Discretionary NAVs *

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Abstract

A relatively recent trend is participation by leading mutual funds and fund families in the private equity market, which has opened the door of this segment of the economy to a broader set of investor participants. Funds have substantial discretion over the pricing of these investments and, in turn, net asset values. We examine strategic pricing behavior in a sample of 152 mutual funds with positions in 314 private firms. We find heterogeneity in fund family discretionary pricing decisions. Some funds are frequent revaluers while others are infrequent. Some funds are leaders, while others are followers. We find that private equity pricing is procyclical with respect to relative fund performance, which is consistent with the theoretical model. We also examine fund flows and observe that investors respond positively to reported private firm performance and appear to rationally anticipate predictable changes in private equity valuations.

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1 Introduction

In recent decades there has been a dramatic decrease in the number of listed securities in the U.S., from more than 7,500 in 1997 to less than 3,500 recently. As documented and discussed in Doidge, Kahle, Karolyi, and Stulz (2018) listing on U.S. publicly traded equity markets is not well-suited to satisfying the financing needs of small and young firms with mostly intangible capital. At the same time, it is clear that private equity has largely taken the place of public markets in terms of startup financing, which often continues for multiple rounds over extended periods of time (Ewens and Farre-Mensa, 2018). Consequently due to this dramatic shift in availability of public equity, passive or even active investing strategies that concentrate solely on listed firms miss a large set of investment opportunities that may represent a sizable fraction of the economy in the future. Private equity has traditionally only been available to wealthy individuals, hedge funds, and large institutions. But, more recently, many mutual funds in the U.S. have begun to engage in active participation in unlisted private equity. In particular, mutual funds have shown considerable interest in the so-called "unicorn startups," firms with post-money valuations on the order of $1 billion or more. There are even examples of private firms with valuations approaching $100 billion, more than all but a few publicly listed firms. Fund families such as Fidelity are now lead arrangers for follow-on venture financing rounds. Because mutual funds have begun to play an important role in financing privately held firms, investing in this increasingly important segment of the economy is now available to a much broader class of the investing public.

While the trend toward broader ownership of private equity has interesting implications for investors and private firms, the interaction of these investments with the regulatory structure of open-end mutual funds is an important consideration for the development of these markets. Specifically, open-end mutual funds may invest up to 15% of their net assets in illiquid securities, including securities issued by private firms. Given that market quotations are not readily available for securities of private firms, the Investment Company Act of 1940 requires that funds value these assets at "fair value as determined in good faith by the board of directors." More recently the U.S. Financial Accounting Standards Board (FASB 157) has promulgated a set of regulations about how assets must be valued. Unlisted private equity falls under the classification of "Level 3" assets, i.e., those that have inactive or nonexistent markets. Mutual funds are given wide latitude to use various approaches involving unobservable assumptions in order to arrive at a valuation for the nontraded securities. For instance they can utilize a market approach in which they attempt to

find a comparable firm with a market price or an income approach based on price/earnings ratios or discounted cash flow analysis. While the individual values are required to be disclosed at a minimum in quarterly statements, the exact methodology employed is not. As a result there is wide discretion given to the valuation of private equity investments. These valuations in turn affect the net asset value (NAV) of the fund, such that discretionary pricing directly impacts the price at which investors can buy or sell fund shares each day.

The purpose of our paper is to explore the *strategic* pricing decisions of mutual funds that hold substantial private equity positions, using a data sample commencing in 2010. It is well-documented that fund flows represent an important component of the size of mutual funds, which is directly relevant to fund managers and fund families in this highly competitive market. Further, there is a significant relation between flows and returns, especially when it comes to returns relative to peers in the same style category. This implies that discretionary pricing decisions for NAVs of funds with substantial private equity stakes can be used to influence returns and thereby interact with fund flows to gain an advantage relative to other funds without such holdings. We begin with a simple theoretical model to frame the basic issues. The model features a fund making a private equity pricing decision based on contemporaneous fund flows, future anticipated fund flows, and returns of the publicly traded part of its stock portfolio. When discretionary pricing decisions are maintained at a constant level for significant periods, as has been documented in the data, we find that pricing of private equity is pro-cyclical with respect to the performance of the fund's non-private holdings and fund flows. This means that price revisions, when observed, are positively correlated with performance and incoming fund flows.

We examine empirically several aspects of mutual fund and investor behavior. We first document that funds tend to report stale prices for their private firm holdings and have considerable discretion over when to revalue these positions. Further, there is substantial heterogeneity across funds in the tendency to report stale prices as well as significant leader-follower relations. For example, the price revisions on private holdings by BlackRock funds significantly lag those by Hartford funds such that the reported private firm returns of BlackRock are predictable. We also show a tendency for mutual funds to report private firm returns that reinforce the returns on their non-private portfolios. Specifically, funds report significantly higher private returns on average in periods in which their non-private holdings outperform the fund's benchmark compared with periods in which they underperform, consistent with model predictions. Finally, we find that fund flows respond to

reported private firm returns in accordance with the traditional positive flow-performance relation. Interestingly, investors also appear to strategically alter flows to follower funds prior to forecastable changes in private firm valuations.

Mutual funds are an interesting vehicle for investing in private firms because of the daily liquidity. Illiquid securities are often held by other types of investment companies, such as hedge funds and private equity funds. These types of funds, however, tend to have specific features that are designed to deal with the issues that are created when investors own illiquid securities through an intermediary. Private equity funds are typically structured to exist for a specific number of years. Investors commit capital up front which is not necessarily immediately invested and investors are restricted from prematurely redeeming their shares, except at times known in advance when capital is liquidated by the fund itself.[1] Hedge funds typically have infrequent redemption periods and may have lock-up periods in which investors are unable to withdraw funds. Hedge funds can also create side pockets in which illiquid assets are held and ownership of the assets in the side pocket is based on a pro rata allocation to existing investors at the time of side pocket creation.[2] Side pockets thus directly prevent any transfer of wealth between existing and new shareholders based on the pricing of illiquid assets. Open-end mutual funds do not have these provisions and must allow for daily trading at the stated NAV, such that funds explicitly offer liquid claims on illiquid assets and there are no specific provisions that prevent transfers of wealth between existing and new shareholders based on the pricing of these assets.[3]

A recent literature examines aspects of mutual fund holdings in the securities of private firms.[4] Chernenko, Lerner, and Zeng (2017) study the contractual features of mutual fund investments in private firms and find that these investments typically have weaker cash flow rights, weaker governance rights, and stronger redemption rights relative to investments by venture capital funds. Kwon, Lowry, and Qian (2018) investigate the potential impact of funding for private firms by

[1] A relatively active secondary market exists for private equity positions with about $58 billion in volume in 2017 (Greenhill Secondary Market Pricing Study), but these transactions typically take place at a value that is different from the NAV reported by the fund. Moreover we do not see evidence in our empirical sample of such trading on the part of mutual funds.

[2] See Aiken, Clifford, and Ellis (2015) for a discussion of the use of side pockets by hedge funds during the financial crisis.

[3] An additional set of issues with holdings of illiquid securities arises because of the possibility of liquidity-related runs (e.g., Diamond and Dybvig (1983) and Brunnermeier and Pedersen (2009)). For example, Ben-David, Franzoni, and Moussawi (2012) study liquidity-motivated trades of hedge funds around the financial crisis. Mutual funds are likely to be insulated from this type of liquidity issue given the relatively tight restriction that illiquid securities cannot exceed 15% of net assets.

[4] Aragon, Li, and Lindsey (2018) study a related issue of securities in private firms that are held by hedge funds that effectively act to provide venture capital.

mutual funds on the public-private decision.[5] They also describe potential benefits for mutual funds of holding private firms from potentially higher returns, better diversification, and more IPO participation. Lawrence, Siriviriyakul, and Sloan (2015) considers the question of whether assets fair values appear related to actual prices using a sample of closed-end funds. They find that the prices of Level 3 assets are just as relevant as those for Level 1 and 2 assets. Huang, Mao, Wang, and Zhou (2018) consider the impact of holdings by mutual funds and hedge funds in a private firm on its initial public offering (IPO) outcomes. In a recent paper Gornall and Strebulaev (2017) consider post-money valuations of subsequent financing rounds for unicorn private firms. They find that differing contractual specifications of subsequent rounds dictate that there should be discrepancies between the pricing of earlier and later rounds. Yet in practice such discrepancies are rarely accounted for and hence post-money valuations are overstated. The most closely related study to our paper is Agarwal, Barber, Cheng, Hameed, and Yasuda (2018), who investigate the pricing of securities of private firms by mutual funds. They document that reported values differ substantially across families, reported values tend to converge around new funding rounds, and there is a tendency for reported prices to become stale. Relative to Agarwal, Barber, Cheng, Hameed, and Yasuda (2018), we consider the question of strategic pricing of private firm holdings by mutual funds as related to fund returns and flows rather than prices and document substantial heterogeneity in the private firm valuation behavior across mutual funds.

Our study is also related to a large literature on the pricing of securities held by financial intermediaries and the potential for discrepancies between reported value and economic value. Cici, Gibson, and Merrick (2011) examine the reported values of corporate bonds held by bond mutual funds, and Bollen and Pool (2009), Cassar and Gerakos (2011), and Cici, Kempf, and Puetz (2016) consider the effects of managerial discretion in the pricing of illiquid securities on hedge fund returns. Phalippou and Gottschalg (2009) and Brown, Gredil, and Kaplan (2017) investigate the impact of managerial discretion in measuring private equity fund performance. Bhargava, Bose, and Dubofsky (1998), Chalmers, Edelen, and Kadlec (2001), Boudoukh, Richardson, Subrahmanyam, and Whitelaw (2002), and Zitzewitz (2006) examine issues related to mutual fund NAVs that are based on stale prices of securities held in the fund portfolio. Related to this, Carhart, Kaniel, Musto, and Reed (2002) and Ben-David, Franzoni, Landier, and Moussawi (2013) contend that funds strategically manipulate the market prices of stocks in their portfolios by trading these stocks

[5] Ewens and Farre-Mensa (2018) also discuss the effect of capital from non-traditional startup investors including mutual funds on the decision to remain private.

late in reporting periods to create favorable price pressure. Agarwal, Daniel, and Naik (2011) and Couts (2018) consider wealth transfers across groups of investors when trading in and out of funds is possible but restricted. Whereas hedge funds and private equity funds often enforce relatively strict trading restrictions (Aiken, Clifford, and Ellis, 2015), the open-end mutual funds we consider are tradeable at the end of each day at the stated fund NAV. Finally Randl, Westerkamp, and Zechner (2014) consider the impact on an asset allocation problem of the existence of non-tradeable assets.

