

A THEORY OF DEBT MARKET ILLIQUIDITY AND LEVERAGE CYCLICALITY*

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Abstract

We analyze determinants of secondary debt market liquidity, identifying conditions under which a large investor can profitably buy stakes from small bondholders and offer unilateral debt relief to a distressed firm. We show endogenous trading by small bondholders may result in multiple equilibria. Some equilibria entail vanishing liquidity and sharp increases in yields absent changing fundamentals. In turn, anticipation of illiquid equilibria induces firms to eschew public debt financing, since such equilibria create higher bankruptcy costs and debt illiquidity discounts. The model thus offers a rational micro-foundation for stylized facts commonly attributed to investor "sentiment" and CFO "market-timing." Finally, we show vulnerability of debt markets to multiple equilibria is highest during downturns, when small bondholders face severe adverse selection.

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Debt markets are prone to sudden bouts of illiquidity. For example, a recent study of investment grade corporate debt by Bao, Pan and Wang (2010) reports that illiquidity increased substantially when Ford and GM were downgraded to junk status around May of 2005 and again even more dramatically subsequent to the collapse of Bear Stearns and Lehman. This spike in illiquidity coincided with sharp increases in bond yields and a significant drop on new debt flotations. While ensuring the liquidity of debt markets remains a priority, effective policymaking has been hindered by limited understanding of the determinants of debt market liquidity. For example, a recent IMF-World Bank-OECD conference concluded: “A key policy problem is that liquidity is not very well-understood in terms of a robust link between theory (analytics) and data.”

The behavioral narrative surrounding such episodes is that liquidity and corporate financing respond to purely exogenous shifts in investor sentiment, with CFOs opportunistically responding by issuing public debt when the market tastes favor it (see e.g. Baker (2010)). In support of this view, behavioralists point to the fact that there are often periods when liquidity, yields and financing move suddenly without any significant change in the parameters underpinning standard rational models, e.g. tax rates and bankruptcy costs.

This paper proposes an alternative, rational, explanation for sudden illiquidity episodes and leverage cycles: *Secondary debt markets have multiple liquidity equilibria, inducing multiple equilibria in primary markets.* A secondary debt market with multiple equilibria superficially resembles a market with fads and bubbles in that observables, such as yields, trading volumes, and financing can change suddenly absent any change in fundamentals. However, multiple equilibria rest upon a rational foundation. In fact, rational trading by small uninformed bondholders, traditionally modeled as pure noise-traders, is shown to be necessary for generating multiple equilibria in our model.

The broad details of the model are as follows. There are two financial market imperfections: debt tax shields and costs of formal bankruptcy. The firm has three financing options: equity, public debt, or a private loan from a large investor. Borrowing from a single lender is attractive since

bilateral renegotiation is ex post efficient, implying formal bankruptcy costs are never incurred with a private loan. However, single-lender debt is priced at a discount to compensate the large investor for his high opportunity cost of funds (stemming from profitable outside options or intermediation costs). To avoid such discounts, the firm may instead borrow from dispersed investors in public debt markets. The focus of our model is on the primary and secondary markets for public debt.

With public debt, costs of formal bankruptcy can still be avoided if creditors grant sufficient voluntary debt relief at the onset of distress. Since small (measure zero) bondholders perceive themselves as non-pivotal, they never grant debt relief. Therefore, in order to avoid formal bankruptcy, the large investor must acquire large stakes via secondary market trading.

The model of the secondary debt market is in the spirit of Kyle (1985), making three critical departures. First, we analyze a concave debt claim. Second, there is a feedback effect from ownership structure to fundamental value since ownership structure influences debt relief. Finally, we depart from the pure noise-trader assumption of Kyle and analyze the incentives of small bondholders who rationally trade-off liquidity preference against adverse selection costs in deciding whether to sell their debt. This last feature of the model is essential for generating multiple equilibria.

A critical element of the model is that small bondholders are exposed to a novel form of adverse selection making them reluctant to sell. This is because the large investor has private information about his own trading strategy, allowing him to confound market makers and causing price to fall below fundamental value at times. Small bondholders know they face underpricing if they sell at the same time the large investor is acquiring large stakes. Intuitively, selling in such states causes them to forego the windfall gain accruing when the large investor's unilateral debt relief renders all remaining debt riskless. In this way, the model can explain how the debt market froze at the same time lenders were facing intense pressure to raise funds via asset sales.¹

Multiple equilibria naturally arise from the fact that adverse selection, and selling by small bondholders, is a non-monotone function of the large investor's buying intensity. Intuitively, if a

¹Of course, another credible element of the puzzle outside the scope of the model is that individual lenders had negative private information regarding their own loan book.

large investor grants debt relief with probability zero, there is no chance of debt being underpriced in the secondary market. Conversely, if a large investor grants debt relief with probability one, there is also no potential for underpricing since market makers will then correctly price the debt to reflect the certainty of debt relief.

In the model's high-liquidity equilibrium, high-volume selling by small bondholders induces the large investor to buy/restructure with high probability. In turn, the high probability of debt relief alleviates adverse selection perceived by small bondholders, since market makers capitalize the near certainty of debt relief into secondary market prices. In turn, this rationalizes the high-volume selling by the small bondholders. Conversely, there is also a socially inefficient equilibrium in which low anticipated selling by small bondholders results in low buying intensity by the large investor. In turn, reductions in the probability of the large investor buying can actually exacerbate the adverse selection problem perceived by small investors, deterring them from selling. Thus, conjectured low-liquidity equilibria become self-fulfilling prophecies.

The second contribution of the paper relates to corporate finance in that we propose a novel illiquidity-augmented trade-off theory based upon trading in secondary debt markets. In contrast to standard trade-off theoretic models, the possibility of endogenous debt relief is factored in. Further, public debt prices contain an endogenous discount for illiquidity. Such discounts reduce total firm value, discouraging the use of public debt financing. *The multiplicity of equilibria in secondary debt markets allows us to explain sharp swings in corporate financing policies absent any change in economic fundamentals.* Specifically, anticipation of the low-liquidity equilibria can lead to a jump in yields and a drastic shift away from public debt finance. Thus, the model offers a liquidity-based explanation for leverage cycles, one in which big fluctuations in aggregate leverage require nothing more than sunspots.

The model provides three other important insights stemming from consideration of the trading incentives of small bondholders. First, even when there is a unique equilibrium, this equilibrium may entail lower liquidity, and lower probabilities of debt relief, than what would be inferred from

a standard noise-trading model which fails to account for small bondholders' willingness to sell. Second, the model highlights a cost associated with policymakers' recent attempts to prop up lenders indiscriminately. Concentrated debt ownership, and voluntary debt relief, is hindered whenever subsidies and lax monetary policy alleviate small bondholders' pressure to sell. Finally, the model shows that debt markets are especially prone to freezes during downturns, since this is the time when small bondholders face the most severe adverse selection.

We turn now to related literature. Morris and Shin (2004) present a model in the spirit of Diamond and Dybvig (1983) with multiple equilibria for default risk and debt prices. Their model is predicated upon imperfect knowledge of fundamentals giving rise to coordination problems across dispersed lenders deciding whether to roll-over debt. The most important difference is that they assume a dispersed debt ownership structure throughout, and do not allow for debt trading. Thus, their model is silent on the question of debt market liquidity, and the source of multiple equilibria differs fundamentally.

Our model is in the spirit of Dow (2004) in showing that endogenous trading by small investors produces multiple equilibria. Our argument rests upon a feedback effect from ownership structure to fundamental value. Further, in our model the supply of liquidity (uninformed selling) is non-monotone in the buying intensity of the large investor, leading to multiple equilibria. In contrast, in Dow's model there is no feedback effect, with increased trading by informed investors always deterring uninformed trade. The source of multiple equilibria in his model is the fact that higher uninformed trade crowds in other uninformed trade by virtue of narrowing bid-ask spreads. This type of strategic complementarity across uninformed traders also underpins the models of Admati and Pfleiderer (1988) and Pagano (1989).

Our paper is also closely related to recent work by Ericsson and Renault (2006) and Duffie, Gârleanu, and Pedersen (2007), both of which analyze debt illiquidity discounts. These papers rationalize debt market illiquidity as arising from imperfect competition, with the latter also incorporating search costs. Our analysis is complementary, since we deliberately abstract from imperfect

competition. Significantly, we show that debt markets can be prone to sudden bouts of illiquidity even if there is perfect competition amongst market makers. Thus, the model alerts policymakers to the fact that increased competition is not a cure-all. Consistent with our model and their own, Ericsson and Renault document empirically a positive correlation between the illiquidity and default components of yield spreads.

Our model shares with that of Dang, Gorton and Holmström (2009) the prediction that increased information sensitivity of debt during recessions leads to illiquidity. However, our model is predicated upon a feedback from ownership structure to fundamental debt value. In their model, no such feedback exists. Another novel feature of our model is that liquidity is fragile in that there are multiple equilibria with varying trade volumes, with conjectured illiquidity being self-fulfilling.

Free-riding in financial markets was first analyzed by Grossman and Hart (1980) in the context of hostile takeovers. Closer to our model is that developed by Shleifer and Vishny (1986), who analyze the interplay between free-ridership and the endogenous ownership structure of equity. Our model is also similar to those of Maug (1998) and Mello and Repullo (2004), who develop pure noise-trader models to analyze takeovers. Aside from the fact that we analyze debt, the key difference is that we allow for endogenous trading by small investors, which gives rise to multiple equilibria.

The remainder of the paper is structured as follows. Section 1 describes the setting. Section 2 analyzes trading in the secondary market. Section 3 shows the possibility for multiple equilibria in the secondary market, while Section 4 discusses implications for the primary market. Section 5 presents implications for corporate financing decisions. We conclude with implications for policymaking.

1. The Economic Setting

There are two dates, t_1 and t_2 , with Figure 1 providing a summary of the timing of events. There are two types of investors: a single large investor and a continuum of ex ante identical small investors N with generic member n having measure zero. All investors are risk-neutral and have access to a riskless government bond with interest rate normalized at zero. The large investor is skilled, being able to invest his own wealth in a scalable investment generating a positive rate of

return from t_1 until t_2 . Therefore, he values competing investments using a positive discount rate, valuing each unit of expected t_2 payoff at $\delta \in (0, 1)$. One can think of the large investor as being a hedge fund with scarce capital. Alternatively, one can think of the large investor as a large bank, with δ capturing intermediation costs. Investors cannot borrow or short sell.

At the start of t_1 , the firm's financial structure is chosen. The firm enters the model unlevered and holding no assets. It has exclusive access to a positive NPV project requiring an up-front investment. The firm is initially owned by an entrepreneur who has enough wealth to fund the project.² The entrepreneur is risk-neutral and discounts at rate zero, just like the small investors. He chooses financing in his self-interest, having the option to use his own funds, i.e. equity or debt. Debt financing terms depend upon a publicly observed macroeconomic state. This macroeconomic state is denoted $\omega \in \{b, g\}$ and remains constant across t_1 and t_2 .

