is sufficient to show that the term is positive for $x = \frac{1}{2}$. Substituting in $x = \frac{1}{2}$ simplifies the term to $2(1+\beta)^3$ which is positive since $0 > \beta > (-1)$.

For $t > \frac{4(1-\alpha)}{3+\beta}$ the oligopoly equilibrium price coincides with the monopolist's optimal price. The monopolist's optimal price results in inefficient overprovision (see Proposition 2). *Q.E.D.*

A.14 Proof of Proposition 11

Denoting the threshold characterising the socially optimal allocation as $y^S(x)$ and the threshold characterising the equilibrium allocation in this case with price discrimination as $y^*_{PD}(x)$, the goal is to show that $y^*_{PD}(x) > y^S(x) \, \forall \, x$.

For $x \in [0, x_L]$, we have $y^*_{PD}(x) = \frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}tx$. Thus, $y^*_{PD}(x) > y^S(x)$ becomes

$$\frac{1+\alpha+\beta}{2+\beta} + \frac{1}{2+\beta}tx > \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}tx \tag{122}$$

which simplifies to $tx < (1-\alpha)$. It is sufficient to show that this inequality holds at $x = \frac{1}{2}$, and $t < 2(1-\alpha)$ indeed holds by the assumption that $c(1) + T(\frac{1}{2}) \leq 1$. Thus, there always exists inefficient underprovision for all $x \in [0, x_L]$.

For $x \in [x_L, \frac{1}{2}]$, we have $y^*_{PD}(x) = \frac{2\alpha+\beta}{2+\beta} + \frac{2}{2+\beta}t - \frac{2}{2+\beta}tx$. Thus, $y^*_{PD}(x) > y^S(x)$ becomes

$$\frac{2\alpha+\beta}{2+\beta} + \frac{2}{2+\beta}t - \frac{2}{2+\beta}tx > \frac{\alpha+\beta}{1+\beta} + \frac{1}{1+\beta}tx \tag{123}$$

which simplifies to

$$(1-2x)2t(1+\beta) > \beta(1-\alpha-tx) \tag{124}$$

Since $(1-2x) > 0$ and $(1+\beta) > 0$, the left side is positive. As $\beta < 0$ and $(1-\alpha-tx) > 0$, the right side is negative. Thus, the inequality always holds.

Thus, there is inefficient underprovision at both $x \in [0, x_L]$ and $x \in [x_L, \frac{1}{2}]$. *Q.E.D.*

A.15 Proof of Proposition 12

Proof: Proposition 11 established that, in equilibrium, there is inefficient underprovision at all x. Since this result holds for any degree of imperfect competition, i.e. any $t > 0$, Proposition 12 (i) holds. Since the result applies at all x for any t, Proposition 12 (ii) holds. *Q.E.D.*

B Microfoundations for Types of Selection and Perfect Competition

B.1 Microfoundations for Selection in Credit Markets

Entrepreneurs seek finance for a project. They have limited liability and raise finance via debt contracts.

Entrepreneurs have private information on the distribution of project returns. The precise assumptions on how the distribution of returns differs across projects determines whether the credit market exhibits adverse or advantageous selection. We consider them in turn.

B.1.1 Assumptions as in Stiglitz and Weiss (1981)

Stiglitz and Weiss (1981) assume that projects differ in their riskiness and that lenders (or banks) cannot distinguish between them. Formally, the distribution of project returns for a riskier project is a mean preserving spread of the distribution of project returns of a safer project. This means that all projects have the same expected return but differ in the dispersion of returns.

For expositional clarity consider the case of binary outcomes, i.e. projects fail or succeed. Normalise the return given failure to zero for all projects. Then the assumption that the returns of a riskier project are a mean preserving spread of the returns of a safer project means that a riskier project has a higher pay-off given success than a safer project, but has a lower probability of success.

Stiglitz and Weiss (1981) establish that under these assumptions there is adverse selection. Entrepreneurs with high risk projects have a higher willingness-to-pay (WTP) for credit (because of limited liability) than entrepreneurs with safer projects. Moreover, lending to riskier projects has lower expected return for the bank. Thus, the borrowers who are willing to borrow at high interest rates are the borrowers who, all else equal, the bank does not like lending to. This means that there is adverse selection.

Our model can be interpreted as capturing such a credit market. Entrepreneurs with risky projects have a high WTP (in our model: high y) and are the agents the bank does not want to lend to, all else equal, as lending to them has a low expected return for the bank. This is analogous to firms having a higher expected cost from selling to agents with a higher WTP, as is the case in our model.