The remainder of the paper proceeds as follows. Section 2 introduces our model for strategic behavior of mutual funds in reacting to and influencing fund flows through the use of discretionary NAV pricing for private equity holdings. Section 3 describes the data. Section 4 examines aspects of the behavior of mutual funds and their investors with regard to private firm returns. Section 5 concludes.

2 A Model of Discretionary Private Equity Pricing

We first provide a simple model of a mutual fund that has holdings consisting of listed publicly traded and fully liquid securities (stock) and private equity. After developing the model, we then derive empirical implications of the key results that are applied to the actual empirical results in the subsequent sections.

The private equity is also considered to be in the form of equity or convertible preferred securities. Private equity is completely illiquid and cannot be traded at all at the initial time, t. Consequently any positive (negative) fund flows cannot be invested in private equity and must instead be invested (sold) using publicly listed stock. We consider the problem from the standpoint of the fund and its existing and new investors at two specific time points, which we label as time t and time $t+1$.

Time t in the model represents a time when the mutual fund has decided to price the private equity held by the fund. Generally this decision is endogenous, so we are considering the process once this decision has already been reached. While there are natural choices for how prices of private equity might be determined, such as at its acquisition price or after a subsequent financing round, note that there is no regulation that forces the mutual fund to use post-money valuations. For instance even if there is a short period of time between the acquisition date and the pricing date, additional information may become relevant to the fund (e.g., news about competitors, overall market returns, etc).

Once the fund has decided to engage in discretionary pricing, we assume for the time being that the private equity price is held constant for the next time period. This is in accord with broad empirical observations. While it is possible to model the optimality of such stale pricing strategies, we have chosen instead to focus on the initial price decision keeping the next period's price fixed to the initial price.

Consistent with the empirical data, we can take the time interval between t and $t+1$ to be a month. The fund comes into time t with holdings from the previous month, $t-1$. That is, the holdings are N_{Lt-1} shares of listed equity and N_{Et-1} shares of private equity. Since the private equity is not traded, the holding in that investment category remains constant: $N_{Et-1} = N_{Et}$. However there is generally an adjustment in the amount of public equity held to N_{Lt}. The price per share at month t of listed equity is p_{Lt}. Based on the information held by the mutual fund the price per share of the private equity is p_{Et}, however the fund may choose *not* to determine the net asset value by using this price; instead it chooses a *discretionary* price equal to p^*_{Et}. This represents the discretionary choice of the fund, and therefore along with the exogenous price of listed equity and the asset composition, this determines the NAV of the fund. Hence the total NAV of the fund is equal to $N_{Lt}p_{Lt} + N_{Et}p^*_{Et}$.

The price of the listed equity and the underlying intrinsic price of private equity follow binomial processes. Namely the listed equity price increases from p_{Lt} to either $u_L p_{Lt}$ or $d_L p_{Lt}$, where u_L and d_L represent the respective gross returns from either a price increase or decrease. Similarly the private equity intrinsic value goes from p_{Et} to either $u_E p_{Et}$ or $d_E p_{Et}$. We assume that the prices are cointegrated and that the probability of simultaneous upward or downward movements in both prices are ρ. Therefore the probability that one price goes up and the other down is $1-\rho$.

At time t there are new fund flows in the amount F_t that flow into the fund. These can be either positive or negative. Consistent with actual practice the fund largely knows the fund flows at the time the private equity pricing decision is made. Therefore, assume that F_t is exogenous from the standpoint of the fund at time t.[6] Since private equity is not traded, the fund flows are all invested in newly purchased shares of listed equity at time t. Hence $F_t = (N_{Lt} - N_{Lt-1})p_{Lt}$. The share of the fund held by new investors or the fraction liquidated by $t-1$ investors is defined

[6]For instance if the pricing decision is made after returns are determined from the previous month, the fund may be able to anticipate what these flows will be for the subsequent month.

to be α_t. We know that

$$\alpha_t = \frac{(N_{Lt} - N_{Lt-1})p_{Lt}}{N_{Lt}p_{Lt} + N_{Et}p^*_{Et}}.$$

After the next period, we assume that the private equity position is maintained and has not gone public at that point. Now there are additional flows into or out of the fund depending on its performance. The fund continues to hold the same number of shares of private equity. Given our assumption that the private equity price is constant, $p^*_{Et+1} = p^*_{Et}$, which is optimally chosen at time t. The overall set of price processes is illustrated in Figure 1.

To see how the discretionary pricing of private equity affects overall fund returns, note that the fraction of the fund invested in private equity, defined as x_t is given by

$$x_t = \frac{N_{Et}p^*_{Et}}{N_{Lt}p_{Lt} + N_{Et}p^*_{Et}}. \tag{1}$$

Since the pricing of private equity is assumed to be static over this time period, the gross returns of the fund are therefore $R_{pt+1} = x_t + (1 - x_t)u_L$ and $R_{pt+1} = x_t + (1 - x_t)d_L$ in the respective cases of increases and decreases in the listed equity prices.[7] Since the pricing of private equity is the primary variable of interest in the model, it is obvious that as the pricing of private equity at time t increases, the weighting of private equity in the fund increases and hence the gross rate of return has lower variation, i.e. it decreases the variance of fund returns, as well as reducing expected returns from time t to time $t+1$, assuming that the listed equity has a positive risk premium. This is because the private equity component is *risk free* with a zero rate of return.

An important aspect of the strategic effect of discretionary private equity pricing is its effect on fund flows. Consistent with the empirical literature, we assume fund flows are convex with respect to fund rates of return. Often it has been found that flows are convex in ranks. Recall that we can interpret fund rates of return to be relative benchmarks or to competitors in the same peer group category, so especially with this specification the convexity of flows is reasonable. We utilize a normalization for fund flows, based on the previous period's listed equity investment, i.e., define the fraction of time ts listed equity holdings in the form of fund flows at time $t + 1$, $f_{t+1} = F_{t+1}/(N_{Lt}p_{Lt})$. We assume this fraction of fund flows is a deterministic function of the *total* fund returns, $f_{t+1} = f(R_{pt+1})$, with the property that $f'(R) > 0$ and $f''(R) > 0$.

In order to model the decision-making process on the part of the fund, we need to formulate an

[7] By gross returns we mean one plus the rate of return.

appropriate objective function. Since fees for most mutual funds are proportional to the assets under management, or at least increasing in assets under management, we assume the fund management wants to maximize the *expected* growth rate of the fund's NAV, i.e., the rate of appreciation of overall fund size. This is an appropriate objective function in the case of the present model since the initial fund flows and previous period's performance are known at the time the discretionary pricing decision is taken. Define g as one plus the growth rate of the fund size from time t to time $t+1$. Given the previous definitions, we know that

$$\begin{aligned} g &= \frac{F_{t+1} + N_{Lt}p_{Lt+1} + N_{Et}p^*_{Et}}{N_{Lt}p_{Lt} + N_{Et}p^*_{Et}} \\ &= \frac{F_{t+1} + (P_{Lt+1}/p_{Lt}) + (N_{Et}p^*_{Et}/N_{Lt}p_{Lt})}{1 + (N_{Et}p^*_{Et}/N_{Lt}p_{Lt})} \\ &= \frac{f(R_{pt+1}) + (p_{Lt+1}/p_{Lt}) + x_t/(1-x_t)}{1 + x_t/(1-x_t)} \\ &= f(R_{pt+1})(1-x_t) + (p_{Lt+1}/p_{Lt})(1-x_t) + x_t. \end{aligned} \tag{2}$$

The objective function is therefore to maximize the expected growth rate:

$$\max_{x_t \in [0,1]} E(g). \tag{3}$$

While the solution to (3) using equation (2) is given in the appendix, we can see from the above the important considerations in determining the optimal pricing of private equity. As the price of private equity increases, the respective holding of private equity, x_t increases. Hence one effect deriving from the size of the fund is to increase the growth rate. However there are two offsetting indirect effects from increasing the pricing of private equity. One effect is that the expected rate of return of the entire fund decreases as long as there is a risk premium to the listed securities, i.e., as long as $u_L > d_L$. The second offsetting effect comes from the convexity of fund flows. As the weighting of private equity goes to one, the probability distribution of the gross return of the fund, R_{pt+1}, approaches one for certain and there are no gains due to the convexity of the fund flow distribution. The optimal solution trades off these three aspects. We derive the first and second order conditions in the appendix and show that a sufficient condition for an interior optimum is that fund flows are sufficiently negative in the event that the overall portfolio return is near the riskfree rate.

Based on the model formulation the key insight from this first order optimality condition is that private equity pricing is uniquely determined at the level of the fraction of the fund invested in private equity, x_t, from equation (1). The important implication is therefore that as the listed equity value increases or the listed number of shares increases, the optimality condition ensures that the private equity price must also increase enough to achieve the appropriate portfolio weight. Corresponding to lower listed equity returns at time t or outgoing fund flows, the mutual fund optimally decreases the price of private equity.

2.1 Implications

We now provide the key testable implications of the structure of the model. First of all, suppose that there are large and positive fund flows at time t. This means that the fund already has an increase in the listed equity allocation percentage. Hence both of the variable effects mentioned above are greater. This supports increased discretionary pricing of the private equity. Similarly when fund flows are negative, we obtain the prediction that private equity prices should be reduced. This implies that when price revisions occur, they should be pro-cyclical with respect to fund flows.

Next with respect to returns at the time of the discretionary pricing decision, whenever returns are greater, so that listed equity makes up an otherwise greater proportion of the fund, the mutual fund optimally raises the price of private equity. Thus we predict pro-cyclicality of private equity returns with respect to listed equity returns. Since fund flows are also typically pro-cyclical with respect to fund returns, especially relative to peers or benchmarks, we predict that upward (downward) revisions in private equity prices are pro-cyclical with respect to both fund flows and performance.