There are two mutually exclusive modes of debt financing. The firm can take a loan from a large investor or can raise the funds by selling public debt in the primary market at time t_1 . Public debt has a face value F due at the end of t_2 while any private loan has face value F due at the end of t_2 . The loan is not registered with the SEC, and cannot be traded. In contrast, public debt can be traded in a secondary market at the start of t_2 , just prior to its maturity date.

Our modeling of the primary and secondary markets for public debt follows the approach of Shleifer and Vishny (1986) and Maug (1998) who analyze equity. The large investor initially buys a commonly observed fraction $s \in [0, 1)$ of the public debt in the primary market at time t_1 . For simplicity, one may think of this fraction as being commonly observed at the time investors trade in this market. Alternatively, one may think of the large investor as trading privately at time t_1 but knowing that his toehold will become known prior to t_2 , so that he chooses his ownership stake optimally in a rational expectations equilibrium. His optimal toehold would be the same.

The public debt is perfectly divisible with quantity normalized at one unit. The primary market price of the public debt is set so that the small investors are just willing to buy the remaining fraction

²Results do not change if external equity is also allowed. An interesting feature of the model is that the equity market would be perfectly liquid since equity value is invariant to the large investor's trades.

$1 - s$. Anticipating, the debt trades at a discount to compensate small investors for exposure to adverse selection and/or illiquidity. Holmström and Tirole (1993) and Maug (1998) present pure noise-trading models of the equity market in which there is also an adverse selection discount. However, illiquidity discounts are necessarily absent from their analyses since agents are essentially forced to trade in such models.

Following Shleifer and Vishny (1986), we assume the large investor has the ability to increase his fractional ownership of public debt via secretive purchases in the secondary market at the start of t_2 . In order to do so profitably, he tries to hide his buy-order behind sales made by the small bondholders who potentially face shocks biasing them toward selling in the secondary market.³

At the start of t_2 a fraction $\gamma \leq 1/2$ of the small investors in N (randomly selected) become vulnerable to a common shock. Vulnerable small investors experience the shock with probability one-half. In this case they bear a carrying cost equal to c times the terminal payoff on the debt if they hold until the maturity date at the end of t_2 . For brevity, we label the small investors who are hit with the shock *impatient investors*. Importantly, and in contrast to a pure noise-trading model, the impatient investors have the discretion to sell in the secondary market rather than bear the holding cost. This setup subsumes pure noise-trading as a special case if one sets $c = 1$. If $c < 1$, impatient bondholders will only sell if perceived underpricing in the secondary market relative to fundamental value is not too severe. In this way, the model makes selling by impatient investors rational. Finally, if the potentially vulnerable investors do not experience a liquidity preference shock, they do not sell their debt holdings.⁴

Duffie, Gârleanu, and Pedersen (2007) also introduce a carrying cost into their search-based model of illiquidity. There are a number of economic rationales that can underpin a cost to holding debt until maturity. The most common motivation for such a cost is that the investor has a pressing

³There are several reasons why we do not consider subsequent sales by the large investor. Most importantly, it is unlikely that an investor who has established a large stake in the primary market would be able to sell anonymously in the secondary market.

⁴ Holding is readily endogenized by assuming a symmetric cost to selling in this state. Such costs could stem from leverage-induced risk-shifting motives, as in Diamond and Rajan (2009).

desire for immediate cash, as in Diamond and Dybvig (1983). In reality, the preference to sell debt immediately can also stem from regulators imposing risk-based capital requirements. This represents a cost for poorly capitalized financial institutions when they fail to reduce risk exposure with the cost being proportional to assets in our specification. To see the connection with risk-capital, note that secondary market trading occurs before the revelation of the project's success or failure (Figure 1). Consequently, the maximum loss and payoff dispersion is higher if the debt is held until maturity rather than sold. Consequently, an investor subject to costly capital requirements may prefer to sell and offload risk. The risk-enhancing effect of holding onto an asset is also a fundamental mechanism in the model of Diamond and Rajan (2009).

Given that they have no private information whatsoever, there is no incentive for the invulnerable small investors to submit any orders to the secondary market. These investors hold an aggregate inventory equal to $(1-\gamma)(1-s)$, which is sufficient to meet any aggregate demand on the equilibrium path. Thus, in our model the market makers are simply the invulnerable small investors, who are willing and able to meet any aggregate demand.

We solve for the perfect Bayesian equilibrium (PBE) of the market making game at time t_2 . The game starts with the vulnerable small investors observing whether they are hit with the shock. Given their respective information sets, investors then simultaneously submit market orders to the continuum of market makers (MM). The aggregate orders of the large investor and vulnerable small investors are denoted x^I and x^N , respectively. Following Kyle (1985), MM only observe the aggregate demand $X \equiv x^I + x^N$. After observing aggregate demand, the market makers engage in a competitive auction à la Bertrand to supply X .

After the secondary market clears, success or failure of the project is observed. If the project is successful it pays H . Cash flow in the event of project failure depends on the macroeconomic state. If the state is good (g), cash flow in the event of failure is $M < H$. If the state is bad (b), cash flow in the event of failure is $L < M$. Intuitively, one can think of it being optimal to sell some assets if the project fails, with asset values being lower during recessions.

To fix ideas, think of the public debt as being risky, with

$$0 < L < M < F < H.$$

If the project fails, each lender has the option of granting unilateral debt relief. If the firm cannot make the required debt payment, even after debt relief is granted, then it enters a costly formal bankruptcy process which wastes a fraction $\alpha \in (0, 1]$ of cash flow. Lenders then have priority in claiming the remaining cash flow, leaving shareholders with zero.

2. Secondary Market Trading

The only difference between the bad and good macroeconomic states is that they imply different cash flows if the project fails. Conveniently, one can describe equilibrium for either state in terms of cash flow in the event of project failure. To this end, let y_ω denote project cash flow in the event of failure, with $y_b \equiv L$ and $y_g \equiv M$. With this notation in hand we characterize the set of equilibria via backward induction.

2.1. Voluntary debt relief

Recall the large investor buys a primary market debt toehold $s \in [0, 1)$ and has the opportunity to trade up in secondary markets, so that his final stake is $S \geq s$. With this in mind, suppose the project has failed, implying debt relief is necessary to avoid formal bankruptcy costs. A large investor holding a stake S after trading in secondary markets is willing to grant debt relief iff

$$y_\omega - (1 - S)F \geq S(1 - \alpha)y_\omega. \quad (1)$$

The following Lemma is a useful summary of the implications of the inequality above.

Lemma 1. *The large investor is willing to grant debt relief iff he emerges from secondary market trading holding a stake at least as large as*

$$\underline{S}(y_\omega, \alpha, F) \equiv \frac{F - y_\omega}{F - y_\omega + \alpha y_\omega}. \quad (2)$$

It is readily verified that the minimum stake \underline{S} is increasing in F and decreasing in both y_ω and α , with

$$\begin{aligned}\lim_{\alpha \downarrow 0} \underline{S}(y_\omega, \alpha, F) &= 1 \\ \lim_{\alpha \uparrow 1} \underline{S}(y_\omega, \alpha, F) &= \frac{F - y_\omega}{F}.\end{aligned}$$

Lemma 1 tells us that the large investor must have a higher debt stake if he is to grant debt relief during a recession since $\underline{S}(L, \alpha, F) > \underline{S}(M, \alpha, F)$. Intuitively, during a recession the large investor must write-down the face value of his own debt by a larger amount if the firm is to avoid costly formal bankruptcy, making the free-riding problem more severe.

It is also apparent that the prospect of incurring high default costs encourages debt relief by the large investor. This incentive channel has the potential to mitigate deadweight bankruptcy costs, especially for firms facing the prospect of very costly formal bankruptcy proceedings.

2.2. Debt pricing

We are primarily interested in PBE such that debt relief occurs with positive probability. It is readily verified that debt relief cannot occur with probability one in equilibrium for if it did the primary and secondary market prices for debt would be F and the large investor would make a sure loss due to the fact that he discounts and takes unilateral write-downs. Further, since his ultimate debt stake S weakly exceeds his primary market toehold s , it must be the case that his optimal toehold (s^*) is strictly less than \underline{S} . Otherwise debt relief would occur with probability one and the large investor would make a sure loss. This leads to the following lemma.

Lemma 2. *In any equilibrium in which debt relief occurs with positive probability, the large investor buys a toehold $s^* \in [0, \underline{S})$ and plays a mixed strategy in the secondary market, placing a buy order with probability $\sigma^* \in (0, 1)$.*

Equilibrium consists of a vector $(s^*, x^*, \sigma^*, \gamma^*)$. The first element denotes the optimal toehold. The second element denotes the size of the buy order placed by the large investor in the secondary market. The third element denotes his probability of placing a buy order. The last element denotes

the measure of impatient investors selling their debt. If impatient bondholders strictly prefer to sell then $\gamma^* = \gamma$. If they are indifferent between selling and holding, then it is possible to support an equilibrium in which only a proper subset of them sells, with $\gamma^* \in (0, \gamma)$.⁵

Recall small investors buy a stake $1 - s$ in the primary market, implying that in equilibrium aggregate selling by the impatient bondholders is given by $(1 - s)\gamma^*$. With this in mind, consider the optimal size of the large investor's buy order. His objective is to hide behind the sell orders of the small investors, so he chooses $x^* = (1 - s)\gamma^*$. This is the only possible buy order size that can create confusion for the market makers.⁶ To see this, turn to Table 1, which depicts equilibrium outcomes of the trading game.

As shown in Table 1, only three aggregate demands occur on the equilibrium path, with

$$X \in \{-(1 - s^*)\gamma^*, 0, (1 - s^*)\gamma^*\}. \quad (3)$$

When there is positive net order flow, the equilibrium is fully revealing and the *MM* know the large investor is buying. The market makers then set the secondary market price (denoted P) equal to F . At the opposite extreme, negative net order flow reveals that the large investor is not buying. In this case the secondary market price is

$$P_{\omega}^{-} \equiv \frac{F + (1 - \alpha)y_{\omega}}{2}. \quad (4)$$

Zero net order flow is non-revealing, forcing the market makers to set a pooling price.⁷ Using Bayesian updating, the *MM* respond to zero net order flow by setting the secondary market price to

$$P_{\omega}^0 \equiv \sigma^* F + (1 - \sigma^*)P_{\omega}^{-}. \quad (5)$$

Consider Table 1 from the perspective of the large investor. He makes a trading loss in the top row, since he buys debt for F despite valuing it at less than face value due to his subsequent

⁵Technically, such equilibria are supported with each impatient atom drawing a random variable according to which they sell with probability γ^* . We assume the strong law of large numbers applies.