Stiglitz and Weiss (1981) show that in credit markets with adverse selection, prefect competition can result in inefficient underprovision. Stiglitz and Weiss (1981) also discuss conditions under which credit rationing arises, i.e. in equilibrium the market does not clear and there is excess demand for credit. However, De Meza and Webb (1987) show that the result of inefficient underprovision does not depend on whether the equilibrium is market clearing or not, but rather depends on the assumption of project returns.

B.1.2 Assumptions as in De Meza and Webb (1987)

De Meza and Webb (1987) assume that projects differ in their quality and that lenders (or banks) cannot distinguish them. Formally, the distribution of project returns of a better project first-order stochastic dominates the distribution of project returns of a worse project. This means that projects differ in their expected return.

As in Appendix B.1.1, consider the case of binary outcomes, i.e. projects fail or succeed. Let all projects have the same outcome given success and the same outcome given failure. Then, a better project (in the sense that its return distribution first-order stochastic dominates the return distribution of a worse project) is a project with a higher probability of success.

De Meza and Webb (1987) establish that under these assumptions there is advantageous selection. Entrepreneurs with high quality projects have a higher WTP for credit. Moreover, lending to high quality projects has a higher expected return for the bank than lending to low quality projects. Thus, the borrowers who are willing to borrow at high interest rates are the borrowers which, all else equal, the bank likes lending to. This means that there is advantageous selection.

Our model can be interpreted as capturing such a credit market. Entrepreneurs with high quality projects have a high WTP (in our model: high y) and are the agents the banks wants to lend to, all else equal, as lending to them has a high expected return for the bank. This is analogous to firms having a lower cost from selling to agents with a high WTP, as is the case in our model.

De Meza and Webb (1987) show that, in credit markets with advantageous selection, perfect competition can result in inefficient overprovision. As in our model, this arises because firms compete for entrepreneurs with high quality projects by lower the interest rate which in turn draws entrepreneurs with lower quality projects into the market.

B.2 Microfoundations for Selection in Insurance Markets

This Appendix provides microfoundations for adverse and advantageous selection in insurance markets, characterises the equilibrium under perfect competition, and discusses the efficiency properties of the perfectly competitive equilibrium. An excellent survey is Einav and Finkelstein (2011). The pictorial exposition in this appendix is borrowed from them, while the pictorial exposition in the remainder of the paper is our own.

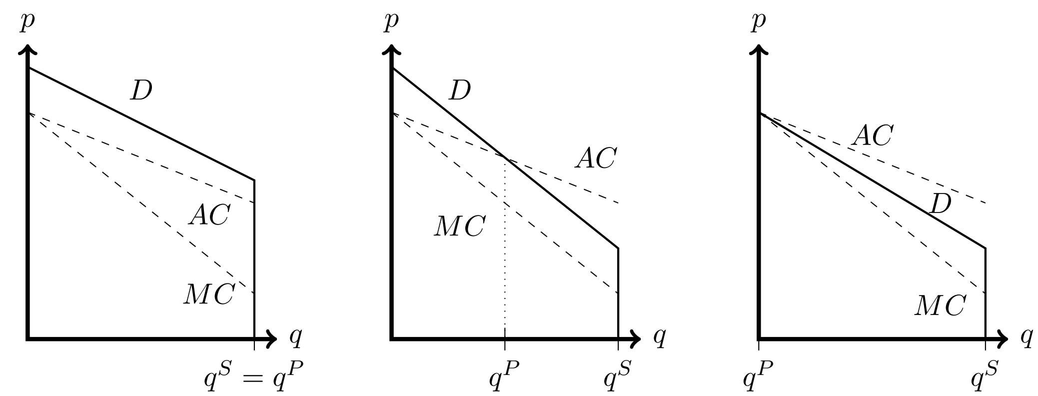
B.2.1 Adverse Selection

A market exhibits adverse selection if marginal costs are decreasing in output. This can be microfounded by consumers having private information on a fixed characteristic that affects the firm's cost. For example, in the market for health insurance, a consumer's health is private information and heterogeneous across agents. Consumers with low health have large expected claims and are therefore expensive for the firm to insure. In insurance markets, the WTP of rational consumers corresponds to the sum of their expected claims and a risk premium. Hence, when consumers have identical risk attitudes, low health consumers have the largest willingness-to-pay for insurance. Thus, at high prices only the low health consumers buy while lower prices draw healthier consumers into the market. Marginal costs are decreasing in output. The market exhibits adverse selection.