We also obtain predictions in the cross section with respect to variation in fund returns. Keeping in mind the optimal tradeoffs, suppose that there is an increase in the variance of listed equity returns, while holding the risk premium constant. This therefore increases the impact of convexity of fund flows. As a result the optimal holding percentage, x_t, must increase in order to counteract this effect. Therefore we predict that on average the discretionary pricing of private equity should be greater.

When it comes to expected returns of listed equity, if anticipated expected returns are greater, offering a larger risk premium to investors, there is reason for the fund to also increase its pricing of private equity. In doing this, the fund is taking advantage of the opportunities afforded by its

anticipated success in active investing by recognizing that it does not need to support the potential fund flow appreciation to as great an extent.

The empirical results we document consider the heterogeneity of discretionary NAV pricing styles. We can derive the impact of more or less frequent discretionary NAV pricing by using comparative statics from the model. More frequent revaluation corresponds to shortening the time interval from t to $t+1$. This therefore implies that the uncertainty in the listed portion of the portfolio is lower, as is the per period risk premium. In order to counteract these two effects the fund should optimally decrease its pricing of private equity, i.e., use less aggressive pricing strategies. Hence we get the empirical prediction that when price revaluations occur, the pricing should be lower for more frequent revaluers and greater for less frequent revaluers.

We now turn to some potential extensions that are presently outside the scope of the model. First of all, again based on fund heterogeneity, suppose that some funds are leaders and make their own discretionary NAV pricing decisions independently, while others are followers and merely adjust their pricing decisions with a lag. The leaders use the model as described above. Since private equity repricing of the follower funds is dependent on decisions of other funds, we would expect the degree of procyclicality with respect to the fund's own listed securities to be lower for the followers.

When it comes to investor behavior, we can distinguish between two competing effects. First of all, suppose that investors rationally anticipate fund behavior. In the context of the model this means that when there is positive or negative fund flow pricing of private equity responds with positive correlation. This implies that the fund moves discretionary NAV pricing to the detriment of new investors and in favor of previous investors. To the extent that this is anticipated, this keeps fund flows from destabilizing the pricing decisions.

On the other hand, suppose that investors react in a behavioral manner. Then, if the fund follows a pro-cyclical pricing strategy, the observed contemporaneous returns will be higher with high fund flows. This reinforces a positive fund flow effect on the part of potential new investors in the fund and tends to exacerbate the pricing decisions.

3 Data

This section introduces our data sample and provides information about the private firm returns that are reported by mutual funds. Section 3.1 describes the process of creating our sample and

measuring private firm returns. Section 3.2 contains an overview of the funds and private firms in our sample. Section 3.3 reports summary statistics for mutual funds and private firm returns.

3.1 Sample Formation

We develop a sample of mutual funds with reported holdings of private firms. The holdings data are from Morningstar Direct. To form our sample, we first identify a set of mutual funds with potential holdings of private firms as of December 2017. We initially screen mutual funds to require that assets under management exceed $50 million and that the fund is classified as U.S. Equity, Global Equity (but not global ex-U.S. equity), or Sector Equity by Morningstar. For each of the 2,017 mutual funds that meet the initial screen, we examine the reported fund holdings that do not have a ticker symbol in the Morningstar Direct database and hand-check whether any of these holdings correspond to private firms. This process identifies a sample of 152 mutual funds from 38 fund families. Funds with private holdings tend to be large with average (median) assets under management of $5.59 billion ($1.46 billion) as of the end of 2017 compared with an average (median) of $2.24 billion ($0.58 billion) across funds that satisfy the initial screen but have no private holdings. A list of families in the sample with the number of funds in each family is available in Table A.I in the Appendix.

We generate a list of private firms held by one or more of the mutual funds in our sample. For each fund, we obtain reported holdings over the period January 2010 to December 2017 from Morningstar Direct. We then examine every reported holding without a ticker symbol to identify potential holdings of private firms. We hand-check potential private firms using the Crunchbase database and other search methods to determine whether the holding is a claim on a private firm.[8] In this process, we exclude private investments in public equity (PIPEs) and other similar situations in which the underlying firm is publicly traded given that mutual funds and investors can directly observe market prices for other claims on the company in these cases. The final sample includes 314 private firms.

To calculate reported returns on private holdings, we use information on the number of shares and the position value in a private firm that is reported by a mutual fund. In the cases in which a fund reports a position in a given firm in two consecutive reporting periods and the number of

[8] BlackRock typically reports holdings of private firms under project codenames in the Morningstar Direct database. For example, "Project Ride" corresponds to Uber Technologies. To identify the underlying private firms, we matched holdings information from Morningstar Direct to information from the fund annual report filed with the SEC to determine the mapping from project codename to company name.

shares is unchanged, we directly calculate a reported return based on the observed change in value. For other cases, we hand-check the observations to compute a return that represents our best guess at the percentage change in firm valuation. For example, we observe several cases in the data in which the number of shares held increases by some proportion of the previous position with a contemporaneous decrease in the price per share that exactly offsets the effect on the total position value. In these situations, we infer that a type of split has occurred and we assign a return of 0%. Finally, we exclude observations that follow the IPO date or date of an acquisition of the private company, and these dates are generally obtained from Crunchbase.

It is reasonably common for a mutual fund to own multiple securities issued by the same private firm. For example, Fidelity Contrafund has Pinterest positions from both the Series E and Series F funding rounds. It is fairly common practice for funds to value all securities for the same private firm equally, but there are many cases in which the reported values differ across securities presumably based on differences in contractual terms. To compute a private firm return for a fund with this situation, we weight the reported returns on the individual securities by their relative weights in the portfolio. Our final sample has 38,332 reported returns of private firms by mutual funds.

In addition to data on private holding returns, we use fund-level information available from Morningstar Direct. Specifically, we examine data on gross and net fund returns as well as benchmark-adjusted returns. We also use information about fund flows and assets under management.

It is important to note for the design of some of our tests that different mutual funds adopt alternative reporting frequencies. Funds in several of the mutual fund families (e.g., Fidelity and John Hancock) report holdings each month. Other funds (e.g., those advised by Morgan Stanley) report holdings once per (calendar or non-calendar) quarter. Finally, a few funds (e.g., Vanguard funds) mix between quarterly and monthly reporting at various times during the sample. We describe how we handle this issue in the design of each test.

3.2 Sample Description

Table I reports the top 10 mutual funds ranked according to two alternative measures of private firm holdings: the number of firms held and the total portfolio weight allocated to private firms. For each fund, we measure the maximum across sample months of the private holdings measures. The Fidelity Growth Company fund holds the most private firms with 34 unique holdings. Six of

the top 10 funds in terms of the number of private firms are Fidelity funds, which reflects Fidelity's position as the most active family with 38 funds in the sample (as shown in Table A.I). Funds from Hartford and T. Rowe Price also rank in the top 10 in firms held. The Morgan Stanley Global Discovery Fund achieves the largest weight in private firms at 11.55%, which approaches the regulatory cap in illiquid securities of 15.00%. BlackRock, Davis, Hartford, John Hancock, and Morgan Stanley each have funds that rank in the top 10 in total weight. The relative lack of overlap between the number of firms held and the total weight in private firms indicates differences across funds in the concentration of holdings within the set of private firms.

Table II shows the private firms that are most held by mutual funds. We rank firms by the number of funds or the aggregate value of fund holdings in the company. Perhaps unsurprisingly, Uber Technologies ranks first in both categories with 39 mutual funds and a maximum aggregate value of nearly $2 billion in holdings (the maximum was achieved in March 2017). Several other well-known companies, including Dropbox, Airbnb, Lyft, and Pinterest, are among the most widely held private firms during the sample. The two rankings reflect differences in the concentration of holdings, as several of the firms with the highest aggregate value such as Pinterest, WeWork, and SpaceX do not appear in the top 10 of number of funds with holdings. On the other hand, The Honest Company ranks 4th with 25 funds but 35th in aggregate value at about $129 million. Finally, several of these companies are no longer private firms. Acerta Pharma was acquired by AstraZeneca in December 2015 and Cloudera, Dropbox, Facebook, First Data, Groupon, Nutanix, and Pure Storage each undertook IPOs during or after our sample. These companies are in our sample during the subperiods in which they were private firms.

3.3 Summary Statistics

Table III reports summary statistics for our sample. Panel A shows properties of mutual fund holdings for the fund-periods in which the mutual fund has at least one private holding. Conditional on having a private holding, the average number of holdings is 6.25 with an average portfolio weight of 0.17% in each holding. The median total fund weight in private holdings is relatively small at 0.54%, but there are several fund-periods with non-trivial weights.

Panel B of Table III provides information about the reported returns of private holdings. The returns are split into groups based on whether the reporting frequency of the fund is monthly or quarterly. There are some observations in the sample in which a fund changes its reporting

frequency and the elapsed time since the previous reporting date is not one or three months, and these return observations are omitted from this table. The table reports the average, standard deviation, and percentiles of the return distribution. The returns are not annualized, such that the returns for the monthly (quarterly) reporting frequency are one-month (three-month) returns. Both the monthly and quarterly returns have large, positive averages of 1.55% and 6.85% and extremely high standard deviations of 62.37% and 208.74%, respectively. These statistics largely reflect the existence of extreme returns in the sample. The minimum return is close to -100% and the maximum return is about 10,500% for each reporting frequency.[9]

Figure 2 provides an additional look at the reported return distribution for private firms. Panels A and B correspond to the monthly reporting frequency and Panels C and D show the quarterly reporting frequency. In Panels B and D, the returns of exactly 0% are omitted from the histograms. All returns that exceed 100% are consolidated into the far-right bar in each panel. Examining the distributions, the modal return is 0%, which occurs in about 53% of monthly reporting observations and 48% of quarterly reporting returns. Importantly, relative to the distribution of returns on public stocks that make up much of the remainder of the mutual fund portfolios in our sample, the distribution of private returns in Panels B and D have very fat tails. Thus, despite the fact that private firms comprise a relatively small portion of a mutual fund portfolio, the extreme returns that are often realized on these holdings can produce non-trivial effects for mutual fund returns.