⁶The appendix discusses market maker beliefs off the equilibrium path.

⁷If market makers could observe the entire order flow table instead of just net orders, the large investor could still mask his trades by placing offsetting buy and sell orders.

granting of debt relief. However, he makes a trading gain in the second row when he buys at the pooling price. Next, consider Table 1 from the perspective of a single impatient investor, who acts as a price-taker. He knows that debt will be priced at fundamental value if the large investor does not buy (bottom row). However, he faces underpricing if the large investor is buying (second row) since the fundamental value of debt to an atomistic debtholder is then equal to F .

The primary market price of debt is denoted p . Recall, the primary market price ensures small investors are just willing to buy. Since in equilibrium impatient investors weakly prefer to sell their debt in the secondary market, the debt can be priced from the perspective of a small investor who knows he will sell if hit with the shock at the start of t_2 . Ex ante, the probability of a small investor being hit with the shock is $\gamma/2$. Conditional upon being hit with the shock, the small investor's expected trading loss is $\sigma(F - P_\omega^0)$, capturing the second row in Table 1. Thus, the primary market price of debt is:

$$\begin{aligned} p_\omega &= \sigma_\omega^* F + (1 - \sigma_\omega^*) P_\omega^- - \frac{\gamma}{2} \sigma_\omega^* (F - P_\omega^0) \\ &= P_\omega^- + \left[1 - \frac{\gamma(1 - \sigma_\omega^*)}{2} \right] [F - P_\omega^-] \sigma_\omega^*. \end{aligned} \tag{6}$$

2.3. The optimal toehold

Consider now the optimal toehold for the large investor. Suppose first the equilibrium is such that debt relief occurs with probability zero. In this case, the primary market price is $p_\omega = P_\omega^-$. However, the large investor would only value each unit of debt at δP_ω^- , implying his optimal toehold is zero. Next suppose the equilibrium is such that debt relief occurs with probability $\sigma_\omega^* \in (0, 1)$, with Lemma 2 having ruled out $\sigma_\omega^* = 1$. Since the large investor plays a mixed strategy, his t_2 continuation value from buying must equal that from not buying, with the latter being equal to sP_ω^- . Thus, his ex ante payoff is:

$$\text{Payoff}(t_1) = (\delta P_\omega^- - p_\omega) s. \tag{7}$$

But we know from equation (6) that $\sigma_\omega^* > 0$ implies the primary market price is strictly greater than

P_{ω}^- . It follows that the optimal public toehold for the large investor is zero. We state this result as Proposition 1.

Proposition 1. *The large investor finds it strictly optimal to avoid buying debt in the primary market ($s^* = 0$). His masking buy order in the secondary market is equal to aggregate selling by impatient bondholders ($x^* = \gamma^*$).*

The intuition for Proposition 1 is straightforward. Buying debt in the primary market is costly for the large investor. First, by virtue of buying debt in the primary market, the large investor reduces the volume of uninformed trading in the secondary market. Second, given that the large investor has an outside technology generating a positive rate of return from t_1 to t_2 , he dislikes spending wealth in the primary market. Finally, buying debt in the primary market would bias the large investor towards trading more aggressively at time t_2 , thus subsequently granting more relief and thereby increasing the amount he transfers to the small investors. Shleifer and Vishny (1986) derive a similar result in relation to public toeholds in equity markets.

Proposition 1 implies that the large investor obtains a final stake $S = \gamma^* \leq \gamma$ when he buys debt in the secondary market. Thus, a *necessary condition* for debt relief is:

$$\gamma \geq \underline{S}(y_{\omega}, \alpha, F). \quad (8)$$

Intuitively, the large investor relies upon broad liquidity shocks to mask his buy orders. If the liquidity shocks are too narrow, the large investor cannot acquire a stake sufficiently large such that he will be willing to grant debt relief. This has important implications for policymaking. In particular, well-meaning attempts to prop up investors via fiscal and/or monetary policy can actually serve to hinder voluntary debt relief if such policies discourage selling by small bondholders.

3. Equilibrium

Equilibrium consists of the vector $(s^*, x^*, \sigma^*, \gamma^*)$. Proposition 1 showed that the optimal toehold $s^* = 0$ implying the masking buy order for the large investor entails $x^* = \gamma^*$. Therefore, the

remainder of the paper turns to determining possible equilibrium values for the large investor's buying probability (σ^*) and selling volume by impatient bondholders (γ^*). Essentially, we are looking for Nash equilibrium values for this pair in the market making game taking place at time t_2 .

We first conjecture an equilibrium in which debt relief occurs with positive probability. To this end, let $G(\sigma, \hat{\gamma}, y_\omega, \alpha, F)$ denote the expected trading gain perceived by the large investor in the event that with probability σ he places a buy order of size $\hat{\gamma}$, where $\hat{\gamma}$ is the conjectured selling volume by impatient bondholders. The gain is equal to his expected payoff net of the expected price paid to acquire the stake $\hat{\gamma}$:

$$\begin{aligned} G(\sigma, \hat{\gamma}, y_\omega, \alpha, F) &\equiv \frac{1}{2}\hat{\gamma}F + \frac{1}{2}[y_\omega - (1 - \hat{\gamma})F] - \frac{\hat{\gamma}}{2}(F + P_\omega^0) \\ &= \frac{1}{2} \left[\frac{\hat{\gamma}(1 - \sigma)(F - y_\omega + \alpha y_\omega)}{2} - (F - y_\omega) \right]. \end{aligned} \quad (9)$$

Differentiating G one finds

$$\begin{aligned} G_\sigma &= -\frac{\hat{\gamma}(F - y_\omega + \alpha y_\omega)}{4} < 0 \\ G_{\hat{\gamma}} &= \frac{(1 - \sigma)(F - y_\omega + \alpha y_\omega)}{4} > 0 \\ G_y &= \frac{1}{2} \left[1 - \frac{\hat{\gamma}(1 - \sigma)(1 - \alpha)}{2} \right] > 0 \\ G_\alpha &= \frac{\hat{\gamma}(1 - \sigma)y_\omega}{4} > 0 \\ G_F &= -\frac{1}{2} \left[1 - \frac{\hat{\gamma}(1 - \sigma)}{2} \right] < 0. \end{aligned} \quad (10)$$

The intuition for each comparative static in (10) is as follows. The large investor's gain to buying is decreasing in σ since P_ω^0 is increasing in σ . The gain is increasing in γ since more selling by small investors serves to reduce subsequent free-riding costs. The gain is increasing in cash flow since higher cash flow reduces the size of the required debt write-down. The gain to buying is increasing in α since P_ω^0 is decreasing in α . Finally, the gain to buying is decreasing in F since the size of the large investor's unilateral write-down is increasing in F .

Since the function G is strictly decreasing in its first argument and increasing in its second

argument, a necessary condition for the large investor to enter the secondary market is that G be strictly positive in the limit as σ converges to zero for $\hat{\gamma} = \gamma$. This implies the following lemma.

Lemma 3. *A necessary condition for the large investor to enter the secondary market and grant voluntary debt relief is that the liquidity shocks hitting small bondholders be sufficiently broad, with*

$$\gamma > \underline{\gamma}_\omega \equiv \frac{2(F - y_\omega)}{F - y_\omega + \alpha y_\omega} = 2\underline{S}(y_\omega, \alpha, F). \quad (11)$$

If $\gamma \leq \underline{\gamma}_\omega$ the unique equilibrium entails $(\sigma_\omega^*, \gamma_\omega^*) = (0, \gamma)$.

Lemma 3 reinforces our argument that propping up investors in an ad hoc fashion impedes voluntary debt relief, since broad selling by small bondholders is a necessary condition for a large investor to find entry/relief profitable. In fact, the necessary condition for large investor entry into the secondary debt market (specified in condition (11)) is twice as stringent as the necessary condition for voluntary debt relief post-trading (condition (8)). The intuition is as follows. The latter condition simply ensures that a large investor would be willing to grant debt relief post-trading, despite the free-riding costs he would bear from that point onward. The former condition ensures the large investor covers all free-riding costs, pre-trade. Since the possibility of debt relief is capitalized into prices, the necessary condition for entry into the secondary market is even more stringent.

The remainder of the paper confines attention to interesting cases where entry and debt relief is actually possible. Therefore, all lemmas and propositions below adopt the following technical assumption:

$$A1 : \gamma > \frac{2(F - y_\omega)}{F - y_\omega + \alpha y_\omega}.$$

3.1. Unique equilibrium under pure noise-trading

A major difference between our model and a traditional noise-trading setup, e.g. Maug (1998), is that selling by the impatient bondholders is rational. In fact, our model subsumes pure noise-trading as a special case if $c = 1$. To facilitate comparison, and highlight the role of endogenous

trading by small investors, this subsection assumes $c = 1$ and evaluates the equilibrium set. If $c = 1$, impatient investors strictly prefer to sell, so $\gamma_\omega^* = \gamma$. Thus, we need only determine σ_ω^* .

Since γ has been assumed to exceed γ_ω (A1), there is a unique $\tilde{\sigma} \in (0, 1)$ satisfying:

$$G(\tilde{\sigma}, \gamma, y_\omega, \alpha, F) = 0. \quad (12)$$

Condition (12) ensures the large investor is indifferent between placing a buy order and not, when he knows that all impatient bondholders will sell. Solving for $\tilde{\sigma}$ in the equation above we arrive at Proposition 2.

Proposition 2. *With pure noise-trading ($c = 1$), the unique equilibrium entails*

$$\begin{aligned} \sigma_\omega^* &= \tilde{\sigma}_\omega \equiv 1 - \frac{2(F - y_\omega)}{\gamma(F - y_\omega + \alpha y_\omega)} = 1 - \frac{\gamma_\omega}{\gamma} \in (0, 1) \\ \gamma_\omega^* &= \gamma. \end{aligned} \quad (13)$$

Proposition 2 provides a convenient benchmark, showing that multiple equilibria never emerge under pure noise-trading. Therefore, multiple equilibria, if they occur, must be due to endogenous trading by small bondholders.

The variable $\tilde{\sigma}_\omega$ measures the buying intensity of the large investor provisional upon all impatient bondholders selling ($\gamma^* = \gamma$). For this reason, below $\tilde{\sigma}_\omega$ is labeled the large investor's *provisional trading intensity*.