In insurance markets with adverse selection, the efficiency of the perfectly competitive equilibrium differs depending on the scale of consumers' risk premia relative to the extent of cost heterogeneity. When consumers' WTP far exceed costs, e.g. because they are very risk averse, the equilibrium achieves the efficient allocation (Figure 8a). When consumers' WTP are lower, the equilibrium exhibits inefficient underprovision (Figure 8b) and the market can even break down completely (Figure 8c).

Figure 8: Adverse Selection: Perfect Competition

(a) Equilibrium is Efficient (b) Inefficient Underprovision (c) Market Breakdown



D denotes demand, MC marginal cost, AC average cost
q^P is the quantity in the perfectly competitive equilibrium
q^S is the efficient or socially optimal quantity

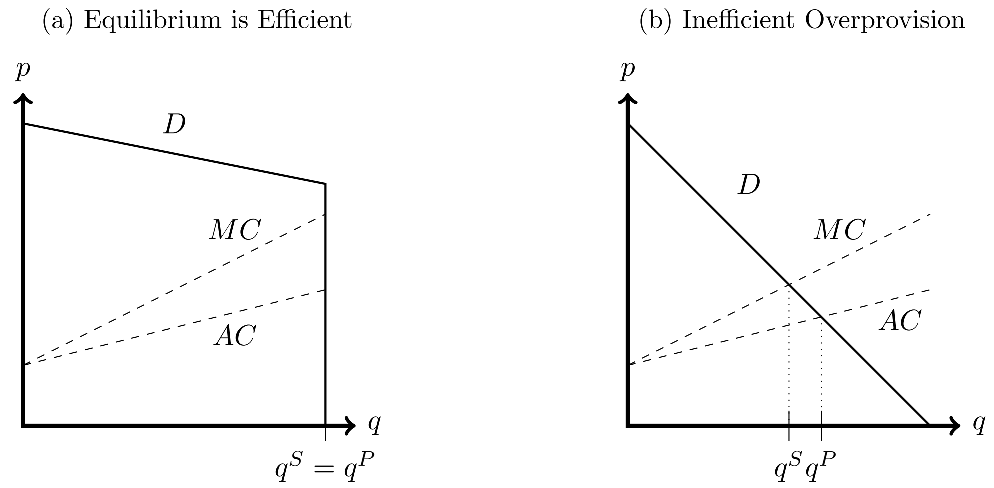
B.2.2 Advantageous Selection

A market exhibits advantageous selection if marginal costs are increasing in output. This arises, for example, when financial risk-taking by consumers is positively correlated with physical risk-taking (Hemenway, 1990). For example, in the market for car accident insurance more risk averse drivers may purchase more generous accident insurance and drive more carefully, thereby reducing the probability of an accident. Thus, it is possible that at high prices only the very risk averse consumers buy - they have low expected costs for the insurer - while lower prices also attract consumers with low risk aversion. Marginal costs are increasing in output.

De Meza and Webb (2001) microfound advantageous selection in insurance markets formally. They consider consumers with private information on either their wealth or their risk-aversion. Moreover, consumers have the possibility to take precautions which reduce the probability of a claim, but which are not observed by the insurance company. They show that more wealthy consumers - modelled as having DARA risk-preferences - or less risk-averse consumer, have a lower benefit from purchasing insurance and from taking precautions. Thus, it is possible that consumers with low wealth (or highly risk-averse agents) buy insurance and take precautions in equilibrium while wealthy consumers (or consumers with a low degree of risk-aversion) neither purchase insurance nor take precautions. The result can be that those who are insured in equilibrium are lower cost to insure than the uninsured. This means that there is advantageous selection.

In insurance markets with advantageous selection, the perfectly competitive equilibrium exhibits inefficient overprovision if for some consumers expected costs exceed their WTP and otherwise results in the efficient allocation. When firms face no administrative costs (then expected costs to the firm are equal to consumers expected claims) and consumers are risk averse (then their WTP exceeds their expected claims by a strictly positive risk premium), all consumers are willing to pay more for insurance than they cost the firm. In this case, the perfectly competitive equilibrium results in the efficient allocation (Figure 9a). When firms have administrative costs or when for behavioural reasons consumers underestimate their future claims, it is possible that some consumers' WTP is below expected cost. In this case, the perfectly competitive equilibrium exhibits inefficient overprovision (Figure 9b).

Figure 9: Advantageous Selection: Perfect Competition



Inefficient overprovision arises because firms undercut each other's price even when the entering consumers are loss making as firms try to steal the rival's existing profitable consumers.