4 Empirical Evidence of Behavior of Funds and Investors

This section characterizes the behavior of mutual funds and their investors. Section 4.1 examines the behavior of mutual funds in reporting private firm returns. Section 4.2 investigates the behavior of mutual fund investors by examining fund flows as they relate to reported private firm returns.

4.1 Mutual Fund Pricing Behavior

We examine three features of fund behavior that are related to the reporting of private firm valuations. First, we investigate the tendency for mutual funds to report stale prices for private

[9]The maximum reported returns correspond to holdings by mutual funds across multiple fund families in Peixe Urbano, which is a Brazilian technology company that was acquired in October 2014 by the Chinese firm Baidu. The consistency in the magnitude and timing of reported returns on Peixe Urbano across mutual fund families suggests that the reported returns do not result from a data error but rather represent a true change in firm valuation by the funds.

holdings. Second, we capture interrelations between the values reported by mutual funds through leader-follower relations. Third, we examine average reported private firm returns across periods in which mutual funds underperform or outperform their benchmark.

An important aspect of discretionary private firm valuation by mutual funds is the extent to which funds allow reported prices to become stale. A simple metric that demonstrates heterogeneity in mutual funds in this dimension is the percentage of reported private firm returns that are non-zero. That is, a fund that reports the same valuation for a private firm over an extended period will be repeatedly reporting returns of 0%, such that a low proportion of non-zero returns indicates that a fund tends to report stale prices.

Table IV reports the percentage of non-zero reported private firm returns aggregated by mutual fund family. Panel A (Panel B) shows results for families with a monthly (quarterly) primary reporting frequency. For each fund family, we report the total number of private firm returns across funds and the percentage of these returns that are non-zero. The table shows that a few of the fund families in the sample have a very limited number of return observations. For example, the Destinations family has a single fund in the sample (Destinations Large Cap Equity) and only two reported returns. The Destinations fund first reported a private firm holding in October 2017 when it established a position in Magic Leap, and the two reported returns are for November and December 2017. Several of the cases with few returns correspond to this situation with private firm holdings that are added late in the sample, which reflects the increasing scope of investments by mutual funds in private firms.

Table IV demonstrates substantial heterogeneity across fund families in the dimension of reporting non-zero private firm returns. For example, whereas Prudential reports a non-zero return in 96% of holding-month observations, Thrivent does so in just 24% of months. Across the three monthly reporting families in Panel A with the most return observations, the percentage of non-zero returns varies widely at 25% for Alger, 44% for Fidelity, and 63% for Hartford. A similar pattern arises in Panel B, as the two families with the most reported returns are T. Rowe Price and Morgan Stanley with non-zero return percentages of 35% and 71%, respectively. The substantial variation in this metric across families indicates that families are adopting different strategies for revaluing private firm holdings.

Figure 3 illustrates differences in revaluation frequency across families. Merrimack Pharmaceuticals was held by three families during its time as a private firm: Prudential, Alger, and Fidelity.

Each family acquired a position at a different time, which is reflected in the figure. Merrimack underwent an IPO on March 29, 2012 and the lockup period expiration date was September 25, 2012. Some disagreement on valuation remains following the IPO during the lockup period, but the valuations of public stock agree across the three families after lockup expiration. Consistent with the aggregated information in Table IV, Prudential reports a non-zero return for each month in the sample, whereas Alger and Fidelity tend to revalue the firm relatively infrequently. The figure also illustrates that substantial deviations can exist across different families that value the same private firm.

Another important aspect of mutual fund behavior is the existence of leads and lags in reported private firm returns across fund families. Figure 4 illustrates leader-follower behavior in the specific example of Uber Technologies. Whereas the general pattern of Uber's valuation is similar across families, there tend to be differences in timing by families around the common revaluation events. For example, each family that holds Uber as of October 2014 proceeds to roughly double its valuation. Hartford, John Hancock, and VALIC each increase Uber's reported value in November 2014, whereas BlackRock responds with a lag and reports a large increase in value in January 2015. Similar patterns occur around other large common revaluations.

We begin our formal analysis the existence of leaders and followers in the pricing of private firms by looking at a set of large fund families with monthly reporting frequency. In particular, we regress reported returns on the contemporaneous and lagged reported returns by funds in another family for the same private firm. This analysis is relatively clean for funds with a monthly reporting frequency given that the lag is only one month compared with a three month lag for funds with a quarterly reporting frequency. We examine the five families that have the most overlap in private firm positions across families with a monthly reporting frequency.

Table V reports results for the relations between reported private firm returns by a particular fund family and the contemporaneous and lagged returns for that private firm reported by another family. Beginning with BlackRock, reported private firm returns are significantly related to contemporaneous returns reported by Fidelity, John Hancock, and VALIC. Perhaps more interestingly, BlackRock reported returns are significantly related to lagged returns on the same private firm from Fidelity, Hartford, and VALIC. In results not reported, BlackRock returns are also significantly related to a second lag of Hartford returns, and the two-month lag has the largest economic magnitude and highest statistical significance. Across the other columns, each family is significantly related

to lagged returns from Hartford, which indicates that Hartford is a leader in the timing of private firm return reporting. Given the BlackRock results discussed above and the fact that no family's returns are significantly related to BlackRock's lagged returns, BlackRock appears to be a follower in private return reporting.

To systematically study leaders and followers in the broader sample of mutual fund families, for each fund family we regress reported private firm returns on lagged, contemporaneous, and lead private firm returns that are reported by other fund families. To calculate the lagged (lead) returns, we first equally weight the reported return for a given private firm across all families with a holding in that firm and then sum these returns across the three months that precede (follow) the current period. Cumulating returns across three months is designed to account for the relatively infrequent reporting of funds with quarterly reporting frequency (both for the family under consideration and other families that are entering the benchmark).

Table VI reports results for each family from a regression of private firm returns on the lagged, contemporaneous, and lead returns reported across other families. A significant positive relation between the family's return and the lagged (lead) return of other families indicates that the family is a follower (leader) in revising private firm valuations. The regressions are pooled across holdings and periods. We only report results in Table VI for families with at least 25 valid observations. The table reports the coefficient estimates with indications of statistical significance, the number of observations in the regression, and the adjusted R^2.

Table VI provides evidence of significant lead-lag relations in reported private firm returns. Most of the fund families have significant coefficients on contemporaneous returns from other families, which indicates substantial commonality in the reported returns. However, several families also have significant lead or lag relations with other families. The coefficients on lagged returns for BlackRock, Macquarie, MassMutual, Morgan Stanley, T. Rowe Price, USAA, Vanguard, and Wasatch are all significantly positive at the 1% significance level, indicating that these families tend to revalue private holdings with a lag relative to other families. The coefficient estimates on lead returns are significantly positive at the 1% significance level for Fidelity, Hartford, John Hancock, and Putnam, which indicates leader status for these families. Finally, it is interesting to note that a few families, including AllianceBernstein and Prudential, report private firm returns that are nearly fully idiosyncratic relative to the reported returns by other families as evidenced by low or negative adjusted R^2. Overall, these tests establish the existence of leaders and followers in private firm

valuations as well as substantial heterogeneity in fund pricing strategies.

Finally, Table VII examines the contribution of reported returns on a fund's private holdings on the fund's monthly return. The private firm return contribution is the reported return for the fund's private holdings for a given month multiplied by the lagged portfolio weight in private firms. We split fund return observations by whether the fund's non-private holdings outperform the benchmark or underperform the benchmark.[10] We calculate the return of non-private holdings by subtracting the contribution of private firm returns from the gross fund return and rescaling the result to full investment based on the percentage of non-private holdings in the fund's portfolio. We then compare this non-private portfolio return to the return of the fund's stated benchmark in Morningstar to classify fund-months as outperforming or underperforming months. We report results across all funds as well as for two cross-sectional splits of funds. Specifically, we classify funds as frequent or infrequent valuers based on whether the percent of non-zero returns for the family reported in Table IV is greater than or less than 50%. We also identify leader and follower funds based on whether the fund family has a significant loading on the lead return or lagged return at the 5% significance level in Table VI. Funds for which neither loading is significant are not classified as either leaders or followers.

The results in Table VII suggest that funds strategically report returns on private firms depending on the performance of their non-private holdings. Across all funds, private firm returns contribute 0.021% to fund returns in months in which the fund's non-private holdings outperform the benchmark compared to 0.004% in underperforming months. The difference is statistically significant at the 1% level. While the magnitudes of these monthly return contributions are relatively small, fund returns tend to closely match their benchmarks such that the private firm returns may affect relative fund rankings. Further, investors respond to relatively long-term performance metrics, and the monthly contributions can accumulate across months to have a larger impact on these measures. The positive relation between performance of the fund's non-private holdings and the reported performance of the fund's private holdings is consistent with the implications of the model in Section 2.

Private firm return contributions for the cross-sectional splits in Table VII suggest that infre-

[10]Reported returns of private returns are much more closely related to returns relative to the benchmark rather than raw returns. This finding is somewhat interesting given that the benchmark often reflects market returns from industries that are similar in nature to the private firms. Rather than reflecting this industry-level information, reported returns are related to the fund's deviation from the benchmark which suggests some strategic element of pricing by the fund.

quent revaluers and leaders are relatively more likely to strategically report returns. For infrequent revaluers, the private firm return contribution average is 0.041% per month when the fund's other holdings outperform versus just 0.006% when the fund underperforms. Finally, leader funds report private returns that generate an average contribution of 0.024% in good months versus 0.008% in bad months. These findings suggest that there exists heterogeneity in the cross section of funds in terms of strategic return reporting behavior. While the model that we introduce in Section 2 focuses on a single fund and does not produce cross-sectional predictions for dynamic strategies, the model could be extended in this dimension to map out strategies for pricing private equity over multiple periods.

4.2 Mutual Fund Investor Behavior

We now investigate how mutual fund investors respond to the private firm return reporting behavior of mutual funds. We specifically examine the net of inflows and outflows of mutual funds as it relates to reported private firm returns. We work with monthly net percentage flows for mutual funds in each analysis.