Differentiating equation (13) reveals:

$$\begin{aligned} \frac{\partial \tilde{\sigma}_\omega}{\partial \gamma} &= \frac{1 - \tilde{\sigma}_\omega}{\gamma} > 0 \\ \frac{\partial \tilde{\sigma}_\omega}{\partial \alpha} &= \frac{(1 - \tilde{\sigma}_\omega)y_\omega}{F - y_\omega + \alpha y_\omega} > 0 \\ \frac{\partial \tilde{\sigma}_\omega}{\partial y_\omega} &= \frac{2}{\gamma(F - y_\omega + \alpha y_\omega)} \left(1 - \frac{\gamma(1 - \tilde{\sigma}_\omega)(1 - \alpha)}{2} \right) > 0 \\ \frac{\partial \tilde{\sigma}_\omega}{\partial F} &= - \left(1 - \frac{\gamma(1 - \tilde{\sigma}_\omega)}{2} \right) \left(\frac{2}{\gamma(F - y_\omega + \alpha y_\omega)} \right) < 0. \end{aligned} \quad (14)$$

The intuition for these comparative statics is identical to those in (10). To illustrate, Figure 2 plots the large investor's provisional buying intensities ($\tilde{\sigma}_b, \tilde{\sigma}_g$) as functions of underlying parameters.

Panel A shows the effect of broader liquidity shocks, as measured by γ . If the breadth of liquidity shocks is too narrow, the large investor does not buy. With sufficient breadth he enters the market, with his provisional trading intensity increasing monotonically in γ . In the bad macroeconomic state, the large investor buys with lower probability, since he must take a larger write-down if the project fails.

Panel B of Figure 2 shows the effect of bankruptcy costs on the large investor's provisional trading intensity. Again we see the large investor buys with higher intensity if the macroeconomic state is good. Since, the large investor can buy at a lower pooling price P^0 if bankruptcy costs are high, his provisional trading intensity is increasing in α . Panel B also shows that increased buying by the large investor can substantially mitigate the costs of formal bankruptcy. For example, as α goes to one, the large investor buys debt and provides debt relief with probability 9/10 in the good state. Thus, high de jure bankruptcy costs need not translate into high costs de facto, since the former motivates out of court restructuring.

3.2. Rational selling by small bondholders

There exists an equilibrium with $\sigma_\omega^* = \tilde{\sigma}_\omega$ iff all impatient investors prefer to sell. To evaluate the viability of such an equilibrium, consider now the trading incentives of an individual small bondholder hit with the shock. Such an investor will sell if the expected value captured by selling immediately is larger than the payoff to unilaterally deviating by holding onto the debt and facing the holding cost c . An individual impatient bondholder weakly prefers to sell iff:

$$\sigma P_\omega^0 + (1 - \sigma)P_\omega^- \geq (1 - c)P_\omega^0 \Leftrightarrow c \geq \underline{c}(\sigma, P_\omega^-, F) = \frac{\sigma(F - P_\omega^0)}{\sigma F + (1 - \sigma)P_\omega^-} = \frac{\sigma(1 - \sigma)(F - P_\omega^-)}{\sigma F + (1 - \sigma)P_\omega^-}. \quad (15)$$

The equation above reveals that small investors have a simple trading rule: sell only if the holding cost c is larger than the cost of adverse selection, as captured by \underline{c} . The expression for \underline{c} is intuitive, equal to expected losses due to underpricing as a percentage of fundamental value, with the expectation being conditioned upon having been hit with the shock.

Importantly, the impatient bondholders are less willing to sell if the interim state is b , since

$$\underline{c}(\sigma, P_b^-, F) > \underline{c}(\sigma, P_g^-, F) \quad \forall \quad \sigma \in (0, 1). \quad (16)$$

The intuition for (16) is as follows. The impatient investors face underpricing of their debt in the second row of Table 1, when the market makers set the pooling price P_ω^0 despite the fundamental value of the debt for atomistic bondholders being equal to F . The gap between the fundamental value and the pooling price is larger in the bad state since $P_b^0 < P_g^0$ for all $\sigma < 1$.

Important for our results is the fact that adverse selection costs, as perceived by the impatient bondholders, are non-monotone in the buying intensity of the large investor. To see this, note that impatient bondholders are always willing to sell for limiting values of σ since

$$\lim_{\sigma \downarrow 0} \underline{c}(\sigma, P_\omega^-, F) = \lim_{\sigma \uparrow 1} \underline{c}(\sigma, P_\omega^-, F) = 0.$$

Intuitively, impatient investors are reluctant to sell due to fear of selling at too low a price relative to fundamental value. If the large investor never buys there is no risk of missing out on the windfall associated with debt relief by the large investor. Conversely, if the large investor buys with probability one then market makers will set the secondary market price at F and there is still no cost to selling prior to maturity.

The adverse selection problem as perceived by small investors is most severe for intermediate values of σ , where market makers are especially confused when they see an order flow of zero. Consistent with this intuition, the function $\underline{c}(\cdot, P_\omega^-, F)$ reaches a unique maximum at an interior point denoted σ_ω^{\max} , where

$$\sigma_\omega^{\max}(P_\omega^-, F) \equiv \arg \max_{\sigma} \underline{c}(\sigma, P_\omega^-, F) \quad (17)$$

$$\Rightarrow \sigma_\omega^{\max}(P_\omega^-, F) = \frac{\sqrt{FP_\omega^-} - P_\omega^-}{F - P_\omega^-} \in (0, 1)$$

$$\Rightarrow \underline{c}(\sigma_\omega^{\max}, P_\omega^-, F) = \frac{(\sqrt{F} - \sqrt{P_\omega^-})^2}{F - P_\omega^-}. \quad (18)$$

Figure 3 plots the functions $\underline{c}(\cdot, P_b^-, F)$ and $\underline{c}(\cdot, P_g^-, F)$, with the three panels considering alternative values for the holding cost parameter c . Consistent with the inequality in equation (16), in

each figure the impatient investors are less willing to sell in the bad macroeconomic state. Panel A depicts $c > \underline{c}(\sigma_b^{\max}, P_b^-, F)$. In this case, each impatient investor strictly prefers to sell regardless of the trading intensity of the large investor, and regardless of the economic state. Panel B depicts an intermediate value for c . In that panel, impatient investors strictly prefer to sell if the state is good, but may prefer to hold if the state is bad. Finally, Panel C depicts low values of the liquidity preference parameter such that the impatient investors may opt for no-trade regardless of the interim state.

3.3. Multiple equilibria secondary markets

The efficiency of an equilibrium is determined by (σ^*, γ^*) with the first element measuring the probability of the large investor placing a buy order (and granting debt relief) and the second representing the measure of impatient bondholders selling. Since the large investor places a buy order of size γ^* to mask his trades, γ^* proxies for trading volume.

Since the large investor plays a mixed strategy, the following indifference condition is satisfied in any equilibrium with debt relief:

$$G(\sigma_\omega^*, \gamma_\omega^*, y_\omega, \alpha, F) = 0. \quad (19)$$

Further, there are two possibilities regarding the actions of the impatient investors. First, they may strictly prefer to sell, in which case $\gamma_\omega^* = \gamma$. Alternatively, it is possible to support equilibria in which only a proper subset of the impatient investors sell, but in order for this to be the case each must be just indifferent between selling and holding. We summarize these two possibilities as follows:

$$\underline{c}(\sigma^*, P_\omega^-, F) < c \Rightarrow \gamma^* = \gamma \quad (20)$$

and

$$\gamma^* < \gamma \Rightarrow \underline{c}(\sigma^*, P_\omega^-, F) = c.$$

From here the analysis is easily followed by referring back to Figure 3. Consider Panel A where $c > \underline{c}(\sigma_\omega^{\max}, P_\omega^-, F)$. In this case, impatient investors strictly prefer to sell regardless of the buying

intensity of the large investor. It follows that $\gamma^* = \gamma$. Further, only the provisional trading intensity $\tilde{\sigma}$, as defined in equation (13), satisfies the large investor's indifference condition (19). Thus, in Panel A of Figure 3 the unique PBE entails $(\sigma_\omega^*, \gamma_\omega^*) = (\tilde{\sigma}_\omega, \gamma)$.

Consider next an arbitrary case where $c < \underline{c}(\sigma_\omega^{\max}, P_\omega^-, F)$, as is possible in the lower two panels of Figure 3. In such cases there is a pair of buying intensities $(\sigma_1^\omega, \sigma_2^\omega)$ such that each impatient investor is indifferent between selling and holding. These points of indifference solve $\underline{c}(\sigma, P_\omega^-, F) = c$, implying

$$\begin{aligned}\sigma_1^\omega &= \frac{1 - c - \sqrt{(1 - c)^2 - 4cP^-/(F - P^-)}}{2} \\ \sigma_2^\omega &= \frac{1 - c + \sqrt{(1 - c)^2 - 4cP^-/(F - P^-)}}{2}.\end{aligned}\tag{21}$$

To pin down the equilibrium set when $c < \underline{c}(\sigma_\omega^{\max}, P_\omega^-, F)$, one must consider alternative ranges for the provisional trading intensity. We begin first with low values of $\tilde{\sigma}_\omega$. Returning to Figure 3 it is readily verified that

$$\tilde{\sigma}_\omega \in (0, \sigma_1^\omega] \Rightarrow (\sigma_\omega^*, \gamma_\omega^*) = (\tilde{\sigma}_\omega, \gamma).\tag{22}$$

And further, the PBE is unique in this case. To see this, note that decreasing σ below $\tilde{\sigma}_\omega$ would result in $G > 0$, and the only way to restore $G = 0$ would be to reduce the measure of investors liquidating. But for $\sigma < \tilde{\sigma}_\omega$ all impatient investors strictly prefer to sell. Conversely, an increase in σ is also not possible since this would result in $G < 0$, with further increases in the measure of liquidating bondholders being impossible.

Considering higher values of $\tilde{\sigma}_\omega$, it can be verified that

$$\begin{aligned}\tilde{\sigma}_\omega &\in (\sigma_1^\omega, \sigma_2^\omega) \Rightarrow (\sigma_\omega^*, \gamma_\omega^*) = (\sigma_1^\omega, \gamma_1^\omega) \\ \gamma_1^\omega &\equiv \left(\frac{1 - \tilde{\sigma}_\omega}{1 - \sigma_1^\omega}\right) \gamma < \gamma.\end{aligned}\tag{23}$$

This PBE is also unique, with the reasoning as follows. Under the maintained condition it is clear that $\tilde{\sigma}_\omega$ cannot occur in equilibrium because the impatient investors are unwilling to sell. Clearly, one can induce the impatient investors to sell with $\sigma \in (0, \sigma_1^\omega]$. However, $\sigma < \sigma_1^\omega$ cannot be an

equilibrium in the present case since then all impatient investors strictly prefer selling and one then obtains $G > 0$. In contrast, it is possible to maintain equilibrium with $\sigma_\omega^* = \sigma_1^\omega$ in which case only a proper subset of the impatient bondholders sells. By construction γ_1^ω maintains the large investor's indifference condition given $\sigma_\omega^* = \sigma_1^\omega$. Finally, under the maintained condition one cannot support a PBE at $\sigma > \sigma_1^\omega$ since any such σ would result in $G < 0$.