The discretionary valuation of private holdings by mutual funds could affect the amounts and economic effects of flows in several potential ways. Stale pricing by mutual funds creates an interesting situation in which investors can freely trade claims on private firm positions at their reported stale prices given that these prices affect funds' NAVs. In the situation in which reported prices are too low relative to economic value, there is a transfer of wealth from existing and remaining investors to new investors in the fund who are able to participate on a pro rata basis in any gains once they are realized. A similar transfer occurs from new and remaining investors to exiting investors when reported prices are too high.

Investors may be sophisticated enough to anticipate revisions in private firm valuations and act in a way to capitalize on stale prices. Investors may, for example, be able to observe the valuation revisions of leader funds and predict that follower funds will take similar actions. Investing in these funds allows the new investors to participate in the potential gains. Alternatively, investors may simply chase reported fund performance without fully understanding fund discretion over its underlying components. In this case, mutual funds could strategically revise private firm valuations to influence reported performance and take advantage of the known convex flow-performance relation.

We provide some preliminary findings on mutual fund flows as they relate to reported private

returns. We first examine how stale pricing affects average flow. We then investigate flows around non-zero reported private firm returns.

Table VIII reports average mutual fund flow across five groups of mutual fund periods sorted by the average months of stale pricing of private firm holdings. For each of a given mutual fund's private holdings, we calculate the number of months that have passed since the last non-zero reported return. We then average this figure across private holdings to find the average months of stale pricing. The "None" category indicates that a mutual fund reported a non-zero return for each of its private holdings in the most recent holdings report. The remaining categories are formed based on the 25th, 50th, and 75th percentiles of the average months of stale pricing measure (i.e., 1.00 month, 2.15 months, and 4.50 months). Finally, within each group we calculate the average net flow and test whether the average flow for the quartiles with positive stale pricing is higher than the average flow for the funds with no stale prices.

The results in Table VIII show a tendency for investors to increase flow to funds that report more stale prices for private firm holdings. For mutual funds with no stale prices, average net flow is -0.63% per month. The categories with non-zero stale pricing measures each have larger average flow. In the group with the most stale pricing (i.e., funds with average months of stale pricing greater than 4.5 months), average flow is -0.27% per month which is significantly greater than flow for the "None" category at the 1% significance level. These findings suggest that investors increase flow to funds that report stale private firm valuations. In turn, this investor behavior may provide mutual funds with incentive to delay revaluations of private firms.

Figures 5 and 6 provide additional evidence on investor responses to reported private firm returns. Each figure shows average fund flows across all funds as well as a cross-sectional split of funds. Specifically, Figure 5 divides funds into frequent or infrequent valuers based on whether the percent of non-zero returns for the family reported in Table IV is greater than or less than 50%. Figure 6 identifies leader and follower funds based on whether the fund family has a significant loading on the lead return or lagged return at the 5% significance level in Table VI. The figures condition on whether the reported return on a fund's portfolio of private firm holdings in a given reporting period was positive or negative. We then plot the average net flow for the reporting month as well as two months prior and two months following the reported return.

Figure 5 demonstrates that the flows of infrequent valuers tend to be much more stable surrounding reported private firm returns compared to frequent valuers. Consistent with the positive

flow-performance relation documented by Chevalier and Ellison (1997), Sirri and Tufano (1998), and others, fund flow tends to increase (decrease) in the months with positive (negative) reported private firm returns. The magnitude of the effects, however, is much greater for frequent valuers. The response of fund flows to reported returns by frequent revaluers are consistent with the view that investors are using this information to assess managerial performance and that the reported returns are not fully anticipated by investors. In contrast, infrequent valuers have more stable flows across the months that surround non-zero reported private firm returns, which suggests that investors may be better able to anticipate the private firm returns that are reported by infrequent valuers based on information from other funds or sources. The infrequent valuers have statistically significantly higher net flow compared with frequent valuers for the two months preceding a positive reported return as well as for the month of and two months following a negative reported return. The stability and magnitude of flows to infrequent revaluers provides some evidence that mutual funds may benefit by reporting stale prices of private firms.

Figure 6 shows that fund flow for leaders tends to respond contemporaneously to reported firm returns, whereas the flows of followers often respond in advance of reporting a non-zero return. When leader mutual funds report a positive private firm return, net flow is higher in the month of the reported return and the subsequent two months compared with flow prior to the reporting period. The opposite pattern occurs around negative private returns as flow drops markedly during the reporting month and remains low for the next two months. These patterns are consistent with the positive flow-performance relation and the use of multiperiod returns by investors to judge performance. Interestingly, the fund flows for follower funds seem to react in advance of the reported returns. Prior to reporting positive returns, the flows of follower funds are significantly higher than the flows of leader funds at the 5% level. After reporting the private firm returns, the two types of funds display similar patterns in net flow. This finding suggests that investors anticipate positive revaluations of private holdings of follower funds and increase flow to these funds in response. Similarly, the flow for follower funds is lower than the flow of leader funds prior to reporting negative returns, whereas flow is similar in the month of and the months following the reported return. As a result of these patterns, the flow for follower funds tends to be more stable surrounding reported firm returns compared with the flow of leader funds.

Overall, the results in this section provide some evidence that investor flows respond strategically to the private firm return reporting behavior of mutual funds. Flow tends to be higher in periods

in which a fund is reporting particularly stale prices for its private holdings. Investors also seem to adjust flow on average in anticipation of valuation changes made by funds that systematically lag behind competitor funds in revaluing private firms. These actions contribute to more stable flow and, in some cases, higher net flow for funds with a greater tendency to report stale prices.

5 Conclusion

An important recent trend in the U.S. is the increase in relative capital being raised by private firms as compared to public firms. Following this trend has been the increasing tendency of leading mutual funds and fund families to become active investors in private equity. This feature has enabled many individuals to participate in this important segment of the growing economy.

By contrast to the situation in private equity and hedge funds, the inconsistency between the daily liquidity of open-end mutual funds and the illiquidity of private equity holdings provides a discretionary opportunity for the fund to make strategic NAV pricing decisions. These decisions can lead to significant impacts on mutual fund returns and flows, especially in relation to competitor funds. The nature of fund strategies is in general of interest and importance to potential investors since stale pricing strategies, as is widely observed, can lead to wealth transfers either to or from new investors vis a vis old investors.

In this paper we consider these strategic aspects from the point of view of the fund as well as the investing community by decomposing the overall fund returns into the private equity returns and the public listed equity returns. We find a number of significant competitive patterns of behavior amongst well-known fund families. There is identifiable heterogeneity in terms of the degree to which certain families revise their private equity prices and respond to pricing changes of other fund families. In fact, we document leader and follower characteristics as well as consistent "styles" concerning pricing revisions.

Importantly we find that private equity pricing revisions tend to be procyclical with respect to fund returns, especially using benchmark-adjusted returns. Our theoretical model justifies this as a rational response by the fund as it trades off gains due to convex fund flows against earning risk premia and increasing fund size directly by upward pricing revisions. This implies that concerns by existing fund investors about new fund investors profiting at their expense are mitigated by this pro-cyclical pricing strategy.

However, when we characterize fund behavior in terms of persistent stale pricing of private

equity we also find evidence in the form of fund flows that investors also exhibit strategic behavior; namely they realize that stale prices do not last forever and use other information, like pricing revisions by competitor funds to attempt to anticipate pricing revisions before they happen. Our paper, therefore, documents that the interesting effects implied by daily NAV pricing by mutual funds holding illiquid assets is something that both fund managers and fund investors realize and have attempted to rationalize through their own behavior. As interest in this form of private equity investing will certainly grow in the future, these issues will continue to be important.

Appendix

A Sample

Table A.I shows the number of mutual funds in our sample from each of the 38 fund families. The table also reports the primary reporting frequency for each family. There are a few families, such as Vanguard, that switch from quarterly to monthly reporting frequencies for at least part of the sample period. We group these families with the quarterly reporting families because they generally report monthly for only a short subperiod which results in a limited number of monthly return observations.

Table A.I: Mutual fund families.

This table reports the mutual fund families that have mutual funds in our sample. The table also reports the number of unique funds for each family.

Family	Number of funds	Primary reporting cycle
Fidelity	38	M
Alger	14	M
BlackRock	10	M
Hartford	9	M
Morgan Stanley	6	Q
Putnam	6	Q
AllianceBernstein	5	M
Janus Henderson	5	Q
T. Rowe Price	5	Q
Baron Capital	4	Q
Franklin Templeton	4	Q
John Hancock	4	M
VALIC	4	M
Wasatch	4	Q
Davis	3	Q
American Funds	2	Q
Eventide	2	Q
Federated	2	Q
Macquarie	2	Q
MassMutual	2	Q
Meridian	2	Q
Neuberger Berman	2	M
Vanguard	2	Q
AMG	1	M
Clipper	1	Q
Destinations	1	M
Great-West	1	Q
Highland	1	Q
Hotchkis and Wiley	1	M
Kontel	1	Q
Legg Mason	1	Q
Miller Value	1	Q
Prudential	1	M
Selected	1	Q
Thrivent	1	M
USA Mutuals	1	M
USAA	1	Q
Victory	1	Q

B Model Derivations

Solving for the growth rate in fund size based on the two point support of the listed pricing as well as the constant value for private equity pricing, and taking expectations, we find that equation (2) can be written as:

$$E(g) = \theta_L(f(x_t(1-u_L)+u_L)(1-x_t)+\theta_L u_L(1-x_t)+(1-\theta_L)f(x_t(1-d_L)+d_L)(1-x_t)+(1-\theta_L)d_L(1-x_t)+x_t.$$

Taking the derivative with respect to x_t and simplifying then yields the necessary condition for optimality:

$$\begin{aligned} 1 = {} & \theta_L(f(x_t(1-u_L)+u_L)+u_L)+(1-\theta_L)(f(x_t(1-d_L)+d_L)+d_L) \\ & -\theta_L(1-x_t)f'(x_t(1-u_L)+u_L)(1-u_L)-(1-\theta_L)(1-x_t)f'(x_t(1-d_L)+d_L)(1-d_L). \quad \text{(B1)} \end{aligned}$$

Analysis of the second order condition shows that a sufficient condition for existence of an unique interior optimum is that

$$\theta_L f'(x_t(1-u_L)+u_L)+(1-\theta_L)f'(x_t(1-d_L)+d_L) > \theta_L f''(x_t(1-u_L)+u_L)(1-u_L)+(1-\theta_L)f''(x_t(1-d_L)+d_L)(1-d_L).$$

It is clear that the second order condition will be satisfied if $u_L - d_L$ is sufficiently close to zero, that is when the time interval between t and $t+1$ is sufficiently small.

In terms of characterizing the optimum, consider (B1). Notice that as $x_t \to 0$, then the right hand side of equation (B1) converges to $\theta_L u_L + (1-\theta_L)d_L > 1$ as long as there is a positive risk premium associated with the listed securities. Hence this is never an optimum. As $x_t \to 1$, then the right hand side of equation (B1) converges to $f(1) - f'(1) + \theta_L u_L + (1-\theta_L)d_L$. A sufficient condition for this to be less than 1, is that the fund flow with a zero rate of return is sufficiently negative. In this case the optimum must be interior.

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Table I: Mutual funds with largest holdings of private firms.

This table reports the mutual funds that rank in the top 10 in holdings of private firms. We rank funds by the number of private firms held as well as the total weight of private firm holdings. For both the number and the total weight of private firm holdings, we examine the maximum achieved by a particular mutual fund during the sample period. The sample period is January 2010 to December 2017.

Number of private holdings		Weight of private holdings	
Fund	Holdings	Fund	Weight
Fidelity Growth Company	34	Morgan Stanley Global Discovery	11.55%
Fidelity Series Growth Company	32	Hartford Growth Opportunities HLS	9.53%
Fidelity Blue Chip Growth	30	John Hancock Mid Cap Stock	8.67%
Fidelity OTC	30	Hartford Growth Opportunities	8.54%
T. Rowe Price New Horizons	30	BlackRock Technology Opportunities	8.46%
Hartford Capital Appreciation	29	Morgan Stanley Mid Cap Growth	8.27%
Hartford Global Capital Appreciation	26	Morgan Stanley Small Company Growth	8.14%
Fidelity Series Blue Chip Growth	24	Davis Opportunity	8.08%
Hartford Growth Opportunities	24	Davis Global	7.15%
Fidelity Select Biotechnology	21	Morgan Stanley Multi Cap Growth	5.97%

Table II: Private firms that are most held by mutual funds.

This table reports the private firms that rank in the top 10 in holdings by mutual funds. We rank firms by the number of mutual funds with a holding in the private firm as well as the aggregate value of holdings of the private firm. For both the number of funds and the aggregate reported value held by mutual funds, we examine the maximum achieved by a particular private firm during the sample period. The sample period is January 2010 to December 2017.

Number of mutual funds		Aggregate value of holdings	
Firm	Funds	Firm	Value
Uber	39	Uber	$1,980M
Cloudera	30	Pinterest	$854M
Dropbox	30	WeWork	$617M
Honest Company	25	SpaceX	$529M
Nutanix	23	Didi Chuxing	$510M
Airbnb	22	First Data	$496M
Pure Storage	22	Groupon	$439M
Palantir	20	Dropbox	$402M
Facebook	19	Cloudera	$361M
Lyft	17	Acerta Pharma	$355M

Table III: Summary statistics

This table presents summary statistics for mutual funds and their portfolios (Panel A) and reported returns on private holdings (Panel B). The average number of private holdings and the average weight per private holding are calculated across fund-periods with at least one private holding. Reported private holding returns in Panel B are separated into groups based on the monthly or quarterly reporting frequency of the fund. The sample period is January 2010 to December 2017.

Panel A: Mutual fund summary statistics	
Number of mutual funds	152
Number of fund families	38
Average number of private holdings	6.25
Average weight per private holding	0.17%
Total fund weight in private holdings:	
Minimum	0.00%
10th percentile	0.05%
25th percentile	0.16%
Median	0.54%
75th percentile	1.40%
90th percentile	2.64%
Maximum	11.55%

Panel B: Private holding return summary statistics		
	One-month reporting horizon	Three-month reporting horizon
Average	1.55%	6.85%
Standard deviation	62.37%	208.74%
Private holding return:		
Minimum	-99.48%	-99.60%
10th percentile	-5.87%	-17.77%
25th percentile	0.00%	-0.14%
Median	0.00%	0.00%
75th percentile	0.00%	0.71%
90th percentile	7.80%	18.77%
Maximum	10510.56%	10499.54%

Table IV: Percentage of non-zero reported private firm returns by family.

This table reports the number of reported private firm returns and the percentage of non-zero returns by mutual fund family. The sample period is January 2010 to December 2017.

Family	Number of returns	Percent non-zero returns
Panel A: Monthly reporting frequency		
Prudential	282	96%
Hartford	6,957	63%
AllianceBernstein	118	56%
John Hancock	2,041	55%
BlackRock	1,971	52%
Fidelity	17,169	44%
VALIC	1,151	28%
Alger	2,830	25%
Thrivent	550	24%
AMG	26	4%
Destinations	2	0%
Hotchkis and Wiley	16	0%
Neuberger Berman	32	0%
USA Mutuals	44	0%
Panel B: Quarterly reporting frequency		
Highland	4	100%
Baron Capital	59	75%
Legg Mason	42	74%
Morgan Stanley	716	71%
Eventide	52	71%
USAA	89	71%
Macquarie	400	70%
Franklin Templeton	124	69%
MassMutual	336	68%
American Funds	246	65%
Federated	22	64%
Wasatch	393	51%
Vanguard	124	49%
Kontel	131	49%
Great-West	96	48%
Putnam	399	47%
Miller Value	57	46%
Selected	11	36%
T. Rowe Price	2,124	35%
Davis	52	33%
Janus Henderson	141	17%
Victory	7	14%
Clipper	8	13%
Meridian	10	10%

Table V: Leader and follower families in reported private firm returns of monthly reporting families.

This table reports results from regressions of reported private firm returns for a given mutual fund family on the contemporaneous and lagged returns reported by another family for the same private firm. The regressions are pooled across all return observations for common private firm holdings across the two fund families. The five fund families in the table are the families with monthly reporting frequency and the highest degree of overlap in private firm holdings with the other families. The sample period is January 2010 to December 2017.

	BlackRock	Fidelity	Hartford	John Hancock	VALIC
BlackRock contemporaneous		0.92	0.12	0.49	0.48
		(5.16)	(1.00)	(1.65)	(1.58)
BlackRock lagged		-0.16	-0.04	-0.01	0.02
		(-1.39)	(-0.73)	(-0.29)	(0.25)
Fidelity contemporaneous	0.50		0.33	0.65	0.63
	(3.00)		(3.61)	(4.05)	(3.72)
Fidelity lagged	0.35		-0.00	0.05	-0.01
	(2.47)		(-0.20)	(1.38)	(-0.27)
Hartford contemporaneous	0.06	0.51		0.32	0.72
	(0.91)	(2.24)		(3.41)	(7.99)
Hartford lagged	0.12	0.48		0.27	0.06
	(2.79)	(2.89)		(2.88)	(2.29)
John Hancock contemporaneous	0.34	0.89	0.51		0.97
	(2.03)	(6.28)	(3.18)		(18.24)
John Hancock lagged	0.19	0.19	-0.01		-0.02
	(1.43)	(1.27)	(-0.51)		(-0.60)
VALIC contemporaneous	0.74	0.95	0.85	0.21	
	(6.30)	(7.73)	(12.45)	(1.62)	
VALIC lagged	0.25	-0.07	0.13	0.09	
	(1.79)	(-2.36)	(2.52)	(1.82)	

Table VI: Leader and follower families in reported private firm returns.

This table reports results from regressions of reported private firm returns for a given mutual fund family on the lagged, contemporaneous, and lead average returns reported by other fund families on the same private firms. All returns are equally weighted across other fund families that report a return on the private firm during a month. The lagged (lead) average returns are calculated as the sum of the three average private firm returns over the past (next) three months. We report results for families with at least 25 valid observations. An * (**) [***] indicates that the coefficient is statistically significantly different from zero at the 10% (5%) [1%] significance level. The sample period is January 2010 to December 2017.

Family	Lagged Return	Contemp. Return	Lead Return	N	Adjusted R^2
Alger	0.09**	0.05	-0.01	98	0.04
AllianceBernstein	-0.01	-0.01	-0.30	29	-0.07
American Funds	-0.08	0.85***	-0.12	65	0.31
BlackRock	0.09***	0.23***	0.02	318	0.14
Fidelity	0.02	0.34***	0.04***	1,028	0.11
Franklin Templeton	0.13	1.08***	0.10	46	0.66
Great-West	0.04	1.38***	-0.14*	44	0.68
Hartford	0.03*	0.36***	0.10***	825	0.14
Janus Henderson	0.04	0.76***	-0.02	39	0.81
John Hancock	0.02*	0.35***	0.04***	924	0.22
Kontel	0.01	1.38***	0.08	57	0.73
Legg Mason	0.65**	0.80*	0.31	25	0.19
Macquarie	0.29***	0.91***	-0.01	188	0.60
MassMutual	0.18***	1.19***	0.05	154	0.59
Morgan Stanley	0.28***	0.62***	-0.00	240	0.38
Prudential	-0.03	0.01	-0.04	53	-0.03
Putnam	0.11	0.37*	0.23***	33	0.36
T. Rowe Price	0.26***	0.74***	-0.00	314	0.53
Thrivent	-0.01	0.35***	0.06**	169	0.25
USAA	0.37***	1.09***	0.01	85	0.54
VALIC	0.00	0.52***	-0.00	848	0.42
Vanguard	0.24***	1.14***	0.01	89	0.59
Wasatch	0.40***	0.08	0.05	35	0.18

Table VII: Contribution to monthly fund return from reported private firm returns.

This table reports the average contribution to the monthly mutual fund return from reported returns on private holdings. Fund periods are divided into those in which the fund's portfolio of non-private holdings outperforms or underperforms relative to the fund's benchmark. Frequent (Infrequent) reporting funds are defined as funds from families that report non-zero private firm returns in more (less) than half of reporting periods. Leaders and followers are defined as funds from families with significant lead (lag) relations in Table VI. We report the t-statistic from the test of whether the private firm return contribution is larger when the fund's non-private portfolio outperforms the benchmark compared with when it underperforms the benchmark. The sample period is January 2010 to December 2017.