Although the equilibrium described in (23) is unique, it differs from what one obtains in a pure noise-trading model with both σ^* and γ^* falling. This equilibrium clearly illustrates one of the model's central messages: *Small investor concern over adverse selection can significantly reduce trading volumes and the probability of debt relief*. In contrast, a model with pure noise-trading ($c = 1$) would predict that the unique equilibrium is always $(\tilde{\sigma}_\omega, \gamma)$.

Our most important finding is that equilibrium is not necessarily unique. Considering even higher values of $\tilde{\sigma}_\omega$ we find it is possible to support three equilibria, with

$$\begin{aligned} \tilde{\sigma}_\omega \in (\sigma_2^\omega, 1) &\Rightarrow (\sigma_\omega^*, \gamma_\omega^*) \in \{(\tilde{\sigma}_\omega, \gamma), (\sigma_2^\omega, \gamma_2^\omega), (\sigma_1^\omega, \gamma_1^\omega)\} \\ \gamma_2^\omega &\equiv \left(\frac{1 - \tilde{\sigma}_\omega}{1 - \sigma_2^\omega}\right) \gamma \in (\gamma_1^\omega, \gamma). \end{aligned} \quad (24)$$

In (24), the equilibrium pair $(\tilde{\sigma}_\omega, \gamma)$ Pareto-dominates the other two, with anticipation of high liquidity ($\gamma_\omega^* = \gamma$) inducing the large investor to buy/restructure with high probability. However, the large investor's indifference condition $G = 0$ can be satisfied at lower σ values by reducing the measure of impatient investors liquidating. Since only a proper subset of them actually sell, it must be the case that each impatient investor is just indifferent between holding and liquidating, which is only possible for $\sigma \in \{\sigma_1^\omega, \sigma_2^\omega\}$. Thus, there are only two points at which indifference can be maintained simultaneously for the large investor and impatient investors: $(\sigma_2^\omega, \gamma_2^\omega)$ and $(\sigma_1^\omega, \gamma_1^\omega)$.

The less efficient equilibria at $(\sigma_2^\omega, \gamma_2^\omega)$ and $(\sigma_1^\omega, \gamma_1^\omega)$ are consistent with the casual intuition that *illiquidity is a self-fulfilling prophecy*. At each of these pairs, anticipation of low liquidity induces the large investor to reduce his buying/restructuring intensity. In turn, the reduction in his buying intensity from $\tilde{\sigma}_\omega$ to $\sigma \in \{\sigma_1^\omega, \sigma_2^\omega\}$ is sufficient to tilt the impatient investors from a strict preference for liquidating to indifference.

Proposition 3 summarizes:⁸

Proposition 3 [Equilibrium]. *If the holding cost is sufficiently high, with $c > \underline{c}(\sigma_\omega^{\max}, P_\omega^-, F)$, then the unique equilibrium entails $(\sigma_\omega^*, \gamma_\omega^*) = (\tilde{\sigma}_\omega, \gamma)$. If $c < \underline{c}(\sigma_\omega^{\max}, P_\omega^-, F)$, equilibrium is not necessarily unique, with*

$$\tilde{\sigma}_\omega \in (0, \sigma_1^\omega] \Rightarrow (\sigma_\omega^*, \gamma_\omega^*) = (\tilde{\sigma}_\omega, \gamma)$$

$$\tilde{\sigma}_\omega \in (\sigma_1^\omega, \sigma_2^\omega) \Rightarrow (\sigma_\omega^*, \gamma_\omega^*) = (\sigma_1^\omega, \gamma_1^\omega)$$

$$\tilde{\sigma}_\omega = \sigma_2^\omega \Rightarrow (\sigma_\omega^*, \gamma_\omega^*) \in \{(\sigma_2^\omega, \gamma_2^\omega), (\sigma_1^\omega, \gamma_1^\omega)\}$$

and

$$\tilde{\sigma}_\omega > \sigma_2^\omega \Rightarrow (\sigma_\omega^*, \gamma_\omega^*) \in \{(\tilde{\sigma}_\omega, \gamma), (\sigma_2^\omega, \gamma_2^\omega), (\sigma_1^\omega, \gamma_1^\omega)\}.$$

The importance of Proposition 3 can be partially appreciated by returning to equation (6) which specifies the primary market debt price. Since p_ω depends upon σ_ω^* it is apparent that the possibility of multiple equilibria in the secondary market opens the possibility for sudden jumps in bond prices (and yields) without any change in fundamentals. Similarly, multiple equilibria can be associated with sharp jumps in trading volumes based upon sunspots. As shown below, this will have important implications for the illiquidity discount on debt, as well as corporate financing decisions.

At this stage it is worth stressing that the possibility for multiple equilibria is quite general. For example, one can derive multiple equilibria even without having the small investors playing a mixed strategy, as they do in Proposition 3. To do so, one can instead assume heterogeneity in the cost parameter c , with the parameter distributed according to some continuous density function. In such a setting, each small investor would play a pure strategy, selling if and only if his idiosyncratic parameter exceeded \underline{c} . The volume of uninformed trading would then be non-monotone in the large investor's buying intensity, again opening up the possibility for multiple equilibria. The disadvantage

⁸If c is just tangent to \underline{c} then $\tilde{\sigma}$ is always a PBE. Further, if $\tilde{\sigma} > \sigma_\omega^{\max}$ then the tangency point σ_ω^{\max} can also be supported as a PBE.

of that modeling approach is that it does not yield closed-form prices.

3.4. Debt market liquidity and macroeconomic conditions

During the credit crisis of 2008 it appeared that debt markets became illiquid at precisely the time when the macroeconomy weakened. This section examines the ability of the model to explain the apparent positive relationship between the macroeconomy and debt market liquidity. For this purpose, we assume that

$$\underline{c}(\sigma_g^{\max}, P_g^-, F) < c < \underline{c}(\sigma_b^{\max}, P_b^-, F),$$

as shown in Panel B of Figure 3. Under this scenario, each impatient investor strictly prefers to sell during the expansion, but may not be willing to sell during the recession given high exposure to adverse selection.

From Proposition 3 we know that:

$$\begin{aligned}\sigma_g^* &= \tilde{\sigma}_g > \sigma_b^* \in \{\tilde{\sigma}_b, \sigma_2^b, \sigma_1^b\} \\ \gamma_g^* &= \gamma \geq \gamma_b^* \in \{\gamma, \gamma_2^b, \gamma_1^b\}.\end{aligned}$$

Thus, we see that during an expansion the large investor trades with higher intensity and unsigned trading volume is weakly higher. Intuitively, the model predicts that the volume of trade in debt markets should fall during recessions because the adverse selection problem as perceived by small investors becomes more severe as the economy cools. In turn, the decline in trade by small uninformed investors reduces the incentive of a large investor to enter the debt market.

Consider now a particularly salient example regarding the state-contingency of debt market liquidity. Suppose $\tilde{\sigma}_b > \sigma_2^b$. In this case, we know that during the recession there is an equilibrium in which the large investor trades with very low intensity (σ_1^b) relative to his equilibrium trading intensity during an expansion, with $\sigma_g^* = \tilde{\sigma}_g > \tilde{\sigma}_b > \sigma_2^b > \sigma_1^b$. Further, in that same recessionary equilibrium, trading volume falls to γ_1^b , which is much lower than trading volume during the expansion (γ). This illustrates that multiple equilibria in debt markets, resulting from the adverse selection problem perceived by small bondholders can explain large declines in trading volumes as

the economy cools. A pure noise-trading model ($c = 1$) would not generate such a prediction since in such models impatient investors always sell, failing to capture macro-contingent liquidity.

4. Illiquidity Discounts in the Primary Market

Equation (6) offers an intuitive expression for the primary market price of debt, showing that debt trades at a discount to compensate small investors for subsequent exposure to adverse selection. Holmström and Tirole (1993) and Maug (1998) present pure noise-trading models of the equity market in which there is also an adverse selection discount. However, illiquidity discounts are necessarily absent from their analyses. In contrast, our model delivers an endogenous discount for illiquidity whenever the equilibrium is such that a proper subset of impatient investors fails to sell in the secondary market.

When a subset of impatient bondholders choose not to sell, it must be the case that $\underline{c}(\sigma^*, P_\omega^-, F) = c$. Substituting this indifference condition into the bond pricing equation (6) one obtains:

$$\gamma_\omega^* < \gamma \Rightarrow p_\omega = [\sigma_\omega^* F + (1 - \sigma_\omega^*) P_\omega^-] \left[1 - \frac{\gamma c}{2} \right]. \quad (25)$$

The first bracketed term in equation (25) captures the fundamental value of the debt and the second term captures the discount resulting from the fact that adverse selection induces a subset of impatient investors to hold onto their debt rather than selling, implying they incur the holding cost c . If $\gamma_\omega^* < \gamma$ each impatient bondholder is just indifferent between selling and holding, so one can simply price the debt from the perspective of a bondholder who knows he will not sell if hit with the shock, which occurs with probability $\gamma/2$.

Duffie, Gârleanu, and Pedersen (2007) also present a model in which there is an illiquidity discount. However, their underlying mechanism differs fundamentally. In their search-based model, the discount arises because an impatient bondholder simply cannot find a counterparty. In contrast, in our model the illiquidity discount arises because the impatient bondholder fails to trade because he thinks he will not receive a good price. There is anecdotal evidence of both of these stories being operative during the recent debt market freeze.

Finally, if one uses the equilibrium condition $G = 0$, then the debt price (6) can also be expressed as:

$$p_\omega = \frac{F}{2} + \frac{y_\omega[1 - \alpha(1 - \sigma_\omega^*)]}{2} - \left(\frac{\gamma - \gamma_\omega^*}{2}\right) [\sigma_\omega^* F + (1 - \sigma_\omega^*) P_\omega^-] c. \quad (26)$$

This expression also implies that there will be an illiquidity discount whenever the equilibrium entails $\gamma > \gamma_\omega^*$. It also vividly illustrates that the primary market value of debt hinges upon the nature of the conjectured secondary market equilibrium. In the good equilibrium where $\gamma = \gamma_\omega^*$ there is no illiquidity discount and the bankruptcy cost is attenuated due to relatively high values for σ_ω^* . However, if one were to jump to bad equilibria with $\gamma < \gamma_\omega^*$ the primary market price would jump down and yields would jump up. We illustrate this next by considering some numerical examples.

Figure 4A shows how debt price varies with the breadth of the preference shocks (γ) and with the particular PBE selected. The numerical example assumes $F = 1$, $y = .9375$, $\alpha = .70$, and $c = .1091$. Panel B of Figure 4 shows the underlying buying intensity of the large investor, while Panel C plots the fraction of impatient investors selling their debt.