Public portfolio outcome	All funds	Frequent	Infrequent	Leaders	Followers
Public return > benchmark	0.021%	0.012%	0.041%	0.024%	0.007%
Public return < benchmark	0.004%	0.003%	0.006%	0.008%	-0.008%
Test for difference = 0	(3.51)	(1.90)	(3.11)	(3.39)	(0.82)

Table VIII: Average flow given the degree of stale pricing.

This table presents average percentage flow for mutual funds with private holdings as a function of the average number of months of stale pricing for a fund's private holdings. The number of months of stale pricing for a private firm is the number of months since a fund reported a non-zero return. The fund-level measure is an average of months across the private holdings of a fund, and we use the lagged staleness measure such that investor flows can respond to the information in reported prices. The cutoffs for the non-zero stale pricing measure categories are based on the 25th, 50th, and 75th percentiles of the stale pricing measure across non-zero measures. The sample period is January 2010 to December 2017.

	Average months of stale pricing				
	None	(0.00,1.00)	[1.00,2.15)	[2.15,4.50)	>4.50
Average percentage flow	-0.628%	-0.504%	-0.497%	-0.538%	-0.265%
t-statistic for difference with None		(0.35)	(0.58)	(0.65)	(2.48)

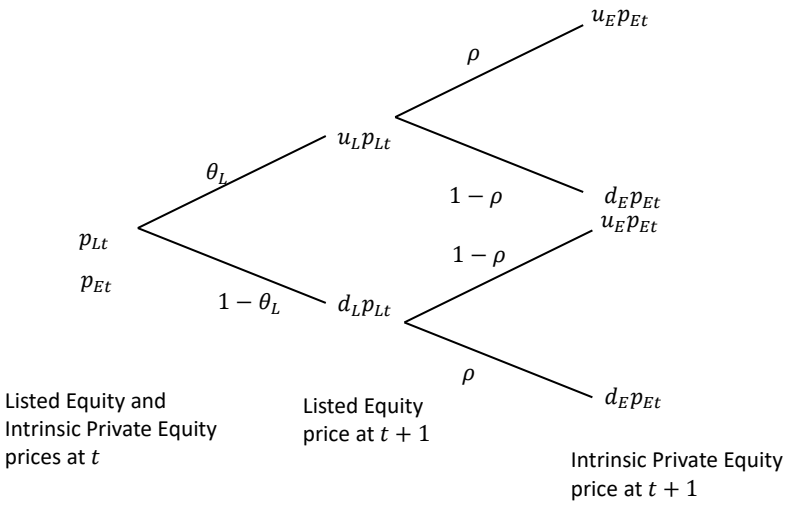

Figure 1: Pricing behavior of listed equity and private equity.

This figure illustrates the pricing processes for the listed equity starting from time t, which is p_{Lt} and private equity, p_{Et}. After a single period the listed equity price increases by either the gross return u_L, or decreases by d_L. The respective probabilities are θ_L and $1-\theta_L$. The conditional return on private equity is correlated with the return on listed equity as indicated using the parameter $\rho > 1$, and the gross rate of returns are u_E and d_E.

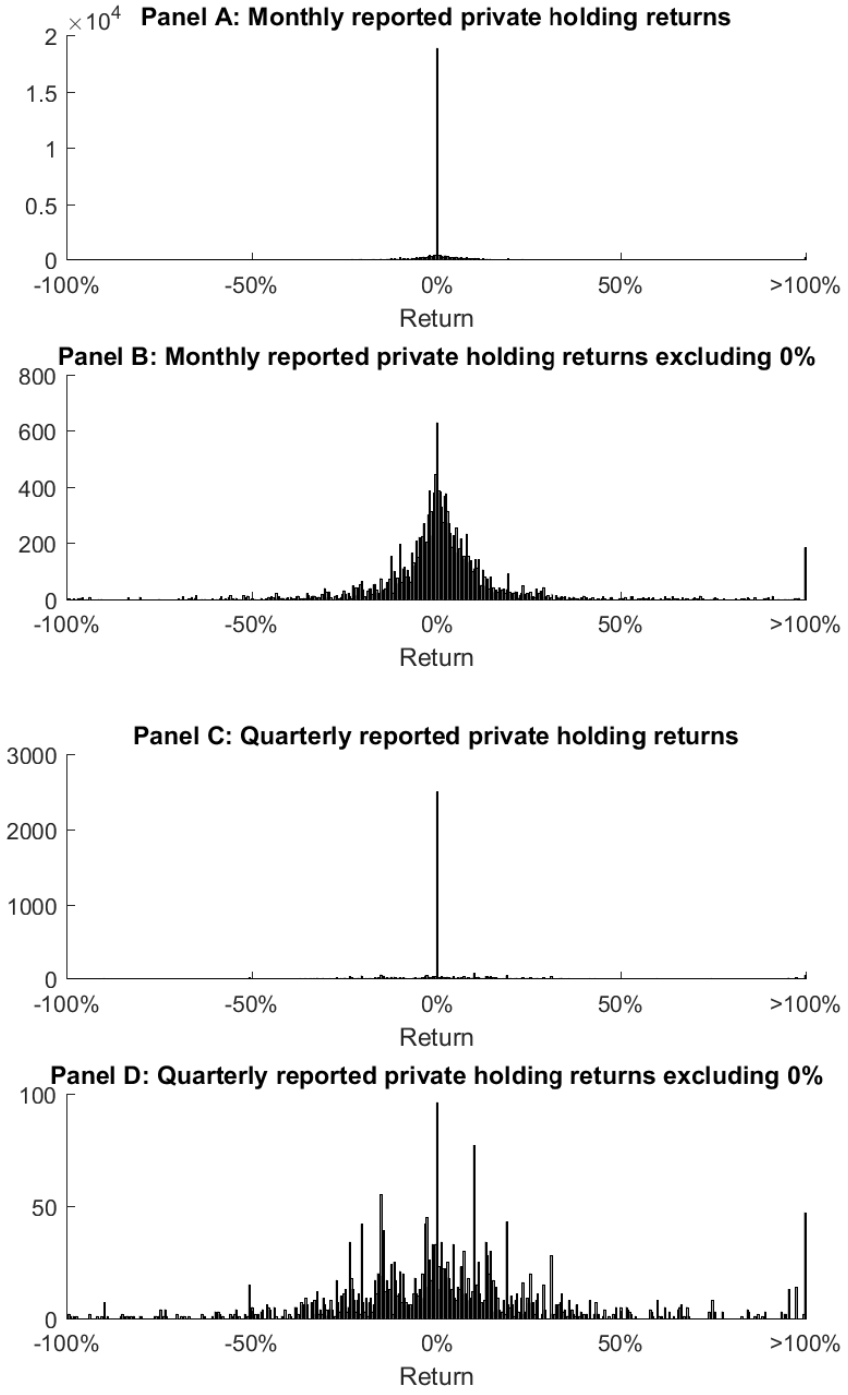

Figure 2: Distributions of reported returns for private holdings of mutual funds.

This figure shows histograms of private firm returns that are calculated based on the reported holdings of mutual funds. Panels A and B contain distributions of private firm returns for mutual funds with a monthly reporting frequency, and Panels C and D report corresponding information for funds with a quarterly reporting frequency. Panels B and D omit all returns of 0%. The distributions are censored at 100% and all reported returns that exceed 100% are combined into the far-right bar of each histogram. The sample period is January 2010 to December 2017.

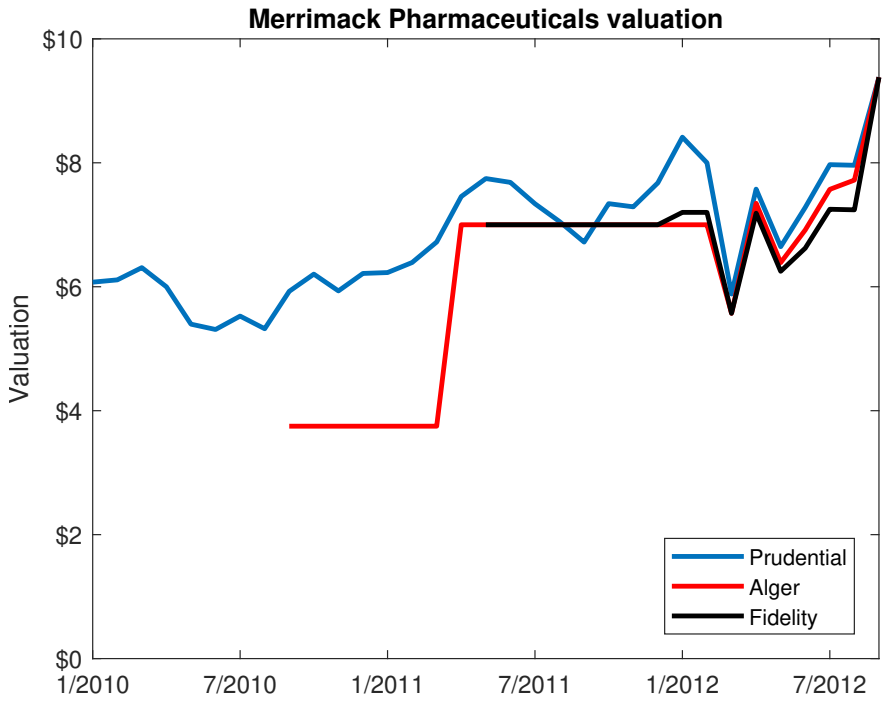

Figure 3: Valuation of Merrimack Pharmaceuticals by mutual fund families.

This figure shows the valuations of Merrimack Pharmaceuticals by Prudential, Alger, and Fidelity over the period January 2010 to September 2012. Merrimack Pharmaceuticals underwent an IPO on March 29, 2012 at a price of $7.00 and the lockup period expiration date was September 25, 2012.

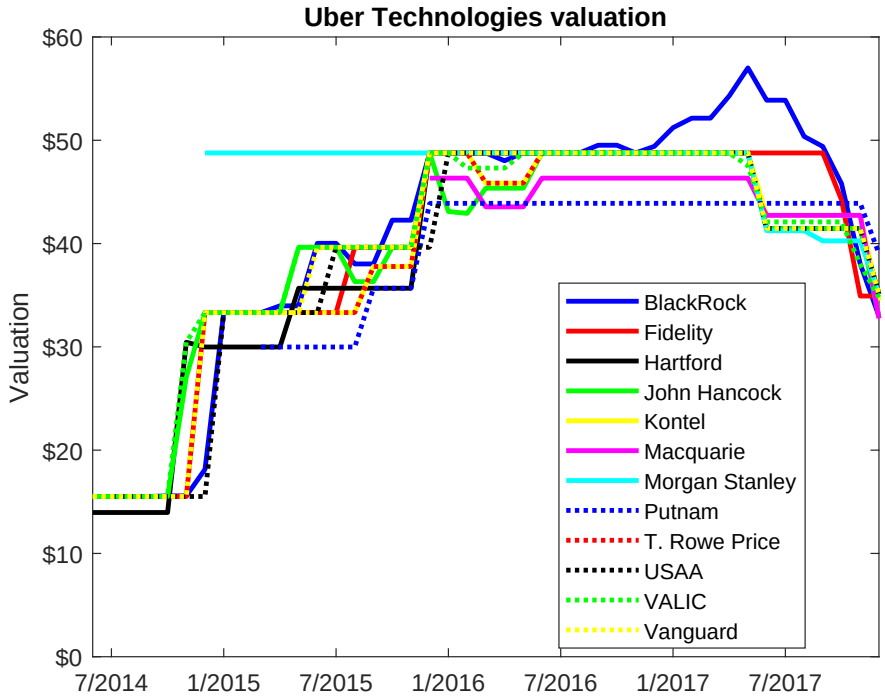

Figure 4: Valuation of Uber Technologies by mutual fund families.

This figure shows the valuations of Uber Technologies by 12 mutual fund families over the period June 2014 to December 2017.

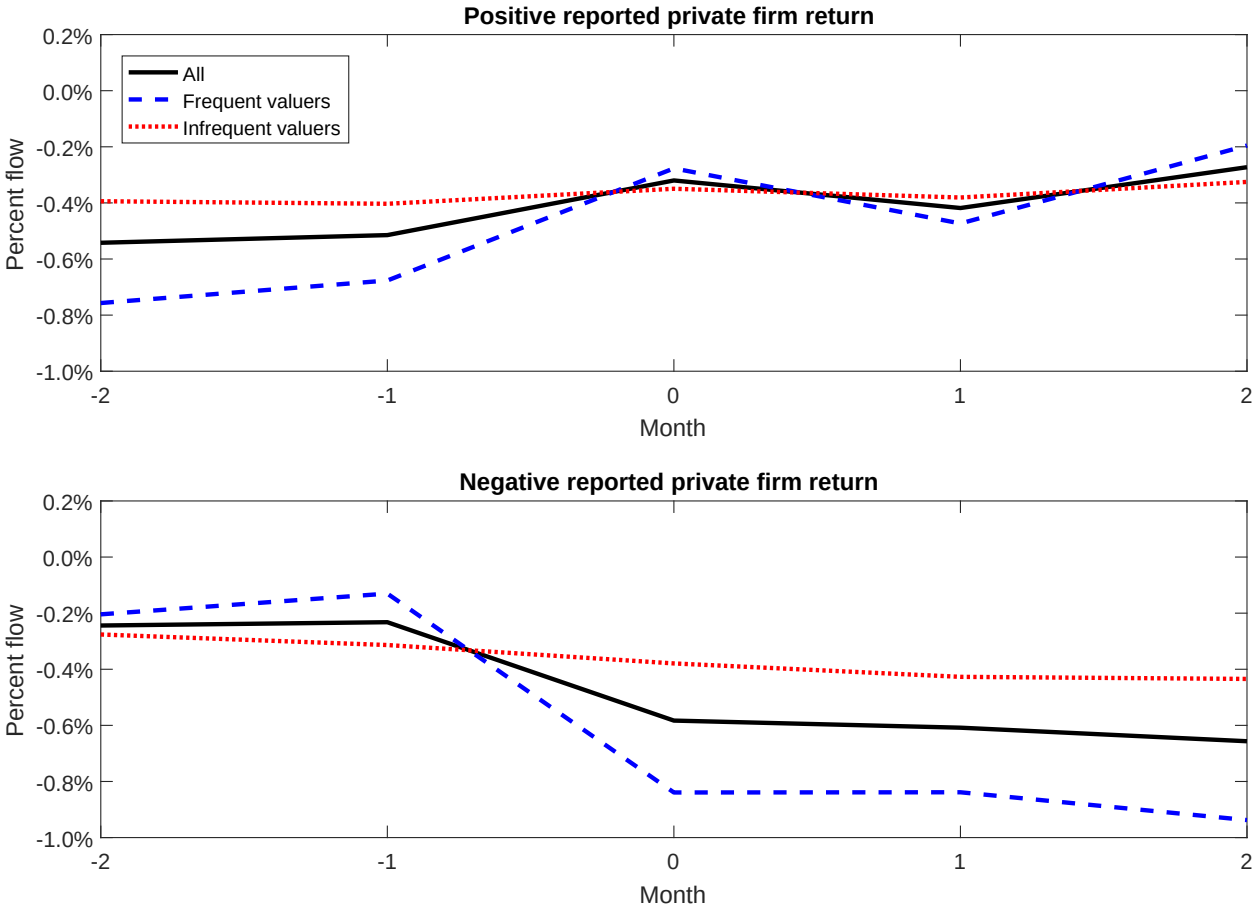

Figure 5: Net flows around reported private firm returns by reporting frequency.

This figure shows the average net fund flows to mutual funds in the months surrounding reported private firm returns. The top and bottom panels show flows for positive and negative private firm returns reported in month 0, respectively. We show average flow for two months before and after the reported return. The black line averages across all mutual funds in the sample, the blue line averages across all funds that report nonzero returns for private firms in more than 50% of reporting periods, and the red line averages across all funds that report nonzero returns in less than 50% of periods. The sample period is January 2010 to December 2017.

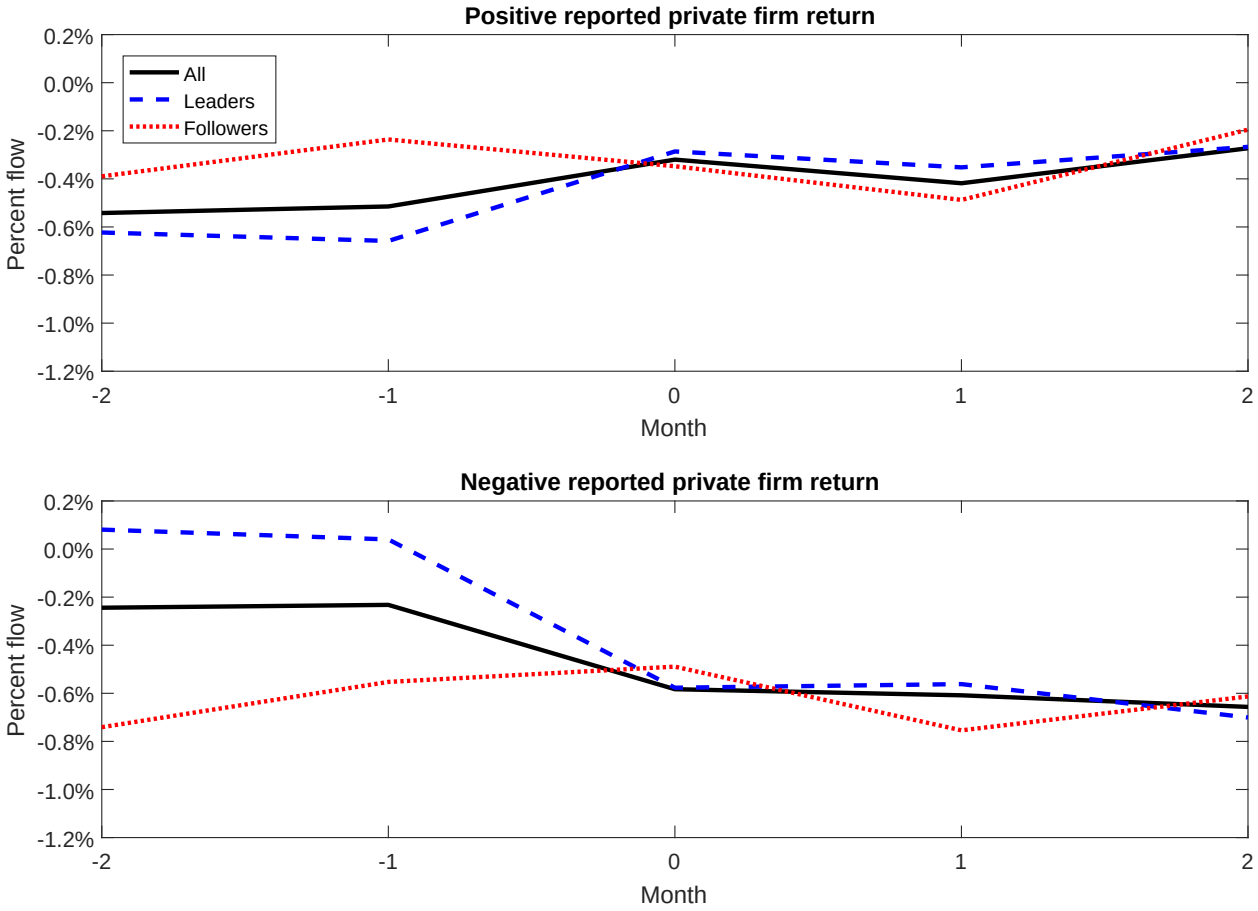

Figure 6: Net flows around reported private firm returns by leader-follower status.

This figure shows the average net fund flows to mutual funds in the months surrounding reported private firm returns. The top and bottom panels show flows for positive and negative private firm returns reported in month 0, respectively. We show average flow for two months before and after the reported return. The black line averages across all mutual funds in the sample, the blue line averages across all funds that are classified as leaders based on Table VI, and the red line average across all funds that are classified as followers. The sample period is January 2010 to December 2017.