For low γ values the large investor does not enter the secondary debt market and the debt trades at a deeply discounted price. As γ increases, the large investor initially increases his buying intensity reflecting the fact that his trading profits are increasing in the measure of impatient investors selling. However, σ^* does not increase in a strictly monotone fashion. As shown in Panel C of Figure 4, this reflects the fact that the small investors may not be willing to sell at the large investor's provisional trading intensity $\tilde{\sigma}$. The resulting endogenous decreases in uninformed selling volume induce endogenous declines in the likelihood of debt relief, resulting in lower debt prices.

The three panels of Figure 5 analyze the effect of changes in formal bankruptcy costs (α). The numerical example assumes $F = 1$, $y = .975$, $\gamma = .50$, and $c = .06$. As shown in Panel A of Figure 5, the sensitivity of debt price to bankruptcy cost depends on how the change in bankruptcy cost influences the trading intensity of the large investor. Recall, increases in bankruptcy costs tend to stimulate debt purchases by the large investor since this results in a lower pooling price P^0 . Thus, endogenous increases in debt purchases by the large investor mitigate the effect of bankruptcy costs

on debt value. For example, when attention is confined to the best PBE at each point, debt value is convex in bankruptcy cost.

Panel A of Figure 5 shows once again that the primary market price of debt is sensitive to which secondary market PBE is conjectured. Examination of Panels B and C of Figure 5 reveals the symbiotic relationship between the large investor's buying intensity and anticipated selling by small investors. When liquidity drops, so too does the buying intensity of the large bondholder. In turn, low buying intensity by the large investor can induce low overall trading volumes since, as shown in Figure 3, the small investors perceive adverse selection to be non-monotone in σ .

5. Corporate Financing Implications

We now close the model by examining the entrepreneur's financing decision. The first subsection analyzes the financing choice assuming that debt and equity are mutually exclusive options. This analysis is useful in that one obtains simple inequalities describing when public debt dominates other sources of funds. The next subsection allows the firm to choose an optimal mix of public debt and equity.

5.1 Equity versus debt

Consider first the choice between public debt and a private loan. The dual to the entrepreneur's financing problem is to choose public debt if it sells for more than the private loan for the same face value. With this in mind, consider pricing of the private loan. The attractive feature of the private loan is that bankruptcy costs are not incurred. However, the loan will trade at a discount to compensate the large investor for his opportunity cost of funds. Consequently, the loan is priced at $\delta(F + y)/2$. Comparing this price with the price of public debt from equation (26), one finds that public debt dominates the privately placed loan if:

$$\alpha(1 - \sigma^*)y + (\gamma - \gamma^*)[\sigma^*F + (1 - \sigma^*)P^-]c \leq (1 - \delta)(F + y). \quad (27)$$

The condition for the dominance of public debt over the private placement is intuitive. The public debt entails bankruptcy costs which are mitigated by the large investor's voluntary debt relief.

Public debt may also entail an illiquidity discount. Public debt dominates the private loan only if these costs are less than the intermediation premium on the private loan.

Equation (27) illustrates that multiple equilibria in secondary markets can lead to sudden jumps in firms' preferred method of borrowing. For example, it is easy to envision cases in which the stated inequality is satisfied in the best possible equilibrium but violated in the worst, implying that there can be cycling between public and private debt contingent upon sunspots in the secondary market.

Consider next the choice between public debt and equity. Assume there is a tax advantage of debt due to the existence of a corporate income tax at rate $\tau > 0$, with shareholders receiving the tax shield value only if they actually deliver the face value F in full. For simplicity, assume for now that a debt obligation with face value of 1 is just sufficient to fund the project.

The value obtained by the entrepreneur if he finances the project with public debt is equal to the value of the debt tax shield plus the expected value of the dividend he receives if the project succeeds. Interest expense on the corporate tax return is computed as bond yield (ψ) times initial loan principal (p). Using the fact that $p = F/(1 + \psi)$ it follows that interest expense is just equal to $F - p$. Thus, under debt finance the entrepreneur captures

$$Debt \Rightarrow \frac{1}{2}[H - 1 + \tau(1 - p)]. \quad (28)$$

Suppose instead the entrepreneur finances the project with equity. Recalling our working assumption that the debt obligation is just sufficient to cover the cost of the project, under equity finance the agent receives a payoff equal to expected cash flow less project cost:

$$Equity \Rightarrow \frac{H + y}{2} - p. \quad (29)$$

Comparing (28) and (29) we see that debt dominates equity finance if:

$$p \geq \frac{\frac{1}{2}[1 - \tau + y]}{1 - \tau/2}. \quad (30)$$

However, we recall from Figure 5 that the primary market price of debt is not unique, with debt value being lower if the market anticipates the low-liquidity equilibrium. Therefore, whether condition

(30) is satisfied depends upon which secondary market equilibrium is anticipated. For example, if the best (worst) PBE is anticipated, then debt (equity) is more likely to be the optimal source of external funds.

To reinforce this intuition, we may compare equations (28) and (29), concluding that debt dominates equity iff:

$$\tau \geq \tau^* \equiv \frac{1 + y - 2p}{1 - p}. \quad (31)$$

Since τ^* is decreasing in p , it follows that debt will be more attractive if the best equilibrium is played. More importantly, the theory once again predicts that the primary market for debt is susceptible to sunspots.

5.2. A liquidity-augmented trade-off theory

This subsection considers a more general setting in which the firm can combine public debt with equity. The entrepreneur's objective is to maximize the total value of marketable claims on the firm. Equity value (E) is

$$E = \frac{1}{2}[H - F + \tau(F - P)]. \quad (32)$$

Adding this expression for equity value to the debt value expression given in equation (26) one obtains

$$\begin{aligned} E + V &= \frac{H}{2} + \frac{y[1 - \alpha(1 - \sigma^*)]}{2} + \tau(F - p)/2 \\ &\quad - \left(\frac{\gamma - \gamma^*}{2}\right)[\sigma^*F + (1 - \sigma^*)P^-]c. \end{aligned} \quad (33)$$

The top line in the total firm value equation (33) is similar to equations derived in a standard trade-off model, with levered firm value typically expressed as expected cash flow plus tax shield benefits less bankruptcy costs. However, standard models ignore the possibility for voluntary debt relief reducing the probability of incurring formal bankruptcy costs.

The second line captures a subtle effect arising in our model: The possibility of a large investor granting debt relief also leads to inferior risk-sharing in the sense that perceived adverse selection costs may induce an impatient investor to hold rather than sell. In particular, if the secondary

market equilibrium is such that a subset of impatient bondholders fail to sell, firm value is reduced by their expected holding costs. That is, there is an endogenous illiquidity discount arising from the prospect of voluntary debt relief.

To clarify the argument, note that the illiquidity discount term in total firm value vanishes if the equilibrium is such that all impatient bondholders sell ($\gamma^* = \gamma$), even though the small investors still face adverse selection in such cases. Thus, adverse selection discounts on debt, which are always present, as shown in equation (6), are insufficient to introduce an illiquidity discount into total firm value. Rather, there is only an illiquidity discount deducted from total firm value if adverse selection causes a proper subset of impatient bondholders to hold rather than sell. Intuitively, the zero profit condition for the large investor ($G = 0$) ensures that if all impatient bondholders sell, then the expected adverse selection costs they bear are just compensated by the expected transfer they receive when the large investor grants debt relief. However, if a subset of impatient investors do not sell, there is an uncompensated deadweight loss.

An interesting direction for future corporate finance research is to consider whether allowing for illiquidity discounts on debt, as we do in equation (33), is helpful in resolving what may appear to be conservative leverage policy from the perspective of standard trade-off models. An attractive feature of our model is that the illiquidity discount is endogenous, whereas some trade-off models have simply tacked on the discount in an ad hoc fashion.

6. Conclusion

This paper has identified conditions under which trading of debt in secondary markets can bring about an efficient shift from dispersed to concentrated ownership. Specifically, we have developed a theoretical framework to analyze a secondary market where endogenously determined ownership structure affects the value of a traded debt security. A novel feature of our model, one which is critical for our findings, is that we allow small investors to consider potential mispricing before trading.

A main obstacle inhibiting ex post efficient ownership concentration is that small bondhold-

ers free-ride off the debt relief granted by the large investor, with free-ridership capitalized into secondary market debt prices. Further, the prospect of a large investor granting debt relief also reduces the willingness of small bondholders to sell, even when they are impatient. We show that the free-ridership problem and small investor concerns regarding adverse selection are both more severe during recessions. Consequently, trading volumes in secondary debt markets, as well as the prospect of debt relief, will fall during downturns.

The most important prediction generated by the model is that small investor concern over adverse selection creates the potential for multiple equilibria in secondary debt markets. For example, there exist low-liquidity equilibria in which the large investor buys debt with low probability because he anticipates that only a low percentage of small bondholders will sell. Conversely, a low buying intensity by the large investor can actually discourage small investors from selling their debt claims due to the fact that adverse selection is non-monotone in the buying intensity of the large investor. We also show that the equilibrium set is contingent on the macroeconomic state. Since small investors face more severe adverse selection during recessions, the secondary debt market is then more likely to be in a low-liquidity equilibrium.

The model offers a novel explanation for leverage cycles, one which is based upon the multiplicity of secondary market equilibria. Anticipation of low-liquidity equilibria in secondary debt markets leads to sharp increases in expected bankruptcy costs and required debt yields, inducing firms to shift away from public debt financing. Significantly, the model predicts that sharp changes in financing patterns can occur absent any change in economic fundamentals such as tax rates and bankruptcy cost parameters. Rather, large fluctuations in aggregate leverage require nothing more than sunspots inducing agents to anticipate high or low trading volumes in secondary debt markets.

Our analysis is consistent with the observation that the recent financial crisis was especially a crisis in debt markets. In our model, the value of debt claims during financial distress depends crucially on how efficiently restructuring can be achieved which in turn depends on the evolution of debt ownership structure. The analysis reveals that the secondary debt market dries up in bad

economic states due to adverse selection. Illiquidity can become dramatically low in poor economic states when “confidence” is lost, i.e. when a self-fulfilling liquidity run takes place. In our model, equity markets are not plagued by such problems.

Several policy implications emerge from the model. First, the model shows that perfect competition is not a panacea for debt market illiquidity. Even with perfect competition between market makers, debt markets can still be prone to pronounced bouts of illiquidity, particularly during economic downturns. When our findings are placed alongside those of Ericsson and Renault (2006) and Duffie, Gârleanu, and Pedersen (2007), which show that imperfect competition can induce illiquidity, one reaches the conclusion that competition is a necessary but insufficient condition for liquid debt markets.

Second, our analysis shows that policies intended to reduce opacity can have an unintended consequence. Specifically, if transparency prevents large investors from trading anonymously, they cannot profitably enter debt markets with the goal of achieving ex post efficient debt restructuring. That is, opacity supports debt relief. On the other hand, opacity comes at the cost of inefficient sharing of risks. In our model, if markets were fully transparent, there would be perfect sharing of risks between impatient investors and market makers, but no debt relief.

Third, our analysis indicates that if policymakers want the private sector to offer debt relief, they must adopt policies that promote concentrated ownership, since only large investors find it optimal to forgive debts. It follows that it does not make sense for governments or lenders of last resort to prop up all bondholders equally during a crisis. Rather, small bondholders should be allowed to fail and liquidate, since this promotes an ex post efficient shift to concentrated ownership. By contrast, it is crucial in our model that large investors have sufficient funds to acquire nontrivial debt stakes. In this sense, our model provides intellectual support for the Geithner Plan’s call for the Treasury to coinvest alongside large private-sector debt investors. Such a policy bolsters large debt investors, increasing the likelihood of concentrated debt ownership.

A fourth implication of our model is that policymakers should try to nudge debt markets towards

the most efficient equilibrium. One potential way for the government to achieve this is by “talking-up” bond markets, making the efficient equilibrium focal in the eyes of market participants. However, this suffers from the cheap-talk critique. A second more credible means of the government pushing the market towards good equilibria is for it to directly participate in the tâtonnement process, perhaps by standing ready to buy small quantities at high prices. This is analogous to deposit insurance in that the hope is that the government never needs to intervene in equilibrium. Third, those pushing the debt market towards the less efficient equilibrium (e.g. short-sellers) could be punished by policies/subsidies for debt restructuring ex post. Finally, we note that the imposition of tighter capital requirements on smaller banks may have an unintended benefit since they are analogous to raising c in our model, with the latter effect often being sufficient to eliminate the possibility of inefficient equilibria.

The recent financial crisis has demonstrated forcefully that much work is still required to understand the dynamics of liquidity – especially in debt markets. Most existing theoretical models predict that equity markets should be more prone to problems of adverse selection and illiquidity. Yet, during the crisis of 2007/2008 we saw debt markets malfunctioning more than equity markets. We therefore need a better understanding of what are the sources of adverse selection in securities markets. Our theoretical analysis illustrates that, compared to equity, debt markets are vulnerable to the presence of large investors who do not have any private information regarding cash flow. While there is by now an extensive theoretical and empirical literature on equity ownership structure, very little is known about debt ownership and the dynamics of debt ownership.

On the theoretical front many questions remain open. For example, a key determinant of debt market liquidity is the trading of small uninformed bondholders. This paper has taken a first step in opening the black-box by making the trading decisions of small bondholders reflect a rational trade-off between an exogenous liquidity preference and endogenous costs of adverse selection. Future theoretical work in this area should focus on understanding the primitive shocks and factors that create liquidity preferences.

Appendix: Beliefs Off the Equilibrium Path

We here briefly discuss market maker beliefs off the equilibrium path. Beliefs and prices in response to order flow X off the equilibrium path are only relevant to our analysis of the large investor's incentive to deviate, since each small investor has measure zero and is thus powerless to push order flow off the equilibrium path. To deter deviations by the large investor we assume the market makers form the least favorable beliefs, in the following sense. Upon observing any order flow X off the equilibrium path, the market makers assume that all impatient investors are liquidating and that the large investor is placing an order to buy $x = X + \gamma$. If the implied $x \geq \underline{S}$ the market makers set $P = F$ and if not they set the price to P^- . Facing such beliefs, the large investor cannot gain if he were to ever deviate by placing a buy order of any size other than γ^* . In our setting, when the market makers see a non-equilibrium order flow, they know the deviator is the large investor. However, the Cho-Kreps refinement has no bite in our setting since the only party capable of pushing order flow off the equilibrium path does not even have a proper "type."

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Table 1: Aggregate Demands and Price Setting

Buy	Shock	$x^I + x^N$	Price	Probability	True Value
Y	N	$(1-s)\gamma^* + 0$	F	$\sigma/2$	F
Y	Y	$(1-s)\gamma^* - (1-s)\gamma^*$	$\sigma+(1-\sigma)P^-$	$\sigma/2$	F
N	N	$0 + 0$	$\sigma+(1-\sigma)P^-$	$(1-\sigma)/2$	P^-
N	Y	$0 - (1-s)\gamma^*$	P^-	$(1-\sigma)/2$	P^-

Figure 1: Timeline

t_1	t_2
Observed state: b or g	Small investor private shocks
Financing: Equity or loan or public debt	Secondary market
Primary market for public securities	Project: Success or Failure
Large investor stake observed	Voluntary relief offered or Not
	Debt maturity
	Formal bankruptcy or Not

Figure 2A: Provisional Buying Intensity and Breadth of Shocks

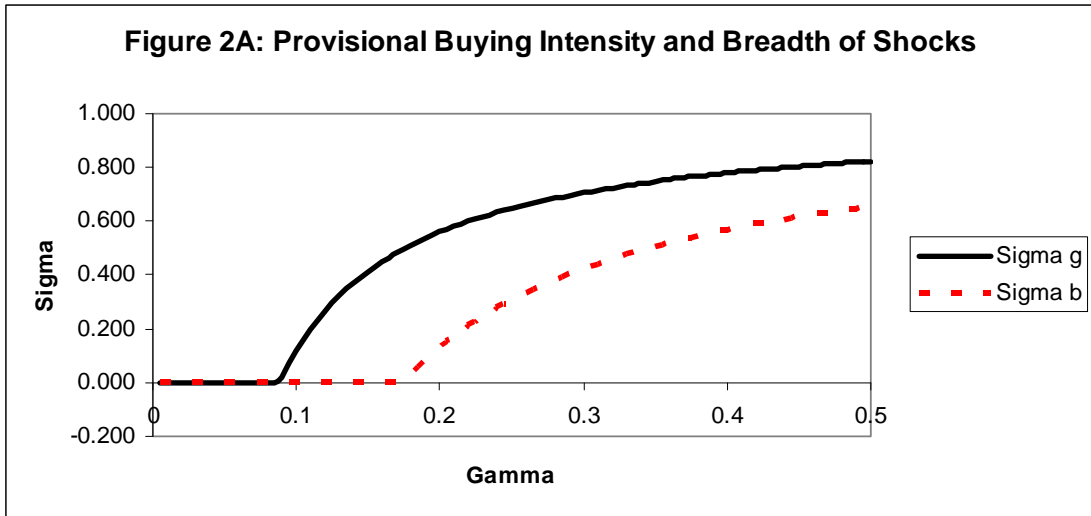


Figure 2B: Provisional Buying Intensity and Bankruptcy Costs

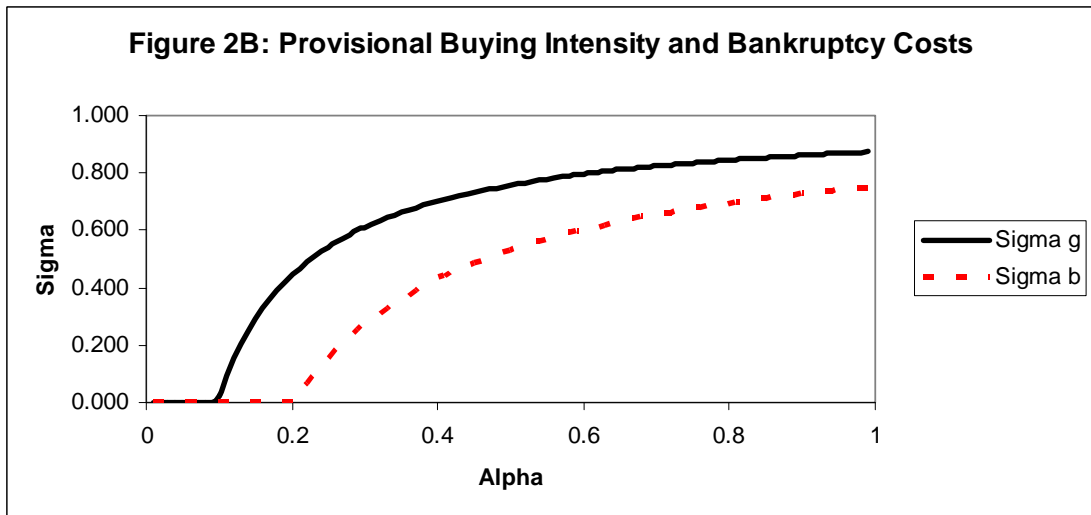


Figure 3A: Sell Decision with High Liquidity Preference

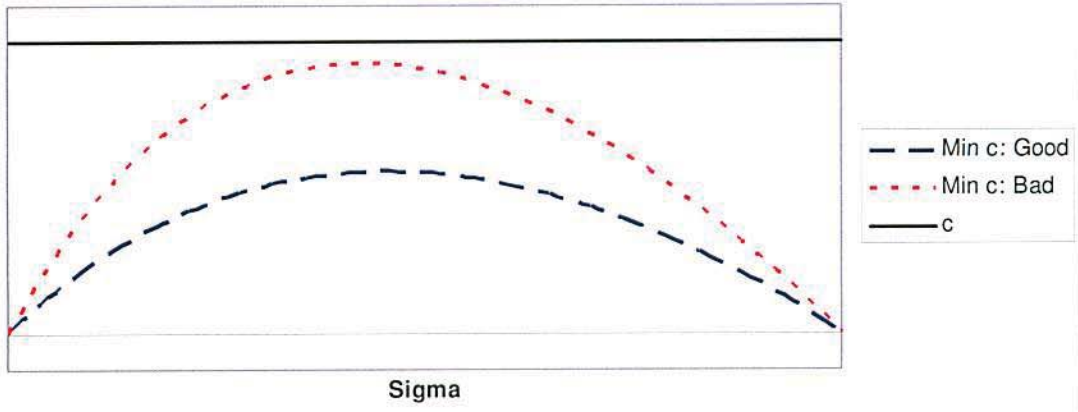


Figure 3B: Sell Decision with Medium Liquidity Preference

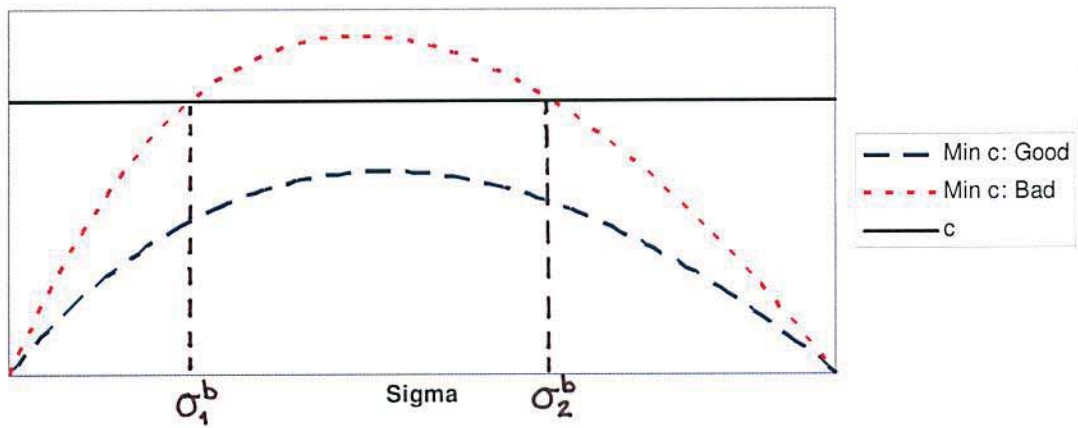


Figure 3C: Sell Decision with Low Liquidity Preference

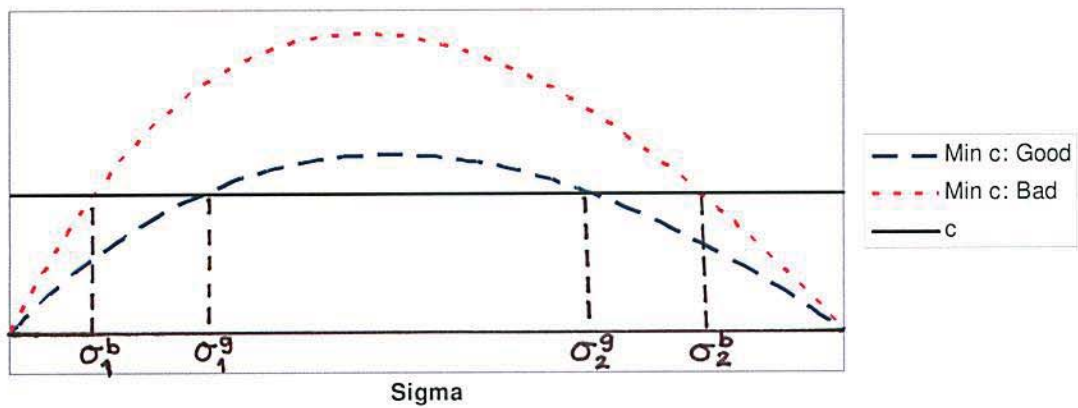


Figure 4A: Debt Price and Breadth of Shocks

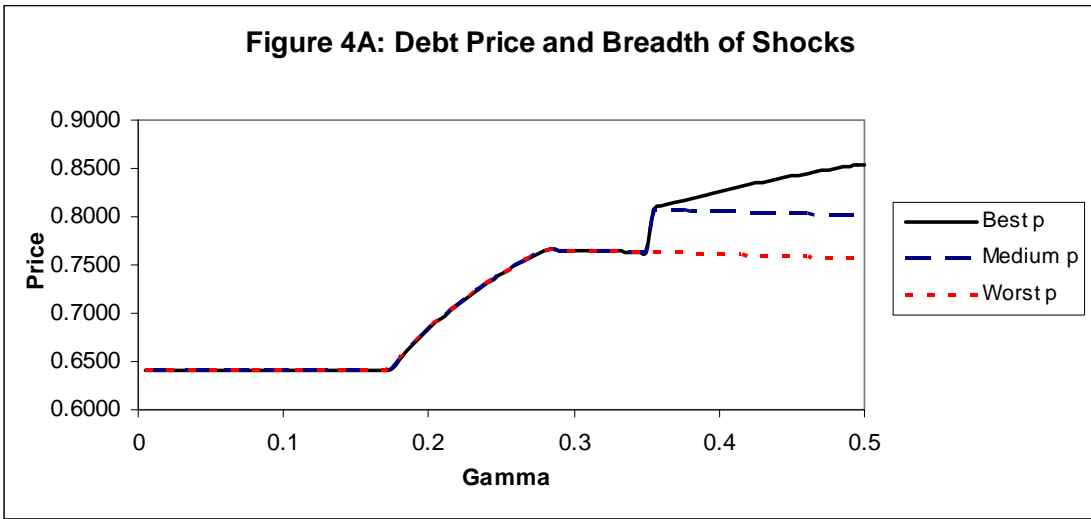


Figure 4B: Buying Intensity and Breadth of Shocks

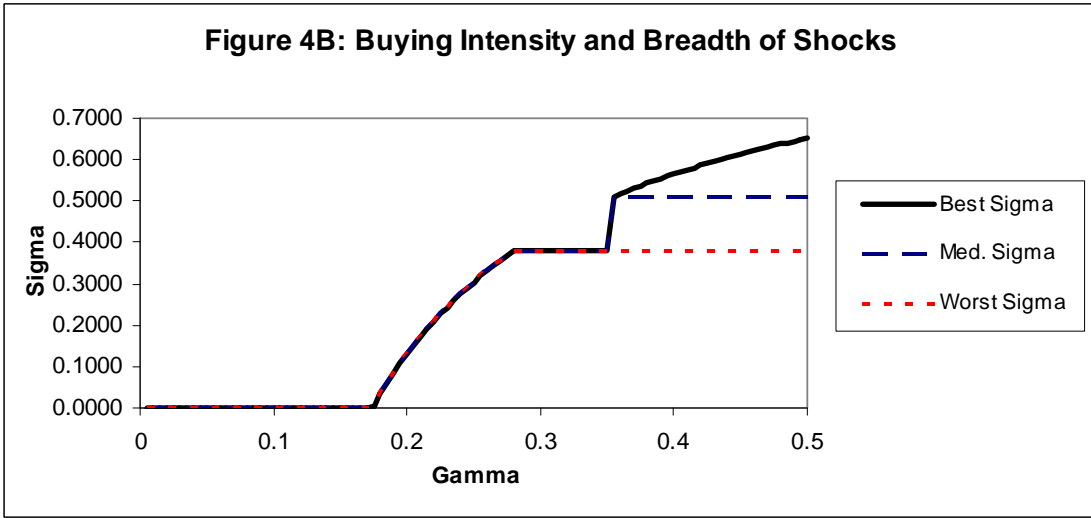


Figure 4C: % Impatient Selling and Breadth of Shocks

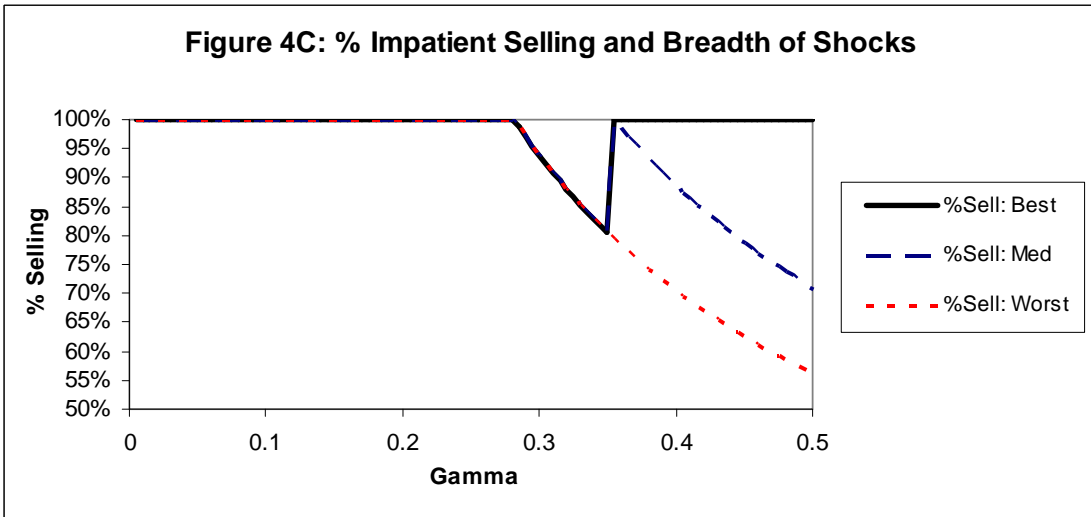


Figure 5A: Debt Price and Bankruptcy Costs

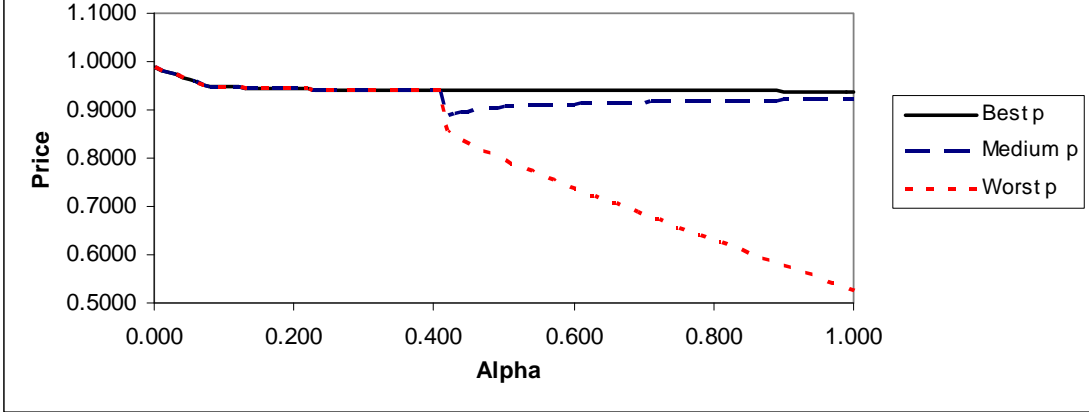


Figure 5B: Buying Intensity and Bankruptcy Costs

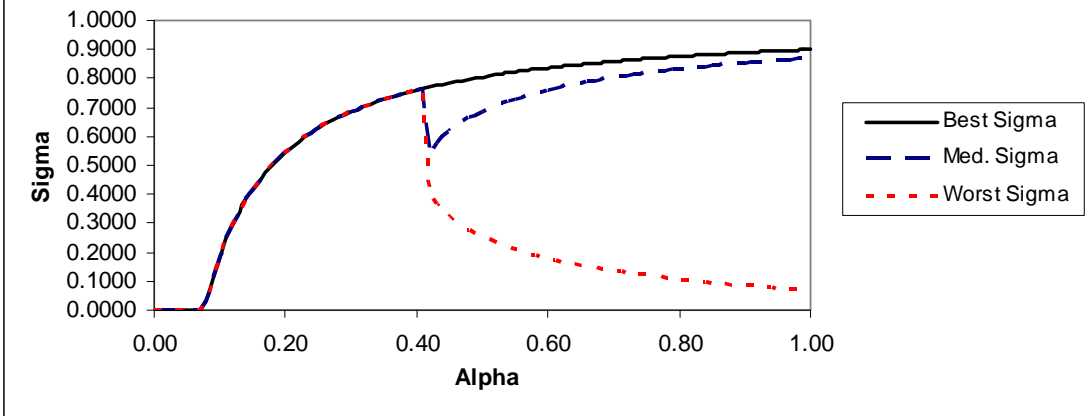


FIGURE 5C: % Impatient Selling and Bankruptcy Costs

